Model Predictive Control of Space Heating and the Impact of Taxes on Demand Response: A Simulation Study

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Abstract
Energy consumption for household HVAC systems constitutes a large demand response potential if it can be made flexible. One way of doing so is through a model predictive control (MPC) scheme that minimizes energy costs by shifting consumption according to a time-varying tariff. However, many studies on price-based demand response use tariffs with little or no taxes even though they often constitute a significant share of the total electricity price. This paper investigates the impact of taxes on the MPC-driven demand response potential for space heating. Simulations were conducted as co-simulations between EnergyPlus and MATLAB coupled by the Building Control Virtual Test Bed software. An economic MPC defined in MATLAB controls an electric radiator in a one-bedroom dormitory apartment. Three electricity tariffs with different taxes were tested as input to the MPC cost function to evaluate the effect on the DR potential: a tariff without taxes, a tariff with constant taxes and a tariff with variable taxes. The results indicated that taxes in general attenuate the load-flattening potential but reduced CO₂ emissions. Constant taxes were also found to reduce both the economic incentives of the end-consumer and the usage of wind power compared to a tariff without taxes while variable taxes did the opposite.

Keywords – economic model predictive control; price-based demand response; space heating; taxes and levies;

1. Introduction
As more and more intermittent renewable energy sources are introduced into the electric system it becomes increasingly difficult to rely solely on supply-side management to ensure grid stability. Price-based demand response (DR) is a demand-side management strategy that is often considered as a promising supplement to help keep balance in the electric system. The idea is to motivate consumers to change consumption pattern through varying electricity tariffs reflecting the state of the electric grid [2-4].
In the household sector, space heating represented approximately 68% of the total household energy consumption in the European Union [5]. Space heating therefore offers a great DR potential if this consumption can be made flexible. One possible approach to accomplish this is through an economic model predictive controller (E-MPC) that utilizes the thermal capacity of the building as an energy storage to be charged in periods with low energy prices and discharged when prices are high [6-10].

1.1 Related work and main objective

Many studies on price-based DR use the electricity spot price and ignore expenses associated with transportation of electricity, taxes and levies [6,8,11-13]. Other studies do recognize the importance of including all cost components but are often conducted in countries with a small share of taxes and levies such as Switzerland [1,7,9] where they represents 5.4% of the total tariff. In other countries such as Italy, Germany and Denmark taxes and levies represents the bulk of the total tariff [14-15] and this paper investigates how this affect the DR potential.

2. Simulation method

This study is based on co-simulations between an EnergyPlus model representing the true building [16] and an E-MPC controller defined in MATLAB [17]. The software environment Building Controls Virtual Test Bed handles the co-simulations [18-19].

A series of simulations have been carried out with different electricity tariffs to investigate the effects hereof. All simulations are performed for a simulation period from January 1 to February 14 and applies Danish electricity prices and system data from 2014 [20], and standard EnergyPlus weather data for Copenhagen [21].

2.1 EnergyPlus model

The simulated test case is a one-bedroom dorm located in Aarhus, Denmark, and its geometry is seen in Fig. 1.

![Figure 1. Geometry of the EnergyPlus model](image-url)
There is an external south-facing wall with a window with a low-e glazing (U=1.1 W/(m²K), g=0.63). All other surfaces are internal and assumed to be adiabatic. Details regarding construction compositions are shown in Table 1. The dorm is equipped with a constant mechanical ventilation rate of 1.1 h⁻¹ and has an infiltration rate of 0.05 h⁻¹. The heat source is an electric radiator and the heat power $\Phi$ is optimized by the E-MPC.

Table 1. Data for constructions used in the EnergyPlus model.

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness [m]</th>
<th>Resistance [m²K/W]</th>
<th>Capacity [kJ/(m³K)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>External wall concrete (ext.) insulation concrete (int.)</td>
<td>0.100</td>
<td>R=0.09</td>
<td>c=736</td>
</tr>
<tr>
<td></td>
<td>0.250</td>
<td>R=6.76</td>
<td>c=52</td>
</tr>
<tr>
<td></td>
<td>0.200</td>
<td>R=0.18</td>
<td>c=736</td>
</tr>
<tr>
<td>Internal wall concrete</td>
<td>0.180</td>
<td>R=0.16</td>
<td>c=736</td>
</tr>
<tr>
<td>Ceiling/Floor wood floor air space concrete</td>
<td>0.025</td>
<td>R=0.17</td>
<td>c=991</td>
</tr>
<tr>
<td></td>
<td>0.050</td>
<td>R=0.10</td>
<td>c=731</td>
</tr>
<tr>
<td></td>
<td>0.220</td>
<td>R=0.20</td>
<td>c=736</td>
</tr>
</tbody>
</table>

2.2 Economic model predictive control

The control objective is to find the optimal heat sequence, $\bar{\Phi}_{OPT}$, defined as the heat sequence that minimize the linear cost function (1) subjected to various constraints (equations 1.a-1.e). The cost $J$ represents the accumulated electricity cost over a prediction horizon of 72 hours and $p_x[k]$ is the electricity tariff in the $k^{th}$ hour. Similar formulations and further details can be found in [6-10].

$$\min_{\Phi} J = \sum_{k=0}^{71} p_x[k] \cdot \Phi[k]$$

s.t. $\bar{x}[k + 1] = A\bar{x}[k] + B\Phi[k] + E\bar{d}[k]$  
$T[k] = C\bar{x}[k]$  
$\bar{x}[0] = \bar{x}_{INI}$  
$0 \leq \Phi[k] \leq 500$ W  
$21.6 \degree C \leq T[k] \leq 25.0 \degree C$  
$-1.0 \degree C / h \leq \frac{\Delta T[k]}{\Delta t} \leq 1.0 \degree C / h$
The first constraint (1.a) represents the system dynamics formulated as a linear state space system, where \( \bar{x} \) are the states of the system, \( \Phi \) is the heat input, \( \bar{d} \) are the disturbances (ambient temperature and solar irradiation) and \( T \) is the room air temperature. \( A, B, E \) and \( C \) are black-box system matrices that captures the thermal dynamics of the EnergyPlus model and are determined via system identification (N4SID) [22]. Constraint (1.b) sets the initial state, (1.c) constrain the heat input according to the physical limitations of the radiator, (1.d) constrain room temperature and (1.e) constrain the rate of change of the room temperature. The solution, \( \Phi_{OPT} \), is the optimal heat sequence over the entire horizon of 72 hours but only the first input, \( \Phi[0] \), is applied. The problem is therefore solved again in the following hour – an approach known as the receding horizon procedure.

3. Construction of tariffs

The different tariffs that have been tested as input, \( p_x \), in the objective function (1) were constructed according to the methodology described by Ulbig and Anderson [1]. However, this method is extended in this paper to also include variable taxes in one of the tested tariffs (see section 3.3).

All test-tariffs are exclusive of VAT but this has no effect on \( \Phi_{OPT} \). This is because the tariff inclusive of VAT, \( p_{x,VAT} \), is calculated from the tariff exclusive of VAT, \( p_x \), as follows:

\[
p_{x,VAT} = f_{VAT} \cdot p_x
\]

where \( f_{VAT} \) is a conversion factor (1.25 in Denmark). If we replaced \( p_x \) in (1) with the expression of \( p_{x,VAT} \) in (2) we would simply get a new objective function similar to (1). The only difference is the constant \( f_{VAT} \), which could be moved outside of the summation and therefore just scales \( J \) without changing \( \Phi_{OPT} \). Furthermore, the end-consumer also pays subscription fees but these are independent of \( \Phi \) and would therefore be added as a constant term in (1). This term would not affect \( \Phi_{OPT} \) and is therefore omitted.

3.1 Baseline tariff (“Today’s tariff”)

The baseline tariff, \( p_{BASE} \), is a constant tariff corresponding to the average tariff that Danish households paid in 2014 [15]. This tariff is comprised of four components:

\[
p_{BASE} = c_{COM} + c_{TRA} + c_{EL,TAX} + c_{PSO}
\]

where \( c_{COM} \) is the average commercial cost of electricity determined by the price on the Nordic spot market [23] plus expenses to the electricity supplier, \( c_{TRA} \) is the average cost due to transmission and distribution of electricity, \( c_{EL,TAX} \) is the average electricity tax and \( c_{PSO} \) is the average public
service obligations (PSO) levy. Table 2 shows the average values of these components in 2014 and their share of the total tariff.

Table 2. Components of the Danish electricity tariff in 2014 [15]

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost [DKK/MWh]</th>
<th>Share [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot</td>
<td>$c_{COM}$</td>
<td>352.8</td>
</tr>
<tr>
<td>Transportation</td>
<td>$c_{TRA}$</td>
<td>221.8</td>
</tr>
<tr>
<td>Tax*</td>
<td>$c_{EL,TAX}$</td>
<td>412.0</td>
</tr>
<tr>
<td>PSO</td>
<td>$c_{PSO}$</td>
<td>190.0</td>
</tr>
<tr>
<td><strong>Total (excl. VAT)</strong></td>
<td>$p_{BASE}$</td>
<td><strong>1176.6</strong></td>
</tr>
</tbody>
</table>

*The electricity tax for electric heating is lower than for other purposes.

3.2 Tariffs with constant taxes

Many studies on price-based DR apply the market spot price directly into the cost function but this is inappropriate as pointed out by Ulbig and Anderson [1] since the spot price does not represent the entire price. They therefore propose the following tariff:

$$p[k] = \frac{\text{Spot}[k]}{\overline{\text{Spot}}} \cdot c_{COM} + \frac{\text{Load}[k]}{\overline{\text{Load}}} \cdot c_{TRA} + c_{TAX}\ (4)$$

where $\text{Spot}[k]$ is the spot price in the k\text{th} hour, $\overline{\text{Spot}}$ is the average spot price, $\text{Load}[k]$ is the grid load [20] in the k\text{th} hour, $\overline{\text{Load}}$ is the average grid load and $c_{TAX}$ is taxes and levies. This way the commercial costs are scaled according to the current spot price, transportation costs are scaled according to the current grid load and taxes and levies are included as a constant term. It is important to realize that a constant electricity consumption under this tariff will result in the same yearly costs as under the constant tariff $c_{COM} + c_{TRA} + c_{TAX}$.

The studies that (to the knowledge of the authors) applies this method are all conducted for Switzerland where taxes and levies represents only 5.4% of the total tariff [1,9]. In many other countries, this percentage is significantly higher [14], e.g. 51% in Denmark (Table 2). To test the effect that taxes and levies have on the performance of the E-MPC the following tariffs are defined:

$$p_{NO\_TAX}[k] = \frac{\text{Spot}[k]}{\overline{\text{Spot}}} \cdot c_{COM} + \frac{\text{Load}[k]}{\overline{\text{Load}}} \cdot c_{TRA}\ (4.a)$$

$$p_{CON\_TAX}[k] = \frac{\text{Spot}[k]}{\overline{\text{Spot}}} \cdot c_{COM} + \frac{\text{Load}[k]}{\overline{\text{Load}}} \cdot c_{TRA} + c_{EL,TAX} + c_{PSO}\ (4.b)$$

where $p_{NO\_TAX}$ is a tariff without taxes and levies, and $p_{CON\_TAX}$ is a tariff that includes the Danish electricity tax and PSO levy. The difference in performance of the E-MPC under these two tariffs will show how taxes and levies can affect the DR potential.
3.3 Tariff with variable taxes

It seems natural to further develop (4) to also include variable taxes as follows:

\[
P_{VAR\_TAX}[k] = \frac{\text{Spot}[k]}{\bar{\text{Spot}}} \cdot c_{COM} + \frac{\text{Load}[k]}{\bar{\text{Load}}} \cdot c_{TRA} + 
\frac{\bar{\text{CO}_2}[k]}{\text{CO}_2} \cdot c_{EL\_TAX} + f_{PSO}[k] \cdot c_{PSO}
\]

where \( CO_2[k] \) is the \( CO_2 \) intensity associated with the electricity production in the \( k^{th} \) hour, \( \bar{CO}_2 \) is the average \( CO_2 \) intensity, \( f_{PSO}[k] \) is the PSO scaling factor in the \( k^{th} \) hour (to be explained). The electricity tax is thus scaled according to the \( CO_2 \) intensity which is aligned with the reasons for introducing electricity tax: to encourage a better usage of resources and to reduce pollution including \( CO_2 \) emissions [24]. Another intention of the electricity tax is to provide revenue for the state. For this purpose, it is an important property of the tariff that a constant electricity consumption results in the same yearly electricity tax as under a constant tax. The end-consumer is able to obtain a tax discount only if consumption is shifted to periods with a \( CO_2 \) intensity below average. Conversely, the end-consumer will get a price surcharge if electricity is used in periods with high \( CO_2 \) intensity.

The scaling of the PSO levy is more complicated but is also based on the intention of the levy: to cover a range of expenses such as subsidies for wind turbines, subsides for decentralized heat and power plants and research activities etc. The subsidies for wind power represented 51% of the total PSO expenses in 2014 [25] and are paid as a supplement to the spot price. They decrease as the spot price increase [26] and are therefore high in periods with low market prices combined with a high wind power production. The detailed calculation of \( f_{PSO}[k] \) is not included here but it is essentially constructed so that it scales the PSO component to be low in periods with high subsidies (see Fig. 2). This motivates the end-consumer to shift consumption towards periods with a combination of low market prices and a high production from wind turbines, thus reducing the need for subsidies.

![Figure 2. PSO Scaling factor as a function of time](image)
4. Results

Fig. 3 is a time sample of the simulation results that visualize the behaviour of the E-MPC when exposed to the different tariffs. It appears that the tariff with no taxes gives rise to the most flexible behavior based on a visual inspection of the number of periods with heat-boots that charge the thermal capacity of the building and hence increase the room temperature. The two tariffs with taxes seems to attenuate this behavior but less so for the tariff with variable taxes.

![Graphs of P_BASE, P_NO_TAX, P_CON_TAX, P_VAR_TAX with P, T, and Price as axes.]

Figure 3. Time sample of the simulation results for the tested tariffs.

Fig. 4 depicts different performance indicators of the three test tariffs compared to the baseline tariff. The bar plot to the left shows that all tariffs resulted in an increased total electricity consumption (blue bar) especially the tariff with no tax, which is in line with the tendencies in Fig. 3. All three tariffs
also increased the usage of wind power but the tariff with variable taxes outperforms the others. Furthermore, the tariff without tax increased the usage of electricity from other sources than wind turbines (red bar) and increased the total CO₂ emissions (black bar). In contrast, both tariffs with taxes reduced the usage of non-wind generated electricity and CO₂ emissions but the variable taxing scheme proved most effective. The bar plot to the right in Fig. 4 shows how the tariffs managed to shift consumption to low load periods (9 PM–6 AM) from peak load periods (8 AM–12 PM & 5 PM–7 PM) and high load periods (the remaining periods). The tariff with no tax reduced consumption in peak and high load periods with 52% and 40%, respectively, and hence contributed significantly to flatten out the overall load in the electric grid. This load-flattening is reduced for the tariffs with taxes but less so for the tariff with variable taxes.

![Bar chart showing electricity and CO₂ emissions for different tariffs.](image)

**Fig. 4** Performance indicators for the tested tariffs evaluated in percentage w.r.t. the baseline tariff (3)

Finally, Table 3 shows the economic performance of the tariffs. The baseline costs are shown in absolute values while the other tariffs are shown relative to this. The tariff without taxes obtained savings on the spot and transport components and pays, of course, no tax or PSO. This gives a total
saving of 8.9% compared to the sum of the baseline spot and transport costs. The tariff with constant taxes obtained smaller savings on the spot and transport component, and end up paying more electricity tax and PSO levies due to the increased electricity consumption. The total saving is therefore only 1.6% compared to the total baseline cost. The tariff with variable taxes obtained savings on all components and end up with a total saving of 4.9%. Although the tariff without taxes obtained the largest saving in percentage the largest saving in absolute values is obtained by the tariff with variable taxes. The economic incentives are thus significantly reduced by constant taxes but increased when they are made variable.

Table 3. Economic performance. Baseline values are absolute and the others are relative to this.

<table>
<thead>
<tr>
<th></th>
<th>( P_{\text{BASE}} ) [DKK]</th>
<th>( P_{\text{NO_TAX}} ) [DKK]</th>
<th>( P_{\text{CON_TAX}} ) [DKK]</th>
<th>( P_{\text{VAR_TAX}} ) [DKK]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Com. (spot)</td>
<td>52.4</td>
<td>-6.1</td>
<td>-4.3</td>
<td>-5.2</td>
</tr>
<tr>
<td>Trans.</td>
<td>33.0</td>
<td>-1.5</td>
<td>-0.7</td>
<td>-0.7</td>
</tr>
<tr>
<td>Tax</td>
<td>61.2</td>
<td>-1.5</td>
<td>1.4</td>
<td>-2.3</td>
</tr>
<tr>
<td>PSO</td>
<td>28.2</td>
<td>-</td>
<td>0.7</td>
<td>-0.3</td>
</tr>
<tr>
<td>Total</td>
<td>174.9</td>
<td>-7.6* (-8.9%)</td>
<td>-2.8 (-1.6%)</td>
<td>-8.6 (-4.9%)</td>
</tr>
</tbody>
</table>

*Compared to a baseline including only commercial and transportation costs.

5. Discussion and conclusion

The results presented in this study must be taken with some reservations. First of all, the simulations applied perfect forecasts of weather and electric grid data, which means that the obtained results should be considered as the performance bound. Secondly, the obtained results depended on a number of factors such as simulation period, building type and heating system, etc. Despite of these reservations, the authors expect that the following tendencies in general will hold: 1) taxes attenuate the load flattening potential but reduce CO\(_2\) emissions, 2) variable taxes perform better than constant taxes on all performance indicators, and 3) constant taxes reduces the economic incentive of the end-customer while variable taxes increases the incentive.

Whether variable taxes should be introduced or not is a political decision. It might therefore be of political interest that variable taxes reduces the revenue for the state slightly but in return the pollution measured in CO\(_2\) emissions is reduced, which is part of the intension of the tax. Furthermore, the PSO revenue is slightly reduced but in return the usage of electricity from wind turbines is significantly increased by variable taxes which would arguably lessen the need for subsidies.

Acknowledgment

The work presented in this paper was supported by The Danish Energy Agency project (no. 12019) - Virtual Power Plant for Smart Grid Ready Buildings (VPP4SGR) and Aarhus University, Denmark.
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