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Step by Step Design of a High Order Power Filter for Three-Phase Three-Wire Grid-connected Inverter in Renewable Energy System

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Abstract—Traditionally, when designing an LCL-filter, a three-phase inverter is simplified as a single-phase inverter for analysis and the output phase voltage is used to calculate the inverter-side current harmonics and to design inverter-side inductor. However, for a three-phase three-wire grid-tied system, the output current harmonics of inverter are directly affected by the output line to line voltage. Hence, this paper proposes a new method to analyze the inverter output current harmonics by using the equivalent phase voltage of the three phase inverter. Based on this, a step by step design method of the high order power filter is introduced. Simulations are carried out to verify the accuracy and the validity of the proposed methods through a 6 kW, 380V/50 Hz grid-connected inverter model with three different types of high order power filters.

Keywords—LLCL-filter; LCL-filter; current harmonics; voltage harmonics; equivalent phase voltage; design procedure; three-phase grid-tied inverter; SPWM

I. INTRODUCTION

Recently, due to the energy crisis, the distributed generation (DG) systems using clean renewable energy such as solar energy, wind energy, etc., have become an important issue in technical research. However, the use of pulse width modulation (PWM) introduces undesirable harmonics that may disturb other sensitive loads/equipment on the grid and also result in extra power losses [1]. Hence, a low-pass power filter is inserted between the voltage source inverter (VSI) and the grid to attenuate the high-frequency PWM harmonics to a desirable limit. Fig. 1 shows the structure of three-phase three-wire grid-connected inverter with different high order filters: LCL-filter, LLCL-filter with one trap [2] and LLCL-filter with two traps [3].

Typically, a simple series inductor \(L\) is used as the filter interface between power converters in the renewable energy system. But a high value of inductance needs to be adopted to reduce the current harmonics around the switching frequency, which would leads to a poor dynamic response of the system and a high power loss. In contrast to the typical L-filter, the LCL-filter can achieve a high harmonic attenuation performance with less total inductance \((L_1 + L_2)\), significantly smaller size and cost, especially for applications above several kilowatts [4]. In order to further reduce the total inductance even more, the LLCL-filter was proposed [2] and the application of the LLCL-filter on the three-phase three-wire Shunt Active Power Filter (SAPF) was analyzed [5]. Compared with the LCL-filter, the total inductance and volume of the LLCL-filter can be reduced which has been exemplified in a single-phase inverter. Since the voltage harmonics spectrums caused by modulation of three-phase inverter are different from that of single-phase inverter, the structure and the parameters of three-phase LLCL-filter should be redesigned.

Ref [3] has analyzed the character of multiple shunt RLC trap filters, but the detail design procedure is not given. Ref [6] presented a design procedure using the trial and error method. Some other LCL-filter design guidelines, criteria and optimizing processes were also proposed in [7]-[9]. However, the design principle and method of the three-phase three-wire power filter need to be further described in detail.

In this paper, the analysis on the output current harmonic of the three phase inverter using SPWM modulation methods is first presented. Secondly, a design procedure of the high order power filter is proposed and the related analysis is carried out. Finally, simulations on the designed inverter cases with three different type of high-order power filter are illustrated to verify the theoretical analysis.
II. **Inverter-side Current Harmonic Analysis For A Three-phase Inverter**

The lower limit of the filter inductance is determined by the harmonic requirement of grid-injected current according to IEEE 519-1992[10], as specified in Table I. $I_g$ is the nominal grid-side fundamental current. $I_{SC}$ is the short circuit current of power system. The harmonic currents can be calculated by the corresponding harmonic voltage amplitudes at different harmonic frequencies.

<table>
<thead>
<tr>
<th>Individual Harmonic Order (Odd Harmonics)</th>
<th>Maximum Harmonic Current Distortion in Percent of $I_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{SC}/I_L$</td>
<td>$&lt;11$</td>
</tr>
<tr>
<td>$11 \leq h &lt; 17$</td>
<td>$2.0$</td>
</tr>
<tr>
<td>$17 \leq h &lt; 23$</td>
<td>$1.5$</td>
</tr>
<tr>
<td>$23 \leq h &lt; 35$</td>
<td>$0.6$</td>
</tr>
<tr>
<td>$35 \leq h$</td>
<td>$THD$</td>
</tr>
</tbody>
</table>

In this paper, only asymmetrical regular sampled Sinusoidal Pulse Width Modulation (SPWM) will be discussed, but the method presented can also be applied to other modulation techniques with slight modifications according to the voltage output characteristics.

**A. Traditional Method**

For an LLCL-filter or LCL-filter, within the low-frequency, the equivalent output impedance of the filter is approximated as the sum of the overall main inductance, while in the high-frequency range, since the capacitor bypasses the high order current harmonics, the output impedance $Z_o$ of inverter is approximated as the inverter-side inductor alone [2], [7], as derived in function (1).

$$Z_o(j\omega) = L_1 j\omega$$

where $L_1$ is the inverter-side inductor. $\omega$ is the frequency in radians per second.

![Fig. 2. Simplified three-phase voltage source inverter with phase voltage in high frequency](image)

**B. Proposed Method**

Considering the high-order current harmonics, Fig. 1 can be simplified as shown in Fig. 4, where $u_{ab}, u_{bc}$ and $u_{ca}$ are three phase line voltages, respectively.

$$u_{an}(n,m) = \frac{2U_{dc}}{\pi} \frac{1}{m} J_m \left(\frac{\pi}{2} M \right) \sin \left(\frac{(m+n)\pi}{2}\right), \quad (2)$$

where $u_{an}(n,m)$ amplitude of the phase voltage harmonic; $M$ the modulation index; $U_{dc}$ the DC link voltage; $m$ carrier band number $[1, \infty)$; $n$ side band number $(-\infty, +\infty)$.

When calculating the amplitude of the inverter current harmonics a three-phase three-line inverter is divided into three same single phase circuits to analyze. Usually, the amplitude of the inverter phase voltage harmonics $u_{an}$ is used, as shown in (3),

$$|I_{o1}|_{\omega=\omega_{ab}} = \frac{|u_{an}(n,m)|}{|Z_o(j\omega)|}$$

Fig. 3 shows the main harmonic current spectrum of the inverter output current under the condition that the grid-voltage (phase to phase) is 50 Hz/380 V, the DC-link voltage of $U_{dc}$ is 700V, the modulation index $M$ is 0.9, converter-side inductance $L_1$ is 2.4 mH and the switching frequency $f_s$ is 10 kHz.

![Fig. 3. Inverter output phase voltage spectrum](image)
According to the inverter three phase voltage functions [12], the output line to line voltage $u_{ab}$, $u_{bc}$ and $u_{ca}$ can be derived as (4):

$$u_{ab} = \frac{\sqrt{3}}{2} U_{dc} M \cos(\omega_0 t + \frac{\pi}{6}) + \frac{4U_{dc}}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m} J_m \left( m \frac{\pi}{2} M \right) \cos \left[ \left( m+n \right) \frac{\pi}{2} \sin n \frac{\pi}{3} \cos \left[ m \omega_0 t + n(\omega_0 t - \frac{\pi}{3}) + \frac{\pi}{2} \right] \right]$$

$$u_{bc} = \frac{\sqrt{3}}{2} U_{dc} M \cos(\omega_0 t - \frac{\pi}{2}) + \frac{4U_{dc}}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m} J_m \left( m \frac{\pi}{2} M \right) \cos \left[ \left( m+n \right) \frac{\pi}{2} \sin n \frac{\pi}{3} \cos \left[ m \omega_0 t + n(\omega_0 t + \frac{\pi}{3}) + \frac{\pi}{2} \right] \right]$$

$$u_{ca} = \frac{\sqrt{3}}{2} U_{dc} M \cos(\omega_0 t + \frac{5\pi}{6}) + \frac{4U_{dc}}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m} J_m \left( m \frac{\pi}{2} M \right) \cos \left[ \left( m+n \right) \frac{\pi}{2} \sin n \frac{\pi}{3} \cos \left[ m \omega_0 t + n(\omega_0 t + \frac{\pi}{3}) + \frac{\pi}{2} \right] \right]$$

where $\omega_0$ and $\omega_o$ are the switching frequency and fundamental switching frequency in radians per second respectively. For a symmetrical three-phase circuit, three-phase line to line voltage can be converted into three-phase phase voltage, as shown in Fig. 5.

Figure 5. Simplified voltage source inverter with equivalent output voltage sources

The equivalent phase voltage can be derived as (5):

$$u'_a = \frac{u_{ab}}{\sqrt{3}} \angle -30^\circ$$

$$u'_b = \frac{u_{bc}}{\sqrt{3}} \angle -30^\circ$$

Note that the neutral point of “N” is the equivalent neutral point which is obtained from balanced three inverter-side line to line voltages and it is different from the neutral point of “N” as labeled in Fig. 1 and Fig. 2.

According to (4), the main harmonics spectrum magnitudes (p.u.) of line to line inverter output voltage by the sinusoidal pulse-width modulated waveform (SPWM) from the voltage source inverter are shown as an example in Fig. 6 under the condition that the modulation index $M$ is 0.9 and $U_{dc}$ is 700V.

Figure 6. Line to line output switched voltage spectrum

The amplitudes of the equivalent inverter output voltage harmonics $|U_A(n, m)|$ and the ideal amplitudes of converter-side current harmonic $|I_{AM}|$ can be derived as in (6):

$$|U_A(n, m)| = \left| \frac{4U_{dc} J_m \left( m \frac{\pi}{2} M \right) \sin \left( m+n \frac{\pi}{2} \right) \sin \left( n \frac{\pi}{3} \right)}{\sqrt{3} m \pi} \right|$$

$$|I_{AM}| = \left| \frac{U_A(n, m)}{Z_c(j \omega)} \right|$$

Since the angle does not change the spectrum of amplitude, the voltage spectrum of the equivalent phase voltage based on (5), $u'_a$, can also be depicted in Fig. 6.
With (6), the calculated harmonic spectrum of output current of inverter is shown in Fig. 7 under the condition that converter-side inductance \( L_1 \) is 2.4mH, rated power is 6 kW and switching frequency \( f_s \) is 10 kHz.

### C. Simulation results of Converter-side Current Harmonics

Fig. 8 shows the simulated harmonics spectrum of the inverter-side current under the same conditions for calculation. Compared Fig. 7 with Fig. 8, it can be seen that the calculated current harmonics spectrum is almost same as the simulated current harmonics spectrum. Hence, the proposed method of equivalent output phase voltage based on line to line voltage spectrum is accurate for designing the output high order filter. It can also be seen from Fig. 8. That inverter-side current harmonics are dominant around the switching frequency and the double of the switching frequency.

#### III. CHARACTERISTICS OF THREE TYPICAL HIGH ORDER FILTERS

According to the converter-side current spectrum, different from single-phase inverter, the current harmonics of three-phase inverter at the double of switching are also dominant. Reference [3] makes use of multiple (n) RLC shunt trap filters, but many traps will increase the size of the filter and bring the extra cost. The LLCL-filter topology with two resonant circuits between the ripple inductor and the grid-side inductor to attenuate the two dominant harmonic currents around the switching frequency and the double of switching frequency can be used, as shown in Fig. 9.

![Equivalent single-phase circuit of the LLCL filter with two traps](image)

\[ Z_1(s) = L_1 s \]
\[ Z_2(s) = L_2 s \]
\[ Z_c(s) = \frac{(L_f_1 C_{f_1})^2 s^2 + 1}{(L_f_1 C_{f_1} C_{f_2} + L_f_2 C_{f_1} C_{f_2}) s^2 + (C_{f_1} + C_{f_2}) s} \]

While all the other parameters of LCL-filter, LLCL-filter with one trap and LLCL-filter with two traps are the same except for resonant circuits.

Fig. 11 shows the transfer functions \( i_g(s)/u_i(s) \) of LCL-filter, LLCL-filter with one resonant circuit and LLCL-filter with two resonant circuits when \( L_1, L_2 \) and the total capacitance are the same and the high order resonant frequencies are set at the switching frequency and the double of switching frequency.

![Bode plots of transfer functions](image)

From Fig. 11, it can be seen that magnitude response and phase response of the LCL-filter and the LLCL-filter in the half of the switching frequency range are similar, so there are no obvious differences during the design of the controller for LCL-filter and LLCL-filter based systems. The resonant peak
of the LCL-filter and LLCL-filter can be attenuated by same active and passive damping methods [13] - [14] and [15].

IV. PARAMETERS DESIGN OF A THREE PHASED HIGH ORDER POWER FILTER

A. Constraints on the Design of a High Order Power Filter

When designing a power filter, the base impedance of the system should be known. Then the base values of the total impedance, inductance, and capacitance are define as (8)

\[
Z_b = \frac{U_n^2}{P_{\text{rate}}} \quad L_b = \frac{Z_b}{\omega_b} \quad C_b = \frac{1}{\omega_b Z_b}
\]

where

- \( U_n \) = the line-to-line RMS voltage;
- \( \omega_b \) = the grid frequency;
- \( P_{\text{rate}} \) = the active power absorbed by the converter in rated conditions.

The following aspects of the design limitation must be addressed [2] and [6]:

1) **Constrain of the total inductor\((L_1+L_2)\):** The maximum value of the total inductance should be less than 0.1pu to limit the ac voltage drop during operation and thereby limit the dc-link voltage.

2) **Resonance frequency of the filter:** The resonance frequency is assumed to be in a range between ten times the line frequency and one-half of the switching frequency to avoid resonance problems.

3) **Design of the filter capacitance:** It is considered that the maximum power factor variation at rated power is less than 5\%, as it is multiplied by the value of base impedance of the system \( C_f \leq 5\%C_b (C_f = C_{f1} + C_{f2}) \).

4) **The value of the inverter-side inductor\((L_f)\):** It is decided by the maximum ripple current.

B. Design Procedure of the High Order Filter in a Three-Phase Inverter

Based on constraints addressed above, then, the current harmonics attenuation around the triple of the switching frequency should be concentrated in the design of the three-phase three-line LLCL-filter design procedures can be derived as:

1) In order to meet a specific current ripple requirement, the inductance can be calculated from the equation [16]:

\[
L_f = \frac{U_{dc}}{8f_s (\alpha I_{\text{ref}}) \omega_f (M_1)}
\]

where, \( I_{\text{ref}} \) is the rated reference peak current, \( \alpha \) is the inverter-side current ripple ratio, which generally is lower than 40\% [2];

2) Select the total capacitance to achieve maximum reactive power absorbed at rated conditions.

\[
(C_{f1} + C_{f2}) \leq 0.05C_b
\]

3) Decide the resonant circuit. Since \( L_{f1}C_{f1} \) and \( L_{f2}C_{f2} \) circuit resonate at the switching frequency and the double of the switching frequency, then, \( L_{f1} \) and \( L_{f2} \) can be calculated as:

\[
\frac{1}{\sqrt{L_{f1}C_{f1}}} = \omega_f, \quad \frac{1}{\sqrt{L_{f2}C_{f2}}} = \omega_{r2}
\]

where, \( \omega_{r2} \) is twice of the switching frequency in radians per second.

4) Selection of \( L_2 \).

For an LCL-filter \( L_2 \) mainly depends on the objective to attenuate each harmonic around the switching frequency down to 0.3\%. Then it can be described in (12):

\[
\frac{4U_{dc}}{3\sqrt{3}\pi} \times \max \left( \left| \frac{J_1(\pi M)}{J_1(\pi 2M)} \right|, \left| J_2(\pi 2M) \right| \times G_{1-\omega_i}(j\omega_1 I_{\text{ref}}) \right) \leq 0.3\% (12)
\]

where \( J_1(\pi M) \) and \( J_2(\pi M) \) are the Bessel functions corresponding to the 2\textsuperscript{nd} and 4\textsuperscript{th} and the sideband harmonics at the switching frequency.

For an LLCL-filter with one trap based three-phase inverter, the uppermost harmonics will appear around the double of the switching frequencies.

\[
\frac{4U_{dc}}{3\sqrt{3}\pi} \times \max \left( \left| J_1(\pi M) \right|, \left| J_4(\pi M) \right| \times G_{1-\omega_i}(j\omega_1 I_{\text{ref}}) \right) \leq 0.3\% (13)
\]

where \( J_1(\pi M) \) and \( J_4(\pi M) \) are the Bessel functions corresponding to the 1\textsuperscript{st} and 5\textsuperscript{th} sideband harmonics at the double of the switching frequency.

5) Verify the resonance frequency obtained. Due to inductors \( L_1 \) and \( L_2 \) are small, the resonant frequency \( \omega_f \) can be derived approximately to:

\[
\omega_f = \frac{1}{\sqrt{\frac{L_{f1}L_{f2}}{L_{f1} + L_{f2}}}} (C_{f1} + C_{f2})
\]

It is necessary to check resonant frequency to satisfy constraint 2). If it is not, the parameters should be re-selected from step 2.

C. Filter Design Example

Under the condition of that \( f_s = 10 \text{kHz}, U_{dc} = 700 \text{V}, P_{\text{rated}} = 6 \text{kW}, R_{\text{R1}} - R_{\text{R2}} = 0.1 \Omega \), grid phase to phase voltage is 380 V/50 Hz, and the sine-triangle, and asymmetrical regular sampled
PWM, design examples of \textit{LCL}-filter and \textit{LLCL}-filter are given as following:

1) Base on the constraint of the total inductor and inverter-side current ripple, a 28% current ripple can be obtained to design $L_1$. Then the inverter-side inductor is selected to be 2.4 mH.

2) The total capacitor value is designed as 4 $\mu$F to limit the reactive power which should meet the constraint of 5%. Then, the capacitance of $C_f1$ and $C_f2$ are set to the same.

3) The grid-side inductor value of $L_2$ can be calculated by (13), (14) and (15) for three types of high order filters. In this paper, $L_2$ is selected to be 0.25 mH for the \textit{LLCL} filter with two traps, 1.2 mH for \textit{LLCL}-Filter with one trap and 2.4 mH \textit{LCL}-filter according to functions.

4) For the \textit{LC} resonant circuits, $L_1$ and $L_2$ can be chosen based on the chosen capacitors and the switching frequency.

TABLE II shows the parameters of the designed filters. Fig.11. shows the value of different inductors in three cases.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>CONVERSOR RATINGS USED FOR SIMULATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements</td>
<td>Parameters</td>
</tr>
<tr>
<td></td>
<td>DC link voltage $U_{dc}$</td>
</tr>
<tr>
<td></td>
<td>Switching frequency $f_s$</td>
</tr>
<tr>
<td></td>
<td>Rated power $P_{rate}$</td>
</tr>
<tr>
<td>AC Grid</td>
<td>Grid phase voltage $U_g$</td>
</tr>
<tr>
<td></td>
<td>Grid frequency $f_o$</td>
</tr>
<tr>
<td>$LLCL$-filter (two \textit{LC} traps)</td>
<td>Converter side inductor $L_1$</td>
</tr>
<tr>
<td></td>
<td>Grid side inductor $L_2$</td>
</tr>
<tr>
<td></td>
<td>Resonant circuit inductor $L_f$</td>
</tr>
<tr>
<td></td>
<td>Resonant circuit inductor $L_2$</td>
</tr>
<tr>
<td></td>
<td>Resonant circuit capacitor $C_f2$</td>
</tr>
<tr>
<td></td>
<td>Resonant circuit capacitor $C_f1$</td>
</tr>
<tr>
<td>$LLCL$-filter (one \textit{LC} trap)</td>
<td>Converter side inductor $L_1$</td>
</tr>
<tr>
<td></td>
<td>Grid side inductor $L_2$</td>
</tr>
<tr>
<td></td>
<td>Resonant circuit inductor $L_f$</td>
</tr>
<tr>
<td></td>
<td>Resonant circuit capacitor $C_f$</td>
</tr>
<tr>
<td>$LCL$-filter</td>
<td>Converter side inductor $L_1$</td>
</tr>
<tr>
<td></td>
<td>Grid side inductor $L_2$</td>
</tr>
<tr>
<td></td>
<td>Filter capacitor $C$</td>
</tr>
</tbody>
</table>

Fig. 12 Comparisons of different inductors in three cases
inverter which has the dominant current harmonics at the switching frequency. Fig. 15 shows the simulated grid-side current waveforms and its spectra of LLCL-filter based inverter with one trap. It has most significant current harmonics at the double of the switching frequency. Fig. 16 shows that LLCL-filter with two LC traps can reduce the grid-side current ripple at the switching frequency and the double of the switching frequency.

All the design of the three case of high order based system can meet the harmonic requirement given in IEEE Standard 519-1992. Note that compared with the LCL-filter, the total inductance of the LLCL-filters with one trap and two traps can be reduced by a factor of 25% and 40% respectively.

VI. CONCLUSION

This paper has introduced a harmonic current calculation method and a step by step design method of the high order power filter in the three-phase three-wire grid-connected inverter. The following can be concluded:

1. Compared with the traditional harmonic current calculation based on the phase voltage, the proposed method based on the equivalent phase voltage is more accurate.

2. The character of LCL filter and LLCL filter in the half of the switching frequency are similar, so the additional inductor of LLCL filter brings no extra control difficulty.

3. Compared with the LCL-filter, under sine-triangle, and asymmetrical regular sampled PWM, the total inductance of LLCL-filters with one trap and two traps can be reduced by a factor of 25% and 40% respectively.

The accuracy of the proposed calculation on the inverter output current harmonics and the step by step parameters design method of high order filters have been verified through the simulation on a 6 kW inverter model with the current controller.

REFERENCES


[9] Limitations of Voltage-Oriented PI Current Control of Grid-Connected PWM Rectifiers with LCL Filters


