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A Broadband Beamformer Using Controllable Constraints and Minimum Variance

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C-LCMV Beamformer

**Introduction**

The minimum variance distortionless response (MVDR) beamformer is an optimal approach to noise reduction:
- Achieves a high output SNR.
- Has degrees of freedom (DOF) corresponding to the number of microphones minus one.
- Its output is contaminated with both residual noise and interference.

The linearly constrained minimum variance (LCMV) beamformer reduces noise, and rejects interferers using linear constraints:
- Achieves a high output SIR.
- The number of constraints may degrade the DOF.
- It may amplify background noise which causes a lower output SNR.

To achieve a trade-off between attenuation of noise and interfering sources, we proposed the controllable LCMV (C-LCMV) beamformer in the frequency-domain.

**Formulation**

Multi-channel observed signals at the frequency $f$ are observed, using an array of $M$ microphones:

$$y(f) = d_x(f)X(f) + \sum_{n=2}^{N} d_y(f)X_n(f) + v(f) = D_y(f)x(f) + v(f),$$

where $d_x(f) \in \mathbb{C}^M$ is the steering vector of signal source $X_x(f)$ (for $n = 1, \ldots, N$), $X_n(f)$ is the signal of interest (SOI), $x(f) \in \mathbb{C}^N$ is the collected $N$ signal sources, $v(f) \in \mathbb{C}^M$ is noise, and $D_y(f) = [d_1(f); d_2(f); \ldots; d_N(f)] \in \mathbb{C}^{M \times N}$.

The correlation matrix of $y(f)$ (assuming uncorrelated signals) is

$$\Phi_y(f) = D_y(f) \Phi_x(f) D_y^H(f) + \Phi_v(f) = d_y(f) \Phi_x(f) d_y^H(f) + \Phi_v(f),$$

where $\Phi_x(f) = \text{diag}(\phi_x(f), \phi_x(f), \ldots, \phi_x(f))$, $\Phi_v(f) = \Phi_v(f)$, and $\Phi_v(f) = \sum_{n=2}^{N} d_y(f) \phi_x(f) d_y^H(f)$ is the interference correlation matrix.

The output variance of the beamformer $h(f)$ is

$$\phi_\ell(f) = h^H(f)d_y(f)\phi_x(f)h^H(f) + h^H(f)[\phi_v(f) + \Phi_v(f)]h(f).$$

With the distortion constraint that $h^H(f)d_y(f) = 1$, we can write

$$o\text{SINR}[h(f)] = \frac{\phi_v(f)}{h^H(f)[\phi_v(f) + \Phi_v(f)]h(f)}$$

and

$$o\text{SIR}[h(f)] = \frac{\phi_v(f)}{h^H(f)\Phi_v(f)h(f)}.$$

**C-LCMV Beamformer**

**MDVR:**

$$\min_{h(f)} \ h^H(f)[\Phi_v(f) + \Phi_y(f)]h(f)$$

subject to $\ h^H(f)d_y(f) = 1,$

**LCMV:**

$$\min_{h(f)} \ h^H(f) \Phi_v(f)h(f)$$

subject to $\ h^H(f) D_y(f) = I_N,$

where $I_N$ is the first column of a $N \times N$ identity matrix.

We divide $N$ signal sources into two sets of $N_s$ sources, containing SOI, and $N_r = N - N_s$ remaining signal sources:

$$x(f) = [x_{1s}(f) x_{2s}(f)]^T,$$

and

$$\Phi_v(f) = D_y(f) \Phi_{v,1s}(f) D_y^H(f) + \Phi_{v,2s}(f),$$

where $\Phi_{v,1s}(f) = D_y(f) \Phi_{v,1s}(f) D_y^H(f) + \Phi_{v,2s}(f)$ is the correlation matrix of $N_s$ signal sources plus background noise.

The C+LCMV beamformer is designed as

$$\min_{h(f)} \ h^H(f) \Phi_{v,1s}(f)h(f)$$

subject to $\ h^H(f) D_y(f) = I_N.$

Then the solution is given like

$$h_{c}(f) = \Phi_{v,1s}^{-1}(f) D_y(f) [D_y^H(f) \Phi_{v,1s}^{-1}(f) D_y(f)]^{-1} I_N.$$

**Properties:**

$$o\text{SINR}[h_{c}(f)] \leq o\text{SINR}[h_{c}(f)] \leq o\text{SINR}[h_{c}(f)],$$

$$o\text{SIR}[h_{c}(f)] \leq o\text{SIR}[h_{c}(f)] \leq o\text{SIR}[h_{c}(f)].$$

**Optimal steering matrix (in practice):**

$$D_y^H(f) = \text{arg max} o\text{SINR}[h_{c}(f)].$$

**Experiment: Real Scenario**

We simulated a room ($6 \times 7 \times 3$ m) with reverberation time $T_{60} = 0.25$ s, and $N = 3$ speech signals, and used a ULA with $M = 5$ hypercardioid microphones, SNR= 20 dB.

**Conclusion**

- The C-LCMV beamformer generalizes the MVDR and LCMV beamformers with the ability to control the quality of the output signal.
- Experiment results indicate that the C-LCMV beamformer using optimal steering matrix gets better results than the other MVDR beamformers.

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