AN OPTIMIZATION APPROACH FOR INTEGRATING DIFFERENT ROOF FUNCTIONS WITH ENVIRONMENTAL IMPACTS CONSTRAINT: “A HYBRID FRAMEWORK”

Nooshafarin Mohammadzadeh*1, Soolyeon Cho2

BETlab, College of Design, NC State University, Raleigh, North Carolina

1 nmohamm@ncsu.edu
2 Soolyeon_cho@ncsu.edu

Abstract
The roof, as part of the building envelope, is one of the most important areas in the development of sustainable buildings. Various roof design strategies have been developed for reducing building energy usage, generating energy, improving water retention, and waterproofing. Integrating different roof functions such as green roofs, cool roofs, and PV systems is of building practitioners’ interests in order to meet certain criteria in terms of energy efficiency, cost effectiveness, and environmental impact. The key question which should be considered is what are the best and/or optimal strategies of utilizing roof areas in commercial buildings to achieve sustainability goals such as NZEBs or Carbon-Neutral buildings. This requires creating a quantitative analysis between spectrums of roof performance factors. The main aim of this research is to develop a framework which includes a mathematical optimization model as a core that is fed primarily by energy, cost, and environmental analyses. It integrates multiple roof functions and optimizes desired roof performances. The hybrid framework, introduced in this paper, is realized as a tool “Roof Function Optimizer (RFO)” as a way of providing useful decision making process for designing sustainable roofs in terms of energy efficiency, cost effectiveness, and environmental impact.

Keywords - Optimization model; Integrated roof; Environmental analysis components

1. Introduction

Major decisions regarding building’s sustainability are made primarily by architects and designers in the early design stages. Improving thermal performance of building envelopes is widely considered in sustainable design from different points of view, i.e. architecture, structure, and construction[1, 2]. The roof, as primary part of the envelope is an important area in the conceptual stage of building design. Roof areas are designed and utilized as an extra space that can improve energy performance and economic aspects of the buildings [3]. Changing roof design parameters such as thermal performance, solar reflectance, and solar absorptance leads to developing sustainable buildings. The roof surface also offers a weather proof barrier for inside of the building [4]. Furthermore, roof system is one of the building’s elements that need frequent replacement. The average lifetime of a conventional roof is between 15 to 20 years before it requires replacement. Thus, changing the characteristics and materials of the roofs is relatively easier than other envelopes [5-7].
Therefore, these give a raise to utilize roof spaces in terms of different roof functions and technologies for sustainable development. Among different roof types of technologies and functions, there are debates and challenges regarding which one should be chosen based on what criteria. The appropriate strategies for the roof functions selection and construction depend on key performance factors, which are considered in sustainable developments such as financial aspect, energy performance, and environmental recognition. In addition, integrating different roof functions also can improve the efficiency of the system and avoid single roof type deficiencies [4, 5].

There are a large number of researches for single roof technology such as green roof, cool roof, and PV systems. Energy savings and temperature reduction by the use of reflective roof surface materials are calculated through actual measurements and simulation [8-11]. Several studies have investigated life cycle costs and compared the economic benefits of green roofs with conventional roofs. However, the maintenance costs in most of these studies were not carefully considered. There is still a lack of quantification for the benefits of vegetated roofs at the point of users, building owners and communities [3, 5, 12-16]. For PV systems, many studies looked at the payback periods and life cycle of photovoltaic panels. However, with the fast improvement in PV technologies and their efficacy, the results are quickly outdated [17-20].

A few researches have been done for integrated roof functions. However, comparing the net present value for energy and cost of different roof technologies are very much lacking in these researches. Most of the net value studies didn’t consider all important criteria (energy, cost, and environment) simultaneously. As a result, there is still lack of quantifiable analyses for integrated roof functions for maximum energy, cost, and environmental benefits [9, 20-22]. Further research is needed to create a framework which addresses the best combination of these roof design strategies in different locations by the use of a comprehensive approach.

This paper proposes a framework that includes an optimization mathematical model as a core that is fed by required data from energy, cost and environmental analyses. It primarily investigates the optimal combination of different roof functions to satisfy different needs such as environmental impact as well as energy consumption while minimizing the financial aspect of project.

2. Methodology

To gain a comprehensive understanding of how different roof configurations interact with different outcomes (i.e., energy savings, environmental impacts and costs), a non-linear mathematical model is proposed. The proposed framework is capable of incorporating behaviors and interactions between several parameters used in the roof function selection. An objective function is employed in this mathematical model to maximize total cost savings associated with different roof type installations and operations. The schematic of the proposed framework is shown in Fig. 1. In other words, the proposed mathematical model predicts the best combination of different roof types in a building to maximize energy savings and satisfy environmental restrictions.
Fig. 2 demonstrates the high level process flow of the methodology and related components. In order to characterize the Cost-Benefit Analysis (CBA) and more specifically the payback periods for different roof functions, parameters such as installation costs, maintenance costs, lifetime, and maintenance periods need to be identified. The cost differences between various roof functions and the conventional roof are defined as the cost gap. Energy saving and generation by each technology are calculated through energy modeling and simulation. Environmental impacts of each technology that is considered primarily as greenhouse gas emissions are obtained through simulation analysis.

All of these information are fed as inputs into the proposed mathematical model in order to provide the optimal solution for roof type’s selection and their combinations. Various criteria including maximum energy benefits, maximum life cost benefits, least payback periods, or maximum environmental benefits can be incorporated in the roof type selection. This can be further adjusted through using certain weighting factors in the objective of the proposed mathematical model.

As shown in fig. 2, three green technologies such as green roofs, cool roofs, and PV systems are considered for the proposed methodology. Moreover, the green roof and cool roof types are considered as energy efficient technologies while the PV systems as energy generation technology. The building’s energy consumptions are decreased by reducing heat transfer through the roof surfaces. They also contribute to the heat island reductions and air quality improvements as their environmental impacts. The green roof has also further environmental benefits in terms of mitigating roof runoff depth. The solar photovoltaic panels are energy generating technologies. The solar panels also reduce energy consumption as a result of providing shade on the roof in summer. The more generalized framework of the proposed mathematical model can also incorporate different parameters associated with various climate conditions. As another benefit of constructing the optimization framework, sensitivity analysis can be performed and provide insights on the design parameters impacts. As a result, parameters with significant impacts on the desired criteria can be identified.
3. Theoretical Aspect of Engineering Economic Analysis

The theoretical aspects of engineering economic analysis are necessary in order to develop the proposed model and address the objective functions of the project. Although annual energy savings can be calculated from the simulation results, the Net Present Value (NPV) analysis is necessary to convert the future electricity costs and actual savings into equivalent dollar amount of present time. The actual cost savings are computed based on predetermined interest and inflation rates. If certain cash flows occur in a multiyear format, then it is needed to convert them into present values. An example of these cash flows is the maintenance costs that could occur every “n” years. The combined interest rate for the duration of n periods can be computed by using annual cash flow to NPV transformation [23].

4. Optimization Model

Every optimization model consists of the following three key parts: Decision variables, Objective, and Constraints. The decision variables represent specific decision which should be made during the process. In the proposed model, the decision variables show the optimal portion of the roof that needs to be constructed by associated roof types.

1.1. Objective Function

The objective function of the proposed mathematical model tries to maximize the total cost savings consisted of net cost savings minus installation, replacement cost and maintenance costs of different roof types.

Equation (1) shows the high level objective function used in this research.

Max [(Energy Savings Costs (NPV)) − (Installation Costs + Replacement Costs (NPV) + Maintenance Costs (NPV))]  (1)
1.2. Variables Definitions

In order to model the problem of interest, the following variables need to be defined:

\[ X_r = \text{Present green roof with roof type } r \] (2)
\[ Y_s = \text{Present cool roof with roof type } s \] (3)
\[ Z_t = \text{Present PV system with roof type } t \] (4)
\[ \text{IM}^1 = \text{Impact of roof types on Annual Energy Savings (EUI = kWh/R^2 \ast yr)} \] (5)
\[ \text{IM}^2 = \text{Impact of roof types on Environment (lb/yr gass emmissions)} \] (6)
\[ L = \text{total surface area of roof} \] (7)

1.3. Constraints

The proposed optimization mathematical model consists of seven sets of constraints. These constraints are defined to model the characteristics of the problem of interest in the real world.

**Constraint Number 1:** This constraint keeps the total environmental impacts of different roof type selection to a sustainable value. What this constraint refers to as an environmental impact is the emission reduction level. The higher this term is more desired, thus the lower bound for this level is provided. This constraint defines that the sum of avoided gas emissions by using different roof functions such as green roof, cool roof and PV systems should be higher than the specified value. The maximized objective function value will guarantee the environmental and energy requirements of the selected sets of roofs. It is evident that the change in the environmental tolerable limits impacts the resulted optimal solution.

\[
\sum_{1}^{n} (X_r \times \text{IM}^2_{Xr} \times L) + \sum_{1}^{n} (Y_s \times \text{IM}^2_{Ys} \times L) + \sum_{1}^{n} (Z_t \times \text{IM}^2_{Zt} \times L) \geq \text{Lower Bound on Environment Effects} \] (8)

**Constraint Number 2:** This constraint covers the entire roof surface with the desired selection of roof types considering the assumption that cool roof and green roof cannot combine together.

\[
\sum_{1}^{n} (X_r + Y_s) \leq 1 \] (9)

**Constraint Number 3:** This constraint set assumes that PV array systems can install above the green roofs and cool roofs. It assures the feasibility of the solution.

\[
\sum_{1}^{n} (X_r + Y_s + Z_t) \geq 1 \] (10)
**Constraint Number 4:** This constraint indicates that PV systems can have only one time overlap with other functions. Furthermore, the PV systems cannot have overlap with themselves. Thus, the total PV surface should not be more than 100% of the roof.

\[ \sum_{1}^{n} Z_t \leq 1 \]  

(11)

**Constraint Number 5:** These constraint sets keep the portions to a logical range avoiding unrealistic solutions such as negative roof portions.

\[ 0 \leq X_r \leq 1 \quad \forall r = 1 \text{ to } n \]
\[ 0 \leq Y_s \leq 1 \quad \forall s = 1 \text{ to } n \]
\[ 0 \leq Z_t \leq 1 \quad \forall t = 1 \text{ to } n \]  

(12)

**Constraint Number 6:** These constraint sets indicate that the summation of all portions of different roof functions should be between 100% and 200%.

\[ \text{Sum (Portions of different Roofs)} \leq 2 \]
\[ \text{Sum (Portions of different Roofs)} \geq 1 \]  

(13)

**Constraint Number 7:** This constraint forces the roof type selection to include PV systems. This is necessary to generate energy when a lower bound on the energy is provided. Note that the PV systems are the only roof functions that can produce energy.

\[ \text{Sum of PV system} \geq \text{Minimum necessary percentage of PV syste} \]  

(13)

This mathematical model assumes \( n \) different types of green roofs, cool roofs and PV systems. It also tends to maximize the cost associated with the installation and operation of different roof types, energy savings, and generations, while it maintains a certain limits on environmental impacts.

It should be noted that the proposed mathematical model can incorporate other factors of roof types as more constraints. The simplicity and size of the proposed model result in very fast modeling and solving in any optimization software.

### 5. Mathematical Model’s Parameters Estimation

In the following sections, the detailed information and steps for engineering economic analyses, energy analyses, and environmental benefits of roof technologies are provided at the building scale.

### 6.1. Cost Analysis

Despite the fact that the financial aspects are affected by the location of the project, there are several key factors for cost analyses that are fairly uniform and predictable. Some examples of these factors are: initial cost, the life span of the roof technologies, and maintenance costs. The Roof’s maintenance time horizon should also be considered in the context of cost analysis. The costs and savings are determined on an area unit basis. The results can be fed into an engineering economic model to estimate the return
investment time using NPV analysis. In this research, the cost analysis includes the total cost required to produce services or project costs associated with the roof’s installation and annual and periodic maintenance. There is a strong relationship between the cost and the dollar value. As a result, the inflation cost and true interest rate are also considered in the NPV analysis. The electricity cost inflation is considered for different sectors (commercial, residential, and industrial). For PV systems, in addition to the annual maintenance costs, the replacement cost for the inverter is also considered to be occurred every 10 years. In the majority of the previous studies, the maintenance costs are ignored in order to simplify the proposed analysis.

6.2. Lifetime

The expected average lifetime of flat roofs (conventional roofs) is considered 20 years. Regarding green roofs, many researches have carried on claiming that the lifetime of the green roofs are roughly two times the average flat roofs. For cool roofs and PV system, the expected time horizon of replacements is considered to be 20 years. Therefore, the period of economic cost benefit analysis of this study is considered to be 40 years which is the biggest common lifetime of the systems. For the PV systems and cool roofs, the NPV of the replacement cost should be considered at the year 20.

6.3. NPV Computation Consideration

In order to perform the NPV analysis for the proposed context, following considerations are also included from economic perspectives:

- For maintenance costs in PV systems cases, since the periods of maintenance are different, the combined interest rate should be calculated for different maintenance periods and then be used in the analysis.
- The lifetime of PV and cool roof systems are considered 20 years. As a result, after year 20 the replacement cost should be incurred for those technologies (NPV cost). The proposed model provides ability for users to consider different lifetime for different types of cool roofs and PV systems.
- In the year 20 of the analysis, due to the replacement of cool roof and PV systems, the annual maintenance and inverter costs should not be included.
- Inflation is only considered for electricity cost in this analysis.
- The emerged benefit of each roof type in terms of NPV is considered to be proportional to the percentage of roof area.
- The structure of the proposed model provides further ability for the analyst to weigh different terms in the objective function. Therefore, different factors such as investment costs or energy saving costs can be prioritized.
- Due to the replacement of PV systems at the end of their lives, an increase in their efficiency is considered to account for future improvements in PV systems performance. According to the National Renewable Energy Laboratory (NREL) report, the efficiency of PV systems will increase about 2% to 3% every year.
6.4. Energy Savings

In order to evaluate various roof configurations in terms of energy consumption, the EnergyPlus simulation program is used in this study. In order to perform the analysis, two input files are needed: 1) a building model and 2) a weather data file. EnergyPlus uses annual weather data file that in our case is Typical Meteorological Year (TMY) and default parameters for each specific roof to simulate the energy performance. It can also calculate the amount of thermal loads to meet occupants comfort conditions. Thermal loads are independent of the HVAC systems; therefore, they are the main priority in roof energy analysis considering their specific characteristics [3, 5, 9, 14, 20, 27].

For industry and commercial sectors, electricity costs are mainly based on two parameters: actual usage and penalty fee. The penalty fee is based on the highest level of demand (peak) during specific period (weekly, monthly, or annually) [28]. The total and peak load reduction for each roof function are calculated and compared to the baseline roof results.

6.5. Environmental Performance

Considering the magnitude of re-roofing projects and generated waste materials, roofing systems will have substantial impacts on the environment. Employing specific roofs such as vegetated systems not only reduces the building energy consumption, but also improves environmental performance measures. Applying green technologies minimizes waste materials and maximizes the environmental benefits. The efficiency of different roof type systems and their environmental impacts in different projects and locations can be quantified through simulations such as EnergyPlus.

Green House Gas (GHG) Emission

Sustainable roof design technologies such as cool roofs, green roofs, and PV systems contribute to building’s energy consumptions or energy generation and as a result GHG generation. They can reduce the energy demands of building and reduce the amount of GHG emissions over their lifecycle [29]. The amount of GHG emissions is also related to the source of energy. Coal combustion generates more greenhouse gas than natural gas or petroleum. The most common method to determine various GHGs effect is considering Global Warming Potential (GWP) which is scribed as Carbon Equivalent of each specific greenhouse gas. The main gases which should be considered in building energy usage are carbon dioxide, carbon monoxide, methane, and nitrous oxide. The EnergyPlus software provides emission for various gases as well as carbon equivalent of overall gases in the facility [30-33].

6. Verification and Validation

6.1 Implementation (Tool Development)

The proposed framework discussed in earlier sections has been coded and implemented in a software tool so called Roof Function Optimizer (RFO) for further verification and validation purposes. Currently, Microsoft Excel/VBA environment has
been selected to develop this tool. Collected data on different roof functions’ characteristics have also been incorporated in this tool. These data includes installation and maintenance costs, energy saving parameters, energy price, interest rate, inflation rate and etc. Fig.3. shows a schematics of this tool. For ease of use and being intuitive, several procedures have also been coded to facilitate the analysis. These procedures perform optimization on the background and generate results.

**6.2. Sensitivity Analysis**

Fig. 19 shows the percent of roof area coverage by different types of green, cool, PV roof functions by changing the tolerable environmental impact sustained within the optimization model. For example, if the sustained environmental impact is 875,000 (lb/Lifetime) then the optimal solution would have 100% PV system on top of green roof type four. The rest of the roof is covered by 80% green roof type 1 and 20% green roof type 8. On the other hand, if the sustained environmental impact decreases to 880,000 (lb) then PV system will cover the roof on top of these combination: 44% green roof type 1 and 56% green roof type 8.

<table>
<thead>
<tr>
<th>Roof Function</th>
<th>870000 (lb/Lifetime)</th>
<th>875000 (lb/Lifetime)</th>
<th>880000 (lb/Lifetime)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green roof 1</td>
<td>100%</td>
<td>80%</td>
<td>44%</td>
</tr>
<tr>
<td>Green roof 2</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Green roof 3</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Green roof 4</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Green roof 5</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Green roof 6</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Green roof 7</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Green roof 8</td>
<td>0%</td>
<td>20%</td>
<td>56%</td>
</tr>
<tr>
<td>Cool roof 1</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Cool roof 2</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Cool roof 3</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Cool roof 4</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>PV System 1</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>PV System 2</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>PV System 3</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Fig. 4 Optimal set of different roof types as a function of different environmental impacts
7. **Summary, Conclusions, and Future Work**

This paper presents a novel approach to select a set of different roof functions to achieve certain objectives. In this analysis, maximizing the energy savings has been considered for the optimization. Appropriate constraints are developed to account for the dynamics of the system in terms of roof coverage and etc. Environmental effects of different roof types are also considered in the proposed approach.

The proposed mathematical model is coded in a computational engine (RFO) for testing and developing case studies. The majority of the input data has been collected by reviewing the literature and necessary energy simulations. The energy simulations are conducted using the EnergyPlus program. The RFO tool has gone through necessary sensitivity analyses in order to uncover the true effects of different parameters such as installation costs, payback periods, and etc. RFO is an interactive tool which can lead designer from sustainable buildings to sustainable cities. RFO needs professional roofing experience and decision for data entry. As a result, it can empower the role of the professional roof consultants in working with the building practitioners who are interested in sustainable development. Further real world tests might be necessary to assure the accuracy of the proposed research.

**Acknowledgment**

This research was funded by Roof Consultant Institute Foundation (RCIF) and the College of Design at North Carolina State University, Raleigh, NC.

**References**


