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Abstract—The next generation of cellular systems are expected to experience a proliferation of the number of emerging use cases alongside supporting high speed mobile broadband services. Massive Machine Type Communication (mMTC), which caters to a large number of low-data rate and low-cost devices, is such an use case. Smart utility meters, automated sensors in farms, and vehicle tracking nodes for logistics monitoring are all examples of emerging mMTC devices. Ensuring efficient mechanisms to access the wireless channel for such a massive number of densely deployed devices is the key challenge posed by mMTC applications. A framework for the analysis of the one-stage massive access protocol is proposed in this paper, which allows to model and evaluate its performance with respect to important performance metrics for mMTC services.

Index Terms—Massive random access, M2M, mMTC, IoT, 5G, one-stage RA.

I. INTRODUCTION

The fifth generation wireless systems (5G) is expected to experience a proliferation in the number of emerging use cases categorized into several broad service groups such as: enhanced Mobile Broadband (eMBB) supporting an evolution of today’s broadband traffic with an increased spectral efficiency, Ultra-Reliable Low Latency Communications (URLLC) where messages need to be transferred with high reliability and low latency, and massive Machine Type Communication (mMTC) catering to a large number of (generally) low-data rate, low-cost devices [1].

One of the main drivers behind mMTC services is the role of Machine Type Devices (MTD) as an enabler for the Internet of Things (IoT). Massive machine type communications are commonly characterized by a large number of MTDs associated with each base station [2], the possibility of asynchronous activation of a massive number of MTDs [3], low payload sizes and traffic asymmetry in which the uplink traffic dominates the downlink [4].

The random access (RA) procedure in Long Term Evolution (LTE) is not suitable for mMTC services due to the associated signaling overhead [5]–[8]. Namely, the LTE access procedure is composed of a contention stage (where the devices contend for access) and a non-contention stage (where the scheduling and the transmission of the uplink payload occurs). However, for small data payloads, the additional signalling overhead incurred by the non-contention stage is extremely sub-optimal in an mMTC context [8].

One-stage access protocols are now being considered as a low signaling overhead alternative for the random access with mMTC services. These types of cellular RA schemes has been put forward first in [9], and has formed the basis of various other similar access protocols [10]–[12]. Specifically, one-stage access denotes the case where the devices contend (i.e., perform random access) directly with their uplink payload. Upon the MTD transmission of the preamble (along with the data), the eNodeB (eNB) acknowledges the reception via an ACK/NACK message. The access itself is performed over a time and frequency resource assigned for RA. Since the access is uncoordinated, it is possible that multiple MTDs will attempt to access the same network resource, resulting in a collision. In case the eNB is not able to decode the collided transmissions, each affected MTD will re-attempt access at a later random access resource.

Ideally, one-stage access protocols should be the preferred access mechanisms for mMTC services due to its low overhead. However, their applicability depends on two main factors: (1) the data payload size, and (2) the device density. Specifically, as the data payload increases the number of required network resources increases proportionally. When collisions occur and the base station is not able to decode the transmitted information, then these resources are wasted. Therefore, for a high number of active MTDs and higher payloads, the access scheme in place in LTE could be preferable. However, for a low number of active MTDs, fewer collisions will occur making the one-stage access protocol desirable.

In this paper our goal is to provide insights for the design of RA protocols for mMTC services in a 5G setting. Specifically, we model the performance of the one-stage access protocol using stochastic geometry tools [13]. To cover the wide variation in the kinds of devices, MTDs capable of adjusting their transmission power to compensate for the path loss are considered alongside simple low-cost fixed transmit power devices. The findings from this contribution will help to identify the network conditions from a physical layer perspective under which the one-stage access protocol can sufficiently meet a desired service target.

The rest of the paper is structured as follows: Section II details the system model and assumptions, the signal and the collision model, and the evaluated performance metrics. The analytical expressions for the considered performance metrics are derived in Section III. Section IV provides numerical and simulation results, and discussions. Concluding remarks are found in Section V.
II. System Model

We consider a single cell of radius $R$ in a multi-cell network. The eNB is located at the cell center, with uniformly distributed MTDs throughout the cells as shown in Figure 1. The number and the location of the MTDs are modelled according to an independent homogeneous PPP $\Phi_u$ with intensity $\lambda_u = \pi R^2 \lambda_u$, where $\lambda_u$ is the device density per square meter. Specifically, for a given PPP, the number of points and their locations are random and respectively follow a Poisson and an uniform distributions [13]. We assume that a device randomly chooses to transmit with probability $\rho (\rho \leq 1)$ at each RA opportunity.

Once a device decides to transmit, it will randomly choose a RA preamble, i.e., a digital signature that the MTD transmits. These preambles are used by the eNB to estimate the channel, and simultaneously decode the MTDs transmitted payload. We assume that there are $M$ orthogonal pseudo-random RA preambles available for the transmitting MTDs to choose from. Information about the available RA preambles is periodically broadcast by the eNB.

Transmission with probability $\rho$ results in independent thinning of the PPP $\Phi_u$. Hence, the number of devices transmitting concurrently is again a PPP $\Phi$, of lower density $\lambda = \lambda_u \rho$ [13]. Due to the orthogonal nature of the RA request preambles, a transmitting device only experiences interference from other concurrently transmitting devices using the same preamble. As a result, the number of interfering devices is again a PPP $\Phi$ with intensity $\lambda = \frac{\lambda_u}{M}$, resulting from the thinning of $\Phi_u$.

\begin{equation}
\label{eq:channel_occupancy}
I_s = \sum_{x \in \Phi} P_T r_x^{-\alpha} g_d,
\end{equation}

where $r_x$ and $g_d$ are the random distance and the channel fading power of the devices to the eNB. Since $\Phi$ is a stationary PPP, the PDF of the distance $r_x$ is given by the Rayleigh distribution, i.e., $f_{r_x}(r) = 2\pi \lambda r \exp\left(-\pi \lambda r^2\right)$ [13], defined for $r \geq 1$. Note that, we explicitly assume $r \geq 1$ to avoid amplification of the signal for $0 < r < 1$, and the singularity at $r = 0$.

B. Collision Model

To cover the wide range of MTD types, we consider two different approaches to determine whether a RA request is successful, namely the singleton approach and the SINR approach, as detailed below.

1) Singleton Approach: Whenever an MTD accesses the channel, it selects one of the $M$ available preambles. A collision is said to occur if two or more devices transmit the same RA request preamble on the same RA opportunity. Different RA request preambles can be detected by the eNB thanks to their orthogonality. However, a collision is not detected by the eNB if the two devices transmitting the same preamble are equidistant from the eNB, resulting in a false ACK. We use the term blocking probability to denote this scenario. In the event of a collision, the MTDs will reattempt access after a random backoff time. The backoff period consists of a fixed backoff time of $b_1$ RA opportunities, followed by a random backoff time uniformly distributed in $[0, b_2]$ slots.

2) SINR Approach: Decaying of the signal power with distance is a fundamental feature of the wireless channel. The effects of channel propagation can be overcome with channel inversion power control, i.e., by transmitting with power $P_T = d^\alpha$, such that all transmissions are received with the same average received power at the eNB. However, this does not account for the power variation due to shadowing and fading. Furthermore, channel inversion may not always be possible due to the maximum transmit power constraints limiting $P_T$ to be below a certain level; and for very simple and low-cost MTDs without this capability.

An alternative approach for the success probability evaluation is through the analysis of the random signal-to-interference plus noise ratio (SINR). In this approach, all devices are assumed to transmit with a constant power $P_T$. A RA transmission can be considered successful if the SINR exceeds a given threshold $\gamma_h$. The SINR for the desired device’s signal at the eNB, $\gamma_h$, is given by $\gamma_h = \frac{S}{I_s + N_0}$, where $I_s$ is the sum interference from other devices as given by Eq. (1), and $N_0$ is the thermal noise power.

C. Performance Evaluation Metrics

The performance of the one-stage access protocol is evaluated in this contribution considering the following evaluation metrics.

1) Channel Occupancy Rate: The channel occupancy rate, $p_{ch}$, is a general network evaluation criteria indicating the number of RA opportunities resulting in successful transmission. This is related to the singleton approach, and is given as the probability that there is exactly one active node in the PPP $\Phi$. Specifically, $p_{ch} = \mathbb{P}[N = 1]$, where the Poisson distributed random number $N$ is the cardinality of the PPP $\Phi$. 

![Fig. 1: System Model showing the random distribution of the MTDs with the cell divided into multiple rings.](image)
2) **Blocking Probability:** To determine the blocking probability, we divide the cell into $K$ equi-radius circular rings as shown in Figure 1. The radius of each ring is chosen such that all nodes within the same ring arrive at the eNB with negligible time difference. The blocking probability is then given by the probability of having two or more devices attempting to simultaneously access the channel using the same preamble from the same ring.

Let $R_k$ denote the $k^{th}$ ring. The active MTDs in $R_k$ choosing the same RA preamble then forms a PPP $\Phi_k$ with density $\lambda_k = \frac{\lambda(2k-1)\pi R^2_k}{K^2}$. The blocking probability for MTDs in the $k^{th}$ ring is therefore given by $p_{\text{block},k} = \mathbb{P}[N_k \geq 2]$, where $N_k$ is the random number of nodes in the PPP $\Phi_k$.

3) **Success Probability:** With the singleton approach, the random number of concurrently transmitting MTDs (irrespective of the selected RA preamble) is $N_s$, which is in fact the cardinality of the PPP $\Phi$. A transmission attempt is said to be successful when the randomly selected RA preamble of an MTD attempting to transmit does not overlap with that of the other $N_s-1$ co-transmitting MTDs. Mathematically, this can be expressed as $p_{\text{single}} = \mathbb{P}[N = 1|N_s \geq 1]$.

On the other hand, the success probability for the SINR approach for a given target SINR $\gamma_{th}$ is given as $p_{\text{SINR}} = \mathbb{P}[\gamma_s \geq \gamma_{th}]$, where $\gamma_s$ is the desired SINR.

### III. PERFORMANCE ANALYSIS FRAMEWORK

The performance metrics of interest, as introduced in Section II-C, are derived analytically in this section.

A. **Channel Occupancy Rate and Blocking Probability**

Owing to the PPP model assumption, the channel occupancy rate and the blocking probability are derived from the Poisson distribution as

$$p_{ch} = \lambda e^{-\lambda} \quad \text{and} \quad p_{bl,k} = 1 - e^{-\lambda} - \lambda e^{-\lambda},$$

where $\lambda = \frac{\lambda_u \rho}{\alpha}$ is the density of the random PPP $\Phi$, with $\lambda_u$ being the number of contending MTDs; and $\lambda_k = \frac{\lambda(2k-1)}{K^2}$.

B. **Success Probability**

1) **Singleton Approach:** The success probability with the singleton approach is given by $p_{\text{single}} = \mathbb{P}[N = 1|N_s \geq 1]$. Using Baye’s rule, $p_{\text{single}}$ can be expressed as $p_{\text{single}} = \mathbb{P}[N = 1, N_s \geq 1]/\mathbb{P}[N_s \geq 1]$. We can evaluate $\mathbb{P}[N = 1, N_s \geq 1]$ by conditioning on $N_s$, followed by taking the expectation over its distribution; i.e., $\mathbb{P}[N = 1, N_s \geq 1] = \mathbb{E}_{N_s \geq 1} [\mathbb{P}[N = 1|N_s]]$, where $\mathbb{E}[\cdot]$ is the expectation operator.

Let us consider a transmitting MTD and $N_s-1$ potential colliding MTDs. Since the RA preambles are chosen randomly with uniform probability, the probability $\mathbb{P}[N = 1|N_s]$ is the probability that none of these $N_s-1$ MTDs chooses the RA preamble selected by the desired MTD. Hence, $\mathbb{P}[N = 1|N_s] = \left(\frac{M-1}{M}\right)^{N_s-1}$. Following the Poisson distributed random variable $N_s$, the expectation over $\mathbb{P}[N = 1|N_s]$ evaluates to

$$\mathbb{E}_{N_s \geq 1} [\mathbb{P}[N = 1|N_s]] = \sum_{n \geq 1} \left(1 - \frac{1}{M}\right)^{n-1} \frac{\lambda_n e^{-\lambda}}{n!}.$$

After some algebraic manipulations, the singleton success probability can be succinctly expressed as

$$p_{\text{single}} = \exp\left(-\lambda + \exp\left(-\lambda\right) \right) \left(1 - \frac{1}{M}\right) \left(1 - \exp\left(-\lambda\right)\right),$$

where $\lambda_t = \lambda_u \rho$ is the density of the PPP $\Phi_t$.

2) **SINR Approach:** Using the definition of the SINR $\gamma_s$, the success probability for the SINR approach $p_{\text{SINR}}$ can be re-expressed as $p_{\text{SINR}} = \mathbb{P}[\gamma_s \geq \gamma_{th}] = \mathbb{P}[S - \gamma_{th} I_s \geq \gamma_{th} N_0]$. Let us define the random variable $u \triangleq S - \gamma_{th} I_s$. Hence, $p_{\text{SINR}}$ can be written as $p_{\text{SINR}} = \mathbb{P}[u \geq \gamma_{th} N_0] = 1 - F_\rho(\gamma_{th} N_0)$, where $F_\rho(.)$ is the cumulative density function (CDF) of $\rho$.

Evaluating the CDF of $\rho$ directly requires an expression for the probability distribution function (PDF) of $S$, which is not readily obtainable. We propose to circumvent this limitation by using the relationship between the CDF and the Laplace Transform (LT), which leads to the SINR success probability being derived as [14, Eq. 19]

$$p_{\text{SINR}} = 1 - \frac{1}{2\pi \rho} \int \frac{M_u(s)}{s} \exp\left(s \gamma_{th} N_0\right) ds,$$

where $M_u(s)$ is the LT of $\mathbb{E}[\exp(-s\rho)]$. By the independence assumption between desired signal $S$ and the sum interference $I_s$, we readily obtain $M_u(s) = M_S(s) M_I(-\gamma_{th} s)$, where $M_S(s)$ and $M_I(s)$ are the LTs of $S$ and $I_s$, respectively.

Assuming Rayleigh fading channels, the desired channel power gain $S$ is exponentially distributed with mean $P_t d^{-\alpha}$. The corresponding Laplace Transform of $S$ is then $M_S(s) = (1 + s P_t d^{-\alpha})^{-1}$ [15]. On the other hand, the Laplace Transform of $I_s$ is obtained as

$$M_I(s) = \mathbb{E}[\exp(-s I_s)] = \mathop{\mathbb{E}}_{g_x} \prod_{x \in \Phi} \mathop{\mathbb{E}}_{g_x} \left[ \exp\left(-s P_T r_x^{-\alpha} g_x\right)\right] = \mathop{\mathbb{E}}_{\Phi} \prod_{x \in \Phi} \left(1 + s P_T r_x^{-\alpha} g_x\right)^{-1} = \exp\left(-2 \pi \lambda \mathcal{F}_1 \left[1, -\frac{2}{\alpha} \left(1 - s P_T d^{-\alpha}\right)\right]\right).$$

The first step in Eq. (5) follows from the independence assumption among the interference from the different sources; the second step is obtained by assuming a Rayleigh fading channel model (i.e., an exponentially distributed $g_x$); the third step is the result of invoking the probability generating functional of the PPP distribution [13]; and the final step is evaluated by carrying out the integration using the distribution of $r$ as given in Section II-A.

### IV. NUMERICAL RESULTS

We now present numerical results based on analytical expressions developed in Section III and investigate the impact of key system parameters on the performance. The simulations adopt the following parameters, unless stated otherwise: number of RA preambles $M = 50$, path loss
exponent $\alpha = 3$, target SINR of 0 dB and transmit power $P_T = 0$ dBm. The channel occupancy rate and blocking probability of the network, and transmission success probabilities for the singleton and SINR approaches are evaluated through MATLAB based Monte-Carlo simulations. The presented results provide insights into the operation of the \textit{one-stage} access protocol and its limitations with respect to the network parameters.

A. Channel Occupancy Rate and Blocking Probability

The channel occupancy rate for different transmission probabilities $\rho$ with number of contending MTDs $\lambda_u \approx [315, 630, 1600]$ is presented in Figure 2a; alongside the blocking probability for different transmission regions $k \in \{1 \ldots K\}$ with $K = 3$ for $\lambda_u = 630$ MTDs in Figure 2b. Analytical findings presented in Section III are found to closely match the simulation results. We can observe that the maximum channel occupancy rate is around 37\%, which is in-line with the well known occupancy figures of framed slotted ALOHA-like RA approaches. It is however interesting to note that a large number of transmissions results in false ACK (corresponding to the blocking probability) with high device density. This results from many devices at similar distances to the eNB having to choose the RA preamble from a fixed set. In fact, this is one of the inherent limitations of the \textit{one-stage} random access procedure. Improved access mechanisms are required to overcome such high blocking probabilities.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Simulated and Analytical Channel Occupancy Rate and Blocking Probability vs. transmission probability ($\rho$) with the singleton approach.}
\end{figure}

B. Success Probability - Singleton Approach

The success probability for an MTD attempting transmission with the singleton approach, as analysed in Section III-B, is illustrated in Figure 3. Contrary to the trends observed in Figure 2, it is always better for an MTD to transmit with a lower probability $\rho$. This is due to the success probability only accounting for the RA opportunities with transmission attempts, and hence idle slots are not accounted for. Therefore, a careful selection of the transmission probability $\rho$ is an important step for the \textit{one-stage} access mechanism, especially at massive MTD densities.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Success Probabilities vs. transmission probability $\rho$ with the singleton approach and $\lambda_u = [314, 628, 1570]$ users.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{Cumulative Success Probabilities after $n^{th}$ re-attempt for $\rho = 0.1$ with the singleton approach and $\lambda_u = [314, 628, 1570]$ users.}
\end{figure}

C. Success Probability - SINR Approach

Complementary to the singleton approach, the success probabilities with the SINR approach for a target SINR $\gamma_{th} = 0$ dB for various transmission probabilities $\rho$ and normalized device distance to the eNB with MTD density $\lambda_u = 500$ MTDs is presented in Figure 5. Since the same transmit power and the same transmission probability is considered for all devices, MTDs closer to the eNB experience a higher success probability compared to those at the cell edge in general.

Similar to the trends in Figure 3, the success rate falls with increasing transmission probability $\rho$. However, devices closer to the eNB are much less affected compared to those further away. In fact MTDs closer to the cell edge experiences a very low success rate as a result of the overwhelming interference with $\rho = 0.5$. To overcome this limitation, we investigated the impact of having a variable transmission probability in Figure 6. First we consider only a distance dependent transmission probability where $\rho \propto \frac{1}{d}$, i.e. the MTDs closer to the eNB transmit with a smaller probability compared to those further away. As a result, MTDs further away from the eNB are less likely to be swarmed with strong interference from MTDs closer to the eNB. Such a distance dependent dynamic $\rho$ is found to provide a success probability improvement of approximately 25\% over having a fixed $\rho$.

In addition, significantly further improvement to the success probability is observed when we incorporate the network information in selecting $\rho$. For example, allowing
\( \rho \propto \frac{1}{d^2} \) results in more than 300% gain for cell edge MTDs, as shown in Figure 6. It must be noted that additional signalling or autonomous \textit{neighbour awareness protocols} are required for MTDs to estimate the device density \( \lambda_u \). Furthermore, the cost of such increased success probabilities is increased delays associated with having a lower \( \rho \).

The evaluation framework presented in this paper has shown its potential in evaluating the advantages and the limitations of the \textit{one-stage} RA protocol for large scale machine type communication. Furthermore, the potential of incorporating location and network awareness into the random access protocol is summarily demonstrated. As part of the future work, we would like to investigate possible enhancements that can improve the channel occupancy rate from the current maximum of \( \sim 37\% \), and perform a comparative analysis of the \textit{one-stage} RA procedure with other RA proposals for massive random access.

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