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Using Two Different ARMA Approaches

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Abstract

In the present investigation, multi-channel response measurements on an offshore platform subjected to wave loads is analysed using Auto Regressive Moving Average (ARMA) models. Two different estimation schemes are used and the results are compared. In the first approach, a scalar ARMA model is used to estimate the poles (eigenfrequencies and damping ratios), and then the mode shapes are found by fitting an analytical form to the empirical estimates of the covariance functions using the estimated poles. In the second approach, a full vector model is used, and the poles and mode shapes are estimated in one step by solving an eigenvalue problem. Both of the models assume 14 modes, some of them are considered non-physical. Results for the 8 most significant modes in the estimations are compared.

Nomenclature

\( y(k), e(k) \): Vectors of response and load
\( x(k), a(k) \): State vectors of response and load
\( \Psi_i \): Eigenvectors of state-space model
\( D_i \): Covariance weight matrices

Introduction

For offshore platforms and other structures subjected to natural loads like wave and wind, the loads are usually unknown, and thus, a traditional modal analysis based on frequency response functions, Ewins [1], cannot be performed. Usually, the analysis must be based on measurements of the response only. A review of the problems of identifying the dynamic properties of offshore structures is given in Jensen[2].

It is usually assumed that the loads might be modelled as stationary white noise. This assumption, however, does not imply any serious limitations. It is possible to show, Ibrahim et al. [3], that if the load process is not white noise, but might be modelled by a set of second order differential equations, i.e. modelled as the response of a mechanical system loaded by white noise, then the structure might be identified using an oversized model to incorporate the non-physical modes describing the load process. This idea is used in the present analysis.
channel measurement of the response of a wave loaded offshore structure. The analysed time series are relatively short, and thus, the modelling of the noise is essential for extracting reliable information about modal parameters.

Two approaches are presented and used for estimation of modal parameters. First, a scalar ARMA model is used to estimate eigenfrequencies and damping ratios. Then this information is used for fitting an analytical form of the covariance functions to the empirically estimated covariance function to obtain the mode shapes. The second approach is based on a more general ARMA model that is formulated directly for multi-channel output systems. Using this approach, the estimation is made in one step solving an eigenvalue problem. Results from the two approaches are compared.

Test Case

The offshore platform is a multi-pile structure located in Lake Maracaibo, Venezuela, and it houses the power plant for a large oil production complex for the Venezuelan oil industry.

The platform consists of a reinforced base structure and a steel superstructure which holds the power generation equipment. This platform was built in 1992, and it experiences continuous vibrations caused by wave and current actions which are to be evaluated in order to determine the effects of this continuous movement on the integrity of the structure.

The reinforced concrete structure is 58 m (195') long and 20 m (66') wide, it is supported by 42 pre-stressed concrete piles, 0.91 m (36") in diameter, 55 m (185') long, precast and driven on site, see figure 1. The steel-framed structure holds 5 turbo-generators and their control rooms.

Water depth at the location of the offshore platform is about 30 m. Wave heights in this zone have been reported to vary between 1.20 m and 2.50 m, with recurrence periods of 3.8 s and 4.9 s respectively. The information reported for current action near the platform shows values in the order of 1.4 m/s in the direction of the waves.

To measure vibrations of the platform due to ambient excitation, the structure was instrumented using 8 seismic accelerometers of the DC type with a maximum frequency of 3500 Hz. The accelerometers were placed at 4 points at the topside deck as shown in figure 1. Signals were amplified 100 times, and low-pass filtered at 5 Hz before sampling at a sampling frequency of 12.8 Hz. Data were recorded simultaneously at all 8 channels using a multi-channel data acquisition system converting from analog to digital with 16-bit accuracy. Data were recorded in blocks with 8192 data points per channel corresponding to a recording time of about 10 min.

Before the data were used for modal analysis, the data were filtered and decimated. Since the physical modes were known to be around 1 Hz, the signals were decimated with a factor 4 reducing the data blocks to 2048 points per channel and the Nyquist frequency to 1.60 Hz. Before decimating, the signals were digitally low-pass filtered to reduce noise. Since the signals had a large long-periodic component, the signals were also digitally high-pass filtered at a cut-off frequency of 0.032 Hz.

Since the accelerometer number 3 some times showed unreliable results, this channel was excluded in the analysis. The modal analysis reported in this paper was based on the data from one data block only, i.e. on 7 channels, each with 2048 data points.

Estimation by Scalar ARMA Model

Let \( y_c(t) \) be a realisation of the continuous stochastic process \( Y(t) \), and let \( y_c(t) \) have the discrete (sampled) representation \( y(k) = y_c(k\Delta t) \), where \( \Delta t \) is the sampling interval. An ARMA model of the order \( (n, m) \), called ARMA\( (n, m) \), for the time series \( y(k) \) is then given by

\[
y(k) = \sum_{i=1}^{n} \Phi_i y(k - i) - \sum_{i=1}^{m} \Theta_i e(k - i) + e(k)
\]  

(1)
where $\Phi_i$ are the auto regressive (AR) parameters describing the response $y(k)$ as a linear regression on the past values, and $\Theta_i$ are the moving average (MA) parameters describing the response $y(k)$ as a linear regression on the past values of an unknown time series, $e(t)$. Now, since the response $y(k)$ might be considered as a linear regression of the past responses and the past unknown loads, the last term $e(k)$ in eq. (1) is called the residue, since it might be considered as the term describing the deviation of the measured time series $y(k)$ from the response predicted by the regression. Thus using minimum least squares, the best fit corresponds to minimising the variance of the residue $e(k)$.

It might be shown, that an ARMA model of order $(2N, 2N-1)$ is the covariance identical discrete model of a continuous system with $N$ degrees of freedom, Pandit [10], Kozin et al. [8], Andersen et al. [11].

When the AR parameters are known, the modal parameters are found from the $2N$ roots $\lambda$ of the characteristic AR polynomial, Pandit [10]

$$\lambda^{2N} - \Phi_1\lambda^{2N-1} - \ldots - \Phi_{2N-1}\lambda - \Phi_{2N} = 0$$

(2)

These roots are called the poles. The roots of the similar characteristic MA polynomial are called the zeroes. The poles and the zeroes are complex numbers. Zeros and poles lying outside the unit circle correspond to unstable systems, and thus, usually zeroes and poles are forced inside the unit circle. Problems with unstable zeroes and poles usually relate to problems with too many degrees of freedom. The poles corresponding to physical degrees of freedom always appear in complex conjugate pairs, one pair for each degree of freedom. The eigenfrequencies $f_i$ and damping ratios $\zeta_i$ are found from the relation between the modal parameters and the $N$ complex conjugate poles

$$\lambda = \exp(2\pi if \Delta t(-\zeta \pm i\sqrt{1 - \zeta^2}))$$

(3)

Since the covariance matrix $C_\theta$ of all the estimated AR and MA parameters $\theta = \{\theta_1, \theta_2, \ldots\}^T$ is easily obtained in the estimation process, Ljung [4], the covariance matrix $C_a$ of any set of physical parameters $a = \{a_1, a_2, \ldots\}^T$ might be estimated by linearisation, see e.g. Kirkegaard [12]

$$C_a = GCC_\theta G^T$$

(4)

where $G$ is a gradient matrix describing the linear relationship of $a$ around its mean value $a = \mu_a + G\theta$, $\mu_a = E[a]$. This technique is used to calculate standard deviations of the estimated eigenfrequencies and damping ratios.

As explained earlier, not all of the estimated modes might correspond to structural modes. A helpful tool in judging to what extent an estimated mode might be physical or not, is to have a measure of its significance for the response. If a certain mode dominates the response, then it is reasonable to assume it to correspond to a structural mode - or to a dominating frequency of the loading system. On the other hand, if the contribution to the response is little, the estimated mode might be describing some non-physical phenomenon. A useful measure might be obtained using that the auto covariance function $R(\tau) = E[y(t) - y(t - \tau)]^2$ can be written as a weighted sum of oscillators, one oscillator for each mode, Pandit [10]

$$R(\kappa\Delta t) = \sum_{i=1}^{N} d_i\lambda_i^\kappa$$

(5)

Now, using an unbiased estimate of $R(\tau)$, e.g. by using the unbiased FFT, see e.g. Bendat & Piersol [13] or Brincker et al. [14], and the poles $\lambda_i$ estimated by the ARMA model, the weights $d_i$ might be estimated by least square regression. Now, since the poles appear in complex conjugate pairs, so does the weights $d_i$. Thus, for each oscillator, the energy content is proportional to $|d_i|^2$, and, thus, $P = \sqrt{\sum|d_i|^2}$ is a measure of the modal amplitude. The participation factors $P_i$ are determined for each estimated mode, and the participation vector is normalised to length one.

For the present multi-channel case, the time series for each channel were stacked to form one long record. In order to weight all channels equally, the time series were normalised to variance one before the merging, and in order to make a smooth transition between the merged time series to prevent transients in the residue $e(k)$, the time series were tapered before high-pass filtering.

Figure 2 shows the variance of the residue and Akaike's Final Prediction Error (FPE) for the different scalar ARMA models estimated for the system. A small variance of the residue indicates a good fit, thus, the smaller values of the variance, the better the fit. However, in order not to over-parameterise the problem, Akaike's Final Prediction Error (FPE), that expresses a statistical trade-off between the variance of the residue, and the number of model parameters, is considered. As it appears from the figure, models were tested with 1 to 15 degrees of freedom. The jump from 5 to 6 degrees of freedom and the flat curve for higher values, indicate that at least 5 physical modes are present (one mode is used to model the high-pass filter). Since the ARMA(28,27) model was the model with the lowest FPE, this model was chosen for the further analysis. The results for the eigenfrequencies and damping ratios with estimated standard deviations and the participation factors for each mode are given in table 1.

The results are shown graphically in figure 3 showing the spectrum and correlation of the residue, a plot comparing the FFT spectrum with the spectrum estimated by the ARMA model, and finally a plot showing zeroes and poles with 99 % confidence regions. The results indicate, that the residue is close to white noise, thus, all the information of the response signals is extracted by...
To estimate the mode shapes, the response of the fully correlated, and the mode shapes would simply be and that all parameters are reasonably well estimated. If the weight matrices were found by estimating the correlations (±1). Thus, the mode shapes would be given by the rows or the columns of the weight matrices. The results are shown in figure 4 together with the modal complexity factor MCF1 and MCF2, Imregun and Ewins [15]. The mode shapes were estimated on the assumption that no longitudinal strain occurs between the measurement points, shear strain however, was allowed. Only one mode is nearly free of complexity, mode 4 at f = 0.69 Hz. All other modes suffer a relatively high complexity. The complexity might be due to for instance non-linearities or non-proportional damping, in this case however, the most likely reason is noise and estimation errors.

**Estimation by ARMAV model**

The ARMAV model is a vector ARMA model. The responses \( y(k) \) for the channels \( i = 1, 2, \ldots, M \) are organised in the vector \( y(k) = \{y_1(k), y_2(k), \ldots\}^T \), and the ARMAV(\( n, m \)) model is a straightforward generalisation of the scalar case

\[
y(k) = \sum_{i=1}^{n} \Phi_i y(k-i) - \sum_{i=1}^{m} \Theta_i e(k-i) + e(k)
\]  

(7)

In this case, the AR coefficients \( \Phi_i \) and the MA coefficients \( \Theta_i \) are (generally full) \( M \times M \) matrices. In the estimation process, Ljung [4] suggest to minimize the determinant of the covariance matrix of the noise time series \( e(k) \). Once the model is estimated, the modal parameters are extracted by formulating the corresponding
discrete time state space model

\[ x(k) = \Phi x(k-1) + \Theta a(k) \]  

(8)

where the state vector \( x(k) \) and the load vector \( a(k) \) are made by stacking the responses and the noise vectors in the following way,

\[
\begin{bmatrix}
    y(k) \\
    y(k-1) \\
    y(k-2) \\
    \vdots \\
\end{bmatrix} 
\begin{bmatrix}
    e(k) \\
    e(k-1) \\
    e(k-2) \\
    \vdots \\
\end{bmatrix}
\]

(9)

and the Auto regressive matrix of the state space model is given by

\[
\Phi = \begin{bmatrix}
    \Phi_1 & \Phi_2 & \cdots & \Phi_n \\
    1 & 0 & \cdots & 0 \\
    0 & 1 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & 1
\end{bmatrix}
\]

(10)

For an ARMAV(2N, 2N-1) with \( M \) channels, the model is a covariance equivalent discrete model of a continuous system with \( NM \) degrees of freedom, Andersen et al. [11]. For this case, the vectors \( x(k) \) and \( a(k) \) both have the length \( NM \), and the autoregressive matrix \( \Phi \) of the state space model is \( NM \times NM \). The poles and the mode shapes are found by solving the eigenvalue problem that naturally arises from eq. (8)

\[
(\lambda_i I - \Phi)\psi_i = 0
\]

(11)

the eigenvalues \( \lambda_i \) constitute the poles, and in this formulation, the mode shapes are the last \( M \) components of the eigenvectors \( \psi_i \). Similar to the scalar case, the eigenvalues and eigenvectors appear in complex conjugate pairs, one pair for each degree of freedom.

For direct comparison of the two ARMA approaches, an ARMAV model was chosen with the same number of degrees of freedom as for the scalar case, thus, an ARMAV(4,3) was estimated for the 7 channels corresponding to 28 poles or 14 degrees of freedom. When the ARMAV model is known, the weight matrices \( \mathbf{D}_i \) of the covariance function matrix is easily determined, Andersen et al. [11], and a participation factor might be defined by taking a scalar measure of \( \mathbf{D}_i \). In this investigation the modal participation factor \( P_i \) were estimated by taking the square root of the sum of the eigenvalues of \( \mathbf{D}_i \). Again, the modal participation vector was normalised to length one.

The results are shown in table 2 giving eigenfrequencies, damping ratios and participation factors for the 14 modes. An uncertainty measure might be estimated using the covariance matrix for the estimated parameters. This was not done however, for the vector ARMA case. The eight most significant modes were selected using the participation factors, though excluding mode 6 \((f = 0.6034 \text{ Hz})\) due to the unrealistically large damping. The eight mode shapes are shown in figure 5.

As it appears from the results, again the mode shape has a relatively large degree of complexity, except the mode at \( f = 0.69 \text{ Hz} \). Comparing the mode shapes with the mode shapes estimated by the scalar ARMA approach it appears that the mode shape at 0.69 Hz is very much the same, whereas some modes at 0.40 - 0.41 Hz, 0.77 - 0.78 Hz are relatively close, the mode shapes at 1.12 Hz is quite different, and the rest of the modes do not seem to correspond. The conclusion is, that the most dominant mode at 0.69 Hz is determined with a good accuracy concerning both eigenfrequency and mode shape, two modes are determined with a good accuracy for the eigenfrequency, and somewhat larger uncertainty on the mode shape, and one mode is determined only with respect to eigenfrequency.

| Table 1. Results from scalar ARMA model (28,27) |
|---|---|---|---|---|
| Mode | Frequency | Standard deviation | Damping ratio | Participation factor |
| 1 | 0.0080 | 0.0010 | 20.58 | 3.14 |
| 2 | 0.1309 | 0.0062 | 19.70 | 3.28 |
| 3 | 0.4980 | 0.0388 | 7.11 | 0.91 |
| 4 | 0.4806 | 0.0337 | 2.87 | 0.78 |
| 5 | 0.5439 | 0.0337 | 2.45 | 0.69 |
| 6 | 0.6905 | 0.0337 | 0.97 | 0.11 |
| 7 | 0.7802 | 0.0337 | 3.31 | 0.49 |
| 8 | 0.8576 | 0.0320 | 0.59 | 0.36 |
| 9 | 0.8641 | 0.0304 | 1.15 | 0.36 |
| 10 | 0.9325 | 0.0306 | 0.15 | 0.06 |
| 11 | 1.1232 | 0.0127 | 5.38 | 0.99 |
| 12 | 1.1549 | 0.0003 | 0.66 | 0.03 |
| 13 | 1.3438 | 0.0000 | 0.69 | 0.00 |
| 14 | 1.4989 | 0.0012 | 0.13 | 0.08 |

| Table 2. Results from ARMAV(4,3) |
|---|---|---|---|---|
| Mode | Frequency | Damping ratio | Participation factor |
| 1 | 0.0577 | 100.0 | 0.0245 |
| 2 | 0.1929 | 62.7 | 0.0651 |
| 3 | 0.2495 | 79.0 | 0.0419 |
| 4 | 0.3707 | 22.6 | 0.0518 |
| 5 | 0.3965 | 37.9 | 0.0789 |
| 6 | 0.6809 | 9.6 | 0.0560 |
| 7 | 0.6934 | 100.0 | 0.0473 |
| 8 | 0.6956 | 1.3 | 0.7330 |
| 9 | 0.7088 | 17.5 | 0.0026 |
| 10 | 0.7751 | 1.4 | 0.4890 |
| 11 | 0.7924 | 1.6 | 0.4312 |
| 12 | 1.0382 | 18.1 | 0.0153 |
| 13 | 1.1190 | 36.9 | 0.0423 |
| 14 | 1.3029 | 33.1 | 0.0143 |
Comparing the differences between the estimated values of eigenfrequencies with the uncertainty measure from the scalar approach, it seems like the standard deviations underestimate the uncertainty. This might be due to bias errors.

An important difference between the results of the two models is, that the ARMAV model estimates three significant close modes, whereas the scalar model only estimates two. The mode at 0.78 Hz in the scalar model is split into two modes at 0.77 and 0.79 Hz. Since the ARMAV model takes advantage of cross information between channels, and thus should be a stronger tool in detecting close modes, and since the mode is split into two modes with a smaller damping ratio around 1.6%, the results of the ARMAV model could be a better estimate of the structural behaviour. On the other hand, since the complexity of the modes did not reduce, it can only be concluded, that the possibly of two close modes exists.
Conclusions

Two ARMA approaches have been compared for estimating eigenfrequencies, damping ratios and mode shapes for an operating offshore platform loaded basically by sea waves. The acceleration response was measured at 7 points at the topside deck, and the responses were analysed without using any information about the load.

Comparing the two approaches, it seems that because of the larger freedom in choosing the degrees of freedom in the model, the scalar model has an advantage in a better understanding of the necessary degrees of freedom needed to have a good estimate. Further, the scalar model gives relatively good estimates of both eigenfrequencies, damping ratios and mode shapes, although the ARMAV model is believed to be better in detecting close modes. The accuracy in determining the modal parameters seems to be comparable for the two approaches. The two models agree quite well on the most significant modes, concerning both eigenfrequencies, damping ratios and mode shapes. The damping ratio of the dominant mode (0.69 Hz) was estimated as 1.3 % for the ARMAV model and 0.9 % for the scalar approach.

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