Identifying a Comfortable Excitation Signal for Generating Building Models for Model Predictive Control: A Simulation Study

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Abstract

Model predictive control (MPC) uses mathematical models of the building to plan HVAC operation. One way of obtaining models is to use statistical methods to derive models from building measurement data. This data is typically collected through excitation experiments that impose temperature fluctuations on the building to reveal information on the building’s thermal dynamics. This paper investigated the trade-off between occupant comfort during excitation experiments and the quality of the resulting model. The results showed no clear tendency of higher model quality with increasing experiment strength. Implementation of models with varying accuracy in an MPC algorithm showed similar heating patterns and achieved cost savings during operation. None of the experiments violated the comfort requirements which indicate that expedient grey-box models for MPC can be obtained without annoying occupants when generating data for calibration of the model.

Keywords - Model Predictive Control, Building models, Occupancy comfort, Excitation signals.

1 Introduction

In the pursuit of increasing energy efficiency and flexibility, a number of model-based control schemes are beginning to emerge in building automation. One of the most prevailing approaches is model predictive control (MPC). An important part of MPC algorithms is the mathematical building model which is used to predict how the building response to stimuli such as changing weather conditions, internal heat loads and the operation of the heating, ventilation and air conditioning (HVAC) system [1] [2]. While these building models can be derived based on knowledge of physics, a more common approach is to derive models using statistical methods. Such methods, also referred to as system identification, are either used to calibrate grey-box models or to create black-box models to reproduce the behavior of the system being modelled [3]. Statistical approaches to building modelling rely on measurement data from the actual building to estimate (black-box) or fine-tune (grey-box) the model parameters. To obtain this data, an experiment that imposes temperature fluctuations on the actual building is carried out. The general theory is that these fluctuations must excite the system in question to a degree that data reveals the dynamic properties of the system.
As stated in [5], fitting models intuitively amounts to explaining variations in the output of the system. This task becomes increasingly difficult as the ratio between the known signal and the unknown noise decreases. This is probably why previous experiments exited buildings with temperature fluctuations far beyond normal indoor environment conditions [6] [7]. Such experiments are, however, infeasible if complex model-aided HVAC controls are ever to enter the residential building sector since the extreme indoor climate would force occupants to leave their homes during experiments. The aim of this study is to investigate whether suitable models for MPC can be obtained through low thermal comfort-impacting experiments.

2 Methods

The following sections present the methodology used to generate and evaluate results as well as important assumptions regarding model structure, input design and the presence of noise. Overall, the study is based on the co-simulation principle. The actual building is modelled in EnergyPlus [8] while a MATLAB program handles the MPC operation of the heater in the EnergyPlus model. The two programs are coupled with the Building Controls Virtual Test Bed [9].

2.1 Model structure

The model used in this study can be categorised as grey-box models. Grey-box models are characterized by having a predefined structure of physically meaningful parameters, such as the U-value of the building envelope or the g-value of the windows. These parameters are coupled with the principles of thermal dynamics to derive differential equations that describe the temperature conditions in the building, and thus how the building responds to the operation of HVAC-equipment, occupants and weather conditions. The model structure used in this study has two lumped capacities: one for the room air and one for the building construction. The model takes solar heat gains, outdoor temperature and heating from the HVAC system as input. Fig. 1 depicts the used model structure, which is a modification of the model presented in [10].

![Model structure and coefficient nomenclature](image)

**Inputs**
- \( T_{\text{ext}} \): Outdoor air temperature.
- \( Q_{\text{sun}} \): Solar heat gains.
- \( Q_{\text{heat}} \): Thermal energy from HVAC system.

**Temperature nodes**
- \( T_m \): Temperature of construction mass
- \( T_s \): Surface temperature
- \( T_a \): Room air temperature

**Parameters** (HTF = Heat Transfer Coefficient)
- \( H_{\text{ea}} \): HTF from room air to ambient air
- \( H_{\text{sa}} \): HTF from surface to room air
- \( H_{\text{ms}} \): HTF from construction mass to surface
- \( H_{\text{em}} \): HTF from ambient air to envelope mass
- \( C_a \): Thermal capacity of room air
- \( C_m \): Thermal capacity of construction mass

Fig. 1 Model structure and coefficient nomenclature.
With the model structure being fixed, the task of the system identification process is to determine the values of the coefficients in the model.

2.2 Artificial noise generation

Excitation experiments are carried out to lower the impact of noise on the system identification process. This study would therefore not be meaningful unless there is some noise involved. A simple method for generating random noise in an expected occupancy-related heat load profile in a building is therefore developed. The noise is generated in two steps:

- The times at which occupants arrive or leave home are randomized using the uniform distribution. Uncertainty varies depending on time-of-day with occupant arrival in afternoons being characterized by the highest uncertainty.
- Random fluctuations are added to emulate the opening of windows and use of electronic equipment. The fluctuations are created using a combination of a random walk and a moving average filter.

Fig. 2 shows an example of how the noise model affects a static occupancy profile. It is clear how the developed noise model retains some of the occupancy patterns that is expected to occur in residential buildings, but alters both the duration and times at which heat loads occur.

![Figure 2](image_url)

Fig. 2 Top: Expected heat load based on 24-hour static schedules. Bottom: Actual heat load, i.e. noise added to the static profile using the developed noise generator.

In this study, a total of five different noise sequences were generated and used in simulations to ensure that conclusions are not based on mere coincidence generated with the noise model. The actual heat load (Fig. 2, bottom) is the one used in the EnergyPlus simulation to represent the actual heat load in the building.
2.3 Experimental Design

The design of the excitation sequence is based on the methodology for designing common input signals in multivariable systems presented by Gaikwad and Rivera [11]. Buildings are good examples of such systems since room air and furniture are characterized by relatively low time constants while the buildings thermal mass can have a time constant spanning over several days. The type of input sequence chosen for this study is the pseudo random binary signal (PRBS). The PRBS signal has advantages that makes it a widely used for excitation experiments. First of all, since PRBS signals are deterministic, they can be designed specifically to fit the system to be identified in terms of the frequency content [11]. Furthermore, PRBS signals are persistently exciting which means that they excite the system on many different frequencies [4]. Because of this, even the properties of complex systems containing several different time constants can be identified. The user-specified parameters of the signal design process are shown in Table 1. For the remaining steps of the signal design process, see the original paper [11].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )-value: Determines signal high-frequency content.</td>
<td>0.25 [-]</td>
</tr>
<tr>
<td>( \beta )-value: Determines signal low-frequency content.</td>
<td>3 [-]</td>
</tr>
<tr>
<td>Sampling time (simulation timestep)</td>
<td>60 [s]</td>
</tr>
<tr>
<td>Lowest (fastest) time constant</td>
<td>6 [min]</td>
</tr>
<tr>
<td>Highest (slowest) time constant</td>
<td>160 [h]</td>
</tr>
</tbody>
</table>

Table 1. Design values used in the design of the input signals

In the following, the term experiment strength will refer to the size of heating fluctuations occurring during the experiment. Four different input signals with identical mean heat loads are generated. The upper and lower bound of each excitation signal and their respective signal-to-noise ratio are shown in Table 2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating load bounds</td>
<td>90 – 150</td>
<td>60 – 180</td>
<td>30 – 210</td>
<td>0 – 240</td>
</tr>
<tr>
<td>Signal-to-noise ratio [dB]</td>
<td>3.4</td>
<td>7.7</td>
<td>10.8</td>
<td>13.2</td>
</tr>
</tbody>
</table>

Table 2. Experiment designs. Mean value of all experiments is 120 W. SNR-values are calculated using the normalized signal (PRBS+Expected occupancy) and the noise (Expected occupancy-Actual occupancy).

2.4 System Identification Methodology

The process of estimating the parameters of the grey-box model is carried out using the System Identification Toolbox in MATLAB. Fig. 3 gives an overview of the methodology to generate data and identify models used in this study.
The method uses one subset of the data to train several models, while the other is used to validate model performance and find the best model. This approach is often referred to as hold-out cross-validation. In this study, five models are identified with each experiment. Both data subsets are affected by the presented noise model.

2.5 Case Building

The analysis is carried out on a dorm apartment built according to the Danish low-energy 2015 building standard. The apartment has a 2.55 m² south facing window with a U-value of 1.1 W/(m²K). One façade is facing the outside, while all other room boundaries are considered adiabatic. The heat source is an electric radiator.

As seen in Table 3, the apartment is characterized by heavy construction elements, most of which consist of concrete. The EnergyPlus model was created by Knudsen in [12].

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness [m]</th>
<th>Resistance [m²K/W]</th>
<th>Capacity [kJ/(m3K)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>External wall</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concrete (ext.)</td>
<td>0.100</td>
<td>R=0.09</td>
<td>c=736</td>
</tr>
<tr>
<td>insulation</td>
<td>0.250</td>
<td>R=6.76</td>
<td>c= 52</td>
</tr>
<tr>
<td>concrete (int.)</td>
<td>0.200</td>
<td>R=0.18</td>
<td>c=736</td>
</tr>
<tr>
<td>Internal wall</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concrete</td>
<td>0.180</td>
<td>R=0.16</td>
<td>c=736</td>
</tr>
<tr>
<td>Ceiling/Floor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wood floor</td>
<td>0.025</td>
<td>R=0.17</td>
<td>c=991</td>
</tr>
<tr>
<td>air space</td>
<td>0.050</td>
<td>R=0.10</td>
<td>c=736</td>
</tr>
<tr>
<td>concrete</td>
<td>0.220</td>
<td>R=0.20</td>
<td></td>
</tr>
</tbody>
</table>
3 Results and Discussion

This section presents the results from the model fitting process using the different PRBS signals and how they affect thermal comfort, model quality, and model effectiveness in relation to MPC.

3.1 Occupant Comfort During Experiments

For simplicity, this section only evaluates the impact on comfort of the low-strength PRBS1 experiment and the high-strength PRBS4 experiment. Fig. 4 shows the amplitude of the excitation signals and their impact on room air and operative temperatures, respectively.

Fig. 4 Comparison of low- and high-strength experiments. Top: PRBS1, Bottom: PRBS4.

Fig. 4 shows how both experiments keep the temperature levels within the typical thermal comfort boundaries used in buildings (20-26 °C). It is thus not the temperature itself that may give rise to occupant discomfort but rather the rate of change of the temperature as the heating power fluctuates.

Several studies have been carried out to determine the impact of transient thermal conditions in the indoor environment, but with contradicting conclusions [13]. ASHRAE requirements for the maximum allowable rate of change of operative temperatures [14] are used in this study, see Table 4. The 15 minute-requirement is considered to be the strictest of the requirements during PRBS experiments. This is because the PRBS signal essentially produces series of step responses with short but high temperature rate of change. Inspection of the simulation data, however, revealed that the high-strength experiment only comes close to violating the 15-minute requirement with a maximum rate-of-change of 0.94 °C.
Table 4. ASHRAE requirements on thermal drifts and ramps compared to experiment data.

<table>
<thead>
<tr>
<th>Duration of temperature increase</th>
<th>0.25 h</th>
<th>0.5 h</th>
<th>1.0 h</th>
<th>2.0 h</th>
<th>4.0 h</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASHRAE limits [K]</td>
<td>1.10</td>
<td>1.70</td>
<td>2.20</td>
<td>2.80</td>
<td>3.30</td>
</tr>
<tr>
<td>PRBS1 max rate-of-change [K]</td>
<td>0.39</td>
<td>0.57</td>
<td>0.68</td>
<td>0.80</td>
<td>0.88</td>
</tr>
<tr>
<td>PRBS4 max rate-of-change [K]</td>
<td>0.94</td>
<td>1.24</td>
<td>1.4</td>
<td>1.49</td>
<td>1.51</td>
</tr>
</tbody>
</table>

As mentioned in section 2.5, the dormitory apartment consists of heavy construction elements, which effectively dampens the temperature fluctuations. The same experiment may impact light-weight buildings more severely.

The results presented here should be interpreted in light of the assumptions that typically apply to the use of building performance simulation programs: The source of heat is modelled as a fully convective source of heat, and the heat is assumed to be distributed evenly throughout the whole apartment. In actual buildings both the radiative and convective contributions from radiators will affect the near vicinity more than more distant parts of the room. Whether this effect is enough to cause discomfort is not treated in this study.

3.2 Model Quality

Model quality is evaluated through the commonly used normalized root mean square error (NRMSE) goodness-of-fit metric. Prior to the analysis two new terms are introduced; the realistic fit and benchmark. Fig. 5 clarifies the terminology and can be seen as an extension to the methodology depicted on Fig. 3.

![Methodology of assessing model quality](image)

The realistic fit is the only of the two measures of model quality that can be obtained in real-world applications. The data used to calculate the benchmark fit does not stem from the experiments simulated, but a noise-free simulation of the EnergyPlus building using several different excitation signals as well as periods of constant heat load. The dataset were designed to expose the models to both high-frequency and low-frequency signals to thoroughly test the models.

Six different occupancy profiles were used in the simulation. All of them are based on the same expected occupancy profile, but five of them have been altered by the noise model (see section 2.2). For each occupancy profile, four different experiments were
carried out and used to identify models. The NRMSE-fits of the resulting models on experiment and benchmark data respectively are shown on Fig. 6.

![Comparison of model accuracy on different noise realizations.](image)

**Fig. 6** Comparison of model accuracy on different noise realizations. Prediction horizon: infinite

The realistic fits show a clear tendency of model quality improvements as the experiment strength is increased. The benchmark fits, which are much better estimates of model quality, showed that only in a few cases the increased experiment strength actually provided better circumstances for deriving models. During identification using one of the occupancy profiles the benchmark fits were even seen to decrease with experiment strength – despite the realistic fit-values indicating the opposite. The decline in benchmark fit is most likely a coincidence but the result is still highly relevant in this discussion as it highlights the fact that using normalized fits during model validation should be carried out with caution.

The NRMSE-fit, or other normalized fit metrics, are often used as they give a dimensionless rating of model accuracy which is easily interpreted. Furthermore, it seems logical to view model residuals in the context of how large of an interval the data spans. The use of such fit metrics, however, becomes problematic when they are used as a basis for designing experiments. This is because using normalized fits essentially favours highly fluctuating experiments because of the way it is calculated [15] – a tendency clearly shown in the results presented here. This characteristic of the NRMSE-fit means that it could easily be interpreted as incentive for increasing the strength of the experiments more than needed. Based on the tendencies of model-fits seen in Fig. 6, low-strength experiments – which are less likely to introduce occupant discomfort – may be sufficient to derive suitable building models.
3.3 Model Effectiveness

In this analysis, a selection of the models estimated in section 3.2 was tested as part of an economic MPC algorithm to investigate the control scheme’s robustness to model inaccuracies. The effectiveness of a model is evaluated as the MPC’s ability to generate savings by exploiting varying electricity prices compared to a traditional PID controller, and ability to maintain room air temperature above a specified set point of 21 °C.

The Nord Pool electricity spot price [16] is used as the cost of space heating. The MPC prediction horizon is six days and the time step is one hour. A Kalman filter is used to introduce feedback in the control scheme. Danish design weather data is used in the simulation [17]. Table 5 presents the results from a 45-day simulation using three of the models generated in the last section. The MPC algorithm used is further presented in [12].

Table 5. Model performance as part of an MPC algorithm: Model fits on noise-free benchmark data.

<table>
<thead>
<tr>
<th>Realistic/Bechmark fits</th>
<th>Time below 21 °C</th>
<th>Min./Mean temp.</th>
<th>Cost reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 / 52%</td>
<td>1.55%</td>
<td>20.94 / 22.42 °C</td>
<td>7.75%</td>
</tr>
<tr>
<td>46 / 66%</td>
<td>26.40%</td>
<td>20.49 / 22.22 °C</td>
<td>9.22%</td>
</tr>
<tr>
<td>77 / 81%</td>
<td>4.87%</td>
<td>20.80 / 22.39 °C</td>
<td>8.15%</td>
</tr>
</tbody>
</table>

It is seen that the model fit value says very little about whether a model is sufficiently accurate to carry out MPC. Visual inspection of model fits during system identification showed that all of the models were able to describe the building’s dynamic behavior reasonable well, but that lower-fitting models had a tendency of slowly diverging from the measurement data. In such scenarios, state estimators such as the Kalman Filter becomes very useful as they keep track of inconsistencies between the temperatures of the model and measurements in the buildings and applies corrections. This ensures that each prediction carried out during MPC operation has a reasonable set of initial conditions which is vital for efficient model-aided control.

The results in Table 5 shows that the model with 66 % fit on benchmark data caused small but frequent violations of the prescribed temperature set point, which in term resulted in higher savings on space heating. These differences are, however, considered to be relatively low – just as the comfort violations are considered to be of too small magnitude and/or duration to be considered critical.

4 Conclusion

This simulation-based study investigated the necessity for conducting high-strength excitation experiments to derive models suited for model predictive control of heating systems in buildings. Mathematical models were identified using data from the simulated experiments. No clear tendency of better model fits with increasing experiment strength was found which suggests that the quality of measurement data was sufficient in all of the experiments. The NRMSE fit, however, was unable to indicate this, as fit values of low-strength experiments were found to be much lower than those achieved using high-strength experiments. This suggests a pitfall in system
identification that may lead to experiments being designed with unnecessarily high temperature fluctuations. All models performed satisfactory in terms of thermal comfort but due to general simplifications made in building performance simulation programs, further research on this topic is needed (e.g. physical experiments in climate chambers). Finally, simulations where low-fit models were used to perform economic MPC showed the control scheme to be robust to model inaccuracies both in terms of the achieved cost savings and in terms of maintaining the prescribed set point. Depending on how realistically the noise model developed in this study reproduced the disturbances occurring in actual buildings, these results suggest that sufficiently accurate models can be derived from relatively subtle excitation experiments. Future research could be to investigate performance of other building scenarios, and carry out similar analysis using an actual building instead of an EnergyPlus model.

References