Tuning of Synchronous-Frame PI Current Controllers in Grid-Connected Converters Operating at a Low Sampling Rate by MIMO Root Locus

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Abstract—Current controller performance is key in grid-connected power converters for renewable energy applications. In this context, a challenging scenario is arising in multi-MW wind turbines, where sampling and switching frequencies tend to be lower and lower as power ratings increase. This strongly affects achievable control time constant. With this perspective, this paper presents a systematic procedure for accurate dynamics assessment and tuning of synchronous-frame PI current controllers, which is based on linear control for multiple input multiple output (MIMO) systems. The dominant eigenvalues of the system are calculated with explicit consideration of time-delay and cross-coupling terms, two factors which clearly impair the system dynamics when considering a low sampling frequency. The proposed methodology is summarized as follows. Firstly, the plant and controller matrices are modeled in state-space. Subsequently, the characteristic polynomial of the closed-loop system is obtained and a computer-aided parametric analysis is performed to calculate the MIMO root locus as a function of the control gain. By its inspection, it is possible to identify the gain which minimizes the current closed-loop time constant. This tuning is suitable for wind turbine applications, taking into consideration cascaded-control structures and grid-code requirements. The validity and accuracy of the analysis is fully supported by experimental verification.

Index Terms—Ac/dc power conversion, current control, pulse width modulation converters, wind power generation.

I. INTRODUCTION

Synchronous-frame proportional-integral (dq-PI) current controllers are widely employed in ac drives [1]–[3] and grid-tied converters [4], [5]. The synchronous-frame axes $d$ and $q$ rotate at the angular frequency $\omega$ of voltage/current waveforms. From the control point of view, this is advantageous since ac voltages/currents are controlled by dc variables, so PI regulators provide their highest performance. In this manner, dq-PI current controllers achieve reference tracking and disturbance rejection with zero steady-state error [1]–[7].

In addition to steady-state features, time-domain dynamics are also very important in some grid-connected applications, such as wind turbines. When cascaded controllers are implemented, special care must be put on tuning the innermost current loops, as they establish the dynamic performance of the overall system (i.e., inner loops should be much faster than the outer loops) [6], [8]–[10]. Furthermore, some grid codes also demand stringent time-domain dynamics in terms of reference tracking when a fault is detected, forcing generation units to supply a certain amount of reactive current within a specific response time [8], [11]. Fulfilling these requirements becomes challenging in a multi-MW context, where switching and sampling frequencies tend to be very low [12], [13].

Closed-loop dynamics mainly depends on the controller gains, so the tuning process is key to optimize the current control loop behavior. Ideally, the PI gains can be tuned so that the system is defined by a single dominant pole, which is set by the controller gain [1], [2] (this gain sets the closed-loop bandwidth, so the term “bandwidth gain” is employed in this paper). However, real systems may differ from ideal ones mainly due to parameter deviations [1], [2] and time-delay effects [3], [7]. Fine analysis for high-bandwidth tuning optimization requires to include the time delay in the circuit modeling [3], [7], [14]–[17].

A big shortcoming of dq-PI controllers arises when considering both the time-delay and the (cross-coupling) decoupling schemes at the same time. The classical dq-PI with feedback axes-decoupling presents important remaining cross-channel effects [14]–[17]. On the other hand, cross-coupling terms are usually neglected for tuning the controllers, so the dq-PI system with delays can be approximated to a second order single-input single-output (SISO) loop [3], [18]. Even though these approaches give useful guidelines for the control parameters, they dismiss cross-coupling effects in the dynamics (root-locus trajectories also depend on cross-coupling terms). The influence of cross-coupling terms in the system dynamics have been previously reported in electric drives applications with high ratios of fundamental-to-sampling frequencies [14]–[17]. Therefore, coupling should be also considered in the grid-connected converters when low sampling frequencies are considered.
This paper contributes a systematic procedure for accurate assessment and tuning of the classic dq-PI current controllers dynamics, which is based on linear control for multivariable, or multiple input multiple output (MIMO), systems. The effect of non-diagonal components, due to remaining cross couplings associated to the system time-delay, is explicitly considered. The MIMO characteristic polynomial is obtained from an accurate modeling of the plant and control matrices, which define the MIMO sensitivities. Its roots (eigenvalues) determine the system closed-loop dynamics [19], [20]. These roots depend on the controller tuning (bandwidth gain). From a given design objective the tuning is obtained. In this work, the design aim is to minimize the current loop time constant, as it provides a good trade-off considering both cascading controllers dynamics [6], [8], [9] and transient response for grid-code compliance [8], [11].

The rest of the paper is organized as follows. Section II shows a basic background in modeling and control from reference papers. Section III develops the plant modeling with explicit consideration of delays, which gives rise to a non-diagonal plant transfer function with explicit time-delay terms. Section IV analyzes a simplified second order standard system, obtained by neglecting coupling terms. The main aim of this section is to provide a qualitative description of the closed-loop dynamics and support the assumptions made in the next section. Section V develops the MIMO method for an accurate calculation (as it includes coupling terms) of system eigenvalues, and shows how to identify the pair of dominant poles by inspection of MIMO root locus diagrams. Section VI provides experimental support to the theoretical approach. The key experimental results prove the higher accuracy provided by the MIMO based approach.

II. BACKGROUND

Fig. 1 represents a power converter connected to the grid and its discrete controller based on a dq-PI with feedback decoupling implementation [1]. By assuming a stiff grid, i.e., $|X_g| << \omega$ [21], [22], the state-space equation of the power circuit (plant) is given by

$$
\frac{d}{dt} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} = \begin{bmatrix} -R/L & -\omega \\ -\omega & -R/L \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} + \frac{1}{L} \begin{bmatrix} U_d(t) \\ U_q(t) \end{bmatrix} + \frac{1}{L} E_d(t)$$

where

$$I \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$  

Equation (1) has cross-coupling terms as a result of the Park transforms [1], [7]. A classical decoupling strategy is given by the following change of variable in the control action [1]:

$$\begin{bmatrix} U_d' \\\nU_q' \end{bmatrix} = I \begin{bmatrix} U_d \\\nU_q \end{bmatrix} + \begin{bmatrix} 0 & -L\omega \\ L\omega & 0 \end{bmatrix} \begin{bmatrix} i_d \\\ni_q \end{bmatrix}.$$  

If the system time delays due to discrete operation and PWM blocks are neglected (see Fig. 1):

$$U_d(t) = U_d'(t) \quad U_q(t) = U_q'(t).$$  

By substituting (4) in (1), the “effective plant” is given by the following diagonal (decoupled) system:

$$\frac{d}{dt} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} = \frac{1}{L} \begin{bmatrix} -R & 0 \\ 0 & -R \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} + \frac{1}{L} \begin{bmatrix} U_d(t) \\ U_q(t) \end{bmatrix} + \frac{1}{L} E_d(t)$$

From (5), the plant matrix transfer function $G(s)$ is given by

$$i(s)|_{E(s)=0} = \frac{1}{L(s+R/L)} I \cdot U(s).$$

This plant modeling should be reconsidered to include the effect of grid impedance when the stiff assumption is not realistic, that is in weak grids [21], [22]. In such a situation, a grid inductive part $L_g$ should be added to L to give the total system inductance and a grid resistive part $R_g$ should be added to $R$ to give the total system resistance [23], [24]. On the other hand, Fig. 1 is also suitable to describe induction motor drives [3]. In that case, both the motor and converter filter impedances should be included in the model [3].

After plant transfer modeling by (6), a diagonal PI controller according to the internal model laws [1], [2], [25] is defined as

$$K(s) = \frac{\alpha L(s+R/L)}{s} I$$

with $\alpha$ being the bandwidth gain, and assuming a good knowledge of the filter parameters. $G(s)K(s) = \alpha/sI$ is the resulting open-loop transfer function matrix. When the closed loop is considered, the time constant is the inverse of $\alpha$ [1], [7]. Since it is a first order diagonal system, each channel is independent from each other, and the control problem is reduced to two independent SISO loops. Such a simple analysis is only accurate for a limited range of $\alpha$ values. In

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high-bandwidth systems, this first order approximation does not provide reliable information about time response, damping or stability [3]. In practice, it is assumed that \( \alpha \) should be ten times smaller than the sampling frequency \( f_s \) [1], [7]. However, second order effects (e.g., overshoot) are noticeable for bandwidth gains even smaller than one-tenth of \( f_s \), [3], [7]. Therefore, as time-delay effects are considered, the dynamics is better defined by second order systems [3], [7], [15]–[17].

III. MULTIVARIABLE PLANT MODELING WITH TIME-DELAY CONSIDERATION

This section details the system modeling taking into account the controller delay. This has relevant effects in the closed-loop dominant roots, as previously discussed.

A time delay \( t_d \) due to discrete-time operation and PWM is considered. The value of \( t_d \) is given by

\[
t_d = 1.5/f_s,
\]

one sample due to discrete-time operation added to half a sample associated to the PWM operation [3], [7], [16]. Thus, the converter output voltage is rewritten as

\[
\begin{bmatrix}
U_d(t) \\
U_q(t)
\end{bmatrix}
= \Phi_{lag} \begin{bmatrix}
U_d^* (t - t_d) \\
U_q^* (t - t_d)
\end{bmatrix}
\]

with

\[
\Phi_{lag} = \begin{bmatrix}
\cos(\phi) & \sin(\phi) \\
-\sin(\phi) & \cos(\phi)
\end{bmatrix}
\]

and

\[
\phi = t_d \omega
\]

being a rotation matrix and its corresponding angle, respectively. These expressions have an insightful physical meaning. The output voltages (control action) in the synchronous frame are delayed \( t_d \) (as in stationary frame analysis [3], [7]), but also rotate an angle \( \phi \) in relation to its reference vector (inputs of the PWM block) [16].

To compensate these effects, a phase-angle lead \( \hat{\phi} \) is included in the inverse Park-transform before the modulation block (see Fig. 1) [7], [16], [17]. This delay-compensation technique can be modeled as

\[
\begin{bmatrix}
U_d(t) \\
U_q(t)
\end{bmatrix}
= \Phi_{lead} \Phi_{lag} \begin{bmatrix}
U_d^* (t - t_d) \\
U_q^* (t - t_d)
\end{bmatrix}
\]

with

\[
\Phi_{lead} = \begin{bmatrix}
\cos(\hat{\phi}) & -\sin(\hat{\phi}) \\
\sin(\hat{\phi}) & \cos(\hat{\phi})
\end{bmatrix}
\]

and

\[
\hat{\phi} = \dot{t}_d \omega
\]

being the delay-compensation matrix and the estimated delay angle, respectively. It is straightforward to notice that an accurate estimation of the delay angle results in \( \Phi_{lead} \) that equals \( \Phi_{lag} \) and, therefore \( \Phi_{lead} \Phi_{lag} = I \). The control action is then simplified to

\[
\begin{bmatrix}
U_d(t) \\
U_q(t)
\end{bmatrix}
= \begin{bmatrix}
U_d^* (t - t_d) \\
U_q^* (t - t_d)
\end{bmatrix}
+ \begin{bmatrix}
0 & -L\omega \\
L\omega & 0
\end{bmatrix} \begin{bmatrix}
i_d(t - t_d) \\
i_q(t - t_d)
\end{bmatrix}
\]

As seen, besides its positive effect in dynamics [7], [16], [17], the delay compensation simplifies the theoretical analysis.

Subsequently, time-domain expressions should be transformed to the frequency-domain. A first order Padé approximation of the time-shifting property is given by

\[
L(f(t-t_d)) = f(s) \cdot e^{-st_d} = f(s) \cdot \frac{e^{-\frac{f}{t_d}}}{e^{\frac{s}{t_d}}}
\approx f(s) \cdot \frac{-\frac{f}{t_d} s + 1}{\frac{f}{t_d} s + 1} = f(s) \cdot \frac{s + 2/t_d}{s + 2/t_d}
\]

which is very accurate at frequencies one decade below \( 1/t_d \), i.e., at the expected bandwidth values [1], [7], [18]. Taking into account this property, (15) is converted to the frequency domain and substituted in (1). Thus, the power converter state-space model results in

\[
s \begin{bmatrix}
i_d(s) \\
i_q(s)
\end{bmatrix}
= \begin{bmatrix}
-\frac{R}{L} & \frac{2\omega_s}{s + 2/t_d} \\
-\frac{2\omega_s}{s + 2/t_d} & -\frac{R}{L}
\end{bmatrix} \begin{bmatrix}
i_d(s) \\
i_q(s)
\end{bmatrix}
+ \begin{bmatrix}
\frac{s + 2/t_d}{L(s + 2/t_d)} \\
\frac{2\omega_s}{L(s + 2/t_d)}
\end{bmatrix} \begin{bmatrix}
U_d(s) \\
U_q(s)
\end{bmatrix}
\]

It should be noted that there are still coupling terms due to time-delay effects in the decoupling paths [16], [17]. Then, from (17), the equivalent plant transfer function is given by

\[
G(s) = \frac{s + 2/t_d}{L[(s + R/L)^2(s + 2/t_d)^2 + 4\omega^2s^2]} \cdot \left[ \frac{s + R/L(s + 2/t_d)}{2\omega_s} \right] + \frac{2\omega_s}{(s + R/L)(s + 2/t_d)}
\]

Thus, in spite of axes-decoupling and delay-compensation schemes, the effective plant still depends on the time delay.

It is interesting to mention that (18) is in agreement with previous works in the field of electric drives, as it shows a coupling dependence with \( \omega \) [15]–[17].

A. Accuracy of the Time-Delay Rational Approximation

Linear control system with delays can be accurately assessed by S-domain analysis methods in the range of frequencies in which the rational approximations of time delays are accurate [19], [20]. In this work, (16) approximates the system delay by a rational function of first order. This approach implicitly assumes its accuracy in the range of frequencies at which the control acts, i.e., the control bandwidth, which is assumed to be limited according to the one-to-tenth rule: \( \alpha \leq 2\pi f_s/10 \) [1], [7]. Fig. 2(a) proves the accuracy of this assumption, as the rational function (modeled in the S-domain) perfectly matches a one sampling delay (modeled in the Z-domain) in the bandwidth range of frequencies. Fig. 2(b) shows the accuracy of the rational approximation in the time-domain by its step response. The rational expression has an initial undershoot and then needs around three samples to track the delayed signal [19].

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The coupling terms in (18), so that $G_{open-loop}$ poles at given by (19)-(20). The closed-loop poles evolve from the system non-minimum phase zeros at $\pm 2/t_d$ and $\infty$ [27]; i.e., as $\alpha$ increases, the non-minimum phase zeros (due to system delays) “attract” the closed-loop poles to the unstable plane.

1) Identification of Dominant Time Constants: For low $\alpha$ values, $p_1$ is closer to the right half plane (RHP), and hence its dynamics is slower than the one given by $p_2$. In such a case, it is possible to neglect the effect of $p_2$ and consider $p_d = p_1$ as a single dominant eigenvalue. Hence, the dynamics of the system is well defined by a single order system, for which the time constant is

$$\tau = \frac{1}{|\Re\{p_d\}|}. \quad (21)$$

At a certain $\alpha$ $p_1$ and $p_2$ converge and then, for larger $\alpha$ values, the system is defined by a pair of complex eigenvalues, such that $p_1 = p_2^\alpha$. As they dominate the system dynamics, the pair of complex eigenvalues can be referred to $p_{d_{1,2}} = [p_{d1}, p_{d2}]$. The dynamics of a second order system is usually defined as a function of the standard parameters

$$\omega_n = \sqrt{\Re^2\{p_{d1}\} + \Im^2\{p_{d1}\}} \quad \text{(natural frequency)}$$
$$\xi = \frac{|\Im\{p_{d1}\}|}{\sqrt{\Re^2\{p_{d1}\} + \Im^2\{p_{d1}\}}} \quad \text{(damping ratio)}. \quad (22)$$

Now, (20) can be written as a standard second order transfer function as

$$T(s) = \frac{\omega_n^2(1-st_d/2)}{s^2 + 2\xi\omega_n s + \omega_n^2}I \quad (23)$$

with

$$\omega_n = \sqrt{2\alpha/t_d} \quad \text{(natural frequency)}$$
$$\xi = \frac{2/t_d - \alpha}{2\sqrt{2\alpha/t_d}} \quad \text{(damping ratio)}. \quad (24)$$

It should be noted that the non-minimum phase zero can be kept as function of $t_d$ (for the sake of clarity, as it does not change with $\alpha$), but also as a function of the standard parameters ($\omega_n$ and $\xi$). The time constant of a standard second order system is given by

$$\tau = \frac{1}{\xi\omega_n} = \frac{1}{|\Re\{p_{d1}\}|}. \quad (25)$$

The settling time to within a tolerance band of the 2% of
the final value is given by the following expression [27]
\[ t_s \approx 3.9\tau. \] (26)

The rise time mainly depends on \( \omega_n \) and can be approximated by [27]
\[ t_r \approx 1.8/\omega_n. \] (27)

B. Tuning by a Second Order Approximation

1) Tuning Objective: In wind turbine applications, the control of the grid-side converter is usually based on cascaded loops: outer loops, with slow dynamics, regulate the reactive power delivered to the grid and the dc-link voltage; they generate the references for the inner current controllers [6], [8]. In this scenario, a suitable design guideline points to minimize the current controller time constant \( \tau \), i.e., to maximize the decay rate of its dominant poles [8].

2) SISO Guideline Tuning Values: From the second order system defined by (20), some reference values can be obtained analytically. When the system is critically damped (i.e., \( \xi = 1 \)) both eigenvalues coincide in a point, at which the real part of analytically. When the system is critically damped (i.e., \( \xi = 1 \)) some stringent grid-code requirements [6], [8], [11]. In this scenario, a suitable design guideline points to minimize the current controller time constant \( \tau \), i.e., to maximize the decay rate of its dominant poles [8].

It is recommended to set each matrix in its Smith-McMillan form to find poles and zeros, and their multiplicities [19], [20]. The Smith-McMillan form of \( G(s) \), obtained according to the guidelines in [19], [20], is
\[ G_{SM}(s) = \begin{bmatrix} -s + \frac{2}{t_d} & 0 \\ \frac{s}{(s \pm j\omega)^2} & -s + \frac{2}{t_d} \end{bmatrix}. \] (31)

From (31), \( G(s) \) has a double (non-minimum phase) zero at \( +2/t_d \). Approximated values for \( G(s) \) poles can be obtained by assuming \( 2/t_d >> R/L \sim \omega \) (circuit time constants are longer than the time-delay [3]), so
\[ p_{p1,p2} \approx -\frac{R}{L}(1 \pm jt_d\omega) \] (plant complex pair of poles)
\[ p_{t1,t2} \approx \frac{2}{t_d} \pm j2\omega \] (time-delay complex pair of poles).

\( K(s) \) is Smith-McMillan by definition: it has a double pole at the origin and a double zero at \( -R/L \).

B. MIMO Eigenvalues Calculation

The sensitivity matrix is defined by
\[ S(s) = [I + G(s)K(s)]^{-1}. \] (33)

The MIMO poles of (33) define the dynamics of the closed-loop system. E.g., all of them should be in the left-half-plane to assure stability.

Since the product \( G(s)K(s) \) does not contain unstable MIMO pole-zero cancellations (this can be checked from MIMO poles and zeros calculation in Section V-A), all the closed-loop system poles defining dynamic responses are given by the roots of the characteristic polynomial [19], [20], i.e.,
\[ \det[S^{-1}(s)] = \det[I + G(s)K(s)] = 0. \] (34)

As in the SISO case, the root-locus given by (34) depends on \( \alpha \) (gain). Therefore, the tuning gain that provides faster decaying dominant roots can be identified by inspection of root-locus diagrams.

To ease the calculations (and avoid any possible unstable pole-zero cancellation), the characteristic polynomial (34) is re-written using Matrix Fraction Description (MFD) matrices [19], as follows:
\[ \det[S^{-1}(s)] = \det[\bar{G}_D(s)K_D(s) + \bar{G}_N(s)K_N(s)] = 0 \] (35)
with
\[
G_D(s) = L[(s + R/L)^2(s + 2/t_d)^2 + 4\omega^2 s^2]I \\
G_N(s) = (-s + 2/t_d) \\
\begin{bmatrix}
(s + R/L)(s + 2/t_d) \\
2\omega s
\end{bmatrix}
\begin{bmatrix}
-2\omega s \\
(s + R/L)(s + 2/t_d)
\end{bmatrix}
\]
\[
K_D(s) = sI \\
K_N(s) = \alpha L(s + R/L)I.
\]
(36)

1) Computer Aided Calculation: In practice, it is very cumbersome to handle (36), since it involves non-diagonal matrix determinant calculations. This means high order expressions, which are very difficult to simplify. On the other hand, “Geometric Analysis” for MIMO systems has also been discarded because of its complexity due to \(\alpha^2\) terms in (34)–(35), as reported in [28].

Thus, in this paper, the problem has been solved by means of a computer-aided methodology. By means of the Matlab Symbolic Math toolbox, the characteristic equation from (34)–(36) results in a sixth order expression as follows
\[
det(S^{-1}) = L^2 t_d^2 s^6 + (-2\alpha L^2 t_d^2 + 4L^2 t_d + 2RLt_d^2)s^5 \\
+ (L^2\alpha^2 t_d^2 + 16L^2 t_d^2\alpha^2 + 4L^2 - 4LR\alpha t_d^2 \\
+ 8LRt_d + R^2 t_d^2)s^4 + (-4\alpha^2 t_d + 8L^2\alpha \\
+ 2LR\alpha t_d^2 + 8LR - 2R^2\alpha t_d^2 + 4R^2 t_d^2)s^3 \\
+ (4L^2\alpha^2 - 8LR\alpha t_d^2 + 16LR\alpha + R^2\alpha^2 t_d^2 \\
+ 4R^2)s^2 + (-4t_d R^2\alpha^2 + 8R^2\alpha + 8LR\alpha^2)s \\
+ 4R^2\alpha^2 = 0.
\]
(37)

Subsequently, the solutions are calculated and depicted as function of \(\alpha\) (parametric analysis) for a wide range of \(\alpha\) values. Fig. 4 shows a generic root locus obtained by solving (37) for increasing \(\alpha\) values. Section V-C details how to interpret the MIMO root-locus diagrams.

It is also interesting to note that (37) is a function of \(\omega\). This dependence comes from the cross-coupling terms in the plant modeling. In a grid-connected converter, \(\omega\) can be assumed to be constant. However, in order to extend the proposed methodology to motor drives, a parametric analysis as a function of \(\omega\) should be also considered, as this parameter depends on the machine speed [15]–[17].

C. Tuning by MIMO Root-Locus Diagrams

Fig. 4 describes a generic root locus obtained by solving (37) for increasing \(\alpha\) values. The following assumptions are made in order to analyze the MIMO root-locus diagram.

- The closed-loop poles are complex pairs. At \(\alpha = 0\) rad/s the poles correspond to \(G(s)K(s)\) in open loop. As \(\alpha\) increases, the pole-trajectories tend to \(-R/L\), (plant), \(2/t_d\) (delay) and \(\infty\), which correspond to MIMO zeros.
- As shown in the zoomed region of Fig. 4, the eigenvalues from trajectory-1 are very close to the double zero at \(-R/L\) [imposed by control design in (7)]. This implies a double pole-zero cancellation around \(-R/L\) [19], [20], and hence, the effect of these eigenvalues is not dominant in the presence of a current step.
- At the range of gains suitable for tuning (i.e., up to \(\alpha_{10}\%\), see shaded region in Fig. 4), the poles of trajectory-3 are faster than the ones of trajectory-2. Therefore, they are less dominant in the time response and hence, they are neglected.
- The trajectory-2 poles set the system dynamics, as they are dominant. Therefore, the dominant poles \(p_{d,2}\) are re-defined accordingly as the eigenvalues given by the trajectory-2 in the MIMO root locus.
- The expressions (22)–(27), as defined in Section IV for a standard second order system (including the non-minimum phase zero at \(-2/t_d\)), are suitable to describe the dynamics, but according to the re-defined MIMO dominant eigenvalues.

As detailed in section IV-B1, the proposed tuning method aims to minimize the current control time constant. This is achieved by maximizing the negative real part of the dominant eigenvalues: by inspection of the MIMO root locus it is possible to select the tuning that places \(p_{d,2}\) (trajectory-2) as far as possible from the RHP. This key tuning is then defined as \(\alpha_{min-t}\).

VI. EXPERIMENTAL VERIFICATION

The theoretical analysis has been validated in a lab-scale prototype. The digital control is executed in a dSpace MABXII DS1401 platform.

Fig. 5 shows the control scheme and a photo of the testbed. The control algorithm is as follows: the \(i_d\) controller is an inner loop of the dc-link voltage \(U_{dc}\) outer loop. The reactive current \(i_q\) is directly set by the dSpace user interface, so the key waveforms are obtained by triggering step changes in the reactive current reference \(i^*_q\) [8]. The power converter is working in rectifier mode of operation, which permits to test the performance of the current controllers (suitable for
any working point of the power converter) without needing an external power source.

Two different sampling frequencies have been considered in the tests: $f_s = 2850$ Hz and $f_s = 1500$ Hz. As the switching carrier frequency $f_c$ is reduced, it causes more significant low order harmonics (in practical applications, $f_s$ is reduced as a result of reducing $f_c$ and switching losses). The setup circuit includes large inductors to better filter switching harmonics. Grid voltage harmonics and dead-time effects are also an important source of distortion at low $f_s$ (i.e., as the control bandwidths are reduced accordingly). Therefore, a selective active filtering was implemented aiming to enhance steady-state waveforms without impairing very much the transient response assessment (see Section VI-C2). For each of the two cases, the equivalent resistance in the plant model of the current loop, which includes the effects associated to the switching and conduction losses, has been previously estimated by the method presented in [29].

Besides current distortion, another practical issue which difficulties the assessment of experimental figures is the inherent low resolution of the problem: the time constant (a parameter to measure) and the sampling period $1/f_s$ are of the same order of magnitude. To support time-domain assessments, the estimated currents $\hat{i}_q^{\text{SISO}}$ and $\hat{i}_q^{\text{MIMO}}$ are depicted. These are the estimated dominant time responses, which are obtained by applying the same current step as that commanded to the actual plant to (23), with the corresponding set of parameters $\omega_n$ and $\xi$, as shown in Fig. 5(a). It should be also noticed that the non-minimum phase zero at the numerator of (23) causes some inaccuracies at the very beginning of the transient [19], as discussed in Section III-A.

### A. Case A: Tests at $f_s = 2850$ Hz

The setup parameters are shown in Table I. Fig. 6 details the position of the dominant poles (the root loci is zoomed at the second quadrant for a better resolution; the MIMO trajectory-3 is not displayed as its eigenvalues are not dominant). Table II summarizes the main parameters in Fig. 6 for both the SISO and the MIMO approaches. The key values (highlighted in the plot) and assumptions to be made are explained below.

- The eigenvalues provided by the SISO approximation are faster and more damped than the ones provided by the MIMO solution, since the latter quantifies the effect of the coupling terms. This cross coupling is expected to be noticeable during transients.
- The tuning at $\alpha_{\text{damp}}$ corresponds to the theoretical value from (28). It theoretically fulfills the tuning objective according to the SISO approximation. However, it corresponds to a quite reduced gain in the MIMO root locus.
- The tuning at $\alpha_{\text{min}}$ is obtained by inspection of the MIMO root locus (the $\alpha$ which minimizes the negative real part of the eigenvalues). This is achieved with a gain higher than $\alpha_{\text{damp}}$.
- The tuning at $\alpha_{10\%}$ is selected according to the one-tenth rule $\alpha_{10\%} = 2\pi f_s/10$.

Fig. 7 shows the test results for the selected tunings. Table III shows the integral absolute error (IAE) from Fig. 7.
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### TABLE I
**Case A: Experimental Setup Circuit Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant Inductance</td>
<td>$L = 12.5$ mH</td>
</tr>
<tr>
<td>Estimated Plant Resistance</td>
<td>$R = 2.2$ kΩ</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>$f_c = 2850$ kHz</td>
</tr>
<tr>
<td>$t_d = 1.5/f_s$</td>
<td>$354$ µs</td>
</tr>
<tr>
<td>$R_L$</td>
<td>$255$ Ω</td>
</tr>
<tr>
<td>DC-link voltage</td>
<td>$U_{dc} = 700$ V</td>
</tr>
<tr>
<td>Fundamental angular frequency</td>
<td>$\omega = 2\pi 50$ rad/s</td>
</tr>
</tbody>
</table>

### TABLE II
**Predicted Dynamics from SISO and MIMO Root Locus Diagrams of Fig. 6**

<table>
<thead>
<tr>
<th>Tuning at $\alpha = \alpha_{\text{damp}} = 652$ rad/s</th>
<th>SISO $p_d = -1574$ rad/s</th>
<th>MIMO $p_d = -857 \pm 4380$ rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 635$ µs (1st order)</td>
<td>$\omega_\text{d} = 982$ rad/s</td>
<td>$\omega_\text{d} = 8.67$ rad/s</td>
</tr>
<tr>
<td>$t_{\text{d}} \approx 2.5$ ms</td>
<td>$t_{\text{d}} \approx 4.6$ ms</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tuning at $\alpha = \alpha_{\text{min}} = 1000$ rad/s</th>
<th>SISO $p_{d1,2} = -1400 \pm 1356$ rad/s</th>
<th>MIMO $p_{d1,2} = -1048 \pm 1127$ rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_\text{d} = 1950$ rad/s</td>
<td>$\omega_\text{d} = 1540$ rad/s</td>
<td>$\omega_\text{d} = 0.68$ rad/s</td>
</tr>
<tr>
<td>$t_{\text{d}} \approx 2.9$ ms</td>
<td>$t_{\text{d}} \approx 3.8$ ms</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tuning at $\alpha = \alpha_{10%} = 1791$ rad/s</th>
<th>SISO $p_{d1,2} = -1006 \pm 2407$ rad/s</th>
<th>MIMO $p_{d1,2} = -848 \pm 2121$ rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_\text{d} = 2009$ rad/s</td>
<td>$\omega_\text{d} = 2284$ rad/s</td>
<td>$\omega_\text{d} = 0.37$ rad/s</td>
</tr>
<tr>
<td>$t_{\text{d}} \approx 4.0$ ms</td>
<td>$t_{\text{d}} \approx 4.7$ ms</td>
<td></td>
</tr>
</tbody>
</table>

Captures, obtained by

$$\text{IAE} = \sum_{t=0}^{t_w} \left| i_q - \hat{i}_q \right| / f_s$$  \hspace{1cm} (38)

with $t_w = 10$ ms being the time window of the oscilloscope captures [18], [29].

From these results, it can be observed that $\hat{i}_q^{\text{MIMO}}$ matches well the measured current $i_q$ in all the cases. More specifically, Fig. 7(e) proves that the minimum time constant is achieved with $\alpha_{\text{min}}$, as correctly predicted by the MIMO root locus. The corresponding IAEs in Table III show that the MIMO prediction matches better than the SISO the one the actual response with $\alpha_{\text{min}}$ and $\alpha_{\text{damp}}$. From Fig. 7(d), and its corresponding IAEs, $\hat{i}_q^{\text{SISO}}$ is less accurate than $\hat{i}_q^{\text{MIMO}}$ with $\alpha_{\text{damp}}$ being the tuning gain. These results using low tuning gains also prove that the SISO approximation is not suitable to find the aimed gain, which minimizes the dominant time constant. As high tuning gains, both SISO and MIMO time-domain predictions become similar [see Fig. 7(f) and the last rows in Table III]. This fact is also as expected from Fig. 6 (SISO and MIMO trajectories tend to be closer as $\alpha$ increases) and their estimated settling times in Table II.

### TABLE III
**IAE Obtained from the Experimental Tests of Fig. 7**

<table>
<thead>
<tr>
<th>Tuning at $\alpha = \alpha_{\text{damp}} = 652$ rad/s</th>
<th>SISO IAE = $4.9E^{-8}$ A·s</th>
<th>MIMO IAE = $4.0E^{-8}$ A·s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuning at $\alpha = \alpha_{\text{min}} = 1000$ rad/s</td>
<td>SISO IAE = $3.8E^{-8}$ A·s</td>
<td>MIMO IAE = $3.4E^{-8}$ A·s</td>
</tr>
</tbody>
</table>

### TABLE IV
**Case B: Experimental Setup Circuit Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant Inductance</td>
<td>$L = 24.3$ mH</td>
</tr>
<tr>
<td>Estimated Plant Resistance</td>
<td>$R = 1.7$ kΩ</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>$f_s = 1500$ kHz</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>$f_c = 1500$ kHz</td>
</tr>
<tr>
<td>$t_d = 1.5/f_s$</td>
<td>$1$ ms</td>
</tr>
<tr>
<td>$R_L$</td>
<td>$255$ Ω</td>
</tr>
<tr>
<td>DC-link voltage</td>
<td>$U_{dc} = 700$ V</td>
</tr>
<tr>
<td>Fundamental angular frequency</td>
<td>$\omega = 2\pi 50$ rad/s</td>
</tr>
</tbody>
</table>

### B. Case B: Tests at $f_s = 1500$ Hz

The setup parameters are shown in Table IV, Fig. 8 displays the root loci from MIMO and SISO approaches, while Table V details the position of the dominant poles for the three key tunings, which have been obtained as in Case A.

Fig. 9 shows the key results for the selected tunings and Table VI their corresponding IAEs. Since the main controller bandwidth is reduced very much, the selective harmonic elimination controller (see Section VI-C2) has some influence on the dynamics: a good filtering is achieved in steady state, but the $6^{th}$ harmonic oscillation arises during the transients. This fact impairs the assessments and significantly increases the IAEs (with respect to the ones of Section VI-A). Despite the $6^{th}$ harmonic ripple, it can be said that $\hat{i}_q^{\text{MIMO}}$ correctly predicts the dominant time constant in all the cases. As expected, the dominant time constant is minimized with $\alpha_{\text{min}}$ being the tuning gain. The experiments using $\alpha_{\text{damp}}$ should be also highlighted: Fig. 9(d) and its corresponding IAEs prove that $\hat{i}_q^{\text{MIMO}}$ fits much better than $\hat{i}_q^{\text{SISO}}$ the actual $i_q$ current. Therefore, the results using $\alpha_{\text{min}}$ and $\alpha_{\text{damp}}$ also show that the SISO approximation is not suitable to find the gain that minimizes the dominant time constant. As also predicted in Fig. 8, SISO and MIMO time-domain predictions become similar when using high tuning gains.

### C. Implementation Issues

1) **Discretization of Integrators:** Similarly to the analysis in section III-A, the accuracy of the time domain implementation in the range of frequencies of the control should be checked. Fig. 10 shows the frequency response of the discrete-time integrator implementation obtained by the Tustin method. It can be observed that it matches the theoretical frequency response up to $f_s/10$ and beyond. At higher frequencies, the magnitude response of the discretized integrator becomes
lower than the one of the ideal integrator, so a good high-frequency noise attenuation is expected. It is worth remarking that (23), which is used to estimate $\hat{i}_q^{\text{SISO}}$ and $\hat{i}_q^{\text{MIMO}}$ in Fig. 5(a), is also discretized by the Tustin method.

2) Active Filtering for Dead-Time and Grid Harmonics: As the sampling frequency and achievable control bandwidth are reduced, grid voltage harmonics and dead-time effects become very significant. In order to deal with current distortion, selective resonant filters for fifth (negative sequence) and seventh (positive sequence) harmonics have been implemented in parallel to the main $dq-PI$ controller [30]. Each resonant filter has been implemented with a very small gain, and each phase-lead compensation angle is calculated according to the Nyquist criterion, as explained in [31]. This implementation eliminates steady-state distortion due to grid harmonics and dead-times. However, some oscillation is present during transients: a sixth order harmonic oscillation in the $dq$-frame (i.e., at resonant frequencies, the Nyquist trajectories are closer to the critical point [31]). This oscillation becomes more significant as $f_s$ is reduced, and hence impairs experimental validations, as shown in Section VI-B. This corresponds with the fact that the disturbance rejection ability decreases with the controller bandwidth [3].

VII. Conclusions

This paper presents a systematic procedure for accurate dynamics assessment and tuning of $dq$-PI current controllers when a low sampling frequency is an imposed constraint.

![Fig. 7. Step responses from $i_{q}^{*} = 0$ to $i_{q}^{*} = -8 \, A$ at $f_s = 2850 \, Hz$ for different tunings. (a) $\alpha = \alpha_{damp}$; (b) $\alpha = \alpha_{\text{min, } r}$; (c) $\alpha = \alpha_{10\%}$; (d) $\alpha = \alpha_{damp}$-zoomed view; (e) $\alpha = \alpha_{\text{min, } r}$-zoomed view; (f) $\alpha = \alpha_{10\%}$-zoomed view. Note that $i_{q}$, $i_{q}^{\text{SISO}}$ and $i_{q}^{\text{MIMO}}$ are inverted.](image)

![Fig. 8. MIMO and SISO Root-loci for $f_s = 1500 \, Hz$ setup.](image)

<table>
<thead>
<tr>
<th>TABLE V</th>
<th>PREDICTED DYNAMICS FROM SISO AND MIMO ROOT LOCUS DIAGRAMS OF FIG. 8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha = \alpha_{\text{damp}} = 343 , rad/s$</td>
</tr>
<tr>
<td>SISO</td>
<td>$p_d = -828 , rad/s$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{\text{min, } r}$</td>
</tr>
<tr>
<td>MIMO</td>
<td>$p_{d1, 2} = -364 \pm j247 , rad/s$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{\text{min, } r}$</td>
</tr>
<tr>
<td>SISO</td>
<td>$p_{d1, 2} = -700 \pm j843 , rad/s$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{10%}$</td>
</tr>
<tr>
<td>MIMO</td>
<td>$p_{d1, 2} = -464 \pm j616 , rad/s$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{10%}$</td>
</tr>
<tr>
<td>SISO</td>
<td>$p_{d1, 2} = -529 \pm j1267 , rad/s$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{10%}$</td>
</tr>
</tbody>
</table>
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Fig. 9. Step responses from $i_d^* = 0$ to $i_q^* = -8$ A at $f_s = 1500$ Hz for different tunings. (a) $\alpha = \alpha_{\text{damp}}$; (b) $\alpha = \alpha_{\text{min}_q}$; (c) $\alpha = \alpha_{10\%}$; (d) $\alpha = \alpha_{\text{damp}}$; zoomed view; (e) $\alpha = \alpha_{\text{min}_q}$, zoomed view; (f) $\alpha = \alpha_{10\%}$, zoomed view. Note that $i_q$, $\hat{i}_q^{\text{SISO}}$ and $\hat{i}_q^{\text{MIMO}}$ are inverted.

This contribution specially aims to grid-connected converters in renewable energy applications, in which a fast dynamics is needed, while the sampling frequency tends to decrease as power ratings increase (e.g., grid code requirements in multi-MW wind turbines). Taking into consideration cascaded-control structures and grid-code requirements, minimizing the current controller time constant is proposed as the tuning objective.

The theoretical approach firstly models the system taking the system delays and coupling terms into consideration. Subsequently, a SISO root-locus approximation is obtained by neglecting the coupling terms, as usually assumed. However, this assumption tends to become inaccurate for low sampling frequency. A MIMO root locus based approach is then proposed to provide a more accurate calculation of the dominant eigenvalues. Then, by inspection of MIMO root locus diagrams, the gain which minimizes the current controller time constant $\alpha_{\text{min}_q}$ is obtained.

Experimental verification validates the high accuracy of the proposed tuning methodology. Experimental figures prove that $\alpha_{\text{min}_q}$ minimizes the settling time, and hence the current loop time constant. Besides the tuning objective is suitable from the MIMO root locus method, it is also important to highlight how real dynamics tends to be slower and more undamped than predictions assuming a negligible cross coupling.

Fig. 10. Accuracy of the discrete-time integrator obtained by the Tustin method. Solid: discrete-time implementation ($H(z) = \frac{z^{-1}}{1 - \frac{z^{-1}}{2}}$). Dotted: ideal integrator ($H(s) = 1/s$).

Table VI

IAE Obtained from the Experimental Tests of Fig. 9

<table>
<thead>
<tr>
<th>Tuning at $\alpha = \alpha_{\text{damp}} = 343 \text{rad/s}$</th>
<th>IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SISO</td>
<td>$13.3 \cdot 10^{-3}$ A-\text{·s}</td>
</tr>
<tr>
<td>MIMO</td>
<td>$4.8 \cdot 10^{-3}$ A-\text{·s}</td>
</tr>
<tr>
<td>Tuning at $\alpha = \alpha_{\text{min}_q} = 600 \text{rad/s}$</td>
<td>IAE</td>
</tr>
<tr>
<td>SISO</td>
<td>$6.6 \cdot 10^{-3}$ A-\text{·s}</td>
</tr>
<tr>
<td>MIMO</td>
<td>$6.4 \cdot 10^{-3}$ A-\text{·s}</td>
</tr>
<tr>
<td>Tuning at $\alpha = \alpha_{10%} = 963 \text{rad/s}$</td>
<td>IAE</td>
</tr>
<tr>
<td>SISO</td>
<td>$7.9 \cdot 10^{-3}$ A-\text{·s}</td>
</tr>
<tr>
<td>MIMO</td>
<td>$8.1 \cdot 10^{-3}$ A-\text{·s}</td>
</tr>
</tbody>
</table>

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REFERENCES


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