Time-Delay System Identification Using
Genetic Algorithm - Part Two:
FOPDT/SOPDT Model Approximation

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Abstract:
The First-Order-Plus-Dead-Time (FOPDT) or Second-Order-Plus-Dead-Time (SOPDT) model
approximation to a complicated process system can be carried out through either a kind of
model reduction approach or a kind of system identification approach. This paper investigates
this model approximation problem through an identification approach using the real coded
Genetic Algorithm (GA). The desired FOPDT/SOPDT model is directly identified based on
the measured system’s input and output data. In order to evaluate the quality and performance
of this GA-based approach, the proposed method is compared with two typical model reduction
methods, namely Skogestad’s rules and Sung et al method. The obtained results exhibit a very
promising capability of GA in handling the data-driven time-delay system approximation.

Keywords: Time-delay system, genetic algorithms, model reduction, FOPDT and SOPDT

1. INTRODUCTION

There is no doubt that a simple but proper mathematical
approximation of a complicated process system can lead to
many positive aspects (Ljung (1999); Skogestad (2003)),
such as to effectively sketch the key dynamic feature(s) of
the considered system so as to make the control development
more efficient and reliable for complicated systems (Åström and Hägglund (1995)).

As a type of most common process models, the FOPDT
and SOPDT have been extensively used in modeling and
control of diverse process systems. For a considered sys-
tem, the FOPDT/SOPDT model can be derived accord-
ing to some analytic rules (Skogestad (2003)) or some
computation algorithms (Sung et al (1998)) based on the
model reduction principle (Mehrmann and Stykel (2005)).
However, these model reduction approaches often require
a reasonably precise model to be known beforehand. Nor-
mally this detailed model is much more complicated than
the desired model, at least in terms of system orders.
Alternatively, the desired FOPDT/SOPDT model can
be estimated through a system identification approach
(Åström and Hägglund (1995); Yang et al (1996)). This is
a straightforward one-step solution, however, the quality
of the obtained model heavily depends on a lot of on-site
issues, such as the excitation condition, measured data
quality, selected identification algorithm, and the type of
desired model etc. (Ljung (1999); Richard (2003)).

The time-delay system identification is always a chal-
 lenging task. Due to the unknown time-delay, this kind
of identification problem often turns to be non-convex
(Richard (2003)). A lot of of interesting research and
results focusing on time-delay systems identification have
been committed in recent decades, such as using the
filter technique for continuous parameter identification
(Wang and Zhang (2001)) and combining Genetic Algo-
rithm (GA) and Recursive Least Square (RLS) for online
identification (Yang et al (1996)). However, the non-
convex issue has not yet been well solved or even discussed
in many existing methods and solutions.

If the time-delay can be discretized as some integer which
value is relevant to the (fixed) sampling frequency pre-
selected for data collection, the time-delay system identifi-
cation could be solved by combining the exhaustive search
(w.r.t. the time-delay coefficient) with some conventional
system identification methods. However, the performance
of this kind of approach is heavily limited by the noise
impact (Yang Seested (2013)). In order to handle the
potential non-convex influence and meanwhile keep some
sense of robustness, the GA technique has caused more
and more attention for time-delay system identification or
model reduction (Pedersen and Yang (2008)). For instance,
the acquisition of FOPDT and SOPDT model from a
system’s step response using GA is proposed in (Shi
et al (2008)), even though this proposed method is only
oriented for using step responses.

In this work, we will investigate the development of
FOPDT/SOPDT model approximation to complicated
process systems through a real encoded GA approach. Di-
ferent evaluation criteria and different excitation are tested
accordingly. Different from the precise FOPDT/SOPDT
estimation case discussed in Yang Seested (2013), where
the estimation quality of the proposed method can be evaluated by comparing the obtained results with the true system values, the approximation quality of the proposed method here is compared with those obtained through two well-known model reduction methods (Skogestad (2003); Sung et al. (1998)) subject to two conditions: (a) the detailed model is known beforehand; and (b) the detailed model is unknown but the measured data is available. The achieved results illustrate that the GA can conduct a quite reliable and efficient FOPDT/SOPDT model approximation to complicated process systems as long as the considered system is reasonably excited.

The rest of the paper is organized in the following: Section 2 formulates the FOPDT/SOPDT approximation into a discrete-time model identification problem; Section 3 briefly describes the chosen real coded GA and two well-known model reduction methods; Section 4 illustrates and discusses the obtained results; and we conclude the paper in Section 5.

2. PROBLEM FORMULATION

Consider the FOPDT and SOPDT models described by their transfer functions as

\[ G_f(s) = \frac{K}{\tau s + 1} e^{-T_ds}, \]  
\[ G_s(s) = \frac{K}{s^2 + as + b} e^{-T_ds}. \]

where \( K \) is the system’s DC-gain, \( T_d \) is the dead-time coefficient, \( \tau \) is the FOPDT’s time constant and \( a,b \) are SOPDT’s parameters.

For a considered system, the FOPDT/SOPDT approximation problem is to determine the parameters \( K, \tau, T_d \) for a FOPDT model (1) or parameter \( K, a, b, T_d \) for a SOPDT model (2) based on either an available system model or the available system’s input and output (sampled) measurements, so as to minimize a predefined approximation criterion.

2.1 Discrete Prediction Models

Denote the sampling period as \( T_s \) and the sampled input and output sequences as \( \{y(k)\}_{k=0}^N \) and \( \{u(k)\}_{k=0}^N \), respectively. The considered FOPDT model (1) can be converted into a discrete prediction model as

\[ \hat{y}(k) = \alpha \hat{y}(k-1) + \beta u(k-l-1). \]  

where \( \alpha \triangleq e^{-T_s \tau}, \beta \triangleq K(1-\alpha) \) and integer \( l \) satisfies \((l-1)T_s \leq T_d < lT_s \).

Similarly, the SOPDT model (2) can also be converted into its equivalent discrete version as

\[ H_s(z) = \frac{m_1 z + m_0}{n_2 z^2 + n_1 z + n_0} z^{-l}, \]

where parameters \( m_0, m_1 \) and \( n_0, n_1, n_2 \) are determined according to original parameters \( K, a, b, \) and \( l \) has the same feature as the discretized FOPDT model does. Thereby, a prediction model can be achieved as well:

\[ \hat{y}(k) = -\frac{n_1}{n_2} \hat{y}(k-1) + \frac{n_0}{n_2} \hat{y}(k-2) + \frac{m_0}{n_2} u(k-l-1) + \frac{m_0}{n_2} u(k-l-2). \]

2.2 Approximation Criteria

Time-Domain Criterion By using (3) and (5) to estimate the system’s output sequence based on measured system input and previous output signals, a quadratic cost function for FOPDT case is defined as:

\[ C_f(\alpha, \beta, l) = \frac{1}{N} \sum_{k=1}^{N} (y(k) - \hat{y}_{\alpha, \beta, l}(y(k-1), u(k-l-1)))^2, \]

where \( l_u \) indicates the maximal potential delay steps and assume \( l_u < N \), \( \hat{y}_{\alpha, \beta, l}(y(k-1), u(k-l-1)) \) is the predicted output at \( k \)-th step, based on measurements \( y(k-1) \) and \( u(k-l-1) \) for \( l_u + 1 \leq k \leq N \) according to (3). Similarly, the cost function for SOPDT case is defined as

\[ C_f(\theta, l) = \frac{1}{N_f} \sum_{k=0}^{N_f} (y(k) - \hat{y}_{\theta, l}(y(k-1), y(k-2), u(k-l-1)))^2, \]

where \( \theta \) is the stack of \( m_0/n_2, m_1/n_2, r_0/n_2, n_1/n_2, \) and \( \hat{y}_{\theta, l}(\ldots) \) represents the estimated system’s output based on measured previous inputs and outputs according to (5).

Frequency-Domain Criterion By converting the measured data into its DFT format using the FFT algorithm \(^1\), denote the unknown parameter \( \theta = (K, \tau) \) for FOPDT case and \( \theta = \theta \) for SOPDT case, then a quadratic cost function in frequency-domain is constructed as

\[ C_f(\theta) = \frac{1}{N_f} \sum_{k=1}^{N_f} |W(k)\hat{Y}_\theta(k)|^2, \]

where \( W(k) \) is a frequency weighting sequence. The amplitude of the \( k \)-th estimated output \( \hat{Y}_\theta(k) \), which is a function of \( \theta \), for FOPDT case, is calculated according to

\[ |\hat{Y}_\theta(k)| = \frac{K}{\sqrt{(\omega_k \tau)^2 + 1}} |U(k)|. \]

For SOPDT case, it is calculated according to

\[ |\hat{Y}_\theta(k)| = \frac{K}{\sqrt{(\omega_k a)^2 + b^2 - 2\omega_k b + \omega_k^2}} |U(k)|, \]

where \( \omega_k = \frac{2\pi k}{N_f} \) for \( 1 \leq k \leq N_f \) and \( N_f \) is the length of signal’s DFT sequence \(^2\).

2.3 Optimal Approximation Problem

The considered optimal approximation problem can be defined as a mixed integer nonlinear programming problem:

\[ \min \{E_{l}f_C_f(\hat{l}, l) + f_C_f(\hat{l})\}, \]

where \( E_{l}f \) represents the expectation operator, \( l \) and \( l_u \) are the potential lower and upper boundaries of delay

\(^1\) The following discussions are also suitable for cases that this frequency information is directly available or it can be derived from some other available resource.

\(^2\) In the frequency domain, the length of DFT sequence \( N \) needs to be large enough in order to avoid aliasing problem, i.e., there is \( N_f \geq 2(N-l_u) - 3 \).
steps, and \( \Theta \) represents the admissible set of the unknown parameters \( \hat{\theta}, \phi_i, \) and \( \phi_j \) are weighting factors for time-domain and frequency-domain costs, respectively.

Once the problem \( (11) \) is solved, the system parameters of original system \( (1)/(2) \) can be derived from the solution of \( (11) \), where the precision of the dead-time estimation is pre-determined by the sampling period. In the following, the GA-based method is investigated to cope with this non-convex optimization problem.

3. SELECTED GA AND TWO MODEL REDUCTION METHODS

3.1 Real-coded GA with Niching

The real coded GA adopted in our Part-one work (Yang Seested (2013)) is employed here again. The binary tournament selection is used according to the combined cost function \( (11) \). The selected chromosomes generate the offsprings according to the Simulated Binary Crossover (SBX) operator and the polynomial mutation (Deb (2000)). The fitness sharing is applied to maintain the population diversity. For further more details about the adopted GA, we refer to Glen (2013) and Yang Seested (2013).

As discussed in Yang Seested (2013), the GA coded variables, except the dead-time coefficient, are the original parameters of \( (1)/(2) \) in the application of GA identification, i.e., to solve the constraint optimization problem \( (11) \) subject to changing the variable vector \( \bar{\theta} \) by \( \hat{\theta} \in \Theta \), where \( \hat{\theta} \) represents the stack of \( (K, \tau) \) for FOPDT case (1) and \( (K, a, b) \) for SOPDT case (2).

3.2 Skogestad's Analytic Model Reduction Rules

A set of simple rules for reducing a high-order LTI system model into a FOPDT or SOPDT model is proposed in Skogestad (2003). In order to keep our paper’s continuity for reading, hereby we summarize these rules in the following:

Consider a high-order time-delay system described as

\[
G_b(s) = \frac{K}{\prod_i^{\tau} (s + \tau_i)} \prod_j^{\tau_s} e^{-T_d s},
\]

that all parameters are positive valued. The first step is that all left-side zero(s) will be one-to-one canceled by their neighboring denominator term(s), according to the following rules:

\[
\begin{align*}
& T_d \geq \tau \geq T_d, \quad \text{Rule T1} \\
& T_d \geq \tau \geq T_d, \quad \text{Rule T1a} \\
& 1 \geq \tau \geq T_d, \quad \text{Rule T1b} \\
& \frac{\tau}{\tau - T_d} \geq \tau \geq 5T_d, \quad \text{Rule T2} \quad (12) \\
& \text{where } T_d \text{ is the final effective time delay, i.e., the time delay } T_d \text{ in the approximated FOPDT/SOPDT model. There are also some tips about how to select the neighboring denominator term and get the initial guess of } T_d, \text{ we refer to Skogestad (2003) for more details.}
\end{align*}
\]

For the FOPDT approximation, the \( K \) of \( G_f(s) \) is the same \( K \) from \( G_b(s) \). Assume the parameters \( \tau_i \) in \( G_b(s) \) has the order: \( \tau_0 \geq \tau_1 \geq \tau_2 \cdots \), then the effective time constant \( \tau \) in FOPDT approximation) can be approximated by \( \tau = \tau_0 + \frac{\tau_2}{2} \), the effective delay by \( T_d = T_d^h + \frac{T_d^j}{2} + \sum_{i=2}^{\tau} \tau_i + \sum_{j}^{T_d} \frac{T_d^j}{2} \). For SOPDT using the formulation

\[
G_s(s) = \frac{Ke^{-T_d}}{(a_1 s + 1)(a_2 s + 1)},
\]

there is

\[
\begin{align*}
& a_1 = \tau_0, \quad a_2 = \tau_1 + \frac{\tau_2}{2}, \\
& T_d = T_d^h + \frac{T_d^j}{2} + \sum_{i=2}^{\tau} \tau_i + \sum_{j}^{T_d} T_d^j + \frac{T_d^i}{2} \\
& \text{for SOPDT using the formulation } (14)
\end{align*}
\]

3.3 Sung et al. Method

A FOPDT/SOPDT model reduction method is proposed in Sung et al (1998). The considered high-order system is described as

\[
G_s(s) = \frac{n_s s^n + n_{s-1} s^{n-1} + \cdots + n_1 s + n_0}{d_s s^d + d_{s-1} s^{d-1} + \cdots + d_1 s + 1}.
\]

A FOPDT approximation (1) of the considered system (15) can be determined according to

\[
K = n_0, \quad \tau = \sqrt{(\Phi^T \Phi)^{-1} \Phi^T \Psi}, \quad T_d = \frac{\pi - \arctan(\tau \omega_u)}{\omega_u},
\]

where \( \omega_u \) is the system’s ultimate frequency, i.e., the phase crossover frequency. \( \{\omega_1, \omega_2, \cdots, \omega_{N_u}\} \) is a stack of frequencies samples within the period \( [0, \omega_u] \) with the property \( 0 < \omega_1 < \cdots < \omega_{N_u} \leq \omega_u \).

Regarding to the specific formula for SOPDT model approximation, we refer to Sung et al (1998) for the details. In general, the Sung et al method is a LS solution by minimizing the accumulated squared amplitude errors between the original system’s and approximated model’s frequency responses over a sampled frequency period up to the system’s ultimate frequency. This is similar to the frequency domain criterion (8), but one is defined on systems’ frequency responses while the other is on signals’ DFTs.

4. TESTING RESULTS AND DISCUSSIONS

One system used in Skogestad (2003) is picked up to test the GA-based FOPDT/SOPDT approximation. The GA-based solution is compared with those obtained from Skogestad’s and Sung et al methods subject to the case, either the original system model is precisely known beforehand, or only the measured data with reasonable SNR is available. Two types of input excitations, named chirp and pulse sequences, are employed, respectively. The considered system has the concrete formulation as

\[
G(s) = \frac{(-0.3s + 1)(0.08s + 1)}{(2s + 1)(s + 1)(0.4s + 1)(0.2s + 1)(0.05s + 1)^2}.
\]

The system bandwidth is about 0.414 rad/sec. The chirp signal uniformly sweeps the frequency from 0Hz to 0.5Hz during the simulation period and the sampling frequency is 20Hz.
If we set the weighting $\phi_f = 0$ in the optimization problem (11), then all methods studied in our part-one work (Yang Seested (2013)) can also be directly employed to handle this FOPDT model identification problem. The obtained results excited by a chirp sequence are illustrated in Table 1 (ELMS, E-IV-LMS and GA). The frequency response comparison within the original system’s bandwidth is illustrated in Figure 1, where clear deviations in amplitudes and phases from all three methods can be easily observed. The deviations of step response of each estimated system from the original system are illustrated in Figure 2. The largest transient deviation is less than 8% from the E-IV-LMS method (abbr. IV), The GA method (abbr. GA) is slightly better than the ELMS (abbr. LS). But all three methods lead to steady state errors. That is mainly due to the imprecise estimation of $K$. These estimation results need to be definitely improved.

### Table 1. Estimated FOPDT parameters via different methods subject to chirp excitation

<table>
<thead>
<tr>
<th></th>
<th>ELMS</th>
<th>E-IV-LMS</th>
<th>GA</th>
<th>GA</th>
<th>Skogestad</th>
<th>Sung</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>1.04</td>
<td>0.98</td>
<td>1.05</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\tau$</td>
<td>3.28</td>
<td>2.99</td>
<td>3.34</td>
<td>2.4</td>
<td>2.5</td>
<td>2.86</td>
</tr>
<tr>
<td>$T_d$</td>
<td>1.4</td>
<td>1.45</td>
<td>1.4</td>
<td>1.55</td>
<td>1.47</td>
<td>1.57</td>
</tr>
</tbody>
</table>

4.2 Combined Freq-Time-domain FOPDT Approximation

By tuning the weighting factors in (11), the FOPDT approximation model of system (17) is obtained by using the GA-based identification and the results are listed in Table 1 (GA column). Correspondingly, the parameters of the reduced FOPDT models obtained through the Skogestad’s method and Sung et al method are also listed in Table 1 (Skogestad, Sung). The frequency response comparison within the original system’s bandwidth of the approximated models with the original system are illustrated in Figure 3. It can be noticed that the GA method results the best FOPDT approximation in terms of frequency fitness within the bandwidth. The comparison of step responses of different obtained models with that of the original system is shown in Figure 4. GA-based approximation also plays quite as good as Skogestad’s model, and both are better than the model derived from Sung et al method in terms of converging rates.

4.3 Combined Freq-Time-domain SOPDT Approximation

The SOPDT approximation using GA, Skogestad’s and Sung et al methods are also studied subject to chirp and (two-different) pulse excitations. In order to make the results comparable, all obtained SOPDT models are reformulated into (13) orientation. The results are listed in Table 2. The (amplitude) frequency response comparison...
Fig. 5. Frequency comparison of different SOPDT models

is illustrated in Figure 5. It is noticed that all concerned methods result a good frequency matching within the system's bandwidth. The comparison of step responses with that of the original system are shown in Figure 6. It can be seen that all GA-induced models also play good jobs in time-domain. From the Nyquist plots shown in Figure 7, all developed models keep the quite similar phase features with the original system.

Table 2. Estimated SOPDT parameters via diff. methods subject to diff. excitations

<table>
<thead>
<tr>
<th>Chirp</th>
<th>Pulse-1</th>
<th>Pulse-2</th>
<th>model</th>
<th>based</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GA</td>
<td>GA</td>
<td>GA</td>
<td>GA</td>
</tr>
<tr>
<td>$K$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.74</td>
<td>1.34</td>
<td>1.29</td>
<td>1.2</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2.2</td>
<td>1.74</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>$T_d$</td>
<td>1.0</td>
<td>0.85</td>
<td>0.85</td>
<td>0.77</td>
</tr>
</tbody>
</table>

4.4 Tuning Weighting Factors

The tuning of weighting factors $\phi_t$ and $\phi_f$ in (11) is very important in acquisition of a good approximation model. A proper selection of the frequency weighting sequence \{W(k)\} used in (8) is also quite necessary. All these tunings depends on how the quality of the measurement data is and how it is distributed, SNR, the type of excitation signals, as well as the purpose of this modeling etc. We refer to Glen (2013) for more details.

Fig. 7. Nyquist curve comparison of different SOPDT models

4.5 Approximation Subject to Unknown Original System

So far, all above results and analysis are based on the assumption that the original system (17) is known beforehand. Of course, there is no influence to GA based method if the original system is unknown, since GA based method directly estimates a FOPDT/SOPDT model based on the measurements. However, the skogestad's and Sung et al methods can not be employed if the original system is unknown. One possible way to solve this problem is to firstly estimate a detailed model based on the measurements by using some standard system identification techniques (Ljung (1999)), afterwards derive the corresponding low-order model using one of these methods based on the estimated high-order system (obtained from first step). This kind of bootstrap approach could have some risks to result in some poor model reduction quality if the high-order system is estimated not precise enough. In the following, we will illustrate this potential problem by using the same exampled system.

By using Matlab System Identification Toolbox based on the measured data excited by the chirp signal, firstly, a high-order system (in transfer function) is estimated. Afterwards, the Skogestad's and Sung et al methods are employed to obtain the corresponding FOPDT/SOPDT models. The results are listed in Table 3. From the bode plots shown in Figure 8, it is quite clear that the GA-based result has a much super frequency fitness than these bootstrapped results do. From the step response analysis shown in Figure 9, it is also observed that both FOPDTs obtained through these bootstrapped approaches can cause much large deviations in time-domain operations. It has been proved that by using the pulse excitation does not help improve the poor approximation situation (Glen (2013)).

Table 3. Estimated FOPDT & SOPDT parameters via model reduction subject to chirp

<table>
<thead>
<tr>
<th></th>
<th>FO- PDT</th>
<th>SO- PDT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>Skog-2</td>
<td>1.0</td>
<td>4.63</td>
</tr>
<tr>
<td>Sung-2</td>
<td>1.0</td>
<td>5.88</td>
</tr>
</tbody>
</table>
5. CONCLUSION

The main benefits of using this GA-based model approximation lie in the following perspectives: (i) the direct model estimation avoids the pre-requisition of a (complicated) detailed model of the considered system, which is often requested by available model reduction methods; (ii) The GA-based approach minimizes the local optimum risk caused by the non-convexity in time-delay system identification; (iii) The GA-based method has much more direct flexibility in handling different cost functions, as well as variable constraints, compared with conventional optimization methods; (iv) The GA’s computation and evolution are direct conducted in the random domain, thereby some robustness subject to noise influence could be kept.

All in all, the so-far achieved results illustrate that the GA can conduct a quite reliable and efficient FOPDT/SOPDT model approximation to complicated process systems as long as the considered system is reasonably excited. Furthermore, some of our undergoing work has already evidenced that once the computation time is not a problem, the proposed method can also be extended to handle online model identification/approximation problem. We will report these results in the near future.

REFERENCES


