On the Applicability of the Surface Equivalence Theorem Inside Enclosures

O. Franek\textsuperscript{1}  M. Sørensen\textsuperscript{1}  H. Ebert\textsuperscript{1}  G. F. Pedersen\textsuperscript{1}

Abstract – A scenario of a generic printed circuit board (PCB) representing an electronic module inside a metallic enclosure is studied numerically. Following the surface equivalence theorem, the PCB is replaced with surface currents running on a Huygens box (HB) inside the enclosure and near-field errors with respect to the full model are observed. In concordance with previous work it is found that leaving the HB empty leads to significant errors. This time, however, countermeasures in the form of including the ground plane or substrate of the PCB inside the HB have the desired effect of reducing the errors at only some of the investigated frequencies.

1 INTRODUCTION

One of the goals of the electromagnetic compatibility (EMC) discipline is assuring compliance of electronic devices in terms of radiated emission, which is accomplished by performing measurements of the tested device in far field. On the other hand, numerical modeling of radiation is preferred in the early design phase of an apparatus, in order to reduce number of physical prototypes. The designed apparatus often consists of several parts or modules, mounted inside a (usually) metallic enclosure, the chassis. Some of the used modules may come from other vendors or subcontractors, their internal structure is unknown and their numerical model is unavailable. It has been suggested that an equivalent model of the unknown module can be used instead, in terms of surface equivalence theorem [1].

The surface equivalence theorem [2] says that, under certain assumptions, an arbitrary source of electromagnetic field can be replaced by a set of electric and magnetic currents flowing over a surface entirely enclosing the source, whereas these currents are obtained from the tangential fields present at the respective surface. The equivalent currents in the form of a Huygens box (HB) will then generate the same fields outside the surface as the source itself. Therefore, it is possible to predict radiated emission of a particular device based on near-field measurements, i.e. on the surface around the device.

However, using the equivalent currents in place of a numerical model of unknown module poses a challenge. One of the assumptions for the equivalence theorem is that the environment outside of the surface is identical in both cases, when measuring and when using the equivalent currents in predicting the surrounding fields numerically. This condition is clearly violated in the presence of obstacles, other modules, and the apparatus enclosure.

In our earlier work we have investigated the influence of nearby obstacles, ground plane and cables in particular, on the applicability of the surface equivalence theorem [3,4]. While the conditions of the theorem are clearly violated, we have proposed a strategy to mitigate the resulting errors: to include an approximated model of the original source inside the HB, even with greatly reduced complexity, e.g. only the ground plane of a PCB [3]. In this paper we would like to show that the same strategy may not work if the equivalent model of the source is placed inside an enclosure. Our numerical experiments with a test PCB in the frequency range 100 MHz–1 GHz show that the errors in near fields inside box-shaped enclosure when using the above mentioned countermeasures may be comparable or even higher than without the countermeasures.

2 NUMERICAL EXPERIMENT

As a typical representative of an electronic module, we have used a generic test printed circuit board (PCB) with dimensions 150 × 100 mm, made of 2 mm thick substrate FR4 with relative permittivity 4.35 and conductivity 10\textsuperscript{-3} S/m (Fig. 1). The PCB contains three traces on the top layer, but only the first trace (denoted by “Trace 1” in Fig. 1) is active, with 50 Ω impedance matching at both ends. The bottom layer of the PCB is formed by a continuous ground plane. In order to evaluate the influence of enclosures, the PCB is placed in the middle of a metallic box with dimensions 450 × 300 x 40 mm, which is open at one side (Fig. 1).

For the numerical experiment we have used the established finite-difference time-domain method [5]. We have assumed that all metals (PCB layers and the enclosure) are made of perfect electric conductor (PEC), i.e. without losses.

In order to quantify the error of the proposed method, we recognize three simulation scenarios:

1. Test PCB in free space
2. Test PCB inside the enclosure
3. Equivalent sources (HB) inside the enclosure.

\textsuperscript{1} APNet Section, Department of Electronic Systems, Faculty of Engineering and Science, Aalborg University, Niels Jernes Vej 12, DK-9220 Aalborg Ø, Denmark, e-mail: \{of,mos,heb,gfp\}@es.aau.dk, tel.: +45 9940 9837, fax: +45 9815 1583.
The first scenario provides the near field values characterizing the PCB, which are then installed on the surface of the HB in the third scenario where these near fields serve as a source replacing the PCB. The HB has a rectangular shape with dimensions $245 \times 170 \times 20$ mm, i.e. with approximately 10 mm gap from the PCB. The second scenario then serves as a reference solution. The peak total error in the near fields between scenarios 2 and 3 is evaluated using the following formula

$$\text{Peak total error} = \frac{\max |F^{(3)} - F^{(2)}|}{\max |F^{(2)}|} \times 100 \ [%]$$

where $F$ stands for either electric (E) or magnetic (H) fields and the superscripts indicate the particular scenario. The maximum values are taken from the vector fields everywhere in the computational domain except inside and just over the surface of the HB.

### 3 RESULTS

The calculated peak errors in E-fields and H-fields are displayed in Table 1 and Table 2, respectively, for three selected frequencies. It can clearly be seen that the error can become quite high (66 % in H-fields) at the upper end of the frequency range, if the HB is left empty. This is caused by the fields reflected from the walls of the enclosure, which penetrate through the HB, but do not find the original structure, the PCB, to be rescattered from.

**Table 1: Peak total error in E-field caused by substituting the test PCB by the HB.**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>100 MHz</th>
<th>900 MHz</th>
<th>1 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>empty</td>
<td>6.2 %</td>
<td>27 %</td>
<td>52 %</td>
</tr>
<tr>
<td>gnd plane</td>
<td>1.3 %</td>
<td>83 %</td>
<td>53 %</td>
</tr>
<tr>
<td>gnd plane, substrate</td>
<td>0.3 %</td>
<td>1.5 %</td>
<td>53 %</td>
</tr>
</tbody>
</table>

**Table 2: Peak total error in H-field caused by substituting the test PCB by the HB.**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>100 MHz</th>
<th>900 MHz</th>
<th>1 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>empty</td>
<td>4.0 %</td>
<td>15 %</td>
<td>66 %</td>
</tr>
<tr>
<td>gnd plane</td>
<td>0.14 %</td>
<td>48 %</td>
<td>66 %</td>
</tr>
<tr>
<td>gnd plane, substrate</td>
<td>0.13 %</td>
<td>0.52 %</td>
<td>67 %</td>
</tr>
</tbody>
</table>

In [3] we have proposed to include some features of the original PCB, namely its bottom layer (ground plane), inside the HB as a countermeasure. This should have the effect of letting the field reflected from the walls rescatter and imitate the original scenario 2. However, in the present case of the enclosure, such countermeasure does not have the desired effect and it makes the situation even worse, as we can see in Tables 1 and 2. In particular, the E-field error rises to 83 % at 900 MHz.

Further attempt at including even more features of the PCB, i.e. adding also the substrate, ended up with significant improvement at 900 MHz, but on the other hand the error remained without change at 1 GHz.

### 4 CONCLUSION

From the numerical experiment it follows that including only the ground plane of the PCB in the HB as a countermeasure against errors is not sufficient when the HB is placed inside a tight enclosure. This is in contrast to our earlier work where similar PCB was positioned next to a metallic plane [3] or a parallel running open-ended cable [4], and inclusion of the ground plane improved the error significantly. One possible explanation is a likely onset of resonances of the metallic enclosure and/or the PCB itself on the observed frequencies. Including the lossy substrate together with the ground plane, intended to damp the eventual resonances, helped at one frequency, 900 MHz, but did not have any effect at another, 1 GHz. Here it is worth noting, however, that including too many features inside the HB goes...
against the very idea of simplification of the problem and in real situation we would not necessarily have the desired parameters of the PCB at hand. In the ongoing work, we intend to find out how close the model of the PCB has to resemble the original in order to reduce the errors to acceptable levels at all frequencies.

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References


