Coordinated Power Dispatch of a PMSG based Wind Farm for Output Power Maximizing Considering the Wake Effect and Losses

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Abstract—The energy loss in a wind farm (WF) caused by wake interaction between wind turbines (WTs) is quite high, which can be reduced by proper active power dispatch. The electrical loss inside a WF by improper active power and reactive power dispatch is also considerable. In this paper, a coordinated active power and reactive power dispatch strategy is proposed for a Permanent magnet synchronous generator (PMSG) based WF, in order to maximize the total output power by reducing the wake effect and losses inside the devices of the WF, including the copper loss and iron loss of PMSGs, losses inside converters and transformers of WTs and the losses along the transmission cables. The active power reference and reactive power reference of each WT are chosen as the optimization variables and a partial swarm optimizing (PSO) algorithm is used for solving the problem. The proposed strategy is compared with traditional strategies in a designed WF. Simulation results show the effectiveness of the proposed strategy.

Keywords—wind farm coordinated power dispatch; energy maximizing; wake effect; loss minimizing; PMSG

I. INTRODUCTION

Wake effect in a wind farm (WF) can cause a high level of energy loss, the study in [1] reports an average energy loss of 12% in an offshore WF caused by the wake. There are two types of wake control method for a WF: redirecting the wakes by yaw control [2-5] and reducing wake interaction by adjusting the axial induction [6-14]. The first method is studied in [2-4] and recently developed by the work in [5], where a control-oriented wake model called FLORIS is proposed. Most of the research works focus on the second method. These works maximize the total captured power of WFs by optimizing the control settings to each WT. The control variables are chosen as the axial induction factors [6], [7] the yaw offset angle, the thrust coefficient [8], the pitch angle [9], the rotational speed (tip speed ratio), or the combination of two of them as the control variables [10-13]. However, WT derating control strategy was not properly considered in these works. In [14], the active power reference of each WT is chosen as the optimization variable, and a Max-Ω WT control strategy proposed in [15] is adopted for derating control. Choosing the active power reference as the control variables is more suitable for the optimal power flow problems in power system engineering.

Besides the energy loss caused by wake effect, the electrical loss inside a WF is also considerable. The electrical losses inside the devices in a WF are related to the active and reactive power flows, which can be controlled by the WF controller or WT controllers. At the WF control level, the dispatch strategies which are used to distribute the demanded active and reactive power will decide the total losses. Proportional dispatch is a commonly used dispatch strategy for active power or reactive power [16-18]. This method is easy to implement and is unlikely to exceed the power limit of each WT. However, it cannot assure a high efficiency of the system. An optimal dispatch strategy for the reactive power is proposed in [19, 20], which includes the losses along the transmission cables and the transformers on WTs into the objective function. However, losses in the WT energy conversion systems were not considered. In [21], losses from wind energy conversion systems were included into the optimization, and an optimal dispatch of reactive power is proposed for total loss minimization. The above strategies only focus on active power dispatch or reactive power dispatch. However, the active power and reactive power are also related by the current and voltage limits of the WT energy conversion systems, so they should be considered together. The combined solution was proposed in [22, 23], where active and reactive power dispatch were optimized together mainly to minimize the electrical power losses in the transmission system.

Most of these works only focus on minimizing the energy loss caused by wakes or minimizing the electrical loss caused by power flows. In [14], these two objectives were combined together to realize an optimized active power dispatch, but only the electrical losses in the transmission cables were considered. However, the electrical losses inside the wind energy generation systems are higher. For a Permanent magnet synchronous generator (PMSG) based WT, which is becoming popular in wind turbine applications, the electrical losses

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include the copper loss and iron loss of PMSGs, losses inside the converters and losses in the transformers.

In this paper, a coordinated active power and reactive power dispatch strategy is proposed for a PMSG based WF, which combined the purpose of minimizing the energy loss caused by wake effects and the purpose of minimizing the electrical losses caused by power flows. The electrical losses include not only the losses along the transmission system, but also the losses inside the energy conversion systems. The models of WTs, wakes and device losses are given and the optimization problem is formulated and solved by partial swarm optimizing (PSO). The proposed strategy is compared with traditional strategies in a designed WF. Simulation results show the effectiveness of the proposed strategy.

This paper is organized as follows: Section II describes the WF model. Section III shows the formulation of the optimization problem and the solving method. The simulation results of proposed strategy are given and discussed in Section IV, and finally conclusions are drawn in Section V.

II. WIND FARM MODEL

The wake model and the power loss model on transmission cables are specified in this section. The WT model using traditional control strategy with no derating is also illustrated and will be used as a baseline for comparison.

A. Wind Turbine Model

The WT can be described using a static model, which is based on the look-up tables of the power coefficient \( C_p(\beta, \lambda) \) and the thrust coefficient \( C_t(\beta, \lambda) \). Then, the WT mechanical power \( P_{mech} \) can be calculated using [24],

\[
P_{mech} = \frac{\pi}{2} \rho R^2 v^3 C_p(\beta, \lambda)
\]

where \( \rho \) is the air density, \( R \) is the rotor radius, \( v \) is the wind speed, \( \beta \) is the blade pitch angle and \( \lambda \) is the tip-speed ratio, which can be expressed by

\[
\lambda = \frac{\omega R}{v},
\]

where \( \omega \) is the rotor rotational speed.

The control strategy decides the steady state values of \( \beta \) and \( \omega \) under a certain \( v \), thus deciding the \( \lambda \), and \( C_p \) and \( C_t \) then. The normal control of WT in the whole wind speed region can be divided into five regions, more details can refer to [26]. However, under derating operation, the control strategies need to be modified. In derating operation, the control target is to maintain the captured power at a reference value, usually lower than the available power. The WT normally uses torque control to regulate the power captured by the rotor, which increases the rotational speed. After the rotational speed is beyond its limits, the WT keeps the rotational speed at its limit and turns to pitch control.

B. Wake Model

The wake models can describe the aerodynamic interaction between WTs in the WF. The multiple wake model based on Jensen model is a common model to simulate the wakes, which can be expressed in the following equations [27], [28]:

\[
v_y = v_0 - v_0^* \left(1 - \sqrt{1 - C_r^2} \right)^2 \left(\frac{R_0}{R_i} \right)^2 \left(\frac{S_{overlap,i}}{S_0}\right)
\]

\[
R_i = R_0 + k \cdot L_{ij}.
\]

All the parameters have the same meaning as in the references. The wind velocity at the WT at row \( n \), column \( m \) can be derived as:

\[
v_{nm} = v_0^* \left[1 - \sqrt{\sum_{i=1}^{N_{row}} \sum_{j=1}^{N_{col}} \left(1 - \frac{v_y}{v_0^*}\right)^2}\right]
\]

C. PMSG Loss Model

The steady state model of the surface-mounted PMSG in the magnet flux reference frame can be expressed as [29]:

\[
\begin{align*}
    u_{ad} &= -R_s j_{ad} + L_s \omega_0 \delta_{ad} + \delta a \psi \\
    u_{aq} &= -R_s j_{aq} - L_s \omega_0 \delta_{aq} + \delta q \psi
\end{align*}
\]

Where \( R_s \) and \( L_s \) are the generator resistance and inductance respectively, \( \omega_0 \) is the generator rotational speed, and \( \psi \) is the amplitude of the flux induced by the permanent magnets of the rotor in the stator phases. The electromagnetic torque can be given by [29]

\[
    T_e = \frac{3}{2} p \omega_0 |I_a|^2
\]

where \( p \) is the number of pole pairs. The q-axis current is proportional to the generator torque. The d-axis current is controlled to be zero to maximize the torque [30]. The copper loss can be calculated using

\[
P_{cu} = 3 R_s |I_a|^2
\]

Where \( R_s \) is the armature phase resistance and \( |I_a| \) is the RMS magnitude of the phase current. And the iron losses is calculated using the relation [30]

\[
P_{ir} = k_1 \omega_0^2
\]

Where \( \omega_0 \) is the electrical speed of the machine and \( k_1 \) is a constant extracted from the iron losses at rated speed, and is found to be 0.1 [30].

D. Converter Loss Model

The losses in the converter, which consists of transistors and reverse diodes, can be divided into switching losses and conducting losses [31]. According to [31], the losses in a converter can be expressed as

\[
P_{con} = a_1 I_{rms}^2 + b_1 I_{rms}^2
\]
where \( I_{rms} \) is the rms value of the sinusoidal current at the converter ac terminal, and \( a_i \) and \( b_i \) are the power module constants and can be expressed as
\[
a_i = \frac{6 V_{KBR}^2}{\pi} \left( V_{KBR} + \frac{E_{ON} + E_{OFF}}{I_{nom}} f_m + \frac{E_m}{I_{nom}} f_m \right)
\]
\[
b_i = 3 r_{KBR}
\]
where \( V_{KBR} \) is the voltage across the collector and emitter of the IGBT, \( E_{ON} + E_{OFF} \) is the total turn-on and turn-off losses of the IGBTs, \( I_{nom} \) is the nominal collector current of the IGBT, \( f_m \) is the switching frequency, \( E_m \) is the turn-off (reverse recovery) loss of the diodes, \( r_{KBR} \) is the lead resistance of the IGBT.

E. Transformer Loss Model

The active power loss in transformers can be calculated using [32]
\[
P_{trans} = P_0 + \alpha^2 P_k
\]
where \( \alpha \) is the load ratio, \( P_0 \) is the no-load loss, and \( P_k \) is the load loss.

F. Cable Loss Model

For a cable connecting two buses \( i \) and \( j \), the cable current, \( I_y \), measured at bus \( i \) and \( j \) and defined positive in the direction \( i \to j \) is given by [33]
\[
I_y = I_i + I_{ij} = y_{ij} (V_i - V_j) + y_{ij} V_i,
\]
where \( y \) and \( I \) mean the admittance and current of each cable, and \( V \) means the voltage on each bus. Similarly, the cable current \( I_y \) is given by
\[
I_y = -I_i + I_{ij} = y_{ij} (V_i - V_j) + y_{ij} V_i.
\]
The power loss in cable \( y \) is the algebraic sum of the complex powers \( S_y \) from bus \( i \) and \( j \) and \( S_y \) from bus \( j \) and \( i \),
\[
S_{y}^{loss} = S_y + S_y = V_i I_y + V_j I_y^*.
\]

III. PROBLEM FORMULATION AND OPTIMIZATION

The optimization problem including the objective function and constraints are formulated and is solved by an improved PSO algorithm in this section.

The output active power of the WF can be calculated by:
\[
P_{\text{out}}^{\text{WF}} = \sum_{k=1}^{N_c} \left( P_{\text{capt}}^{k} - P_{\text{Cu}}^{k} - P_{\text{Fe}}^{k} - P_{\text{con}}^{k} - P_{\text{trans}}^{k} \right) - \sum_{i=1}^{N_{\text{WT}}} \sum_{j=1}^{N_{\text{ref}}} P^{\text{ref},i,j}
\]
where \( P_{\text{capt}}^{k} \), \( P_{\text{Cu}}^{k} \), \( P_{\text{Fe}}^{k} \), \( P_{\text{con}}^{k} \), and \( P_{\text{trans}}^{k} \) are the captured power, copper loss, iron loss, losses of converters and transformer loss from WT \( k \), respectively, \( N_{\text{WT}} \) is the number of WTs. \( P^{\text{ref},i,j} \) is the active power loss in cable \( ij \), \( N_{\text{ref}} \) is the number of buses.

The reactive power at the PCC of the WF is expressed by:
\[
Q_{\text{PCC}}^{\text{WF}} = \sum_{k=1}^{N_c} Q_{k}^{\text{ref},i,j} - \sum_{i=1}^{N_{\text{WT}}} \sum_{j=1}^{N_{\text{ref}}} Q_{ij}^{\text{loss}}
\]
where \( Q_{k}^{\text{ref},i,j} \) is the reference reactive power of WT \( k \) and \( Q_{ij}^{\text{loss}} \) is the active power loss in cable.

Then, the optimization problem can be expressed as:

Objective: \[
\max_{P_{\text{out}}, Q_{\text{PCC}}} \frac{P_{\text{out}}^{\text{WF}}}{P_{\text{out}}^{\text{max}}} P^{\text{ref},i,j}
\]

Constraints:
\[
P_i = \left| V_i \right| \left| \sum_{i=1}^{N_{\text{WT}}} \left| S_{\text{ref},i,j} \right| \cos \left( \theta_{\text{ref},i,j} + \delta_{\text{ref},i,j} \right) \right|
\]
\[
Q_i = -\left| V_i \right| \left| \sum_{i=1}^{N_{\text{WT}}} \left| S_{\text{ref},i,j} \right| \sin \left( \theta_{\text{ref},i,j} + \delta_{\text{ref},i,j} \right) \right|
\]
\[
Q_{\text{PCC}} = Q_{\text{PCC}}^{\text{WF}}
\]
\[
V_{\min} \leq V_i \leq V_{\max}
\]
\[
I_{\text{GSC}}^{\text{nom}} \leq I_{\text{GSC}}^{\text{max}}
\]
\[
\beta_{\min} \leq \beta \leq \beta_{\max}
\]
\[
0 \leq P_{\text{ref},i,j} \leq P_{\text{rated}}
\]
\[
\frac{\partial C_{\beta}^{\text{loss}}}{\partial \lambda} \leq 0
\]
\[
\frac{\partial C_{\beta}^{\text{loss}}}{\partial \beta} \leq 0
\]

where \( P_i \) and \( Q_i \) are the active power and reactive power injected at bus \( i \), \( V_i \) and \( \delta_i \) are the voltage and angle of each bus. \( Y_{\text{ref}} \) is the entry in the \( j^\text{th} \) row \( i^\text{th} \) column of the admittance matrix. \( Q_{\text{PCC}}^{\text{WF}} \) is the reactive power reference of the WF, \( I_{\text{GSC}}^{\text{nom}} \) is the rated current of the Grid Side Converter (GSC). The optimization variables used here are active power reference \( P_{\text{ref},i,j} \) and reactive power reference \( Q_{ij}^{\text{ref},i,j} \) of each WT.

The constraints include the power flow balance limits (19), (20), the WF reactive power constraint (21), the bus voltage limit (22), the current constraint of the GSC (23), which is used to limit the reactive power, the pitch angle limit (24), the active power limit (25), and the WT operation region constraints (26), (27). In the power flow problem, the point of common coupling is treated as slack bus and all the other buses are treated as PQ buses. A full Newton–Raphson method is used to solve the power flow equations. Since the problem is nonlinear and non-convex, the PSO algorithm [34] is adopted to solve the optimization problem.

IV. CASE STUDY

In this paper, the chosen WF has 5 rows, 5 turbines each, with 882 m (seven times the WT diameter) between the turbines. The layout of the WF is shown in Fig. 1. The cables in the WF are XLPE-Cu and operated at 34 kV nominal
The parameters are shown in Table I. The WT parameters are shown in the appendix.

![Diagram](image)

*Fig. 1. The layout of the wind farm*

It can be seen that using Strategy C, the WF output power is increased and the WT loss is reduced. The cable loss is increased at direction 90°, whereas it decreased at direction 270°. The reason is that the cable loss is related to the power circulation distance along the cables, and when wind direction is 90°, the WT produce more power is nearer to the PCC, so the power circulation distance is less than the solutions where upwind WTs are derated. However, the total loss is reduced because the WT loss decrease is more than the cable loss increase.

### V. Conclusion

The coordinated active power and reactive power dispatch strategy proposed in this paper shows the potential to improve the output power of a WF. Comparing with the traditional active power and reactive power dispatch strategy, the loss reduction of an optimized dispatch strategy depends on if the loss model is accurate and complete, also depends on the wind direction, i.e., the original active power distribution pattern. Models of all the devices shall be considered in the objective function. Otherwise, reducing losses in part of the devices will increase the losses in the other part. The cable loss is related to the wind directions, so it may be increased at some wind directions. The proposed dispatch strategy can be used in WF energy management systems or wind power dispatch centers.

### APPENDIX

#### A. Wind Turbine

The 5 MW NERL WT is adopted as the reference WT [36]. The parameters are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut-in, Rated, Cut-out Wind Speed</td>
<td>3 m/s, 11.4 m/s, 25 m/s</td>
</tr>
<tr>
<td>Rotor, Hub Diameter</td>
<td>126 m, 3m</td>
</tr>
<tr>
<td>Rated Power</td>
<td>5 MW</td>
</tr>
<tr>
<td>Cut-In, Rated Rotor Speed</td>
<td>6.9 rpm, 12.1 rpm</td>
</tr>
</tbody>
</table>

#### B. Converters

The converters are chosen based on the method in [30]. Two IGBT modules (ABB 5SNA 2000K451300) are series connected on each bridge. Based on the data for the IGBT module on the data sheet [37], the power module constants \( a_i = 7.0252 \) and \( b_i = 0.0087 \), and \( f_{sw} \) is chosen as 800 Hz.

#### C. Transformer

The Siemens GEAFOL cast-resin transformer rated at 8000 kVA is chosen as the transformer set in the WT, with no-load loss of 13.5 kW and load loss of 36 kW [38].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Output Power (kW)</td>
<td>5000</td>
</tr>
<tr>
<td>Number of Poles</td>
<td>8</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>50</td>
</tr>
<tr>
<td>Armature Phase Resistance (Ohm)</td>
<td>0.0375</td>
</tr>
<tr>
<td>D-Axis Main Reactance (Ohm)</td>
<td>2.93</td>
</tr>
</tbody>
</table>

### Table I. Parameters of Cables [35]

<table>
<thead>
<tr>
<th>Cross section (mm²)</th>
<th>Resistance (Ω/km)</th>
<th>Capacitance (μF/km)</th>
<th>Inductance (mH/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>0.1842</td>
<td>0.18</td>
<td>0.44</td>
</tr>
<tr>
<td>150</td>
<td>0.1167</td>
<td>0.21</td>
<td>0.41</td>
</tr>
<tr>
<td>240</td>
<td>0.0729</td>
<td>0.24</td>
<td>0.38</td>
</tr>
</tbody>
</table>

### Table II. Comparison of these strategies at different operation conditions

<table>
<thead>
<tr>
<th>Wind direction</th>
<th>Strategy</th>
<th>Output Power (MW)</th>
<th>Total loss (MW)</th>
<th>Cable loss (MW)</th>
<th>WT loss (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>Strategy A</td>
<td>49.59</td>
<td>2.49</td>
<td>0.21</td>
<td>2.28</td>
</tr>
<tr>
<td></td>
<td>Strategy B</td>
<td>52.23</td>
<td>2.45</td>
<td>0.23</td>
<td>2.22</td>
</tr>
<tr>
<td></td>
<td>Strategy C</td>
<td>52.29</td>
<td>2.44</td>
<td>0.22</td>
<td>2.21</td>
</tr>
<tr>
<td>270°</td>
<td>Strategy A</td>
<td>49.53</td>
<td>2.56</td>
<td>0.27</td>
<td>2.28</td>
</tr>
<tr>
<td></td>
<td>Strategy B</td>
<td>52.27</td>
<td>2.50</td>
<td>0.26</td>
<td>2.23</td>
</tr>
<tr>
<td></td>
<td>Strategy C</td>
<td>52.33</td>
<td>2.47</td>
<td>0.26</td>
<td>2.21</td>
</tr>
</tbody>
</table>
Q-Axis Main Reactance (Ohm) 2.93
Fundamental Impedance Factor Line Voltage (Vrms) 1.005759
Stator Phase Current (Amps) 743.962
Power Factor 0.809748
Rated Rotational Speed (rpm) 750

REFERENCES


