Bayesian Ranging for Radio Localization with and without Line-of-Sight Detection

Lishuai Jing, Troels Pedersen, and Bernard Henri Fleury
Section Wireless Communication and Networks (WCN), Dept. of Electronic Systems, Aalborg University, Denmark
Email: {lji, troels, fleury} @es.aau.dk

Abstract—We consider Bayesian ranging methods for localization in wireless communication systems. Based on a channel model and given priors for the range and the line-of-sight (LOS) condition, we propose range estimators with and without LOS detection. Since the pdf of the received frequency-domain signals is unknown, we approximate the maximum-a-posteriori (MAP) and the minimum mean-squared error (MMSE) estimators. The promising ranging accuracy obtained with the proposed estimators is demonstrated by Monte Carlo simulations. We observe that the approximate MMSE estimators outperform the approximate MAP estimators. In addition, we find that including LOS detection in the approximate estimators, while adding a higher computational complexity, has no major impact on the ranging performance.

I. INTRODUCTION

Having accurate localization capability is increasingly important for wireless communication systems [1] [2]. One approach to increase localization performance is to rely on high precision ranging techniques [3]. State-of-the-art ranging techniques based on, for example, the received signal strength, angle-of-arrival, time-of-arrival, time-difference-of-arrival, etc., may be sensitive to line-of-sight (LOS) conditions [2] [4]. Therefore, accounting for the unknown LOS or non-LOS (NLOS) conditions is an issue considered in many ranging and localization techniques [5] [6].

To tackle this issue, one approach is to rely on LOS identification techniques. Such a approach is reliable provided that the signal bandwidth and signal-to-noise ratio (SNR) are sufficiently large [4] [7] [8] [9] [10] [11]. Existing LOS identification techniques include methods based on machine-learning [10] [12] and hypothesis-testing [5] [9] [2]. The LOS identification step labels range estimates as “LOS” or “NLOS” to facilitate the localization algorithms [13] [12]. The rational is that if the LOS condition can be correctly identified, this information can be used to improve the ranging and localization accuracy. However, in communication systems with limited bandwidth and SNR, the LOS detector may be unreliable [4].

Instead of identifying and mitigating NLOS range estimates, direct ranging, which infers the range parameter directly from the received signal, can be potentially applied. Direct ranging methods have been proposed in [14], [15] for bypassing a related problem, i.e. first-path detection. These methods rely on a channel model formulated via a point process to compute the required moments of the received signal. In [14], an approximate maximum-likelihood ranging method using the first- and second-order moments of the received signal has been presented. Using the prior distribution of the range, Bayesian estimators, including approximate maximum a posteriori (MAP) and minimum-mean-square-error (MMSE) estimators and a pth-order MMSE polynomial estimator, are proposed in [15]. In contrast to the methods in [16] [17], direct ranging operates without knowledge of the number of multi-path components in the channel response and separability condition on these components. Although the methods in [14], [15] still rely on LOS state information, the principle of ranging without estimating intermediate parameters seems promising.

In the present contribution, we propose Bayesian ranging methods with and without LOS detection for multi-path channels. Inspired by the direct ranging principle, we make use of a channel model to approximate pdfs of the received signal. In addition, we incorporate prior information on the range and the LOS condition on these components. Although the methods in [14], [15] still rely on LOS state information, the principle of ranging without estimating intermediate parameters seems promising.

We address the problem of estimating the range parameter \( r \) directly from the received signal vector \( y = [y_1, \ldots, y_N]^T \) obtained at frequencies \( f_1, \ldots, f_N \). We follow the Bayesian approach and consider the range \( r \) to be a random variable with a priori pdf \( p(r) \). Assuming the channel to be time-invariant with additive noise, we write

\[
y = Ah(r) + n, \tag{1}
\]

where \( A = \text{diag}\{a_1, \ldots, a_N\} \) is a diagonal matrix containing the known pilot symbol, \( h(r) \) denotes the range-dependent frequency-domain channel response, and \( n \) is a white circular complex Gaussian noise vector with component variance \( \sigma^2 \).

As in [14] and [15], we decompose \( h(r) \) as the Hadamard product of a range-dependent factor \( \varphi(r) \) and a range-independent factor \( \xi \):

\[
h(r) = \varphi(r) \odot \xi \tag{2}
\]

with

\[
\varphi(r) = [\varphi_1, \ldots, \varphi_N]^T, \quad \varphi_n = e^{-j2\pi f_n r},
\]

\[
\xi = \left[\begin{array}{c}
\xi_1 \\
\xi_2 \\
\vdots \\
\xi_N
\end{array}\right], \quad \xi_n = \sqrt{\frac{\sigma^2}{\mathbb{E}[\varphi_n]}}
\]

II. SIGNAL AND CHANNEL MODEL

\[
\mathbf{x}(\mathbf{u}) \approx \mathbf{A}(\mathbf{u}) \mathbf{y} + \mathbf{n},
\]

where \( \mathbf{A}(\mathbf{u}) = \text{diag}\{a_1, \ldots, a_N\} \) is a diagonal matrix containing the known pilot symbol, \( \mathbf{y} \) denotes the range-dependent frequency-domain channel response, and \( \mathbf{n} \) is a white circular complex Gaussian noise vector with component variance \( \sigma^2 \).

As in [14] and [15], we decompose \( \mathbf{h}(\mathbf{r}) \) as the Hadamard product of a range-dependent factor \( \varphi(\mathbf{r}) \) and a range-independent factor \( \xi \):

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\mathbf{h}(\mathbf{r}) = \varphi(\mathbf{r}) \odot \xi \tag{2}
\]

with

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\varphi(\mathbf{r}) = [\varphi_1, \ldots, \varphi_N]^T, \quad \varphi_n = e^{-j2\pi f_n \mathbf{r}},
\]

\[
\xi = \left[\begin{array}{c}
\xi_1 \\
\xi_2 \\
\vdots \\
\xi_N
\end{array}\right], \quad \xi_n = \sqrt{\frac{\sigma^2}{\mathbb{E}[\varphi_n]}}
\]
where \( j = \sqrt{-1} \) and \( c \) is the speed of light. Unlike [14] and [15], we here consider the case of a multi-path channel in which LOS propagation occurs with probability \( p_{\text{LOS}} \). Thus we write \( \xi \) as a superposition of a LOS term and a multi-path term

\[
\xi = qo_01 + \varepsilon, \quad (3)
\]

where \( 1 \) denotes an all-ones vector. The random LOS indicator \( q \) takes value one with probability \( p_{\text{LOS}} \) and zero otherwise. The complex gain of the LOS term is denoted by \( \alpha_0 \). The multi-path term \( \varepsilon = [\varepsilon_1, \ldots, \varepsilon_N]^T \) has entries

\[
\varepsilon_n = \sum_{l=1}^{L} \alpha_l e^{-j2\pi f_n\tau_l}, \quad n = 1, \ldots, N, \quad (4)
\]

where \( \alpha_l \) is the complex gain and \( \tau_l \) is the excess delay of path \( l \) with respect to the LOS delay \( \tau_c \). The random excess delays form a point process \( T = \{ \tau_1, \tau_2, \ldots \} \) with intensity function \( \rho(\tau) \) whose shape controls the average number of points in \( T \) per unit time. By convention, we set the delay associated to the LOS component to be zero, i.e. \( \tau_0 = 0 \). Depending on the specific point process assumed, the number \( L = |T| \) of multi-path components may be random and potentially infinite. We further assume that

\[
E[\alpha_l|\tau_l] = 0, \quad E[\alpha_l\alpha_{l'}^*|\tau_l, \tau_{l'}] = \begin{cases} \sigma^2_\alpha(\tau_l), & l = l' \\ 0, & \text{otherwise} \end{cases}, \quad (5)
\]

where \( \sigma^2_\alpha(\tau_l) \) denotes the expected power of a path component delay \( \tau_l \). With these definitions, the delay power spectrum of the considered channel model is of the form

\[
P(\tau) = E[P(\tau|q)|q] \quad (6)
\]

where \( P(\tau|q) \) is the conditional delay power spectrum [14]

\[
P(\tau|q) = \sigma^2_\alpha(\tau)(\rho(\tau) + q^2\delta(\tau)) \quad (7)
\]

with \( \delta \) denoting the Dirac delta function. Thus, \( P(\tau) = \sigma^2_\alpha(\tau)(\rho(\tau) + p_{\text{LOS}}\delta(\tau)) \).

III. ESTIMATION OF RANGE

A. Approximate Likelihood Function

Standard Bayesian estimators such as MAP and MMSE estimators necessitate the computation of the posterior pdf \( p(r|y) \). For a specific estimation problem, this pdf may be known directly or alternatively computed via Bayes Theorem, provided that the likelihood function \( p(y|r) \) is known. For the problem described in Section II, it is most convenient to work with the likelihood function, which can be expressed as

\[
p(y|r) = \sum_{q=0}^{1} p(y|r, q)p(q), \quad (8)
\]

where \( p(q) \) denotes the probability mass function of \( q \). Unfortunately, for the case considered, the two likelihood functions \( p(y|r, q) \) and \( p(y|r) \) are unknown and therefore we resort to approximations. Here, we consider two different approximations for \( p(y|r) \).

To derive the first approximation, we follow the same approach as in [14] and [15]: we approximate the likelihood function \( p(y|r, q) \) as a Gaussian pdf \( p_G(y|r, q) \) with the same first and second moments, i.e. with mean zero and covariance

\[
C_{y|r,q} = E[yy^H|r, q] = \Phi(r)C_{\xi|q}\Phi^H(r) + \sigma^2I, \quad (9)
\]

where \( \Phi(r) = \text{diag} \{ \Phi(r) \} \), \( I \) denotes the identity matrix, and \( C_{\xi|q} = E[\xi\xi^H|q] \) with the \( (m,n) \)th entry computed as

\[
[C_{\xi|q}]_{mn} = \mathcal{F}\{P(r|q)\}(f_m - f_n). \quad (10)
\]

Here, \( \mathcal{F} \) denotes the Fourier transform. Inserting \( p_G(y|r, q) \) for \( p(y|r, q) \) in (8), we obtain the Gaussian mixture

\[
p_{GM}(y|r) = \sum_{q=0}^{1} p_G(y|r, q)p(q). \quad (11)
\]

In the second approximation, we replace \( p(y|r) \) directly by a Gaussian \( p_G(y|r) \) with the same first and second moments as \( y|r \), i.e. with mean zero and covariance

\[
C_y|r = E[C_{y|r,q}] \quad (12)
\]

in which \( C_\xi = E[C_{\xi|q}] \) can be straightforwardly computed.

Evaluation of \( p_G(y|r) \) and \( p_{GM}(y|r) \) requires calculation of determinants and inverses of the matrices defined in (9) and (12). Following the same line of arguments as in [14], these computation tasks simplify since the determinants do not depend on \( r \) and inversion of the involved matrices can be carried out efficiently.

The accuracy of the above approximations depends on the specific parameter settings of the channel model. As an example, the Gaussian approximation may be inaccurate if the average number of path components in the multi-path channel, see (4), is small or the delay power spectrum exhibits a fast exponential decay. In the other extreme where the delay power spectrum is a constant and the average number of path components is high, the Gaussian approximation is well justified. Consequently, the accuracy of the estimators derived from the proposed approximations should be assessed, e.g. via Monte Carlo simulations.

B. Approximate MAP Ranging

The MAP estimator for \( r \), defined as

\[
\hat{r}_{\text{MAP}}(y) = \arg \max_r p(y|r)p(r), \quad (13)
\]

cannot be computed since \( p(y|r) \) is unknown. Therefore, we propose to approximate it by replacing \( p(y|r) \) with either \( p_{GM}(y|r) \) or \( p_G(y|r) \) defined above. Accordingly, we define two approximate MAP estimators:

\[
\hat{r}_{\text{AMAP},GM}(y) = \arg \max_r p_{GM}(y|r)p(r), \quad (14)
\]

\[
\hat{r}_{\text{AMAP},G}(y) = \arg \max_r p_G(y|r)p(r). \quad (15)
\]

In (14) and (15), we marginalized over \( q \) and therefore LOS detection is not needed. Alternatively, we can obtain the
range by detecting the LOS condition first. This results in an approximate MAP estimator for $r$:

$$\hat{r}_{\text{AMAP,Dec}}(y) = \arg\max_r p_G(y|r, \hat{q}) p(r),$$

(16)

with $\hat{q}$ denoting the approximate MAP decision rule

$$\hat{q}(y) = \arg\max_q p(q) \int p_G(y|r, q) p(r) dr.$$  

(17)

Computation of (16) and (17) is a two-step procedure with a LOS detection step followed by a ranging step. However, it is unclear if this additional complexity due to the LOS detector translates into improved ranging accuracy since the involved Gaussian approximations may undermine the performance of (16) and (17). In Section IV, we carry out a simulation study to answer this question.

Depending on the choice of prior and delay power spectrum, the optimization in (14)–(17) may require numerical procedures. We remark that to numerically evaluate the objective functions, it is necessary to invert the corresponding covariances defined in (9) and (12) for each value of $r$. As already shown (see [14]), this inversion can be simplified using eigenvalue decomposition.

C. Approximate MMSE Ranging

For the ranging problem, the MMSE estimator is given by

$$\hat{r}_{\text{MMSE}}(y) = \arg\min_{r'} E[(r - r')^2 | y] = E[r | y],$$

where the expectation is taken over the unknown pdf $p(r | y)$.

Using the approximations for $p(r | y)$ in Section III-A, we obtain approximate MMSE estimators:

$$\hat{r}_{\text{AMMSE,GM}}(y) = E_{p_{\text{GM}}}(r | y),$$

(19)

$$\hat{r}_{\text{AMMSE,G}}(y) = E_{p_G}(r | y),$$

(20)

where the expectations are taken over $p_{\text{GM}}(r | y)$ and $p_G(r | y)$ respectively.

The performance of the estimators (19) and (20) is essentially limited by the involved approximations. These estimators are therefore not optimal in a particular sense. Better performing estimators could potentially be obtained by invoking more accurate approximations. One candidate improvement provided a reliable detection of the LOS condition is to use separate approximations for the LOS and NLOS cases. Here, we consider Bayes’ decision rule in combination with the approximate MMSE estimator defined in (20) where $p_{\text{LOS}} = 1$ when LOS is detected and zero otherwise:

$$\hat{r}_{\text{AMMSE,D}} = E_{p_G}[r | y, \hat{q}].$$

(21)

Bayes’ decision rule for $q$ reads

$$\hat{q}(y) = \begin{cases} 1; & C_{11} p(q = 1 | y) + C_{01} p(q = 0 | y) \\ < C_{10} p(q = 1 | y) + C_{00} p(q = 0 | y) \quad \text{(22)} \end{cases}$$

where $C_{qq'}$ is the cost resulting from the MSE of the estimator (21) with LOS decision $q'$ applied under the true LOS condition $q$.

Implementation of the approximate MMSE estimators requires, in contrast to the approximate MAP estimators, evaluation of certain integrals. In case no closed-form expression can be obtained, this can be done fairly accurately by using standard numerical integration methods. We remark that the cost functions in (22) can be computed using Monte Carlo methods and stored for each considered parameter setting of the power delay profile. Therefore, range estimators with LOS detection require additional computational effort and storage compared to the estimators without LOS detection in (19) and (20).

IV. NUMERICAL PERFORMANCE EVALUATION

The invoked approximations of the likelihood function naturally impair the estimation performance. It is, however, unclear which of the estimators suffers the most. Note that the theoretical result that the MMSE estimator achieves lower MSE than all other estimators, e.g., the MAP estimator, does not hold for the approximate MMSE estimators. Thus, we rely on Monte Carlo simulations for assessing which of the above estimator yields the lowest MSE.

We compare the performance of the proposed estimators in terms of root-mean-squared-error (RMSE) and probability of LOS detection error. In addition, we compare them to “ genie-aided” estimators obtained from (16) and (21) by inserting the true $q$ value for $\hat{q}$. The genie-aided estimators provide lower bounds on the RMSE. As a study case, we simulate an OFDM communication system operating in the channel defined in the next subsection. Table I reports the parameter settings used for the simulations.

A. Simulation Scenarios and Related Analytical Results

To reflect the situation where the user terminal (to be localized) can appear at any distance within an interval, we assume that the prior of range $r$ is uniform on $[0, r_{\text{max}}]$. Inspired by Turin’s channel model, we assume that the random excess delays form a Poisson point process. For simplicity, we assume that the process is homogeneous, i.e. $\rho(\tau) = \rho_0$. The conditional second moments of the path gain are modeled as

$$\sigma^2_{\alpha}(\tau) = \begin{cases} C \kappa; & \tau = 0 \\ C \exp(-\frac{\tau}{\kappa}); & 0 < \tau < T_{\text{cp}} \\ 0; & \text{otherwise} \end{cases}$$

Table I

<table>
<thead>
<tr>
<th>OFDM system:</th>
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<tbody>
<tr>
<td>Bandwidth: 9 MHz, $N = 100$.</td>
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<tr>
<td>$\Delta f = 15$ kHz, $T_{\text{cp}} = 5.4$ µs, SNR = $E[</td>
</tr>
<tr>
<td>Equal power and equal spacing pilot signal is used.</td>
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</table>

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<tr>
<th>Channel parameters:</th>
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<tr>
<td>Homogenous Poisson point process: $\rho(\tau) = \rho_0$, $\lambda = 360$ m, $\kappa = 2$. Average no. of paths: $\mu_L = \rho_0 T_{\text{cp}}$, $\tau \sim U([0, 100]$ m).</td>
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Results obtained from 3000 Monte Carlo trials are displayed.
where parameter $\kappa$ determines the power of the LOS component, $\lambda$ denotes the root-mean-square (RMS) delay spread of the multi-path term, and $C$ is selected to normalize the channel power gain. The cyclic prefix length, $T_{cp}$, is assumed to be long enough such that the power of the path components with excess delays larger than $T_{cp}$ becomes negligible. Accordingly, the delay power spectrum reads

$$P(\tau) = C \exp(-\frac{\tau}{\lambda}) (\rho_0 1 (0 < \tau < T_{cp}) + \kappa \rho_{LOS} \delta(\tau))$$  \hspace{1cm} (23)$$

with $1$ denoting indicator function and the conditional delay power spectrum is given by

$$P(\tau | q) = \begin{cases} 
C \exp(-\frac{\tau}{\lambda}) (\rho_0 1 (0 < \tau < T_{cp}) + \kappa \delta(\tau)); & q = 1 \\
C \exp(-\frac{\tau}{\lambda}) \rho_0 1 (0 < \tau < T_{cp}); & q = 0.
\end{cases}$$  \hspace{1cm} (24)$$

For the simulation, it is necessary to compute the covariance matrices $C_\xi$ and $C_\xi[q]$:

$$[C_\xi]_{mn} = \kappa \rho_{LOS} C + \rho_0 g_{mn}$$  \hspace{1cm} (25)$$

and

$$[C_\xi[q]]_{mn} = \begin{cases} 
\kappa C + \rho_0 g_{mn}; & q = 1 \\
\rho_0 g_{mn}; & q = 0
\end{cases}$$  \hspace{1cm} (26)$$

with

$$g_{mn} = C \frac{1 - e^{-(j2\pi (f_m - f_n) + \frac{1}{2}) T_{cp}}}{j2\pi (f_m - f_n) + \frac{1}{2}}.$$  \hspace{1cm} (27)$$

B. Evaluation of Ranging Accuracy

In the simulation, we obtain similar RMSEs for the approximate MAP estimators (14) and (15). The same observation holds for the approximate MMSE estimators (19) and (20). Therefore, we omit reporting the performance of (14) and (19).

Figs. 1 and 2 report the simulated RMSE versus SNR of the approximate MAP and MMSE estimators respectively for different values of $p_{LOS}$. It is apparent that the approximate MMSE estimators outperform the approximate MAP estimators. We observe that as $p_{LOS}$ increases, the ranging accuracy improves.
To investigate the impact of the average number of path components on the estimators performance, we plot simulated RMSE versus $\mu_L$ in Figs. 3 and 4, again when $p_{LOS}$ is varied. We observe that, overall, the RMSE decreases with increasing $\mu_L$. Similarly, the RMSE decreases as $p_{LOS}$ increases as expected. As it is also observed in Figs. 1 and 2, the approximate MMSE estimators exhibit a higher ranging accuracy than the approximate MAP estimators.

Figs. 1–4 indicate that including the LOS detector somewhat improves the ranging accuracy for low and medium values of $p_{LOS}$. However, for large $p_{LOS}$, the trend is different. For the approximate MAP estimator, there is no noticeable performance gain, while including the LOS detection in the approximate MMSE estimator degrades the performance. To investigate the cause of this behavior, we turn our attention to the performance of the detectors, see Fig. 5. The probability of detection error seems rather high considering the prior information. This high value may be due to either the considered multi-path channel or the pdf approximations applied to the design of the detectors.

Given these results, it seems obvious to ask whether or not the accuracy of the proposed methods can be improved by using better pdf approximations. Due to the fact that we cannot access the likelihood functions, lower bounds such as the Cramér-Rao bound, are not available. It is, therefore, unclear how much the estimation accuracy can be improved. To evaluate the importance of the impact of the approximations on the performance of the detectors, we applied them to signals generated according to their respective approximate pdfs. The results, not reported here, show error probabilities less than 7% for all considered detectors with $\mu_L$ settings as given in Fig. 5. We thus conclude that the accuracy of the pdf approximations indeed plays a major role. The potential performance gain in ranging accuracy obtained by better pdf approximations in the detector can be assessed by comparing the RMSE curves to those of the genie-aided methods as done in Figs. 1–4. We conjecture that better pdf approximations can also increase the ranging accuracy of the estimators without detection.

V. CONCLUSION

We have proposed approximate MAP and MMSE estimators of the range with and without LOS detection. These estimators are derived by approximating the pdf of the received signal vector. The approximate MMSE estimators outperform the approximate MAP estimators in terms of RMSE. Using the proposed pdf approximations, we observe that including LOS detection in the estimators, while adding complexity, has no major impact on the ranging performance. Our simulation study indicates that there is a potential for improving the ranging performance by relying on better pdf approximations.

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