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Convection Oriented Heat Exchanger Model - Identification

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Abstract

A convection oriented heat exchanger model, proposed in [1], features the ability to advect sharp moving fronts in the water stream as opposed to a traditional finite difference numerical solution scheme. The model is capable of predicting heat exchanger output temperatures subject to simultaneous dynamic input changes for a wide range of flow rates including a zero flow. This paper extends the convection oriented heat exchanger model proposed in [1] by an identification of heat exchange coefficient functions and presents a successful validation of the method by experimental data.

Keywords – heat exchanger; fan coil unit; distributed parameter system; identification

1. Introduction

A heat exchanger (HX) is - in general - a distributed parameter system described by a set of partial differential equations. Analytic solutions are often not available and therefore numerical procedures are employed. A typical numerical solution method is the method of lines (MOL) [2]. There the spatial variable is discretized into a finite number of elements, the spatial derivatives are approximated by finite differences and the resulting system of ordinary differential equations is integrated in time by standard integration solvers.

Finite difference approximations, however, introduce artificial numerical diffusion to the solution, which for convection driven processes, like the HX, results in an “information flow” in the integration scheme being faster than speed of the convection itself. This approach is unable to advect sharp moving fronts as demonstrated in [4].

Literature offers many alternative ways of modelling transient behavior of heat exchangers; many of them however treat step changes in only one input, like in [5], where only step changes in inlet water temperature are considered. Or they describe dynamic behavior around a single operating point as in [3]. Analytical solution to water mass flow step changes from a selected steady state, while all other inputs are kept constant, is treated [6].

The paper uses the HX simulation model proposed in [1], having the information propagation speed matched to the speed of convection. The model is able to predict water outlet and air outlet temperatures subject to dynamic changes in inlet water temperature, water flow rate, air inlet temperature and air flow rate combined. This paper extends [1] by a description of heat exchange coefficient model identification, which effectively sets the range of flow rates for which the predictions are accurate; zero flow included.

The paper is organized as follows. PDE description of heat exchanger is given in the first section. Then a discrete-time convection model from [1] is revisited, followed by the experimental setup description. The heat exchange coefficient fitting procedure is presented in Section 3 and validation against experimental data closes the paper.

Heat exchanger description

A water-to-air heat exchanger is for the simulation purposes considered to be a tubular single-pass cross-flow heat exchanger with the following assumptions

- A1 Heat transfer rates are constant with length, time and temperature.
- A2 There is only one phase in both fluids.
- A3 Fluids is incompressible and density and specific heat capacity of the fluids and wall are constant.
- A4 Axial heat diffusion is negligible in both fluid and wall.
- A5 Water flow is radially coherent.
- A6 Air flow has no dynamics and is ideally mixed at inlet and outlet.

The model equations are derived using the first principles, by the conservation of mass and energy. The resulting system of partial differential equations (PDE) is of a first order hyperbolic form [8], concretely

$$\begin{aligned} C_w \frac{\partial}{\partial t} T_w(t,x) + \dot{m}_w(t) c_{pw} \frac{\partial}{\partial x} T_w(t,x) &= h_{wb}(\dot{m}(t))\big(T_b(t,x) - T_w(t,x)\big) \\ C_b \frac{\partial}{\partial t} T_b(t,x) = h_{ba}(\dot{V}_a(t))\big(T_a(t,x) - T_b(t,x)\big) &+ h_{wb}(\dot{m}(t))\big(T_w(t,x) - T_b(t,x)\big), \end{aligned} \tag{1}$$

where $T_w(t,x)$ [°C] and $T_b(t,x)$ [°C] are water and body temperatures at time t and position x, $\dot{m}_w$ [kg/h] and $\dot{V}_a$ $[m^3/h]$ are water mass flow and air volumetric flow, C_w [J/K] and C_b [J/K] are heat capacities of water in the pipes and the HX body. h_{wb} [W/K] and h_{ba} [W/K] are water-to-body and body-to-air heat transfer coefficients and c_{pw} $[J/(kg\ K)]$ is the specific heat capacity of water.

Boundary conditions (BC) of the system are

$$\begin{aligned} T_w(t,0) &= T_{wi}(t) \\ T_a(t,x) &= T_{ai}(t) \quad, x \in \langle 0, L \rangle, \end{aligned} \tag{2}$$

initial conditions (IC) are

$$\begin{aligned} T_w(0,x) &= T_{w0} \\ T_b(0,x) &= T_{b0} \quad ,x \in \langle 0, L \rangle \end{aligned} \tag{3}$$

where $T_{wi}(t)$ [°C] and $T_{ai}(t)$ [°C] are inlet water and inlet air temperatures and T_{w0} [°C] and T_{b0} [°C] are initial temperature distributions in water and body, respectively.

Outputs are the outlet water temperature $T_{wo}(t) = T_w(t, L)$, the outlet air temperature $T_{ao}(t)$, and the heat exchange rate $Q_{hx}(t)$.

2. Convection based heat exchanger model

In [1] a convection heat exchanger model is derived. It uses spatial discretization of water and body temperature profiles, both into N elements ($T_w^{1:N}$, $T_b^{1:N}$). Then the water elements are progressed along the body elements by a speed of water flow and theirs ongoing heat rate is being integrated. When an element travels its own length, the heat exchanger outputs are sampled. Illustration of the process is given in Fig.Figure *1*.

Furthermore in [1] a discrete-time state-space representation of a heat exchanger model is given. It is a convenient form for implementation and it is revisited in the next section.

Discrete-time state-space HX representation

Under additional assumptions of constant inputs and fixed element temperatures in between consecutive sampling instances and by an explicit Euler discretization of time, the water and body element temperature updates can be explicitly stated by the following set of equations

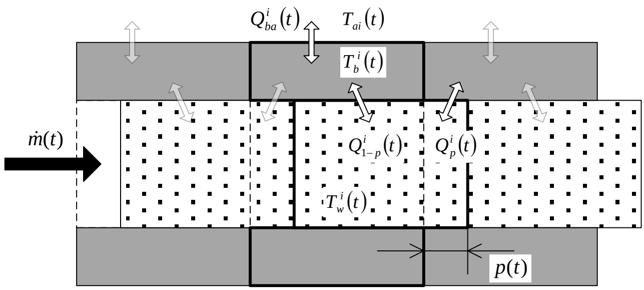


Figure 1: Water element advection and ongoing heat exchange between elements.

$$
\begin{aligned}
T_w^1(k+1) &= T_{wi}(k) \\
T_w^{i+1}(k+1) &= T_w^i(k) + \tau_s(k)\frac{N}{C_w}\left(-Q_{wb}^i\right) \qquad ,i = 1..N \\
T_b^i(k+1) &= T_b^i(k) + \tau_s(k)\frac{N}{C_b}\left(Q_{wb}^i - Q_{ba}^i\right),
\end{aligned} \tag{5}
$$

where the heat flows Q are defined as

$$
\begin{aligned}
Q_{wb}^i &= \frac{h_{wb}(\dot{m}_w(k))}{N}(T_w^i(k) - T_b^i(k)) \\
Q_{ba}^i &= \frac{h_{ba}(\dot{V}_a(k))}{N}\left(T_b^i(k) - T_{ai}(k)\right).
\end{aligned}
$$

The output set of equations is

$$
\begin{aligned}
T_{wo}(k) &= T_w^N(k) \\
T_{ao}(k) &= T_{ai}(k) + \frac{1}{\dot{V}_a c_{pa}} \sum_{i=1}^N Q_{ba}^i(k) \\
Q_{hx}(k) &= \sum_{i=1}^N Q_{ba}^i(k)\,,
\end{aligned} \tag{6}
$$

where T_{wo} [°C] and T_{ao} [°C] are the outlet water and outlet air temperatures, respectively, Q_{hx} [W] is the heat flow to air and $c_{pa}[J/(m^3K)]$ is a specific heat capacity of air.

By inspection of the system equations we reveal that, except outlet air equation, all the equations are linear. When also the air flow is held constant the outlet air equation results linear and the system description can be summarized into a familiar linear state-space form

$$
\begin{aligned}
x(k+1) &= \mathbf{A}x(k) + \mathbf{B}u(k) \\
y(k) &= \mathbf{C}x(k) + \mathbf{D}u(k).
\end{aligned} \tag{7}
$$

The state, input and output vectors are $x = [T_w^{1:N}, T_b^{1:N}]^T, u(k) = [T_{wi}, T_{ai}]^T, y(k) = [T_{wo}, T_{ao}, Q_{hx}]^T$ and the system matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are such that they mimic system description defined above.

The sampling interval is given by

$$
\tau_s(k) = \frac{m_w}{N}\,\frac{1}{\dot{m}(k)}\,, \tag{8}
$$

where $\tau_s(k)$ is a sampling interval at discrete time k and m_w [kg] is the mass of water present in the HX pipe. The sampling time fully depends on the water mass flow rate and the number of elements. When sampling interval rises above a desired level, the model governing equations are switched to a finite difference model where the sampling does not depend on the flow.

Experimental setup

A GEA UW11 2-pipe heat exchanger (Fig. 2) was installed and measured. The HX was subjected to water and air flow steps as well as to inlet water temperature steps and the water flow rate, inlet and outlet water and air temperatures, and the air flow rate were recorded.

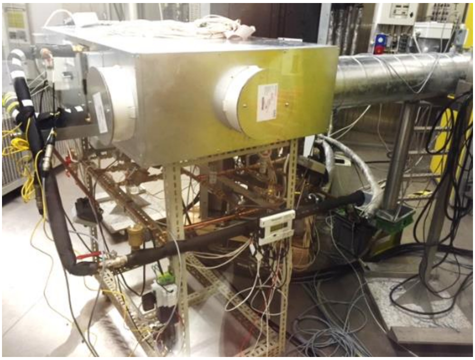
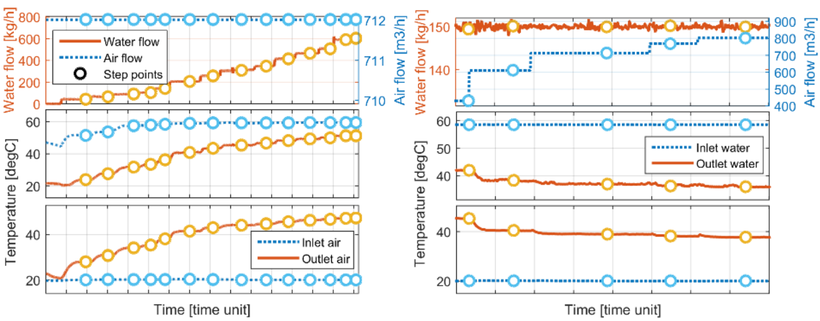


Figure 2 GEA UW11 2-pipe heat exchanger experimental setup.

The data set obtained was first filtered to remove a high frequency noise and a set of steady-state points was identified. The set of steady state points was clustered into groups, where each group represents one step of the test.

Each group was then represented by an average *step point*, averaging all points in a group. The procedure was repeated for all water flow steps and air flow steps and the outcome is shown in Fig. 3.



(a) (b)

Figure 3 Water flow steps and corresponding step points (a). Air flow steps (b).

3. Heat exchange coefficient fitting

The total heat transfer coefficient between water and air is composed of the water-to-body and body-to-air heat exchange coefficients and a thermal conductivity of the HX wall. The thermal conductivity of the wall is constant and it is for simplicity omitted as it can be included into the body-to-air heat transfer coefficient for example.

The total heat exchange coefficient (conductivity) is expressed as

$$h_{tot}(\dot{m}_w, \dot{V}_a) = \left(h_{wb}^{-1}(\dot{m}_w, \theta_w) + h_{ba}^{-1}(\dot{V}_a, \theta_a)\right)^{-1}, \qquad (9)$$

where h_{wb} [W/K] is a water-to-body heat transfer coefficient that depends on the water flow $\dot{m}_w$ and parameters θ_w. h_{ba} [W/K] is a body-to-air heat transfer coefficient that depends on the air flow $\dot{V}_a$ and parameters θ_a.

The water-to-body heat transfer coefficient is the following laminar-transient-turbulent-region function

$$h_{wb}(\dot{m}_w, \theta_w) = \begin{cases} h_w^{lam} & Re < 1800 \\ h_w^{lam} \cdot \left(\frac{3000-Re}{1200}\right) & \\ \quad + h_w^{turb}(\dot{m}_w|_{Re=3000})\left(\frac{Re-1800}{1200}\right) & 1800 < Re < 3000 \\ h_w^{turb}(\dot{m}_w) & Re > 3000, \end{cases} \qquad (10)$$

where the laminar region heat exchange coefficient, independent of water flow, is given by a constant Nusselt number [7] as

$$h_w^{lam} = \theta_{w1}\left(3.66\frac{k_w}{D}\right),$$

where the parameter θ_{w1} [m^2] represents the total inner area of the pipe, k_w [W/(mK)] is the water thermal conductivity, and D [m] is the inner diameter of the pipe. The diameter was measured to be $D = 7$ mm.

The turbulent region heat exchange coefficient is

$$h_w^{turb}(\dot{m}_w) = \theta_{w1}\left(\theta_{w2} \cdot (\dot{m}_w)^{\theta_{w3}}\right),$$

where the term $\theta_{w2} \cdot (\dot{m}_w)^{\theta_{w3}}$ mimics the dimensionless analysis fitting function $Nu = c_1 Re^{c_2} Pr^{c_3}$ [7]. The water flow is in [kg/h].

The body-to-air heat transfer coefficient $h_{ba}(\dot{V}_a, \theta_a)$ is approximated by a second order polynomial function

$$h_{ba}(\dot{V}_a, \theta_a) = \theta_{a1} + \theta_{a2}\dot{V}_a + \theta_{a3}(\dot{V}_a)^2, \qquad (11)$$

where the air flow is in [m^3/s].

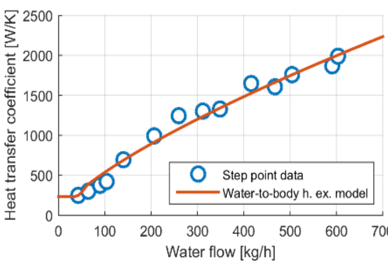
The parameter vectors $\theta_w = [\theta_{w1}, \theta_{w2}, \theta_{w3}]^T$ and $\theta_a = [\theta_{a1}, \theta_{a2}, \theta_{a3}]^T$ are found by minizing the following least squares criteria

$$\min_{\theta_a,\theta_w} \quad \left\| \left(h_{tot}^{-1}(\dot{m}_w^i, \dot{V}_a^i) - h_{wb}^{-1}(\dot{m}_w^i, \theta_w)\right)^{-1} - h_{ba}(\dot{V}_a^i, \theta_a) \right\|_2^2 + \left\| \left(h_{tot}^{-1}(\dot{m}_w^i, \dot{V}_a^i) - h_{ba}^{-1}(\dot{V}_a^i, \theta_a)\right)^{-1} - h_{wb}(\dot{m}_w^i, \theta_w) \right\|_2^2, \quad \forall\, i \in S \tag{12}$$

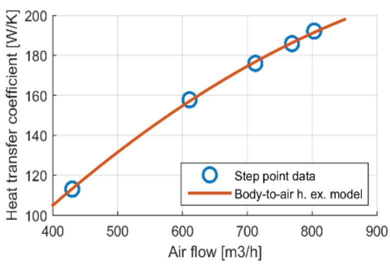
where $\dot{m}_w^i$, $\dot{V}_a^i$ are the flow rates at a step point *i* out of the group of all step points *S*. The total conductivity in the step points $h_{tot}(\dot{m}_w^i, \dot{V}_a^i)$ is obtained from (9) by finding some h_{wb}^i and h_{ba}^i such that the steady state solution to the model (7) conforms to the step point data.

Solving (12), the water-to-body and body-to-air models (10, 11) were obtained and the parameter values are $\theta_w = [0.7483, 24.6712, 0.7321]^T$, $\theta_a = [-35.1735, 1504.3, -2188.7]^T$. The fits of the model to the data set are plotted in Fig. 4. It is worth to note that even though the water-to-body and body-to-air heat transfer coefficients are independent of each other, the measured data we have only give the total heat exchange rate (9). The presented data set, where the interpolating function cross only in one point ($\dot{m}_w^i = 150$ kg/h, $\dot{V}_a^i = 712$ m^3/h), does not yield a unique solution of (12). The solution selected is the one where the parameter θ_{w1} is equal to a calculated real inner tube area. Then the laminar flow region of (1) is fully determined by first principles, (12) loses a degree of freedom and the solution is unique.

The total heat exchange coefficient of the model and its fit to the data is depicted in Fig. 5.



(a)



(b)

Figure 4 a) water-to-body conductivity function fitting, b) body-to-air heat exchange coefficient fitting.

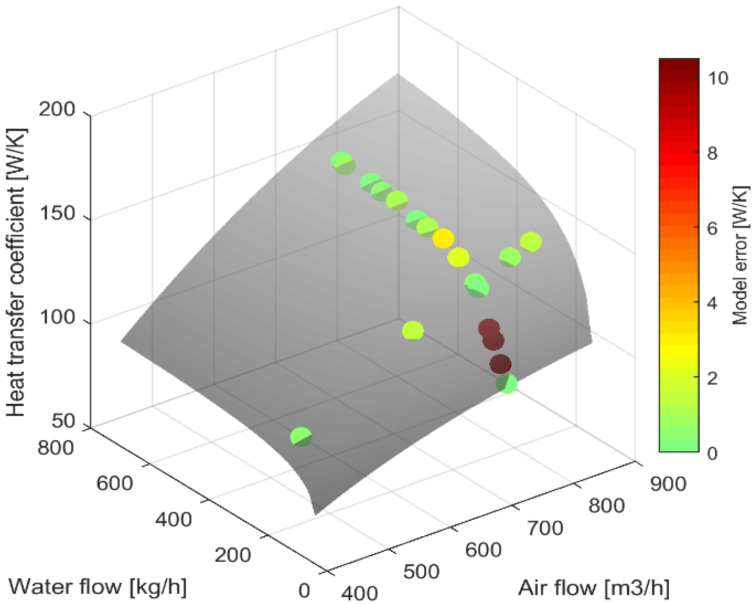


Figure 5 Heat exchange coefficient model and its error to the measured step points (dots).

The heat exchange coefficients are the only parameters to seek, when the steady state solutions are concerned. For correct dynamic response the heat capacities of water C_w and body C_b also need to be identified. C_w was calculated from the physical dimensions of the HX. C_b was first approximated from physical dimensions and then manually fitted to the measured data. Resulting values are $C_w = 5360\,\frac{\mathrm{J}}{\mathrm{K}}$ and $C_b = 8000$ J/K.

Validation

A Simulink heat exchanger model block was built using Level-2 Matlab S-function and was validated against the measured data. Additional offset of +3.5°C had to be added to the outlet air temperature output of the model in order to match the data. This static offset may have arisen by the positioning of the air temperature measurement or by the difference between the real and simulated air specific heat capacity; also the fan power may play its role. Validation plots are presented in Fig.6 and Fig. 7, mean square error measure of the outputs is given in Tab. 1.

Table 1 Convection based heat exchanger model MSE data fit

MSE	Outlet water fit	Outlet air fit
Water steps data	0.3938	0.9748
Air steps data	0.4051	0.1796

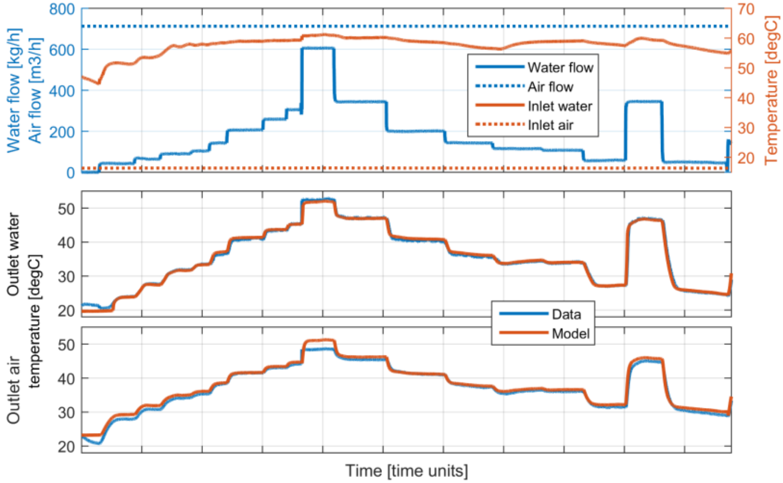


Figure 6 Experimental data fit: water mass flow steps.

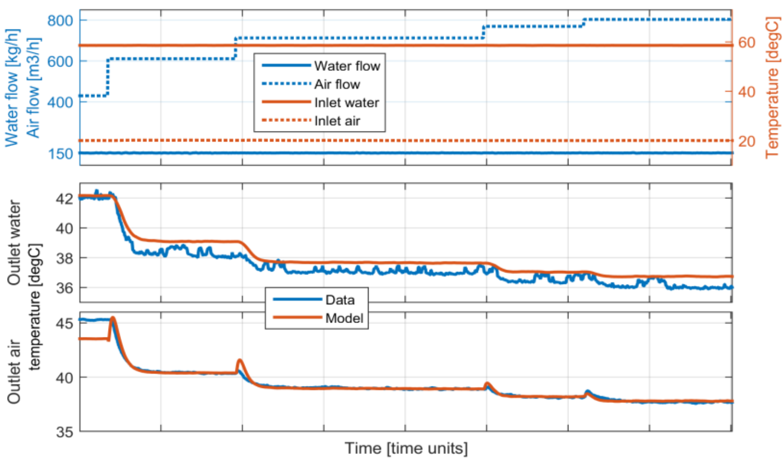


Figure 7 Experimental data fit: air flow steps.

4. Conclusion

A water-to-air heat exchanger simulation model with a focus on a true convection behavior, proposed in [1], has been implemented. The heat transfer coefficient functions were identified using model structure, known fluid flow principles and least squares optimization. Validation against the laboratory data shows model's good ability to predict outlet water temperature and, with minor inaccuracies, also the outlet air temperature.

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