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# Disturbance Modeling with Subspace Identification Method

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## **Abstract**

*Real-time optimal control of heating, ventilation and air-conditioning (HVAC) systems needs to regularly reset the set points of the local control loops that have a significant influence on the energy performance of the whole system. However, resetting a set-point for a local control loop introduces disturbance within the control loop, as well as disturbance in other local control loops that interacts with this loop, which may significantly affect the stability of the system operation. Traditional reset methods follow a step or a rate-limited change. They present similar problems because they lack methodical study.*

*In order to identify a better way to reset the set-points (other than step or rate-limited change), it is necessary to build disturbance models which are able to describe the transient behaviors of the affected control loops triggered by resetting the set-points. To study transient behavior of dynamic system due to resetting of decision variables, disturbance models are developed with a subspace identification method (SIM) using canonical variant analysis (CVA) approach. The SIM is a black box system identification technique that has tendency to model SIMO systems without structural parameterization. Because of its intelligibility and intrinsic capability, it can be used in the synthesis of model based optimization techniques. Data for training of SIM disturbance model is generated from HVAC test bed build in TRNSYS simulation environment. Step reset is introduced in chilled water set point temperature (classic finite step response method) and the tracking errors (disturbance) introduced in the control loops for the decision variables were recorded until the whole system settled. Next, the recorded data (input: reset in chilled water set point temperature and output: disturbance introduced in control loops) is used as training data (data-driven), to identify state space parameters for SIM disturbance model.*

*Comparative analysis of developed disturbance model with respect to the process data (from test bed) was conducted, which demonstrated fairly good performance of the identified models based on accuracy.*

**Keywords - real-time optimal control, air conditioning, subspace identification method, set-point reset**

## 1. Introduction

Energy conservation has become an important issue, due to world wide increase in energy consumption. Energy consumption of building has scaled over recent years. For instance, building energy consumption in EU was approximately 37% of total energy consumption [1], In US building consumes 41% of total energy consumption [2] and china has seen increase from 10 to 25 % in building consumption of total energy consumption in recent years [3]. For both residential and commercial buildings HVAC systems have been recognized as major concern for energy consumption [1]. HVAC systems makes up almost 50% of energy demand by building [1]. Therefore real time optimization has been developed for the optimal operation of HVAC systems. It has been in practice since 1980s. Brawn et al. [5] developed a nonlinear optimization algorithm to obtain the optimal control settings for chilled water systems [4]. It has been demonstrated that significant amount of energy savings are possible through optimal control. Gibson [6] proposed a supervisory control strategy using artificial neural network (ANN) to model the dynamic behavior and genetic algorithm (GA) for optimization. Cumali 1994 demonstrated the application of real-time optimization for building systems [7].

In real time optimization, optimization of decision variables [8,9] is carried out regularly, resetting the set points of the local control loops with respect to thermal comfort and energy use. On the other hand, resetting the set point of a local control loop introduces artificial disturbance not only with in the control loop with change in decision variable, but also other interacting control loops with in HVAC systems. This affects the overall stability of HVAC systems. The former method has been found impractical as a step change, especially when the magnitude is large, will introduce significant disturbance into the whole system; while latter rate limited reset method [10] can overcome this problem to a certain extent, but the rate is always predefined using a rule of thumb and lacks a systematic and scientific study.

It is necessary to study transient behaviors of the affected local control loops triggered by resetting the set points of decision variables, in order to develop a new resetting approach for enhanced system stability. This paper therefore presents disturbance estimation model for transient behaviors triggered by resetting of decision variables. The subspace identification method (SIM) using CVA (i.e. a black box system identification approach) has been employed for this study. This study is organized as follows: Section 2 presents the subspace identification method (SIM) for identification of disturbance model, using data generated from TRNSYS test bed. Section 3 describes the evaluation tests used for the validation of SIM disturbance model. Section 4 presents performance of developed SIM disturbance model

with respect to process data from test bed. Conclusion is presented in Section 5.

## 2. Subspace identification method

The subspace identification method (SIM) is used to develop disturbance estimation model in order to study transient behavior of disturbance introduces due to reset in set point of decision variables in HVAC system. SIM is a black box system identification approach that has tendency to model SIMO systems without structural parameterization. SIM comprises of following steps,

**Step 1** Formulation of extended state space model, i.e. model representation [11]. Consider linear discrete state space description,

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (1)$$

$$y_k = Cx_k + Du_k + v_k \quad (2)$$

For above state space description Kalman filter could be formulated as

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (3)$$

$$y_k = Cx_k + Du_k + e_k \quad (4)$$

where,  $e_k = y_k - Cx_k - Du_k$ . State space description (3 & 4) can be expressed as,

$$x_{k+1} = A_k x_k + B_k z_k \quad (5)$$

$$y_k = Cx_k + Dz_k + e_k \quad (6)$$

Extended state space description from (3 & 4) ,

$$Y_f = \Gamma_f X_k + H_f U_f + G_f E_f \quad (7)$$

where,

$$\Gamma_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix}, H_f = \begin{bmatrix} D & 0 & \dots & 0 \\ CB_k & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \dots & D \end{bmatrix}, G_f = \begin{bmatrix} I & 0 & \dots & 0 \\ CK & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & \dots & I \end{bmatrix}$$

where,  $A_k = A - KC$  and  $B_k = [B - KD, K]$ . By iterating description (5 & 6),

$$x_k = \bar{L}_p z_p(k) + A_k^p x_{k-p} \quad (8)$$

Extended state space description could be formulated from (5 & 6) [12],

$$y_f(k) = \bar{\Gamma}_f x_k + \bar{G}_f z_{f-1}(k) + D_f u_f(k) + e_f(k) \quad (9)$$

where,

$$\bar{\Gamma}_f = \begin{bmatrix} C \\ CA_k \\ \vdots \\ CA_k^{f-2} \end{bmatrix}, \bar{G}_f = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB_k & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA_k^{f-2}B_k & CA_k^{f-3}B_k & \cdots & CB_k \end{bmatrix}, D_f = \begin{bmatrix} D \\ D \\ \vdots \\ D \end{bmatrix}$$

where,

$$\bar{L}_p = \begin{bmatrix} B_k & A_k B_k & \cdots & A_k^{p-1} B_k \end{bmatrix}$$

Substituting (8) in (9),

$$y_f(k) = \bar{H}_{fp} z_p(k) + \bar{\Gamma}_f A_k^p x_{k-p} + \bar{G}_f z_{f-1}(k) + D_f u_f(k) + e_f(k) \quad (10)$$

where,  $\bar{H}_{fp}$  is Hankel matrix that is product of observability and controllability matrix,

$$\bar{H}_{fp} = \bar{\Gamma}_f \bar{L}_p = \begin{bmatrix} CB_k & CA_k B_k & \cdots & CA_k^{p-1} B_k \\ CA_k^2 B_k & CA_k^3 B_k & \cdots & CA_k^p B_k \\ \vdots & \vdots & \cdots & \vdots \\ CA_k^{f-1} B_k & CA_k^f B_k & \cdots & CA_k^{f+p-2} B_k \end{bmatrix}$$

Substituting (8) in (7),

$$Y_f = H_{fp} Z_p + H_f U_f + G_f E_f \quad (11)$$

**Step 2** Obtain projections and then reduce the order of model (i.e. observable). Multiply (11) by  $\Pi^{\frac{1}{U_f}}$  i.e. uncorrelated with  $U_f^T$  [13].

$$Y_f \Pi^{\frac{1}{U_f}} = H_{fp} Z_p \Pi^{\frac{1}{U_f}} + G_f E_f \quad (12)$$

Multiply (12) by  $Z_p^T$  i.e. uncorrelated with  $E_f$ ,

$$Y_f \Pi^{\frac{1}{U_f}} Z_p^T = H_{fp} Z_p \Pi^{\frac{1}{U_f}} Z_p^T \quad (13)$$

$$H_{fp} = Y_f \Pi^{\frac{1}{U_f}} Z_p^T \left( Z_p \Pi^{\frac{1}{U_f}} Z_p^T \right)^{-1}$$

SVD form is given by (14), where smaller singular value states can be ignored. The weighted form (for unified formulation) of Hankel matrix can be given by (15) and weighting function [14] for CVA approach (of exact solution) is given in (16).

$$\hat{H}_{fp} Z_p = \hat{\Gamma}_f \hat{L}_p Z_p = U_n S_n V_n^T \quad (14)$$

$$W_1 \hat{H}_{fp} W_2 = U_n S_n V_n^T \quad (15)$$

$$W_1 = \left( Y_f \Pi \frac{1}{U_f} Y_f^T \right)^{1/2}, W_2 = \left( Z_p \Pi \frac{1}{U_f} Z_p^T \right)^{1/2} \quad (16)$$

**Step 3** State space parameters A, B, C, D and K are estimated through reduced observability matrix obtained in previous step [13,15].

$$\hat{x}_k = \hat{L}_p z_p(k) \quad (17)$$

$$[\hat{C} \quad \hat{D}] = \arg \min \left\{ \sum_{k=1}^N \left\| y_k - [C \quad D] \begin{bmatrix} \hat{x}_k \\ u_k \end{bmatrix} \right\|^2 \right\} \hat{e} \quad (18)$$

$$[\hat{A} \quad \hat{B} \quad \hat{K}] = \arg \min \left\{ \sum_{k=1}^N \left\| x_k - [A \quad B \quad K] \begin{bmatrix} \hat{x}_k \\ u_k \\ \hat{e}_k \end{bmatrix} \right\|^2 \right\} \hat{e}_k$$

### 3. Evaluation Test

The developed SIM disturbance model is subjected to several tests in order to judge the accuracy of estimated output. Three tests are performed: best fit (BF), mean square error (MSE) and Akaike Information Criterion (AIC) to check the reliability and accuracy of estimates.

- Best fit

$$BF = \left( 1 - \frac{|y - \hat{y}|}{|y - \bar{y}|} \right) \times 100\% \quad (19)$$

- Mean square error

$$MSE = \frac{1}{N} \sum_{i=1}^N |y - \hat{y}|^2 \quad (20)$$

- Akaike Information Criterion

$$AIC = N \times \log \left( \det \left( \frac{1}{N} \sum_1^N \varepsilon(t, \theta_N) (\varepsilon(t, \theta_N))^T \right) \right) + 2n_p + N \times (n_p \times \log(2\pi) + 1) \quad (21)$$

where,  $y$  is process data,  $\hat{y}$  is the estimated output,  $\bar{y}$  is means output,  $N$  is number of values in the estimated output,  $\varepsilon$  is vector of prediction error,  $\theta_N$  is estimated parameters,  $n_p$  is number of estimated parameters and  $n_y$  is number of model outputs.

#### 4. Performance of SIM Disturbance Model

The HVAC test bed build in TRNSYS simulation environment is used to generate training data. In order to investigate disturbance induction and propagation in HVAC system due to change in set point of decision variable. Step change (7°C to 5°C) is introduced in chilled water temperature; respective change in chilled water temperature is shown in Figure 1.

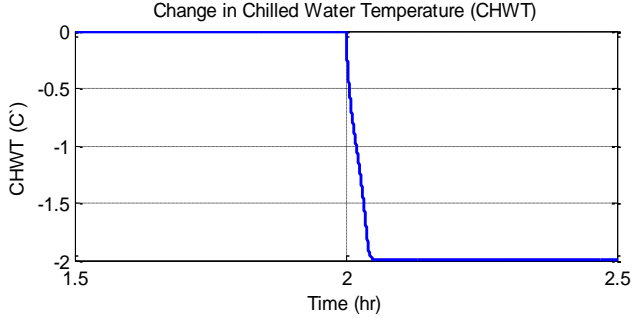


Figure 1. Change in chilled water temperature due to reset in chilled water temperature

Disturbance modeling is conducted in two parts. First the error introduced in chilled water temperature due to reset in set point in same local control loop is considered. SIM disturbance model is trained with input: magnitude of reset in chilled water temperature and output: dynamic response of chiller plant as training data. Then subtracting the estimate from desired reset in chilled water temperature will give tracking error in chilled water temperature (ECHWT). State space model developed with SIM is presented below.

$$\begin{aligned} x_1(t+Ts) &= A_1 x_1(t) + B_1 u(t) + K_1 e_1(t) \\ y_1(t) &= u(t) - [C_1 x_1(t) + D_1 u(t) + e_1(t)] \end{aligned} \quad (22)$$

where,

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.9854 & -0.0044 \\ -0.0404 & 0.9253 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0001 \\ -0.0130 \end{bmatrix}, K_1 = \begin{bmatrix} 0.1057 \\ -14.26 \end{bmatrix} \\ C_1 &= [13.06 \quad -0.0337], D_1 = [0] \end{aligned}$$

The disturbance model results are compared with process data (from test bed) in Figure 2. That showed fairly good performance with fitting (BF) between estimate and process data of 99.97%, mean square error (MSE) of 7.795E-8 and Akaike Information Criterion (AIC) of -16.3658.

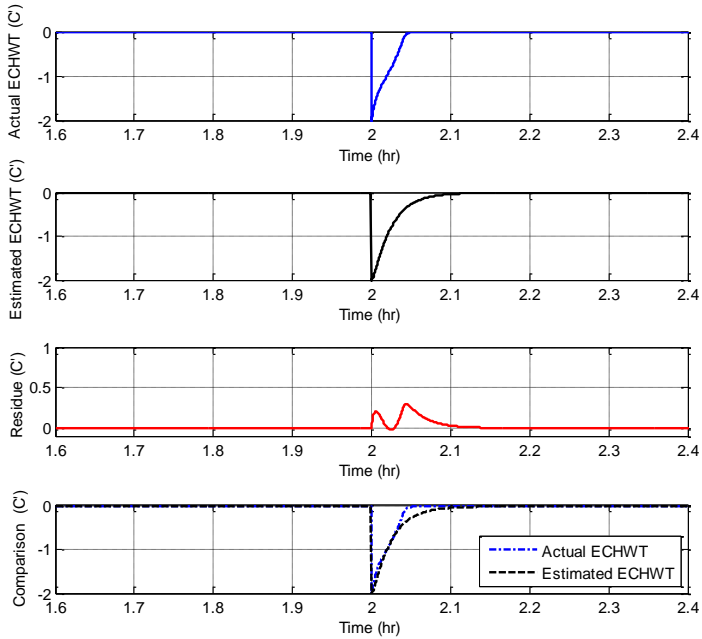


Figure 2. Performance of actual ECHWT and estimated ECHWT

For second part other affected local control loops must be considered. The disturbance in condenser water temperature (DCWT) and disturbance supply air temperature (DSAT) are under scope in Figure 3, these changes can be seen as disturbances caused by step change in chilled water temperature.

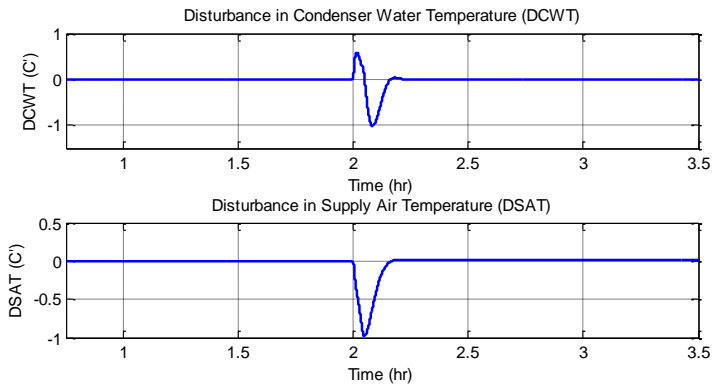


Figure 3. Disturbance introduced in other interacting control loops



In order to study transient behavior of interacting control loops, SIM disturbance model (SIMO) is trained with one input i.e. magnitude of reset in chilled water temperature and two outputs that are disturbance in condenser water and supply air temperature,

$$x_2(t+Ts) = A_2x_2(t) + B_2u(t) + K_2e_2(t) \quad (23)$$

$$y_2(t) = C_2x_2(t) + D_2u(t) + e_2(t)$$

where,

$$A_2 = \begin{bmatrix} 0.9972 & -0.0033 & 0.0013 & -0.0015 & 0.0002 \\ 0.0130 & 0.9919 & 0.0022 & 0.0062 & 0.0007 \\ -0.0250 & 0.0387 & 0.8998 & 0.2538 & -0.1146 \\ 0.0531 & -0.0947 & 0.1468 & 0.5835 & 0.2771 \\ 0.0111 & -0.0190 & 0.1447 & -0.2804 & 0.4899 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0122 \\ -0.0358 \\ -0.6797 \\ 0.0854 \\ 7.752 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0467 & 0.0017 \\ -0.0347 & 0.0403 \\ 1.531 & 0.5115 \\ -0.4299 & 0.9089 \\ 2.324 & 0.0765 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 12.22 & -0.1906 & -0.0012 & 0.0047 & -0.0031 \\ 8.332 & 10.37 & 0.0105 & -0.0353 & -0.0008 \end{bmatrix}, D_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Figure 4 shows comparison between actual DCWT and estimated DCWT i.e. output channel 1. Small values of residue justifies the accuracy of estimator.

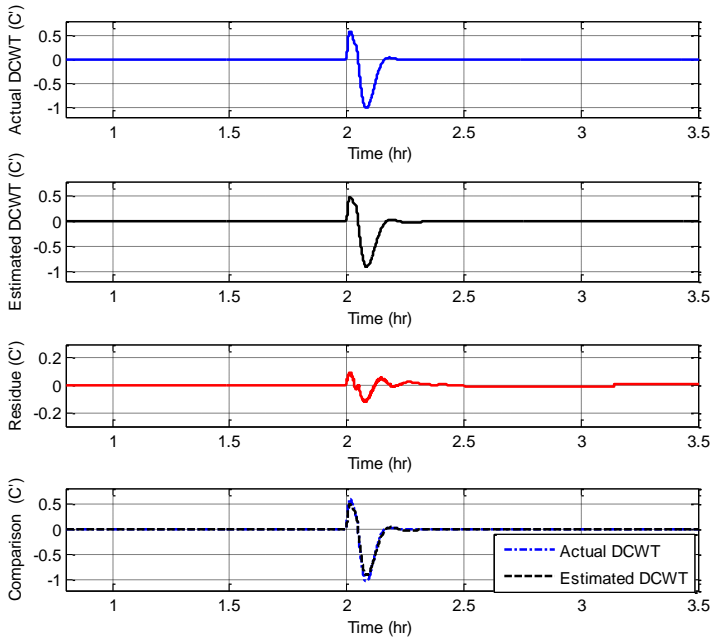


Figure 4. Performance of actual DCWT and channel 1 estimate

Figure 5 shows comparison between actual DSAT and estimated DSAT i.e. output channel 2. Based on evaluatry tests demonstrated in Table 1, performance of both channels DCWT and DSAT can be considered as fairly accurate.

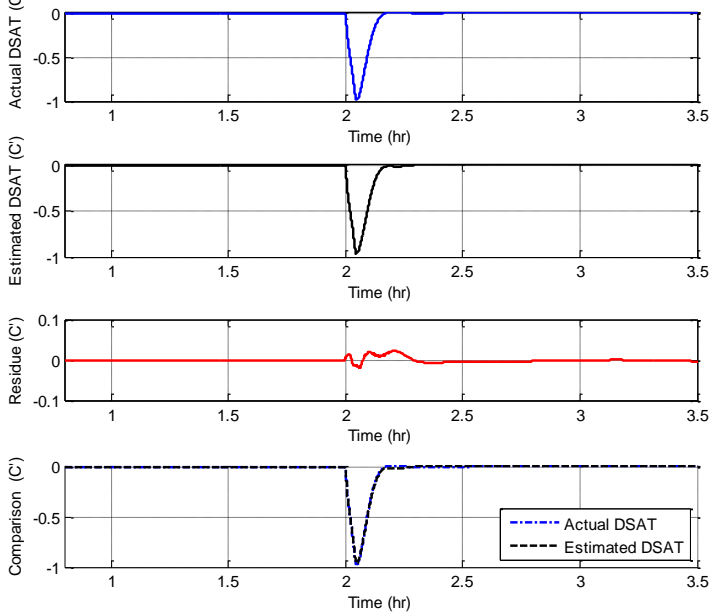


Figure 5. Performance of actual DSAT and channel 2 estimate

	Best Fit (%age)	MSE	AIC
O/P Channel 1	99.8800	2.79E-05	-32.2358
O/P Channel 2	99.2900	1.74E-04	

Table 1. Performance of disturbance model

## 5. Conclusion

In this paper model of transient behaviour of local control loop with reset and interacting control loops has been developed with SIM. The aim has been to obtain state space parameters  $\{A, B, C, D, K\}$  of SIM disturbance model triggered by reset in decision variable. Performance of this approach for given application is demonstrated through three evaluation tests, that showed good performance based on accuracy. Because SIM is

numerically stable and simple, this approach can be adopted easily for process industry. In the future work, a better way to reset set-points with the help of identified disturbance model will be developed, which can be used to reduce the disturbances in the local control loops due to set-point reset and hence to enhance the stability of overall system.

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