Modelling and control of Wave Energy Converters
By Morten Kramer

PhD course
"Numerical and experimental modelling and control of Wave Energy Converters"

Tuesday 1 September 2015
Location: Ecole Centrale Nantes, Nantes, France
Energy production by wave energy converters

### Wave climate

#### Hanstholm site. Wave probability [%]

<table>
<thead>
<tr>
<th>Wave height $H_{m0}$ [m]</th>
<th>Wave period $T_{0.2}$ [s]</th>
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#### Wave height $H_{m0}$ = 3.5 m. Electrical power [kW]

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#### Wavestar. Predicted energy production at Hanstholm [MWh/year]

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#### Yearly production: 1650 MWh

(Excluding: Periods out of operation due to maintenance or faults, and background consumption when in idle mode)
How can we model this?
Wavestar extracts the wave power and converts it to electricity.
Today you will learn how to calculate the optimum force to apply on the Wavestar absorbers in order to maximize the average power production in terms of absorbed mechanical power.

Focus is on linear damping control and reactive control for the point absorbers, but the methods apply for most types of WECs.
**Kinematics of the Wavestar floats**

The Wave Star floats rotate around a bearing: The motion is controlled using the force from the hydraulic cylinder $M_c = R F_H$:

![Diagram showing the rotation of the Wavestar floats and the force $F_H$.]

Control problem: To maximize the power transfer (by controlling $M_c$)

The instantaneous absorbed power is calculated by:

$$P_a(t) = M_c(t) \cdot \dot{\theta}_A(t)$$

Maximal power absorption is achieved when $M_c(t)$ and $\dot{\theta}_A(t)$ is in phase!
Mathematical model of the Wavestar floats (force balance)

Newton’s second law:
\[ J \ddot{\theta}_A = M_d - M_g - M_c \]

**Symbols and Definitions:**
- \( J \): Mass inertia moment of the moving body.
- \( \dot{\theta}_A \): Angular acceleration.
- \( M_d \): Hydrodynamic moment (from water pressure).
- \( M_g \): Gravitational moment.
- \( M_c \): Control moment from Power Take Off.

We would like to choose the control moment \( M_c \) in a way to take out as much energy as possible. The method for choosing the moment is referred to as “control strategy”.

Pivoting motions are described by angular rotations \( \theta_A \).
Two simple control strategies with constant gain factors

**Resistive control** (linear damper-system)

\[ M_c = c_c \dot{\theta}_A \]

**Reactive control** (linear spring-damper-system)

\[ M_c = c_c \dot{\theta}_A + k_c \theta_A \]

Purpose of gains:
- Resistive part \((c_c)\): Power absorption
- Reactive part \((k_c)\): Change system frequency

Demonstration with GUI
Example of laboratory tests with Wavestar floats

Linear damping control $c_c = 4.0 \text{ Nm/(rad/s)}$, wave dir $\theta^*$, H81 (H$_{\text{m0}} = 0.051 \text{ m}, T_p = 1.0 \text{ s})$

Resistive control (linear damper-system)

$M_c = c_c \dot{\theta}_A$
Power absorption depends strongly on the choice of control strategy

Example of measurements from Wavestar at Hanstholm

\[ M_c = c_c \ddot{\theta}_A + k_c \theta_A \]
Forces depends strongly on the choice of control strategy.
Constraints on eg. forces are more important when applying advanced control

![Graph showing resistive and reactive control with pressure limit at 180 Bar]
Motions depends strongly on the choice of control strategy
How do we perform numerical calculations?

In the following we will only consider a linear potential wave model as this is still today the only practical method to perform optimization.
Description of harmonic oscillations using complex and real notation

Definition of a complex number:
A complex number $A$ is an ordered pair of real numbers $[a, b] \equiv [a + ib]$. $a$ is the real part of $A$, $a = Re(A)$, $b$ is the imaginary part, $b = Im(A)$.

Complex and real representation of wave elevation:
$$\eta(t) = Re(A_w e^{i\omega t}) = |A_w| \cos(\omega t + \delta_w)$$
Equation of motion in frequency domain

Newton’s second law:
\[ j \ddot{\theta}_A = M_d - M_g - M_c \]  \hspace{1cm} (1)

Equation 1 is expanded to:
\[ j \ddot{\theta}_A = M_{hs} + M_r + M_{ex} - M_c \]  \hspace{1cm} (2)

Hydrostatic moment: \[ M_{hs} = M_b - M_g = -k_h \cdot \theta_A \]
Radiation moment: \[ M_r = -m_h \cdot \ddot{\theta}_A - c_h \cdot \dot{\theta}_A \]  \hspace{1cm} (3)
Wave excitation moment: \[ M_{ex} = \text{Re}(M_e \cdot e^{i\omega t}) \]
Control moment: \[ M_c = m_c \cdot \ddot{\theta}_A + c_c \cdot \dot{\theta}_A + k_c \cdot \theta_A \]

where:

- \( k_h \): Hydrostatic stiffness coefficient
- \( m_h \): Hydrodynamic added mass coefficient
- \( c_h \): Hydrodynamic damping coefficient
- \( M_e \): Complex amplitude for wave excitation force, \( M_e = H_{en} \cdot A_w \)  \hspace{1cm} (4)
- \( H_{en} \): Frequency response function for wave excitation moment (complex)
- \( A_w \): Wave amplitude (complex)
Equation of motion and solution in frequency domain

By inserting (3) into (2) and re-arranging the following is obtained:

\[ M\ddot{\theta}_A + C\dot{\theta}_A + K\theta_A = Re\left(M_e e^{i\omega t}\right) = |M_e|\cos(\omega t + \delta_e) \]  

(5)

where:

\[ M = J + m_h + m_c \]
\[ C = c_h + c_c \]
\[ K = k_h + k_c \]
\[ M_e = H_{e\eta} \cdot A_w \]  

(6)

The solution to (5) is a harmonic motion:

\[ \theta_A(t) = Re\left(A_A e^{i\omega t}\right) = |A_A|\cos(\omega t + \delta_A) \]
\[ \dot{\theta}_A(t) = Re\left(i\omega A_A e^{i\omega t}\right) \]
\[ \ddot{\theta}_A(t) = Re\left(-\omega^2 A_A e^{i\omega t}\right) \]  

(7)

where the motion amplitude is given by:

\[ A_A = \frac{M_e}{K - \omega^2 M + i\omega C}, \quad |A_A| = \frac{|M_e|}{\sqrt{(K - \omega^2 M)^2 + \omega^2 C^2}} \]  

(8)
Power absorption in regular waves

The instantaneous absorbed power is calculated by:

\[ P_a(t) = M_c(t) \cdot \dot{\theta}_A(t) \]  

(9)

And the average absorbed power:

\[ \bar{P}_a = \frac{1}{T} \int_0^T P_a(t) \, dt = \frac{1}{2} \omega^2 c_c |A_A|^2 \]  

(10)

If using reactive control the optimum value of (10) may be found by choosing coefficients such that:

\[ K - \omega^2 M = 0 \]

\[ c_c = c_h \]  

(12)

In case of reactive control when (12) is fulfilled it can be shown that the optimum absorbed average power is:

\[ \bar{P}_a = \frac{1}{8} \frac{|M_e|^2}{c_h} \]  

(13)
Equation of motion in time domain

Fixed support

Moving rigid body

Newton’s second law:

\[ J \ddot{\theta}_A = M_d - M_g - M_c \]  \hspace{1cm} (1)

Equation 1 is expanded to:

\[ J \ddot{\theta}_A = M_{hs} + M_r + M_{ex} - M_c \] \hspace{1cm} (2)

Hydrostatic moment:

\[ M_{hs} = M_b - M_g = -k_h \cdot \dot{\theta}_A \]

Radiation moment:

\[ M_r = -m_{h\infty} \ddot{\theta}_A - \int_{-\infty}^{t} h_r(t - \tau) \dot{\theta}_A(\tau) d\tau \] \hspace{1cm} (14)

Wave excitation moment:

\[ M_{ex} = \int_{-\infty}^{\infty} h_{e\eta}(t - \tau) \eta(\tau) d\tau \]

Control moment:

\[ M_c = m_c \cdot \ddot{\theta}_A + c_c \cdot \dot{\theta}_A + k_c \cdot \theta_A \]

where:

- \( k_h \): Hydrostatic stiffness coefficient
- \( m_{h\infty} \): Hydrodynamic added mass coefficient at infinite frequency
- \( h_r \): Impulse response function for wave radiation moment
- \( h_{e\eta} \): Impulse response function for wave excitation moment
- \( A_w \): Wave amplitude (complex)
Solving the equation of motion in time-domain

By inserting (14) into (2) and re-arranging the following is obtained:

\[
(J + M_{h\infty})\ddot{\theta}_A(t) + \int_{-\infty}^{t} h_r(t-\tau)\dot{\theta}_A(\tau)\,d\tau + k_h\dot{\theta}(t) + M_c(t) = M_e(t) \tag{15}
\]

The convolution integral represents the memory effect in the radiation force. \(M_{h\infty}\) is the limiting value of the added mass for \(\omega \to \infty\). The coefficients may be calculated by:

\[h_r(\tau) = \frac{2}{\pi} \int_{0}^{\infty} C_h(\omega) \cdot \cos(\omega \cdot \tau) \,d\omega\]

\[M_{h\infty} = M_h(\omega) + \frac{1}{\omega} \int_{0}^{\infty} h_r(\tau) \cdot \sin(\omega \cdot \tau) \,d\tau \tag{16}\]

The equation is integrated numerically from initial conditions of position and velocity.
A simple Power Take Off efficiency model

Repetition: The instantaneous absorbed power is calculated by: \( P_a(t) = M_c(t) \cdot \dot{\theta}_A(t) \)

The generated electrical power is calculated from the harvested absorbed power by:

\[
P_e = \eta_{PTO}^+ \cdot P_a^+(t) + \frac{1}{\eta_{PTO}^-} \cdot P_a^-(t)
\]

(17)

\( P^+ \) is the positive power and \( P^- \) is the negative power. The efficiency is normally considered to be independent of the power direction, i.e. \( \eta_{PTO}^+ = \eta_{PTO}^- = 0.7 \) (in case of PTO efficiency of 70 %).

The effect of the formula on negative power flow may also be explained by the following two examples:

- If \( \eta_{PTO}^- = 0.5 \) and \( P_a^- = -10 kW \) then \( P_e = \frac{1}{0.5} P_a^- = -20 kW \). In this case the double amount of power is drawn from the grid (through the generator) compared to what we get out to the mechanical system.

- If \( \eta_{PTO}^- \to 0 \Rightarrow \) It is necessary to draw infinitely large amounts of power from the grid (through the generator) to get any power out in the mechanical system.
Message:

1) Be aware of limitations in linear potential wave theory, and small amplitude approximations.

2) Applying advanced control methods in computer models may show very high increases in power production. However, in practice such methods are extremely difficult to apply in real irregular 3D waves. Wrong timing of latching/reactive controls is destructive for the buoy motion giving a lower power output.

3) Increase in power from Wave Energy Converters typically also increases forces and thereby costs. The optimal design is a balance between costs and production.

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