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# Performance evaluation for the optimal design of chiller plants concerning uncertainty and reliability

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## Abstract

*Conventional design of chiller plant is typically based on the peak cooling loads of buildings, while the cooling load reaches its peak level for only a small proportion of time in a year. This results in the oversizing of chiller plant and thus causes significant energy wastes. In this paper, a robust optimal design based on performance evaluation is proposed to optimize the design of chiller plant concerning impacts of uncertainty in the design input data and the system reliability in operation. Monte Carlo simulation is used to generate the cooling load distribution and Markov method is used to obtain the probability distribution of system state. A case study of a building in Hong Kong is conducted to demonstrate the design process and validate the robust optimal design method. Comparisons are made among conventional design, uncertainty-based optimal design and robust optimal design. The results show that the system could operate at a relatively high efficiency and the minimum annual total cost (including annual operational cost, annual capital cost and annual availability risk cost) could be achieved under various possible cooling load conditions considering the uncertainties and system reliability.*

**Keywords:** *performance evaluation; optimal design; chiller plant; uncertainty; reliability*

## 1. Introduction

The building sector is the largest energy consumer in most countries and regions worldwide, especially in the metropolis such as Hong Kong [1]. In commercial buildings, about 40-60% of the total electricity consumption is consumed by the heating, ventilation and air-conditioning (HVAC) systems [2]. It is found that a significant saving of HVAC system can be achieved by optimal design and energy efficient operation [3].

The sizing and selection of chiller plant plays the most important role in determining the overall energy performance of the HVAC system. Conventional design of chiller plant is usually based on sizing the components individually to meet a peak duty of a nominal operating point [4]. Due to the inevitable uncertainty of weather data, indoor occupants and internal heat gain, designers tend to select a much larger system capacity than the peak duty (e.g., multiply a safety factor) for safety [5]. This may result in significant oversizing of chiller plant and thus cause a large amount of energy wastes. Some measures, such as using a detailed simulation method, statistic and realistic weather data and model calibration, have been recommended to reduce the oversizing problems to a certain degree caused by uncertainties [6]. Different from previous design methods that only address the peak cooling load on the design day, few studies also has taken part-load conditions into account in order to achieve a high efficiency in most of operating time throughout the cooling seasons [7]. Nevertheless, considering that conventional design is based on the predefined conditions and subject to a deterministic model-based simulation, chiller plants are very likely to operate at a low efficiency when the actual conditions are different from predefined conditions [8].

In this paper, a robust optimal design based on performance evaluation is proposed to optimize the design of chiller plant concerning impacts of uncertainty in the design input data and the system reliability in operation. Monte Carlo simulation is used to generate the cooling load distribution and Markov method is used to obtain the probability distribution of system state. It can ensure that the selected chiller plant could operate at high efficiency and the minimum total cost could be achieved considering the uncertainties of design parameters and reliability of the systems. Monte Carlo simulation is used for obtaining the accurate cooling load distribution and Markov method is used to obtain the steady probability distribution of each state of the system considering the reliability. In order to achieve the minimum total cost, trials of simulations on different design cooling capacity are conducted to obtain the optimum chiller plant. The concept of robust optimal design and the procedure of implementing this method are introduced. A case study on the application of this method for the chiller plant in Hong Kong is also presented.

## 2. Robust optimal design method of chiller plant

### 2.1 Concept of robust optimal design

The objective of robust optimal design is to achieve a cost-benefit design option which provides the system with the capability to operate at relatively high efficiency at various possible conditions considering the uncertainties of input parameters and system reliability in design and operation. Fundamental difference between the robust optimal method and other methods can be illustrated in Fig.1 [9]. Conventional optimal design in HVAC field guarantees the optimization over predefined conditions (without considering the uncertainties and reliability). It can be seen that the conventional method determines the HVAC system without quantitative uncertainty and reliability analysis. Uncertainty-based method determines the size of the systems or investigates the building performance considering uncertainties in design. Reliability-based method ensures the system capability by minimizing the effect of sources of design parameters or process variables, which is rarely studied in HVAC field. Robust optimal design method concerns quantitative uncertainty and reliability analysis as well as quantitative performance optimization simultaneously.

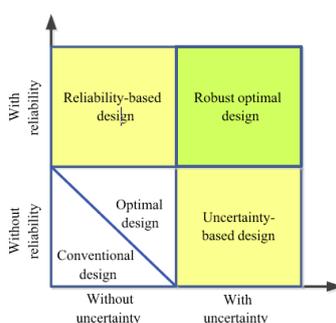


Fig.1 Illustration of different design methods [9]

### 2.2 Implementation of robust optimal design of chiller plant

The procedure of the proposed chiller plant design method can be divided into three steps as follows:

**Step 1: Obtain the cooling load distribution considering the uncertainty of inputs.** In order to generate the cooling load distribution considering uncertainties and then obtain the design flow, Monte Carlo simulation is employed. In this study, the uncertainties of the input parameters are computed by Matlab. Combining the output uncertainties from Matlab, the TRNSYS building model is used to generate the building cooling load distribution considering the uncertainties based on the determined simulation

number. After conducting the required trials of Monte Carlo simulations, the cooling load distribution involving uncertainties is determined. In this paper, about 780 times of Monte Carlo simulations are used to generate the cooling load distribution [10]. Table 1 shows an example of the settings of uncertainties of the variables.

Table 1. Settings of stochastic input parameters

Parameters	Distributions
Outdoor temperature (°C)	$N(0,1)$
Relative Humidity (%)	$N(0,1.35)$
Number of Occupants	$T(0.3,1.2,0.9)$
Infiltration rate (m <sup>3</sup> /s)	$U(2.7, 3.3)$
Equipment rejection load (kW)	$U(376, 464)$

*Remarks:*  
 $N(\mu, \sigma)$  represents normal distribution with mean value  $\mu$  and standard deviation  $\sigma$ .  $U(a, b)$  represents uniform distribution between  $a$  and  $b$ .  
 $T(a, b, c)$  represents triangular distribution with lower limit  $a$ , upper limit  $b$  and mode  $c$ .

**Step 2: Determine the range of design cooling capacity.** To determine the range of design cooling capacity, it is essential to obtain the design capacities with numbers of hours when the cooling demand cannot be met (marked as unmet hours). Based on the cooling load distribution, the “max” value represents the maximum value among all the simulation trials. Commonly, the design capacity is based on that the number of unmet hours should be no more than 50. In this study, the design cooling capacities are assumed to be 0~10 unmet hours (i.e., the interval is 1 unmet hour) and 10~50 unmet hours (i.e., the interval is 10 unmet hours).

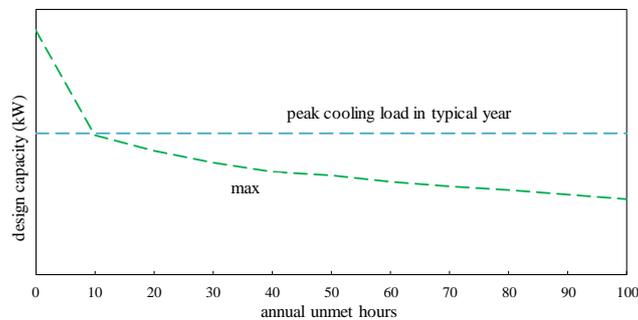


Fig.2 Design capacity vs. annual unmet hours

**Step 3: Determine the probability distribution of system state.** Markov method is used in this study because of for its wide application in reliability analysis of multi-state system. It is assumed that the state probabilities at future instant do not depend on the states occupied in the past considering the present state of the process. The chiller plant is comprised of  $n$  chillers and it is assumed that the chiller has two states only: normal (0) and failure (1). Totally the plant has  $n$  states (i.e., state 0 symbolizes that no chiller fail and state  $n$  symbolizes that all the chillers fail. From state 0 to state  $n$ , the failure rate  $\lambda$  is used to represent the probability from one state to another. From state  $n$  to state 0, the repair rate  $\mu$  is used to represent the probability from one state to another. The transition probability is determined by a state transition density matrix  $A$  (Equation (1) & (2)), which only involves the repair rate and failure rate of chillers. It can be deduced from the initial state by Equation (3) & (4). When the time approaches to infinity,  $P(\infty)$  will keep stable (Equation (5)). Then the steady state probabilities can be obtained by solving the linear algebraic equations (Equation (6) & (7)).

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} & \dots & a_{0n} \\ a_{10} & a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n0} & a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \quad (1)$$

$$a_{ij} = \begin{cases} C_n^i \cdot C_{n-i}^{j-i} \lambda & , \quad i < j \\ 1 - a_{i0} - \dots - a_{i(j-1)} - a_{i(j+1)} - \dots - a_{in} & , \quad i = j \\ C_n^i \cdot C_{n-i}^{i-j} \mu & , \quad i > j \end{cases} \quad (2)$$

$$P(0) = [1, 0, 0, \dots, 0]^n \quad (3)$$

$$P(n) = P(n-1)A = P(0)A^n \quad (4)$$

$$P(\infty) = \lim_{n \rightarrow \infty} P(n) = \lim_{n \rightarrow \infty} P(0)A^n \quad (5)$$

$$P(\infty) = P(\infty-1)A = P(\infty)A \quad (6)$$

$$\begin{cases} p(0) = a_{00}p(0) + a_{10}p(1) + \dots + a_{n0}p(n) \\ p(1) = a_{01}p(0) + a_{11}p(1) + \dots + a_{n1}p(n) \\ \vdots \\ p(n) = a_{0n}p(0) + a_{1n}p(1) + \dots + a_{nn}p(n) \\ \sum_{i=0}^n p(i) = 1 \end{cases} \quad (7)$$

**Step 4: Optimize the chiller number/nominal capacity to achieve minimum total cost under each design cooling capacity.** The annual total cost  $TC_n$  contains annual capital cost  $CC_n$ , annual operation cost  $OC_n$  and annual availability risk cost  $RC_n$ . Fig.3 visually illustrates the concepts of cost related reliability analysis. It is well-known that large system capacity means well system reliability. The capital cost normally increases with the system capacity. Under the optimal configuration of chiller plant, the operation cost may change slightly as the system capacity increases. On the other hand, the availability risk cost decreases as the system total capacity increases. The total cost is comprised of the capital cost, operation cost and availability risk cost, as shown in Equation (8). According to Fig.3, there should be a comprised system capacity to achieve the minimum total cost, at which a comprised level of reliability is achieved [11].

$$TC_n = CC_n + OC_n + RC_n \quad (8)$$

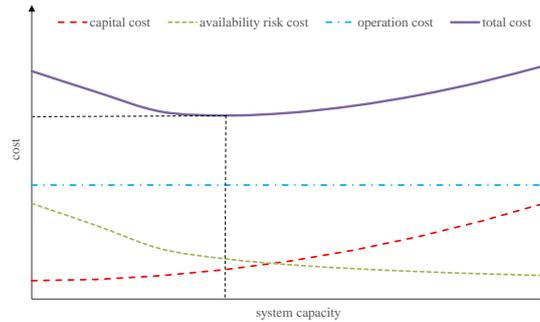


Fig.3 Total cost as a function of system capacity

The energy performance of chiller, usually evaluated by COP (coefficient of performance), is strongly dependent on the PLR of chiller, as shown in Equation (9):

$$COP = C_0 + C_1 \cdot PLR + C_2 \cdot PLR^2 + C_3 \cdot PLR^3 \quad (9)$$

where,  $C_0$ - $C_3$  are the correlation coefficients that can be identified from chiller catalogues or field measurement data. PLR is the part-load ratio. For engineering application, PLR can be simply defined as the ratio of the required cooling load ( $CL_R$ ) to the available cooling capacity ( $CL_A$ ) as shown in Equation (10).

$$PLR = \frac{CL_R}{CL_A} = \frac{CL_R}{N_{operating} \cdot CL_{Nominal}} \quad (10)$$

where,  $CL_{Nominal}$  is the nominal cooling capacity of each chiller, and  $N_{operating}$  is the number of operating chillers. This equation is based on the assumption that identical chillers are used in the same building, which is very common in practice. It clearly shows that the PLR is not only determined by the actually cooling load but also determined by the number of operating chillers and nominal capacity. Fig. 4 shows the relationship between the number of chillers and operating COP. It can be seen that the operating COP increases when the number of chillers increases to certain value and it reduces when the number of chillers increases further. Because when the number of chillers increases, the PLR increases sharply and thus the operating COP also increases. When the number of chillers increases further, the PLR does not increase obviously and the rated COP of chillers reduces sharply, and the operating COP decreases.

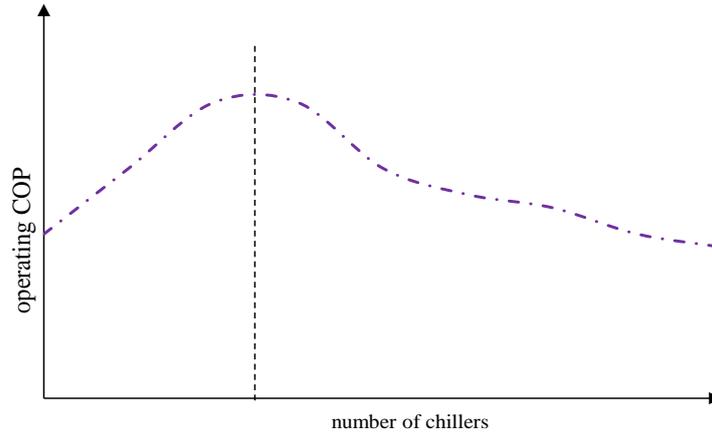


Fig. 4 Number of chillers vs. operating COP

At the same time, the operation cost, capital cost and availability risk cost can be obtained. After conducting the trials under each design cooling capacity, the options which have the lowest total cost under each design cooling capacity can be determined. Then it is worth noticing that the option which has the minimum total cost is selected as the optimum option applied in buildings.

### 3. A case study in a building in Hong Kong

A case study is presented to demonstrate the implantation of the proposed robust optimal design method in a building in Hong Kong. According to the aforementioned procedures of the proposed method, the first step is to obtain the cooling load distribution considering the uncertainty of inputs. After conducting 780 times of Monte Carlo simulations [10], the cooling load distribution is obtained, as shown in Fig.5. The reference case is the normal cooling load distribution without considering the uncertainties. It can be seen that the cooling load distribution profile of 780 simulation trials is smoother than that of reference case because more cooling load conditions are considered.

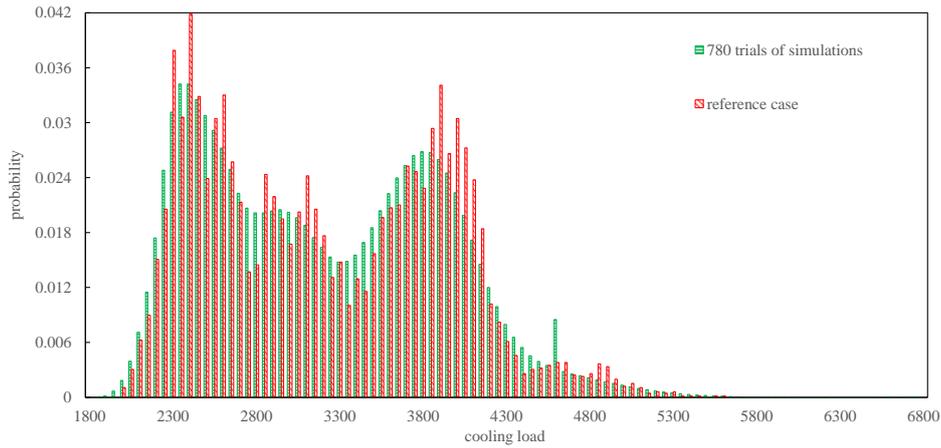


Fig.5 Distribution of cooling load considering uncertainties

The second step is to determine the range of design cooling capacity. Then it is necessary to determine the total design capacity based on annual cooling load distribution. The design capacities corresponding to different unmet hours are presented in Fig.6. Commonly, the design capacity is based on that the number of unmet hours should be no more than 50. In this study, the design cooling capacities are assumed to be 0~10 unmet hours (i.e., the interval is 1 unmet hour) and 10~50 unmet hours (i.e., the interval is 10 unmet hours), such as 5100kW (50 unmet hours) and 6600kW (0 unmet hours).

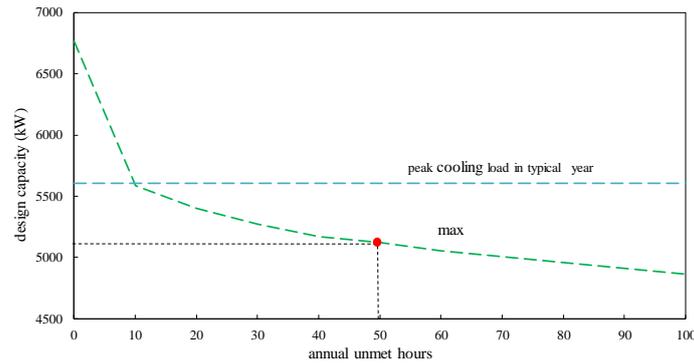


Fig.6 Design capacity vs. number of annual unmet hours

Followed is to determine the probability distribution of system state. In this paper, the chiller plant is assumed to be comprised of about 3~8 chillers and the chiller plant has 3~8 states correspondingly considering the reliability of chillers. It is assumed that the failure rate of 0.0001/hour and repair rate of 0.002/hour. Then, it is significant to obtain the probability distribution of each steady state under various chiller number. Table 2 shows the probability distribution of each steady state under different chiller number. It can be observed that the probability of state 0 decreases as the increase of chiller number. Moreover, state 0, 1, 2 and 3 account for at least 98% of the total conditions. To enhance the computation efficiency, only four conditions are considered, which include that no chiller fail, one chiller fails, two chillers fail and three chillers fail respectively.

Table 2 Probability distribution of each steady state

state	chillers			
	3	4	5	6
0	0.8906	0.8494	0.7951	0.7289
1	0.0809	0.1114	0.1517	0.1973
2	0.02	0.0247	0.0327	0.0462

3	0.0085	0.0096	0.0115	0.0146
4	0	0.0049	0.0057	0.0068
5	0	0	0.0033	0.0039
6	0	0	0	0.0024

The last step is to achieve the minimum total cost through the optimal selection of chiller number /nominal capacity. It is assumed that the penalty ratio is 2HKD/kW. Five combinations of chillers with different chiller number/nominal capacity are proposed under the design capacity 6300kW for comparison, as listed in Table 3. It can be observed that the option comprised of 6 identical chillers (1050kW) has the lowest annual total cost compared with other options and it can be selected as the best option under the design capacity 6300kW. Table 4 lists the best options under the corresponding design capacity. It can be seen that the operation cost under robust optimal design and uncertainty-based design is lower than conventional design and the availability risk cost under robust optimal design conventional design and is lower than uncertainty-based design. Compared with conventional design and uncertainty-based optimal design, the total cost under robust optimal design is reduced by about 4.5%. To achieve the minimum annual total cost, the option with 4 chillers (1425kW) can be considered as the optimum selection for the design.

Table 3 Total cost of different options under design capacity 6300kW

Options	Operation cost (k HKD)	Capital cost (k HKD)	Availability risk cost (k HKD)	Total cost (k HKD)
3*2100kW	3391	439	606	4436
4*1575kW	3327	530	398	4255
5*1260kW	3310	615	290	4215
6*1050kW	3287	694	224	4106
7*900kW	3250	770	178	4199

Total cost of different options under different design capacities

Options (kW)	$OC_n$	$CC_n$	$RC_n$	$TC_n$	
	(k HKD)	(k HKD)	(k HKD)	(k HKD)	
Robust optimal design	4*1350	3174	503	354	4032
	<b>4*1425</b>	<b>3189</b>	<b>513</b>	<b>296</b>	<b>3998</b>
	4*1500	3253	522	245	4019
	4*1575	3327	530	199	4056
	4*1260	3411	539	159	4109
Uncertainty-based optimal design	4*1275	3261	494	418	4172
Conventional design	3*2300	3501	453	233	4187

#### 4. Conclusion

This paper presents a robust optimal design method to ensure the high performance of chiller plant and achieve the minimum annual total cost concerning uncertainties of inputs and system reliability. Compared with conventional design and uncertainty-based design, the proposed robust optimal design could be employed to achieve a relatively low operation cost and have the sufficient robustness towards the uncertainties and system reliability.

To facilitate the proposed optimal design method, the capacity and type of chillers are not be considered, which could obviously improve the operating part load ratio and COP. Besides, if a larger penalty ratio is used (i.e., the penalty ratio in this study is assumed to be 2HKD/kW), the design capacity may be larger to reduce the total availability risk cost and thus the optimum result may be different.

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