A Virtual Inertia Control Strategy for DC Microgrids Analogized with Virtual Synchronous Machines

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Abstract—In a DC microgrid (DC-MG), the dc bus voltage is vulnerable to power fluctuation derived from the intermittent distributed energy or local loads variation. In this paper, a virtual inertia control strategy for DC-MG through bidirectional grid-connected converters (BGCs) analyzed with virtual synchronous machine (VSM) is proposed to enhance the inertia of the DC-MG, and to restrain the dc bus voltage fluctuation. The small-signal model of the BGC system is established, and the small-signal transfer function between the dc bus voltage and the dc output current of the BGC is deduced. The dynamic characteristic of the dc bus voltage with power fluctuation in the DC-MG is analyzed in detail. As a result, the dc output current of the BGC is equivalent to a disturbance, which affects the dynamic response of the dc bus voltage. For this reason, a dc output current feed-forward disturbance suppressing method for the BGC is introduced to smooth the dynamic response of the dc bus voltage. By analyzing the control system stability, the appropriate virtual inertia control parameters are selected. Finally, simulations and experiments verified the validity of the proposed control strategy.

Index Terms—DC microgrid, bidirectional grid-connected converter, power fluctuation, virtual inertia control, small-signal modeling, disturbance suppressing.

I. INTRODUCTION

DC microgrids (DC-MGs) have been developed rapidly due to the penetration of distributed generations (DGs), energy storage, and the local dc loads [1]-[3]. As the interface between the DC-MG and the utility grid, bidirectional grid-connected converters (BGCs) play a significant role in controlling the energy exchange between the DC-MG and the utility grid, maintaining the dc bus voltage stability, and improving the system efficiency [4], [5]. However, the DC-MG is a low-inertia grid dominated by power electronic converters. The frequent switching loads and intermittent DGs (e.g., PV source, wind resource) can give rise to large volatility of the dc bus voltage [6], and reduce the efficiency and stability of the DC-MG [7]. Introducing the virtual inertia control into the BGC is a promising way to increase the inertia of DC-MG, to diminish fluctuation of the dc bus voltage, and to enhance the stability of the DC-MG.

Currently, researches about the virtual inertia control of power electronic converters mainly focus on the active support to the utility grid or the AC microgrid (AC-MG). A common virtual inertia control strategy is to operate converters as virtual synchronous machines (VSMs) [8]-[19]. The rotor inertia of the synchronous machine (SM) is emulated by combining the energy storage with the converters in [8]. The concept of the synchronverter is firstly proposed in [9], [10], which is similar to the SM in mechanical and electrical characteristics by establishing the electromagnetic and mechanical equations of the SM. Due to its superior control performance, the VSM control is successfully applied to modular multilevel converters (MMCs) [11], doubly fed induction generator (DFIG)-based wind turbines [12], voltage source converter (VSC) stations [13], and energy storage systems [14]. In order to suppress power fluctuation of the VSM, J. Alipoor et al. [15] propose the bang-bang control method used on virtual rotor inertia of the VSM by adaptively changing its inertia parameters. In [16], [17], an oscillation damping method is proposed to avoid the low-frequency oscillation of the VSM. In [18], a small-signal model of the VSM is built, and its control parameters design method is also provided by analyzing the system stability and dynamic performance. In [19], the comparisons of the dynamic characteristics between the VSM control and droop control are analyzed in detail, and prove that the VSM control owns more advantages.

However, researches on the virtual inertia control for DC-MG are hardly reported. The DC-MG often adopts power-
current-based droop controller to stabilize the dc bus voltage [20]. However, there exists a trade-off between the voltage deviation and current sharing accuracy. By the low bandwidth communication, the improved droop control method can restore the dc bus voltage and enhance the current sharing accuracy [21]. DC bus signaling (DBS) is presented as a prominent decentralized coordination method for DC-MG [22]. Using the DBS approach, the different operating modes of the converters can be coordinated according to the magnitude of dc bus voltage to maintain the dc bus voltage. In [23], a soft-start voltage control strategy is proposed to improve the transient response during the initial startup of power converters. In [24], a ripple elimination method is proposed to restrain the dc bus voltage ripples. The tradeoff between the current distortion and bandwidth of dc bus voltage control of the converter is analyzed and solved in [25]. In large scale dc distribution systems with high penetration of DG units, a multi-agent distributed voltage regulation scheme is proposed to mitigate the voltage regulation challenges [26]. However, the above dc voltage regulation methods are not to discuss how to enhance the inertia of DC-MG. To this issue, an approach using super-capacitors to suppress the dc bus voltage fluctuation is presented to improve the inertia of DC-MG when the local loads or DGs suddenly change [27]. But the costs of super-capacitors are relatively high. Moreover, when the DC-MG is in the steady-state operation, these super-capacitors are idle, which causes resource waste. In [28], a virtual inertia control strategy of the wind-battery-based islanded DC-MG is presented. By adding a high-pass filter into the additional inertia control loop, these converters can keep the power balance of the DC-MG when the dc bus voltage suddenly changes. But the high-pass filter may bring in the high-frequency disturbance.

In this paper, the idea of migrating the relatively mature VSM control strategy into the BGC for improving the inertia of the DC-MG is studied. The remainder of this paper is organized as follows: the structure of the inertia-enhanced DC-MG with a BGC is introduced, and a virtual inertia control strategy for the BGC analogized with VSMs is proposed in Section II. Then, the small-signal model of the BGC system is built in Section III. In Section IV, the dynamic performance of the BGC system is analyzed, and the dc output current feedforward disturbance suppressing method for the BGC is introduced to further improve the dynamic response of the dc bus voltage. The BGC control system stability analysis and parameters selection are discussed in Section V. Simulations and experiments verify the theoretical analysis in Section VI. Finally, some conclusions are given in Section VII.

II. VIRTUAL INERTIA CONTROL STRATEGY OF THE BGC

A. DC-MG structure with a BGC

As shown in Fig. 1, the inertia-enhanced DC-MG consists of a BGC, DGs, energy storages, loads and relevant power electronic converters. The main circuit of the BGC adopts the three-phase full-bridge converter, where \( C \) is the dc-link output capacitor, \( L \) is the input filter inductor, \( r \) is the equivalent series resistance of the \( L \), \( u_j \) (j=a, b, c) is the utility grid voltage, \( i_j \) is the grid-connected current, \( e_j \) is the voltage of the BGC on the ac side, \( v_{dc} \) is the dc bus voltage of the DC-MG, \( i_{dc} \) is the dc-link current of the full-bridge converter, and \( i_o \) is the dc output current of the BGC.

When the DC-MG is operating in grid-connected mode, the BGC is responsible for keeping the power balance of the DC-MG through the bidirectional energy exchange with the utility grid, which ensures the stability of the dc bus voltage. The maximum power point tracking control is used in the DGs. The energy storage units are charged with the rated current when they are not completely charged. Loads connected to the dc bus are mainly constant power loads. In this mode, the DGs, energy storage units and loads can be regarded as current sources connected to the dc bus, and the dc bus voltage is regulated by the BGC. When the DC-MG is in the islanded mode, the BGC is out of service.

B. Inertia analogy between AC-MG and DC-MG

In an AC-MG, the active power-frequency (\( P-\omega \)) control of the VSM emulates inertia, damping characteristic and the primary frequency regulation of the SM. In this paper, it is assumed that the number of pairs of poles for the VSM is 1, thus the mechanical equation [16] can be described as

\[
P_{in} - P_e - D_p (\omega - \omega_n) = J \frac{d\omega}{dt} \approx J \omega_n \frac{d\omega}{dt} \quad (1)
\]

where \( P_{in}, P_e, D_p, \omega, \omega_n \) are the active power reference, the electromagnetic power, the damping coefficient, the angular frequency of the VSM, and the rated angular frequency of the utility grid, respectively. \( J \) is the virtual moment of inertia.

When the AC-MG is in steady state, (1) can be rewritten as

\[
\omega = \omega_n - m P_e \quad (2)
\]

where \( \omega_n = \omega_n + P_{ref}/D_p \) is the no-load angular frequency, and \( m = 1/D_p \) is the droop coefficient. It can be known from (1) and (2) that the \( P-\omega \) control of VSM is a modified droop control.

In the DC-MG, the voltage-current droop control is usually adopted for the BGC. There are three advantages as follows:

1) System stability of the DC-MG with constant power loads can be improved [29], [30].
2) It is easy to expand the system capacity by paralleling BGCs [31].
3) Coordinated control based on the bus-signaling for the DC-MG is easy to be realized [32].

The voltage-current droop control of the BGC can be expressed as

\[
u_{dc} = U_{dc\_ref} - R_i i_o \quad (3)
\]
where $U_{dc\_ref}$ is the no-load dc output voltage reference of the BGC, $R_e$ is the droop coefficient, and $u_{dc}$ is the reference value of the dc bus voltage of the DC-MG.

For the AC-MG, inertia of the system manifests the ability to prevent sudden changes of the frequency, and thereby leaving SM enough time to regulate the active power $P_e$, then rebuilding balance of the active power. For the DC-MG, its inertia manifests the ability to prevent sudden changes of the frequency, and thereby leaving SM enough time to regulate the active power to prevent sudden changes of the frequency, and thereby leaving SM enough time to regulate the active power.

Comparing with the droop expressions (2) and (3), it can be shown that $\omega$ and $u_{dc}$, $P_e$ and $i_e$ are comparable in form. It is also mentioned in [33] that the current sharing (the voltage-droop current control) of the DC-MG is similar to the droop expressions (2) and (3), it can be shown that $\omega$ and $u_{dc}$, $P_e$ and $i_e$ are comparable in form.

When the frequency of an AC-MG suffers from disturbance, the rotor can quickly provide active power support. Similarly, when the dc bus voltage of a DC-MG suffers from disturbance, the capacitors can quickly provide active power support.

From the above analysis, it is obvious that many variables and characteristics are mutually corresponding between the AC-MG containing a VSM and the DC-MG containing a BGC, as shown in Table I.

### Table I

<table>
<thead>
<tr>
<th>Analogies between AC-MG and DC-MG</th>
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<td>Droop relation</td>
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#### C. The virtual inertia control strategy for DC-MGs

As shown in Fig. 2, the virtual inertia control equation of the BGC similar to (1) is proposed via analogizing with the VSM, where $I_{set}$ is the dc output current reference of the BGC, $D_h$ is the droop coefficient, $U_u$ is the rated dc bus voltage, and $C_v$ is the introduced virtual capacitance. Obviously, it is reasonable to enhance the inertia of the DC-MG by introducing the virtual inertia control into the BGC.

![Fig. 2. Virtual inertia equation of the BGC via analogizing with the VSM.](image)

In terms of the virtual inertia equation of the BGC, the virtual inertia control is presented to enhance the inertia of the DC-MG, and to restrain the dc bus voltage fluctuation. The output of the virtual inertia control is regarded as the dc bus voltage reference $u_{dc}$.

The dc output current feed-forward control is introduced to improve the dynamic performance of the dc bus voltage, and to suppress the disturbance derived from the dc output current of the BGC. $G_{ff}(s)$ is the transfer function of the dc output current feed-forward control.

The voltage and current dual-loop control consists of the PI voltage outer loop control to track accurately the dc bus voltage reference, and the current inner loop control based on synchronous reference frames (SRFs) to realize active power exchange with the utility grid. $u_d$ and $u_q$ are the d-axis and q-axis components of $u$ in the SRFs, respectively. $i_d$ and $i_q$ are the d-axis and q-axis components of $i$ in the SRFs, respectively. $i_{d1}$ and $i_{q1}$ are the current reference of $i_d$ and $i_q$, respectively.

### III. SMALL-SIGNAL MODELING OF THE BGC SYSTEM

In order to find out the relation between the dc bus voltage and power demand of the DC-MG, it is necessary to build the BGC small-signal model. From Fig. 1, the mathematical expression of the BGC in the SRFs is described as

\[
\begin{align*}
\frac{du_d}{dt} &= (Lp + r)i_d - \omega_L i_q + e_d \\
\frac{du_q}{dt} &= (Lp + r)i_q + \omega_L i_d + e_q
\end{align*}
\]  

where $p$ is the differential operator, and $\omega_L$ is the angular frequency of the utility grid voltage. $e_d$ and $e_q$ are the d-axis and q-axis components of $e$ in the SRFs, respectively.

![Image 3](image)

**Fig. 3.** Block diagram of the virtual inertia control strategy for DC-MGs analogized with virtual synchronous machines.
From (5), it is clear that the BGC variables in the d-axis and q-axis are mutually coupling, which makes it difficult to design the controller. Here, the SRFs current control is adopted. PI controller is used for the grid-connected current control, which is represented as $G(s) = k_p + k_i/s$, thus the inner current control equation can be expressed as

$$
\begin{cases}
e_q = -G_i(s)(i_q^* - i_q) + \omega_s L_i + u_d \quad \text{(6)}
e_d = -G_i(s)(i_d^* - i_d) - \omega_s L_i + u_q
\end{cases}
$$

Substituting $e_d$ and $e_q$ from (6) to (5), the following expression can be obtained:

$$
\begin{cases}
(L_p + r) i_d = (i_d^* - i_d) G_i(s) 
(L_p + r) i_q = (i_q^* - i_q) G_i(s) \quad \text{(7)}
\end{cases}
$$

Assume that the state variables in (7) are written as the sum of steady-state variables and their small perturbations ($i_d = I_d + \Delta i_d$ and $i_q = I_q + \Delta i_q$). Ignoring the second-order perturbations, and applying the Laplace transform, the corresponding small-signal equation can be expressed as follows

$$
\begin{align}
\Delta i_d(s) &= \Delta i_d^*(s) - \Delta i_d(s) G_i(s)/(L_p + r) 
\Delta i_q(s) &= \Delta i_q^*(s) - \Delta i_q(s) G_i(s)/(L_p + r) \quad \text{(8)}
\end{align}
$$

Considering symmetry of the grid-connected current inner loops in d-axis and q-axis, the q-axis is taken as an example in order to simplify the analysis. Ignoring the influence of the current sampling delay, PWM control delay, and the perturbation component $\Delta i_d$, the small-signal model of the q-axis grid-connected current inner loop is obtained based on (8), as shown in Fig. 4, where $K_{PWM}$ is the equivalent gain of pulse width modulator.

Then the control structure of the dc bus voltage outer loop is built. And the small-signal model of the BGC system can be derived as shown in Fig. 4, where $K_{PWM}$ is the equivalent gain of pulse width modulator.

The state variables in (9) are written as the sum of steady-state variables and small perturbations ($u_d = U_d + \Delta u_d$, $u_q = U_q + \Delta u_q$, $u_{dc} = U_{dc} + \Delta u_{dc}$, $i_d = I_d + \Delta i_d$). If the second-order perturbations are ignored, the small-signal equation of (9) can be described as

$$
1.5(u_{dc,1} + u_{dc,2}) = u_{dc,1} K_1 = u_{dc,1} (C \cdot \frac{d \Delta u_{dc}}{dt} + i_d) \quad \text{(9)}
$$

The state variables in (9) are written as the sum of steady-state variables and small perturbations ($u_d = U_d + \Delta u_d$, $u_q = U_q + \Delta u_q$, $u_{dc} = U_{dc} + \Delta u_{dc}$, $i_d = I_d + \Delta i_d$). If the second-order perturbations are ignored, the small-signal equation of (9) can be described as

$$
1.5(U_d \Delta i_d + U_q \Delta i_q + \Delta u_d I_d + \Delta u_q I_q) =
CU_{dc} \frac{d \Delta u_{dc}}{dt} + U_{dc} \Delta i_d + \Delta u_{dc} I_o \quad \text{(10)}
$$

When the grid-voltage-oriented control is used in the BGC, $U_d$ is equivalent to zero in steady state. BGC is only used to control the dc bus voltage, and does not provide reactive power to the utility grid, so the reactive current component $I_q$ is zero. Thus, the expression (10) can be simplified as

$$
1.5(U_d \Delta i_d + \Delta u_q I_o) = CU_{dc} \frac{d \Delta u_{dc}}{dt} + U_{dc} \Delta i_d + \Delta u_{dc} I_o. \quad \text{(11)}
$$

According to the superposition theorem, ignoring the perturbation components $\Delta i_d$ and $\Delta i_q$, and applying the Laplace transformation to (11), the relation between $u_{dc}(s)$ and $i_d(s)$ is obtained as follows:

$$
\Delta u_{dc}(s)/\Delta i_d(s) = 3U_{dc}/2(CU_{dc}s + I_o) = G_i(s). \quad \text{(12)}
$$

Similarly, the relations between $\Delta u_d(s)$ and $i_d(s)$, $\Delta u_d(s)$, and $\Delta u_q(s)$ are respectively obtained as the follows:

$$
\Delta u_d(s)/\Delta i_d(s) = -U_{dc}/(CU_{dc}s + I_o) = G_i(s), \quad \text{(13)}
$$

$$
\Delta u_{dc}(s)/\Delta i_q(s) = 3I_o/2(CU_{dc}s + I_o) = G_i(s). \quad \text{(14)}
$$

Performing the small-signal decomposition, the virtual inertia equation of the BGC in Fig. 2 can be expressed as

$$
-\Delta i_d - D_i \Delta u_{dc} = C_s U_{dc} \frac{d \Delta u_{dc}}{dt} \quad \text{(15)}
$$

where $\Delta u_{dc}$ is the small-signal perturbation of $u_{dc}$. Then applying the Laplace transform, (15) can be rewritten as

$$
\Delta u_{dc}^*(s) = (-\Delta i_d(s) - D_i \Delta u_{dc}^*(s))/sC_s U_{dc} \quad \text{(16)}
$$

According to (12)-(14), (16) and Fig. 3, the small-signal model of the BGC control system can be derived as shown in Fig. 4, where $G_i(s)$ is the transfer function of dc bus voltage regulator using the PI regulator ($G_i(s) = k_p + k_i/s$).

**IV. DYNAMIC PERFORMANCE ANALYSIS AND THE DC OUTPUT CURRENT FEED-FORWARD CONTROL**

In Fig. 4, ignoring the influence of the utility grid voltage, and if the dc output current feed-forward control is not adopted, the small-signal closed-loop transfer function $TF(s)$ between $\Delta u_{dc}(s)$ and $\Delta i_d(s)$ is obtained as follows:

$$
TF(s) = \frac{\Delta u_{dc}(s)}{\Delta i_d(s)} = \frac{G_i(s)G_v(s)G_i(s)G_v(s) + G_z(s)}{1 + G_i(s)G_v(s)G_i(s)} \quad \text{(17)}
$$

where $G_i(s) = -1/(D_i + C_s U_{dc})$, $G_v(s) = G_v(s)K_{PWM}/(G_i(s)K_{PWM} + L_s + r)$.

Fig. 5 shows the unit-step responses of $TF(s)$ for various values of $C_v$ when $D_i$ is 5 or 1. The physical meaning of the unit-step response is the dc bus voltage change when the BGC output current suddenly increases. As is known to all, the dc output current of the BGC is relevant to the power demand of the DC-MG which is decided by the power consumption of the loads and the output power of DG sources. In Fig. 5(a), the
red solid points represent the positions where the step response curve reaches the 95% of the steady state value.

It can be seen from Fig. 5 that the dc bus voltage would change more mildly when \( C_i \) increases if the initial stage of the step response is neglected, which indicates that the inertia of the DC-MG is larger. When \( C_i \) is less than 0.04mF, the dc bus voltage changes quickly, and the inertia of the DC-MG is very small. As a result, the value of \( C_i \) directly determines the inertia of the DC-MG.

From Fig. 5, it is clear that the dc bus voltage at the initial stage of all step response with different \( D_b \) and \( C_i \) has a serious impact. The abrupt voltage changes at the beginning of the unit-step response process would do harm to the realization of virtual inertia of the DC-MG, jeopardizing the dynamic performance of the dc bus voltage.

As shown in Fig. 4, \( \Delta i_d(s) \) can affect \( \Delta u_{dc}(s) \) through two loops (loop1 and loop2). Loop1: \( \Delta i_d(s) \) affects \( \Delta u_{dc}(s) \) through the virtual inertia control and the voltage control. Ignoring the influence of the ESR of the filter inductor, the transfer functions of the two loops \( TF_{loop1}(s) \) and \( TF_{loop2}(s) \) are derived as:

\[
TF_{loop1}(s) = \frac{(G_i(s)K_{pwm} + Ls)G_v(s)}{G_i(s)K_{pwm}G_v(s) + G_v(s)K_{pwm} + Ls} \tag{18}
\]

\[
TF_{loop2}(s) = \frac{-G_i(s)K_{pwm}G_v(s)G_i(s)/(D_b + C_iU_{dc})}{G_i(s)K_{pwm}G_v(s)G_i(s) + G_v(s)K_{pwm} + Ls} \tag{19}
\]

Clearly, the transfer function \( TF(s) \) of \( \Delta u_{dc}(s) \) and \( \Delta i_d(s) \) can be split into two parts:

\[
TF(s) = TF_{loop1}(s) + TF_{loop2}(s) \tag{20}
\]

Fig. 7 illustrates the unit-step responses of \( TF_{loop1}(s) \) and \( TF_{loop2}(s) \). A notch appears at the beginning of the unit-step response of \( TF_{loop1}(s) \). Over time, this response would finally reach zero. However, the unit-step response of the \( TF_{loop2}(s) \) would smoothly decline to the steady state value. So, the serious impact at the initial stage is caused by \( TF_{loop1}(s) \), thus \( \Delta i_d(s) \) passing through loop1 is equivalent to perturbation, which affects the smoothness of the dc bus voltage dynamic response.

Here, dc output current feed-forward control for the BGC is introduced to eliminate the serious impact from the disturbing loop1, which can improve the dynamic performance of the dc bus voltage. The transfer function \( G_f(s) \) of dc output current feed-forward control in Fig. 4 is expressed as follows:

\[
G_f(s) = \frac{-G_i(s)(K_{pwm}G_i(s) + Ls + r)}{K_{pwm}G_v(s)G_i(s)} \tag{21}
\]

It is considered that in the voltage and current dual-loop control, the bandwidth of the grid-connected current inner loop is usually designed to be wide \( (K_{pwm}G_v(s) \gg Ls+r) \). So, the expression \( (21) \) can be simplified as the practical feed-forward transfer function as follows:

\[
G_f(s) \approx -G_i(s)/G_v(s) = 2U_{dc}/3U_{dc} \tag{22}
\]

Combining the dc output current feed-forward control with the virtual inertia control, the closed-loop transfer function \( TF_f(s) \) between \( \Delta u_{dc}(s) \) and \( \Delta i_d(s) \) can be expressed as:

\[
TF_f(s) = \frac{\Delta u_{dc}(s)}{\Delta i_d(s)} = \frac{G_i(s)G_v(s)G_i(s)}{1 + G_i(s)G_v(s)G_i(s)} \tag{23}
\]

Fig. 8 shows the unit-step responses of \( TF_f(s) \) for various values of \( C_i \) when \( D_b = 5 \). The red solid points represent the positions where the step response curve reaches the 95% of the steady state value. Obviously, introducing the dc output
current feed-forward control and the virtual inertia control for the BGC, the unit-step responses of the dc bus voltage all smoothly decline to the steady state value, so the dynamic process of the dc bus voltage is improved.

Fig. 8. Unit-step responses of \( TF_{ff}(s) \) for various values of \( C_v \) when \( D_b = 5 \).

V. SYSTEM STABILITY ANALYSIS AND PARAMETERS SELECTION

Fig. 9 shows the zeros and dominant poles distribution diagram of \( TF_{ff}(s) \). The symbol “×” and “+” represent poles, the symbol “o” represents zero. It can be seen that two pairs of dipoles are in each sub-graph of Fig. 9. These dipoles are far away from the origin point so their impact on the control system can be neglected. In Fig. 9(a), a sole dominant pole moves with \( C_v \) changing. If \( C_v \geq 0 \), the pole moves toward the imaginary axis with \( C_v \) increasing but it will not cross the imaginary axis. If \( C_v < 0 \), the sole mobile pole is on the right side of the system so that the system is unstable. Therefore, for the stability of system, it is satisfactory enough when \( C_v \geq 0 \). In Fig. 9(b), a sole dominant pole moves away from the imaginary axis with \( D_b \) increasing.

Seen from Fig. 9(a) and Fig. 9(b), there is only one dominant pole. Therefore, the dynamic performance of the high-order system can be estimated by the dynamic performance index of the first-order system. Using MATLAB, the sole dominant pole is calculated to be \( -D_b/(C_v U_n) \) and the final value of the dynamic response is determined by \( D_b \). Therefore, \( TF_{ff}(s) \) is approximately equal to:

\[
TF_{ff}(s) \approx -1/(D_b(1 + \tau s)) \tag{24}
\]

where \( \tau = C_v U_n/D_b \). The time constant of the first-order system is \( T = 3 \tau = 3C_v U_n/D_b \) which directly reflects the inertia of the first-order system. Assuming that \( D_b \) remains unchanged, \( T \) gets larger with \( C_v \) increasing, inertia of the DC-MG becomes larger, and the process of the dynamic response slows down, which makes it better to suppress the impact of the dc bus voltage caused by power fluctuation of the DC-MG. Conversely, when \( C_v \) is smaller, inertia of DC-MG becomes smaller and its ability to suppress the dc bus voltage fluctuation weakens. Therefore, the value of \( C_v \) should be reasonably set according to the above analysis and inertia requirement of the DC-MG.

In Fig. 9(c), when \( C_v \) is fixed, with \( C \) increasing, a dominant pole of the \( TF_{ff}(s) \) (surrounded by dotted green circle) is fixed. Its value is \( -D_b/C_v U_n \) which is related to \( C_v \) not \( C \). But there are two changing poles when \( C \) changes. When \( C < 50mF \), those poles are not the dominant ones. When \( C > 50mF \), those poles are getting closer to the imaginary axis with \( C \) increasing and become the dominant poles apart from the fixed one.

And the system damping decreases and is prone to cause the dc bus voltage oscillation when the power of the DC-MG suddenly changes. Those changing poles are called low frequency oscillatory modes [34].

Fig. 9. Zeros and dominant poles distribution of \( TF_{ff}(s) \). (a) Arrow direction : \( C_v \) changes from 0mF to 10mF. (b) Arrow direction : \( D_b \) changes from 1 to 15. (c) Arrow direction : \( C \) changes from 0. 5mF to 250mF.

Fig. 10. Unit-step responses of \( TF_{ff}(s) \) for various values of \( C_v \). (a) \( C_v = 0.28mF \). (b) \( C_v = 1.4mF \).

Fig. 10 shows the unit-step response of \( TF_{ff}(s) \) for various
values of $C$ when $C_{v} = 0.28$mF or 1.4mF. In Fig. 10(a), $C_{v} = 0.28$mF, when $C > 50$mF, the dc bus voltage oscillation is rather obvious in the unit-step response of $TF_{b}(s)$. In Fig. 10(b), $C_{v} = 1.4$mF, when $C > 50$mF, the dc bus voltage oscillation is less obvious in the unit-step response of $TF_{b}(s)$, but the oscillation still exists. In Fig. 9(c), when $C < 29.5$mF, all the poles are located on the real axis and the $TF_{b}(s)$ damping is above 1, thus, no low frequency oscillation will happen. Therefore, from the view of low frequency oscillatory modes, the maximum value of $C$ has the corresponding limit.

In the VSM control, the small-signal closed-loop transfer function of the inverter output active power is a typical second-order system and it is described as follows:

$$\frac{\Delta P_{s}(s)}{\Delta P_{intr}(s)} = \frac{\omega_{na}^2}{s^2 + 2\xi \omega_{na} s + \omega_{na}^2}$$  \hspace{1cm} (25a)

$$\xi = \frac{D_{p}}{2\sqrt{J_{F}} P_{max}}$$  \hspace{1cm} (25b)

$$\omega_{na} = \sqrt{\frac{P_{max}}{J_{F} S}}$$  \hspace{1cm} (25c)

where $P_{max}$ is the maximum output power of the inverter. From (25b), it can be seen that the damping factor $\xi$ of the second-order system is proportional to $D_{p}$. However, it is known from (2) that $D_{p}$ is inversely proportional to the droop coefficient of the $P$-to droop. When an AC-MG operates in the islanded mode, the active power sharing between inverters is better if the droop coefficient of the $P$-to control is larger. On the other hand, the larger damping factor $\xi$ is, the better oscillation suppression effect is. Therefore, the damping factor $\xi$ of the second-order system and the droop coefficient of the inverter are mutually constrained by $D_{p}$, making it difficult to set both the damping factor and the droop coefficient at their optimal values.

$$1 \leq \frac{1}{D_{b}} \leq \frac{\Delta U_{dmax}}{2i_{c}}$$  \hspace{1cm} (27)

where $i_{c}$ is the rated dc output current of the BGC. $\Delta U_{dmax}$ is the difference between the maximum and minimum dc bus voltages when the DC-MG is in grid-connected mode. When $1/D_{b}$ is larger, the stability range of the BGC with constant power loads is larger. In the grid-connected system, the larger $1/D_{b}$ means the better power sharing among each BGC. So $1/D_{b} = \Delta U_{dmax}/(2i_{c})$, thus $D_{b} = 2i_{c}/\Delta U_{dmax}$.

Fig. 12 shows the unit-step responses of $TF_{b}(s)$ for various values of $D_{b}$ when $C_{v} = 1.4$mF.

Noticeably, from Fig. 12, when $C_{v}$ is determined, the change of $D_{b}$ would influence the inertia. The smaller $D_{b}$ means the larger system inertia, and vice versa. Therefore, $D_{b}$ should be firstly selected before $C_{v}$.

The parameter design of voltage and current dual-loop control is decided by stability margin and system bandwidth. In order to attain better stability margin, satisfied anti-noise ability and quick dynamic response, the phase margin and bandwidth of the current inner-loop controller are 90° and 1 kHz, respectively. And those two parameters of the voltage outer-loop controller are 80° and 85Hz. According to the above performance indexes, the relevant controller parameters are shown in Table II. The switching frequency $f_{s}$ of BGC is 10 kHz. The rated dc output current $i_{c}$ of the BGC is 70A. The rated dc bus voltage $U_{c}$ is 700V. In order to confine the bus voltage fluctuation within 2% of the rated value in the grid-connected mode, the $\Delta U_{dmax}$ needs to be 28V, thus, $D_{b} = 2i_{c}/\Delta U_{dmax} = 5$. The fluctuation component of loads in DC-MG is often at millisecond level. Hence, in order to suppress the impact from this kind of fluctuation component, the time constant $T$ sets to be 0.6s, then $C_{v} = TD_{b}/(3U_{c}) = 1.4$mF. Considering the bidirectional energy flowing of the BGC, $i_{c}$ is set to be 0. The function of the dc-side capacitor is to be as the buffer of energy exchange between AC and dc side of the BGC and to suppress the harmonic voltage on the dc side. And the filter inductor is used to restrict the grid-connected harmonic current. Based on engineering experience, $C$ and $L$ can be set at 5740μF and 1mH, respectively. The electrolytic capacitor bank $C$ on the dc side is 5740μF, adopting the integrated
structure which has 14 parallel branches with 2 series electrolytic capacitors. Each capacitor is 820 μF and its withstanding value is 450V.

VI. SIMULATIONS AND EXPERIMENTS

In order to verify the validity of the proposed control strategy, a DC-MG simulation platform is built in Psim according to Fig. 1. In this system, the maximum output power of the DGs is 50kW. The maximum power of the loads is 50kW. The rated dc bus voltage $U_n$ is 700V. The system parameters of the BGC are listed in Table II.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>SYSTEM PARAMETERS OF THE BGC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Values</td>
</tr>
<tr>
<td>$U_{dc, ref}$ (V)</td>
<td>700</td>
</tr>
<tr>
<td>$L$ (mH)</td>
<td>1</td>
</tr>
<tr>
<td>$C$ (μF)</td>
<td>5740</td>
</tr>
<tr>
<td>$C_v$ (mF)</td>
<td>1.4</td>
</tr>
<tr>
<td>$D_{bj}$</td>
<td>5</td>
</tr>
</tbody>
</table>

Fig. 13(a) shows the simulation waveforms of the dc bus voltage $u_{dc}$ when the power demand of the DC-MG $p_{dc}$ suddenly changes. When $t = 2s$, $p_{dc}$ suddenly declines from 41kW to −43kW. Adopting virtual inertia control without output current feed-forward, $u_{dc}$ smoothly increases to the steady-state value. On the other hand, adopting virtual inertia control with output current feed-forward, $u_{dc}$ smoothly increases at first, and then drops down, and finally smoothly increases to the steady-state value. In this case, the dynamic performance of the dc bus voltage can be improved by introducing the dc output current feed-forward control into the proposed BGC virtual inertia control.

Fig. 13(b) is the simulation results with or without ($C_v = 0$) the virtual inertia control when the BGC already adopts the dc output current feed-forward control, where $p_{ Randall}$ is the output power of the BGC. When $p_{dc}$ abruptly changes, without the virtual inertia control, $u_{dc}$ quickly reaches the steady state value. Using the virtual inertia control, $u_{dc}$ arrives at the steady state value in a slower speed. When $t = 2s$, $p_{dc}$ suddenly drops, without the virtual inertia control, the BGC transmits the spare power of DC-MG to the utility grid. When the virtual inertia control is employed, the BGC transmits less power toward the utility grid than that without the virtual inertia control. When $t = 4s$, $p_{dc}$ suddenly increases. Using the virtual inertia control, the BGC can provide more power to the DC-MG than that without the virtual inertia control, immediately compensating the power shortage of the DC-MG. Therefore, no matter from the perspective of the dc bus voltage or the BGC output power, inertia of the DC-MG can be improved when the BGC uses the virtual inertia control.

In order to further verify the theoretical analysis and the simulation results, the DC-MG experiment platform is built, as shown in Fig. 14. The BGC system parameters are identical with the simulation parameters in Table II.

Fig. 15 shows the experimental waveforms of the grid-connected current $i_L$ and the dc bus voltage $u_{dc}$ with different control strategies, when $p_{dc}$ suddenly drops from 28kW to 4.7kW. In Fig. 15(a) $u_{dc}$ soon reaches the steady state value after $p_{dc}$ suddenly drops and system inertia is very small. In Fig. 15(b), at the moment when $p_{dc}$ abruptly drops, $u_{dc}$ experiences a sudden increase and then slowly arrives at the steady state value. The dynamic process of the dc bus voltage is not smooth. In Fig. 15(c) and Fig. 15(d), after the sudden drop of $p_{dc}$, $u_{dc}$ slowly and smoothly reaches the steady state value, the...
inertia of the DC-MG is enhanced. As a result, the larger $C_v$ is, the stronger inertia of the DC-MG is.

![Fig. 15. Experimental results during sudden load step-down. (a) No virtual inertia control with dc output current feed-forward. (b) Virtual inertia control without dc output current feed-forward. (c) Virtual inertia control with dc output current feed-forward when $C_v = 0.28\text{mF}$. (d) Virtual inertia control with dc output current feed-forward when $C_v = 1.4\text{mF}$.]

Comparing the waveforms of the grid-connected current $i_a$ of the BGC in Fig. 15 and Fig. 16, the inertia of the DC-MG is actually enhanced by increasing the energy exchange speed between the DC-MG and the utility grid. When $p_{dc}$ decreases, $i_a$ soon drops, thus the power quickly decreases. Similarly, when $p_{dc}$ increases, $i_a$ can increase quickly, thus the power immediately increases, restraining the dc bus voltage fluctuation and maintaining the stability of dc bus voltage.

![Fig. 16. Experimental results during sudden load step-up. (a) No virtual inertia control with dc output current feed-forward. (b) Virtual inertia control without dc output current feed-forward. (c) Virtual inertia control with dc output current feed-forward when $C_v = 0.28\text{mF}$. (d) Virtual inertia control with dc output current feed-forward when $C_v = 1.4\text{mF}$.]

![Fig. 17. Experimental results of the proposed control strategy during DC-MG operation mode changes when $C_v = 0.28\text{mF}$. (a) From rectification mode switch to inversion mode. (b) From inversion mode switch back to rectification mode.]

![Fig. 18. Experimental results of the proposed control strategy during DC-MG operation mode changes when $C_v = 1.4\text{mF}$. (a) From rectification mode switch to inversion mode. (b) From inversion mode switch back to rectification mode.]

It is obvious from Fig. 17 and Fig. 18 that when the power flow direction changes, the phase of the current wave suddenly changes $180^\circ$ and the waveform of the voltage $u_{dc}$ smoothly arrives at the steady state value so the proposed control strategy still can maintain the DC-MG inertia.

**VII. CONCLUSION**

Problems like the low inertia and the drastic dc bus voltage fluctuation jeopardizes the safe and steady operation of the DC-MG and reduces the system efficiency. In order to solve this problem, in this paper, a virtual inertia control strategy for DC-MG through the introduced BGC analogized with the VSM in AC-MG is proposed to enhance the inertia of the DC-MG, and to restrain the dc bus voltage fluctuation. The small-
signal model of the BGC system is built and the dynamic transfer function between the dc output current of the BGC and the dc bus voltage is deduced. It is also found that the dc output current of the BGC is equivalent to disturbance, which will have an impact on the dynamic process of the dc bus voltage. The dc output current feed-forward disturbance suppressing method for the BGC is introduced to smooth the dynamic response of the dc bus voltage. By analyzing the system stability, the proper control parameters of the BGC are selected. Moreover, the proposed control idea can also be extended to the control strategies of various kinds of converters in the DC-MG.

REFERENCES


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