Reducing the Energy Consumption of HVAC Systems in Buildings by Using Model Predictive Control

Saeed Sayadi\textsuperscript{1}, George Tsatsaronis\textsuperscript{2}, Tatiana Morosuk\textsuperscript{3}

\textit{Technische Universität Berlin, Institute for Energy Engineering}
Marchstr. 18, D-10587, Berlin, Germany
\textsuperscript{1}s.sayadi@tu-berlin.de
\textsuperscript{2}tsatsaronis@iet.tu-berlin.de
\textsuperscript{3}morozyuk@iet.tu-berlin.de

Abstract
This article demonstrates the advantage of model predictive control (MPC) as an alternative to an already-existing conventional control system, in terms of total energy consumption and comfort criteria. In our case study, we consider one office in the main building of the E.ON Energy Research Center in Aachen, Germany. A dynamic heat transfer model for building thermal elements using the lumped-capacitance method has been formulated. Then unknown parameters in this model have been estimated by minimizing the errors between measured and simulated temperatures. Finally, the model has been linearized and used in our proposed MPC to predict the future states of the system. MPC can, in contrast to the current control system, predict the future changes in the system and consequently makes wise control decisions before the system faces new conditions. The first results of this study show approximately 43\% and 31\% reduction in energy use after implementation of MPC during the estimation (01-05 December 2014) and validation (11-15 February 2015) periods, respectively.

Keywords - model predictive control; energy consumption; thermal modelling; parameter estimation; HVAC

1. Introduction

Nearly one-third of the global energy consumption comes from the building sector, which makes it one of the principal contributors to the world’s total greenhouse gas emissions. In cold climate countries around 50\% of this energy demand is directly associated with space Heating, Ventilation and Air Conditioning (HVAC) [1]. On the other side, as the world’s population and also the tendency towards urbanization grow, even a higher energy demand is expected in the future. Thus, development of energy efficient buildings represents a great concern and has become the focus of many research activities. As a result, in addition to the use of high-performance construction materials for new buildings and retrofitting the old ones, significant efforts have also been made to the optimization of the
operation of HVAC systems using smart controllers instead of conventional ones.

Developing a reliable dynamic model is of crucial importance to improving the energy efficiency of buildings, as it provides a very good platform to test different control strategies and select the most efficient one. A large number of studies has been carried out on simulation platforms and dynamic models that can be used to analyze HVAC control systems in buildings.

In [2] a simulation framework including basic modular HVAC components in Simulink is proposed. The simulation results show a significant possibility to save energy through the optimal control of temperature and damper position.

A simplified approach to develop a low-order linear time invariant (LTI) state-space model in Simulink is presented in [3]. This model, with the advantage of simplicity and computational efficiency, is aimed to be in excellent agreement with the field monitoring data of a building. Another approach to building space modelling is described in [4], and is based on parameter optimized second-order descriptions of each building envelope element. The result of this work is a detailed dynamic model, which enjoys flexibility, transparency and computational efficiency.

A procedure to formulate a dynamic model for an HVAC system consisting of a zone, heating and cooling coils, humidifier and dehumidifier, duct work, fan and mixing boxes is shown in [5]. The results imply that the system is capable of rejecting disturbances more effectively (e.g. less energy consumption) after implementation of the dynamic model in the control system.

Reference [6] demonstrates the modelling and simulation of the entire high-temperature heating circuit of the same building as in our case study here. The model has been developed in the equation-based object-oriented language Modelica and the maximum deviation between simulated and measured data is reported below 20% for boiler subsystems and below 10% for all other subsystems.

Reference [7] illustrates a detailed model for the room air temperature. Computational Fluid Dynamics (CFD) has been applied in this study to show the effect of turbulence and movement of air molecules on the heat exchange between different air zones in a room. This highly accurate model is then transferred into the state-space format to decrease the computation time.

Different approaches have been applied for the parameter identification. Use of Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) for simultaneous state estimation and parameter identification of building predictive model is reported in [8] and the results show that UKF outperforms EKF as it yields a more realistic estimation of wall temperatures. A Building Data-Dependent Identification (BDDI) algorithm is proposed as an alternative method for parameter identification of multi-
zone building models in [9]. The authors concluded that an experimental design is necessary for proper identification of real systems.

Regarding control strategies for HVAC systems numerous studies have been conducted, either in theoretical or in practical fields. A comprehensive comparison between different advanced and classical control techniques has been made in [10,11], where model predictive control is reported as an appropriate choice for buildings applications as it improves thermal comfort mainly by reducing overheating (because of including a model for future disturbances). However, complexity of the mathematical modelling as well as the expensive installation of MPC, especially in old buildings, are mentioned as its drawbacks.

In [12] MPC and PID controllers have been embedded in a Building Energy Management System (BEMS) to compare their performances. It is reported that MPC reduces energy consumption up to 18% as compared to PID controllers. In addition, the number of switching cycles (on-off) of the heat pump is decreased from 144 to 35 cycles in case of using MPC. The prediction of disturbances, however, is considered without any uncertainties.

Besides the appealing advantages of MPC, several problems and difficulties related to the design and implementation of MPC for buildings have been presented in [13]. Ensuring stability and feasibility of MPC, uncertainties involved in the disturbance prediction, convergence of nonconvex MPC problems to suboptimal solutions, computational complexity and equipment retrofitting in sense of adding more sensors or the need to replace existing control systems with digital versions are some of these issues.

The major challenge of using MPC lies in the prediction of disturbances (e.g. weather and internal gain) due to the stochastic nature of them. The impacts of model uncertainties on MPC controllers as well as methodologies for handling them are addressed in many papers. Stochastic Model Predictive Control (SMPC) is proposed as a well-suited approach for building climate control in [14], where a chance-constrained formulation of comfort bounds is employed to cope with the uncertainties. References [15,16] present two approaches to minimize model uncertainty: (1) using a Parameter-Adaptive Building (PAB) model to capture system dynamics through an online estimation of time-varying parameters of a model; and (2) proposing two robust MPC frameworks against additive uncertainties: Open-Loop Robust Model Predictive (OL-RMPC) and Closed-Loop Robust Model Predictive (CL-RMPC). The latter is capable of maintaining room temperature within the comfort range for model uncertainties up to 75%.

The objective of this work is to demonstrate advantages of model predictive control as an alternative to the existing conventional control systems, in terms of total energy consumption and comfort criteria. In our case study, we consider one office in the main building of the E.ON Energy Research Center in Aachen, Germany, which is introduced in section 2.
Section 3 represents a dynamic heat transfer model formulated for this case study. The methodology for estimation and validation of model parameters is described in section 4. The mathematical model has been then linearized and transferred into the MPC framework as shown in sections 5 and 6. Results from the comparison between the existing control system and our proposed MPC as well as some discussion about influence of prediction horizon on the performance of MPC are presented in section 7. Finally, conclusions are drawn in section 8.

2. The Case Study

The main building of the E.ON Energy Research Center has a net floor area of 7222 m² located in the Campus Melaten of RWTH Aachen University in Germany. Its state-of-the-art building technologies, multi-level usage and complex HVAC equipment makes it an ideal case study for various control and energy related researches. Detailed information about this building is given in [6, 17–19]. Besides laboratories and conference rooms, the building mainly consists of offices for more than 200 occupants equipped with the Concrete Core Conditioning (CCC) system and Façade Ventilation Units (FVU) as the sources of energy supply.

The aim of this study is to develop a detailed procedure for design and implementation of MPC for one of the offices, which is introduced as our case study (illustrated in Fig. 1). As the types of offices are very similar, the same approach can be applied to the rest of the offices with minor modifications.

![Fig. 1 Layout of the 2nd floor of E.ON ERC main building and location of our case study [19].](image)

3. Mathematical Model

A dynamic heat transfer model for building thermal elements using the lumped-capacitance method has been formulated, where the air and walls are assumed to have uniform temperatures across their volumes. This approach
helps analysts to obtain a fast and low-order model, which is appropriate for
the control purposes. The model takes following terms into account: (1)
conduction through the window and walls, (2) convection due to the air
movement, (3) solar radiation through the window, (4) absorption of solar
radiation in external walls, (5) heat supplied by HVAC components (CCC
& FVU), (6) internal heat gain, (7) internal surface radiation between walls, and
(8) heat storage capacity of the room air and walls. For a better illustration of
the heat transfer phenomena in the system, the thermal equivalent circuit
model is represented in Fig. 2.

\[ \text{Fig. 2} \quad \text{Thermal equivalent circuit model for heat transfer phenomena in the case study.} \]

Dynamic thermal behavior of the system is obtained from (1) – (17). For
the sake of simplicity and in order to keep the circuit model clear and
understandable, the internal radiative heat exchanges between surfaces of the
internal walls \( (Q_{w1, \text{rad,in}}) \) are not shown in Fig. 2. This term, however, is
considered in the model and is calculated using (11) – (13) from Ref. [20].
Equation (8) determines the supply heat from façade ventilation unit.

\[
\begin{align*}
C_{r1} \frac{dT_{r1}}{dt} &= \Sigma(T_{s1w1j} - T_{r1})/R_{i,j} + (T_{\text{amb}} - T_{r1})/R_{\text{win}} + \dot{Q}_{\text{int}} + \dot{Q}_{\text{FVU}} + \dot{Q}_{\text{sol,win}} \quad (1) \\
C_{w12} \frac{dT_{w12}}{dt} &= (T_{s1w12} - T_{w12})/R_{12} + (T_{s2w12} - T_{w12})/R_{12} \quad (2) \\
C_{w13} \frac{dT_{w13}}{dt} &= (T_{s1w13} - T_{w13})/R_{13} + (T_{s3w13} - T_{w13})/R_{13} \quad (3) \\
C_{w14} \frac{dT_{w14}}{dt} &= (T_{s1w14} - T_{w14})/R_{14} + (T_{s4w14} - T_{w14})/R_{14} \quad (4) \\
C_{w15} \frac{dT_{w15}}{dt} &= (T_{s1w15} - T_{w15})/R_{15} + (T_{s5w15} - T_{w15})/R_{15} + \dot{Q}_{\text{ccc-w15}} \quad (5) \\
C_{w16} \frac{dT_{w16}}{dt} &= (T_{s1w16} - T_{w16})/R_{16} + (T_{s6w16} - T_{w16})/R_{16} \quad (6) \\
C_{w17} \frac{dT_{w17}}{dt} &= (T_{s1w17} - T_{w17})/R_{17} + (T_{s7w17} - T_{w17})/R_{17} + \dot{Q}_{\text{ccc-w17}} \quad (7) \\
\dot{Q}_{\text{FVU}} &= m_{sa} c_{pa} (T_{sa} - T_{r1}) \quad (8) \\
\dot{Q}_{\text{sol,win}} &= \tau_{\text{win}} A_{\text{win}} q_{\text{rad,t}} \quad (9)
\end{align*}
\]
\[
\begin{align*}
(T_{r1} - T_{w1j})/R_{i,j} + (T_{w1j} - T_{w1j})/R_{i,j} + Q_{w1j}\text{rad,in} = 0 & \quad (10) \\
Q_{w1j}\text{rad,in} = A_{w1j} \varepsilon_{w1j} \Sigma \Phi_{w1i,w1j} [\dot{e}_{w1j} + (1 - \varepsilon_{w1j}) I_{ave}] - \dot{E}_{w1j} & \quad (11) \\
\dot{E}_{w1j} = A_{w1j} \dot{e}_{w1j} = A_{w1j} \varepsilon_{w1j} \sigma (T_{w1j} + 273.15)^4 & \quad (12) \\
I_{ave} = \Sigma \dot{E}_{w1j}/\Sigma (A_{w1j} \varepsilon_{w1j}) & \quad (13) \\
(T_{amb} - T_{w1j})/R_{o,i,j} + (T_{w1j} - T_{w1j})/R_{o,i,j} + \dot{Q}_{w1j,}\text{sol} - \dot{Q}_{w1j,}\text{sky} = 0 & \quad i=6,7 \quad (14) \\
\dot{Q}_{w16,}\text{sol} = \alpha A_{w16} q^\text{rad,t} & \quad (15) \\
\dot{Q}_{w17,}\text{sol} = \alpha A_{w17} q^\text{rad,h} & \quad (16) \\
\dot{Q}_{w1j,}\text{sky} = \varepsilon \sigma A_{w1j} ((T_{w1j}+273.15)^4 - (T_{sky}+273.15)^4) & \quad , i=6,7 \quad (17)
\end{align*}
\]

The following 11 parameters are disturbances to the model. Note that in this work the prediction of disturbances is assumed to be perfect and without uncertainties:

- Temperature of adjacent rooms (\(T_{r2} - T_{r5}\)).
- Ambient temperature (\(T_{amb}\)).
- Solar radiation on horizontal and tilted surfaces (\(q^\text{rad,h} & q^\text{rad,t}\)).
- Internal heat gains due to presence of people in the room as well as operation of electronic devices (\(Q^\text{int}\)).
- Supply heat from concrete core conditioning system to the floor (\(Q^\text{ccc-w15}\)) and ceiling (\(Q^\text{ccc-w17}\)), because the control system of CCC is separate from the office and the local controller in the office has no influence over it.
- Mass flow rate of the high-temperature air stream from FVU (\(m^\text{sa}\)). This value is defined according to the ventilation requirements and the control system cannot change it.

Besides the above-mentioned disturbances, the following parameters are categorized as known parameters in our model:

- Surface area of the window and walls (\(A_{w1j} & A_{win}\)).
- Shape factors for radiative heat exchange between wall i and j (\(\Phi_{i,j}\)), obtained from [21].
- Emissivity of the walls (\(\varepsilon_{i}\)).
- Heat capacity of air at constant pressure (\(c_{pa}\)).
- Stefan Boltzmann constant for radiation (\(\sigma\)).

And finally 23 unknown parameters, which need to be estimated, are all thermal resistances (\(R_{ij}, R_{i,j} & R_{o,j}\)), the heat storage capacities of the room and walls (\(C_{r1} & C_{w1j}\)), the transmissivity of the window (\(\tau_{win}\)), and absorptivity coefficient of the external walls (\(\alpha\)). The procedure to estimate these parameters and validate the estimation is explained in details in the next section.

4. Parameter Estimation and Validation of Estimated Parameters

Estimation of unknown parameters has been done through formulation of an optimization problem in Simulink to minimize the error between measured and simulated temperatures of the office with respect to the upper and lower bounds of these parameters, as shown in (18). The monitored data
(all known inputs and disturbances as explained in section 3) for 5 days during a heating period (from 01.12.2014 to 05.12.2014) is used for the estimation of parameters. The final results of the parameter estimation are listed in Table 1.

\[
\min S = \Sigma (T_{sim} - T_{mes})^2 \\
\text{subj. to} \\
\text{lower bounds} \leq \text{estimated parameters} \leq \text{upper bound}
\]  \hspace{1cm} (18)

Table 1. Results of parameter estimation.

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</thead>
<tbody>
<tr>
<td>(R_{i,12})</td>
<td>6.804</td>
<td>(R_{12})</td>
<td>6.598</td>
<td>(C_{r1})</td>
<td>0.691</td>
<td>(\tau_{\text{win}})</td>
<td>0.889</td>
</tr>
<tr>
<td>(R_{i,13})</td>
<td>6.804</td>
<td>(R_{13})</td>
<td>6.598</td>
<td>(C_{w12})</td>
<td>2.901</td>
<td>(\alpha_{\text{wall}})</td>
<td>0.870</td>
</tr>
<tr>
<td>(R_{i,14})</td>
<td>9.071</td>
<td>(R_{14})</td>
<td>8.797</td>
<td>(C_{w13})</td>
<td>2.901</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_{i,15})</td>
<td>3.828</td>
<td>(R_{15})</td>
<td>16.387</td>
<td>(C_{w14})</td>
<td>2.176</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_{i,16})</td>
<td>11.658</td>
<td>(R_{16})</td>
<td>215.582</td>
<td>(C_{w15})</td>
<td>84.418</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_{i,17})</td>
<td>4.396</td>
<td>(R_{17})</td>
<td>92.150</td>
<td>(C_{w16})</td>
<td>3.293</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_{o,16})</td>
<td>3.264</td>
<td>(C_{w1})</td>
<td>23.313</td>
<td>(C_{w17})</td>
<td>23.313</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_{o,17})</td>
<td>1.854</td>
<td></td>
<td></td>
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</tr>
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</table>

In order to validate the estimated parameters shown in Table 1, the thermal behavior of the same office has been simulated with respect to the estimated parameters. This time, however, different monitored data again during another heating period (11.02.2015 to 15.02.2015) have been used. Simulation results from the validation, as depicted in Figures 3 and 4, are in great agreement with the measured room temperature.

5. **Linear State-Space Model**

The mathematical model, which is developed in section 3, is non-linear on two accounts: Firstly, the forth power of temperature in (12) and (17). Secondly, the control input (\(T_{sa}\)) and one of the states of the system (\(T_{r1}\)) are multiplied by a disturbance variable (\(m_{sa}\)) as seen in (8), which also forces a non-linearity. After linearization around an equilibrium point and using Euler's discretization method, the dynamic model of the system can be simplified as shown in the following equation:

\[
x_{t+1} = Ax_t + Bu_t + Ed_t
\]  \hspace{1cm} (19)

Where x is the state vector (temperature of the room and walls), u is the control input (temperature of supply air from FVU), d is the disturbance vector (temperature of adjacent rooms, ambient temperature, solar radiation,
internal heat gain, heat from CCC and airflow from FVU) and t represents the time step. \( A, B \) and \( E \) are matrices of proper dimensions.

As the full range of variation in temperature of the room and walls is not very wide, a linearization of the model about an equilibrium point is fairly accurate and does not introduce a significant error [22], which is also proved in Fig. 3. On the other side, dealing with a linear model decreases the computation time dramatically.

According to Fig. 4, the simulation error for 95% of the data-points falls into the range of ±3.0% and ±3.5% for nonlinear and linearized models, respectively. This means the linearization results in a slightly less accurate, but acceptable model, as discussed earlier.

![Fig. 3 Measured room temperature vs. nonlinear and linearized models.](image1)

![Fig. 4 A graphical representation of distribution of errors for nonlinear and linearized models.](image2)

### 6. Design of Model Predictive Control

The linear state-space model from the previous section is used to formulate a model predictive control problem with the objective of minimizing the total energy consumption as seen in (20) and (21).

\[
F = \sum \{ \dot{m}_{sa}(t) c_{pa} [T_{sa}(t) - \frac{1}{2}(T_{r1}(t) + T_0(t))] \} \Delta t \\
\min \{ |F|_1 + \rho (|c_{lb}|_1 + |c_{ub}|_1) \} \\
subj. to \\
x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} + Ed_{t+k|t} \\
T_{lb} - c_{lb,t+k|t} \leq T_{r1,t+k|t} \leq T_{ub} + c_{ub,t+k|t}
\]

(20)  
(21)
\[ \epsilon_{lb,t+k|t} \& \epsilon_{ub,t+k|t} \geq 0 \]
\[ U_{lb,t+k|t} \leq u_{t+k|t} \leq U_{ub} \]
\[ \delta U_{lb} \leq u_{t+k|t} - u_{t+k-1|t} \leq \delta U_{ub} \]

Here \( T_{lb} \) and \( T_{ub} \) are the lower and upper comfort bounds, \( U_{lb,t+k|t} \) and \( U_{ub} \) correspond to the lower and upper limits of supply air temperature (the lower limit depends on the ambient temperature and changes with time, but the upper level is constant and equals to 34°C), \( \delta U_{lb} \) and \( \delta U_{ub} \) are the lower and upper limits of the change of supply air temperature due to the dynamics of the FVU heat exchanger.

In order to guarantee constraints satisfaction at all times (i.e. feasibility) and to penalize comfort bounds violations, soft constraints denoted by \( \epsilon_{lb} \) and \( \epsilon_{ub} \) as well as a comfort penalty factor (\( \rho \)) are considered in the formulation of MPC as well. We use the \textit{YALMIP} toolbox and \textit{Gurobi} solver to set up and solve the MPC optimization problem in \textit{Matlab}.

7. Results and Discussion

The measured room temperature, obtained from the current control system, together with the results of MPC are shown in Fig. 5. As the current controller, in contrast to our proposed MPC, does not predict the future changes in the system, it makes control decisions shortly after the system faces new changes. This results in an overreaction of the control system (i.e. overshooting) and waste of energy. In addition, it is not able to satisfy comfort bounds at all times as it becomes evident from this figure.

![Fig. 5 Results of the current control system and MPC. Blue dotted lines are lower and upper comfort bounds (21.5°C and 25°C) during the working hours (7:00 AM to 7:00 PM).](image)

Cumulative energy consumptions during the estimation and validation periods are illustrated in Fig. 6. We consider \( N = 9 \) hours as the prediction horizon for our model predictive controller. Larger values might result in less energy consumption, but increase the computational time needed for solving the optimization problem. Smaller values, on the other hand, do not provide enough time for the controller to adjust control input and consequently it might lead to a violation of comfort constraints.

From these figures it can be seen that the cumulative energy consumption for the estimation period (01-05 December 2014) falls from
57.25 to 32.76 kWh (i.e. around 43% reduction in energy consumption) as a result of implementing a model predictive controller instead of the already-existing control system. Similarly, energy consumption during the validation period decreases from 5.46 to 3.76 kWh (i.e. around 31% reduction) after implementation of MPC.

Another major conclusion can be drawn from Fig. 6: MPC decreases the energy consumption during the estimation period much more than during the validation period. The reason lies in the fact that the heat load of the building through the estimation period becomes larger due to the lower ambient temperature and solar radiation (see Fig. 7). In general, as the energy demand of a zone increases, the control system must deal with larger amounts of energy flows. In such cases, energy loss in conventional control systems becomes noticeable and a dramatic improvement can be achieved by using optimal control techniques such as MPC.

8. Conclusions

In summary, we have introduced a model predictive controller for an office building. The advantage of MPC over a conventional controller is prediction of disturbance load to the building, which is obtained from weather forecast and occupancy schedules of the building. This results in optimal control inputs and less energy consumption compared to the conventional controllers. Finally, by replacing the current controller with a
model predictive controller, we achieved about 43% and 31% reduction in the overall energy use during the estimation and validation periods, respectively, while the comfort parameters were also kept within an acceptable range. In this paper, the prediction horizon of MPC is considered to be 9 hours and the prediction of disturbances is assumed to be perfect and without any uncertainties.

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