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Ad Hoc Microphone Array Beamforming Using the Primal-Dual Method of Multipliers

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Abstract—**In the recent years, there have been an increasing amount of researches aiming at optimal beamforming with ad hoc microphone arrays, mostly fusion-center-based schemes. However, huge computational complexities and communication overheads impede many of these algorithms from being useful in practice. In this paper, we propose a low-footprint optimization approach to reduce the convergence time and overheads for the distributed beamforming problem. We transcribe the pseudo-coherence-based beamforming which is insightful for taking into account the nature of speech. We formulate the distributed minimum variance distortionless response beamformer using the primal-dual method of multipliers. Our experiments confirm the fast convergence using the proposed distributed algorithm. It is also shown how a hard limit on the number of iterations affects the performance of the array in noise and interference suppression.**

Index Terms—**Speech enhancement, ad hoc microphone array, distributed beamforming, primal-dual method of multipliers.**

I. Introduction

As part of the ongoing digital revolution, the use of portable devices is continuously increasing. Pervasiveness of such devices provides resources for new strategies in signal processing, since they are equipped with powerful embedded processors, fast wireless network adapters, several microphones, auxiliary sensors, and long life batteries. The so called ad hoc microphone array signal processing is an emerging approach to enhance the quality of captured acoustics using the available resources in an acoustic enclosure.

Traditional array signal processing approaches that are presented in [1]–[3], may be extended and used with ad hoc microphone arrays with precaution since they have challenging characteristics. Apparatus such as the minimum variance distortionless response (MVDR) beamformer [4], [5], the linearly-constrained minimum variance (LCMV) beamformer [6], [7], the speech-distortion weighted multi-channel Wiener (SDW-MWF) filter [8]–[10], the transfer function generalized sidelobe canceler (TF-GLC) [11]–[14], and alike, can be used for distributed microphone arrays; however, the dominant trend is to implement these algorithms in a centralized approach. The centralized approach is not useful in ad hoc arrays for its huge communication and computational overheads. Another shortcoming arises in situations where the fusion center is not a remote compute server, but is one of the wireless nodes that may leave the ad hoc array at any time, so that a reconfiguration step is required to setup a new fusion center.

Distributed beamforming is a solution to the aforementioned shortcomings. In this scheme, the beamforming technique of interest is carried out in a way that each node is responsible to process its own signal, with possible communications between neighboring nodes for updating mutual data. Distributed adaptive node-specific methods are proposed in [15], [16] to remove the redundant communications and reduce the computational load. This approach is extended to cooperative adaptive [17], and greedy algorithm [18], as well as the distributed MVDR beamformer [19]. Clique-based [20], diffusion-based [21], and message passing [22] algorithms are among other distributed approaches. The clique-based approach has been recently used in estimation of error covariance matrices for multichannel noise reduction [23]. Alternative distributed approaches take into account the convexity of beamforming optimization problems, and solve them with distributed convex optimization techniques. Alternating direction method of multipliers (ADMM) and its variants [24] have been widely applied to such problems. A problem with some of these approaches is that they follow the Gauss-Seidel method which restrict them to synchronous updating schemes.

In this work, we propose a distributed beamforming technique based on the primal-dual method of multipliers (PDMM), formally BiADMM [25]. The PDMM method has been shown to converge with both synchronous and asynchronous updating regimes (with a predefined node activation strategy) [26]. This makes the PDMM method applicable to ad hoc microphone arrays. The proposed technique is derived from the signal model based on the speech pseudo-coherencies. This model is very flexible and is able to track and respond to changes in the geometry.

The rest of this paper is organized as follows. The method of interest of this paper is described in Section II. At first, the signal model is defined, and the optimization problem is introduced. Then, the required steps to set up the distributed beamforming approach using PDMM are described. Convergence of the primal and dual variables and performance of the proposed distributed enhancement technique are studied with experiment on random graphs in Section III. The paper is concluded by discussions in Section IV.

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II. Method

A. Signal Model

The problem of interest is interference/noise suppression with an ad hoc microphone array. We assume that the array is composed of M omni-directional microphones in a random geometry. The acoustic enclosure is assumed to be reverberant with P sources. We formulate the problem in the short-time Fourier transform (STFT) domain with a sufficiently long analysis window. At time-frame t and frequency index k, we denote the STFT-domain signal for the p-th source by $S^p(k,t)$. For the sake of readability, we will omit the time-frequency indices from this point forward, when there is no ambiguity. The clean but convolved signal received by microphone m can be expressed as

$$X_m^p = G_m^p(k) S^p, \tag{1}$$

where $G_m^p(k)$ is the acoustic transfer function (ATF) from the p-th source to the m-th microphone, and is assumed to be time-invariant. We select one microphone as the "reference node" for the p-th source, and denote its clean but convolved signal as the "reference signal", X_{ref}^p. As a rule $G_{\text{ref}}^p \neq 0$.

By superposing all signals captured by microphone m and considering the microphone self-noise, V_m, the received signal at this node, Y_m, is expressed in STFT-domain as

$$Y_m = \sum_{p=1}^{P} G_m^p(k) S^p + V_m. \tag{2}$$

Alternatively, the m-th received signal can be expressed using the relative transfer function (RTF) signal model as

$$Y_m = \sum_{p=1}^{P} D_m^p(k) X_{\text{ref}}^p + V_m, \tag{3}$$

where $D_m^p(k)$ is the relative transfer function regarding the m-th microphone and the reference node for the p-th source,

$$D_m^p(k) = \frac{G_m^p(k)}{G_{\text{ref}}^p(k)}. \tag{4}$$

As discussed in [27], [28], the pseudo-coherence-based signal model is another alternative, which considers the natural coherency of speech signals. The complex pseudo-coherence between the m-th clean signal and the reference signal for the p-th source is defined as

$$P_{X_m^p, X_{\text{ref}}^p} \triangleq \frac{E\left[X_m^p X_{\text{ref}}^{p*}\right]}{E\left[\left|X_{\text{ref}}^p\right|^2\right]}, \tag{5}$$

where superscript * represents complex conjugate and $E[\cdot]$ denotes mathematical expectation. Consequently, the received signal can be restated using the pseudo-coherence as

$$Y_m = \sum_{p=1}^{P} P_{X_m^p, X_{\text{ref}}^p} X_{\text{ref}}^p + V_m. \tag{6}$$

Then, the pseudo-coherence-based signal model for the whole ad hoc array would be

$$\bar{\mathbf{y}} = \sum_{p=1}^{P} \bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}^p, X_{\text{ref}}^p} X_{\text{ref}}^p + \bar{\mathbf{v}}, \tag{7}$$

where $\bar{\mathbf{y}} = [Y_1 \quad \cdots \quad Y_M]^T$, $\bar{\mathbf{x}}^p = [X_1^p \quad \cdots \quad X_M^p]^T$, and $\bar{\mathbf{v}} = [V_1 \quad \cdots \quad V_M]^T$ are the stacked vectors for the received, clean, and noise signals, respectively, and

$$\bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}^p, X_{\text{ref}}^p} = \left[P_{X_1^p, X_{\text{ref}}^p} \quad \cdots \quad P_{X_M^p, X_{\text{ref}}^p}\right]^T$$

is the pseudo-coherence vector for source p.

In the following subsection, we formulate the beamforming approach to recover one desired signal among the P sources, and we omit the superscript p for simplicity.

B. Centralized Optimization Problem

In this section, we formulate the optimization problem for the minimum variance distortionless response (MVDR) beamformer. The output of this beamformer is obtained by

$$Z = \bar{\mathbf{h}}^H \bar{\mathbf{y}}, \tag{8}$$

where $\bar{\mathbf{h}} = \left[h_1 \quad \cdots \quad h_M\right]^T$ is the vector of weights for enhancing the desired source using the whole ad hoc array, and superscript ^{H} denotes the hermitian transpose.

The MVDR beamformer minimizes the variance of its output, $\phi_Z = E[ZZ^*]$, such that

$$\begin{aligned} \min_{\bar{\mathbf{h}}} \quad & \bar{\mathbf{h}}^H \boldsymbol{\Phi}_{\bar{\mathbf{y}}} \bar{\mathbf{h}} + \epsilon \bar{\mathbf{h}}^H \bar{\mathbf{h}} \\ \text{s.t.} \quad & \bar{\mathbf{h}}^H \bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}, X_{\text{ref}}} = 1, \end{aligned} \tag{9}$$

where $\boldsymbol{\Phi}_{\bar{\mathbf{y}}} = E[\bar{\mathbf{y}}\bar{\mathbf{y}}^H]$ is the covariance matrix of the received signals, and ϵ is the Tikhonov regularization factor. A closed form solution for (9) exists and weights can be obtained in a fusion center by

$$\bar{\mathbf{h}} = \frac{\left(\boldsymbol{\Phi}_{\bar{\mathbf{y}}} + \epsilon \mathbf{I}\right)^{-1} \bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}, X_{\text{ref}}}}{\bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}, X_{\text{ref}}}^H \left(\boldsymbol{\Phi}_{\bar{\mathbf{y}}} + \epsilon \mathbf{I}\right)^{-1} \bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}, X_{\text{ref}}}}, \tag{10}$$

where $\mathbf{I}$ is the identity matrix of proper size.

C. Distributed Optimization with PDMM

In order to use PDMM, microphones in the ad hoc array are first mapped to a graph. We denote the graph $G = (\mathcal{V}, \mathcal{E})$ with vertexes $\mathcal{V}$ (nodes) and edges $\mathcal{E}$. Then, the distributed optimization problem is described as

$$\begin{aligned} \min_{\bar{\boldsymbol{\omega}}} \quad & \sum_{m \in \mathcal{V}} f_m(\bar{\boldsymbol{\omega}}_m) \\ \text{s.t.} \quad & \mathbf{A}_{mn} \bar{\boldsymbol{\omega}}_m + \mathbf{A}_{nm} \bar{\boldsymbol{\omega}}_n = \bar{\mathbf{c}}_{mn} \quad \forall (m,n) \in \mathcal{E}, \end{aligned} \tag{11}$$

where $\bar{\boldsymbol{\omega}} = \{\bar{\boldsymbol{\omega}}_1, \ldots, \bar{\boldsymbol{\omega}}_M\}$ is the set of optimization parameter vectors, $f_m(\bar{\boldsymbol{\omega}}_m)$ is a closed, proper and convex fraction of the global cost function which is computable locally, and the constraints are defined for all sets of neighboring nodes connected with edges.

The optimization problem in (9) should be modified to comply with the canonical form in (11). In the remaining of this section we reformulate (9) to obtain a special form of (11) which is known as the consensus problem, in which matrices $\mathbf{A}_{mn} = -\mathbf{A}_{nm}$ and vector $\bar{\mathbf{c}}_{mn} = \bar{\mathbf{0}}$.

We construct our distributed optimization problem in a similar manner to the approach in [29]. First thing to notice is that the enhanced signal would not be accessible in a distributed manner unless the locally calculable output, Z_m, becomes a side product of the algorithm or an optimization parameter itself. We define the sequence

$$Z_m(t-l) = \bar{\mathbf{h}}^H \bar{\mathbf{y}}(t-l) \qquad l \in \{0, \ldots, L-1\}, \tag{12}$$

such that $Z_m(t-l)$ be equal to $Z(t-l)$, where $t-l$ refers to the l-th time-frame before the current one. Then, by accommodating the outputs from all nodes and expanding the inner product we have

$$\begin{aligned}\sum_{m\in\mathcal{V}} Z_m(t-l) &= M\bar{\mathbf{h}}^H \bar{\mathbf{y}}(t-l) \\ &= M \sum_{m\in\mathcal{V}} h_m^* y_m(t-l). \end{aligned}\tag{13}$$

Assuming signals are ergodic, the output variance can be estimated with smoothing over the past L time-frames as

$$\begin{aligned}\hat{\phi}_Z(t) &= \frac{1}{L}\sum_{l=0}^{L-1} Z(t-l)Z^*(t-l) \\ &= \frac{1}{LM}\sum_{l=0}^{L-1}\sum_{m\in\mathcal{V}} Z_m(t-l)Z_m^*(t-l). \end{aligned}\tag{14}$$

Further modifications are required for (9) to obtain a distributed prescription. The vector inner product in the constraint of (9) can be expanded as

$$\bar{\mathbf{h}}^H \bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}},X_{\mathrm{ref}}} - 1 = \sum_{m\in\mathcal{V}} \Big(h_m^* P_{X_m,X_{\mathrm{ref}}} - \frac{1}{M}\Big), \tag{15}$$

where $P_{X_m,X_{\mathrm{ref}}}$ is the m-th element of the pseudo-coherence vector, $\bar{\boldsymbol{\rho}}_{\bar{\mathbf{x}}^p,X^p_{\mathrm{ref}}}$. The squared ℓ^2-norm of the weight vector is

$$\|\bar{\mathbf{h}}\|_2^2 = \bar{\mathbf{h}}^H\bar{\mathbf{h}} = \sum_{m\in\mathcal{V}} h_m^* h_m. \tag{16}$$

Using (13) as an auxiliary constraint, (14) and (16) to modify the objective function, and (15) to modify the main constraint, we can rewrite the optimization problem in (9) as

$$\begin{aligned}\min_{\{\bar{\mathbf{h}}^H, Z_m(t)\}} \quad & \sum_{m\in\mathcal{V}} \Big(\frac{1}{ML}\sum_{l=0}^{L-1} \big|Z_m(t-l)\big|^2 + \epsilon h_m^* h_m \Big) \\ \text{s.t.} \quad & \sum_{m\in\mathcal{V}} \Big(h_m^* P_{X_m,X_{\mathrm{ref}}} - \frac{1}{M}\Big) = 0 \\ & \sum_{m\in\mathcal{V}} \Big(Z_m(t-l) - M h_m^* y_m(t-l)\Big) = 0. \end{aligned}\tag{17}$$

It is possible to restate the optimization problem in (17) as

$$\begin{aligned}\min_{\bar{\boldsymbol{\omega}}_m} \quad & \sum_{m\in\mathcal{V}} \bar{\boldsymbol{\omega}}_m^H \mathbf{Q} \bar{\boldsymbol{\omega}}_m \\ \text{s.t.} \quad & \sum_{m\in\mathcal{V}} (\mathbf{A}_m \bar{\boldsymbol{\omega}}_m - \bar{\mathbf{b}}) = 0, \end{aligned}\tag{18}$$

where

$$\begin{aligned}\bar{\boldsymbol{\omega}}_m &= \Big[Z_m(t-L+1), \ldots, Z_m(t), h_m^* \Big]^T, \\ \mathbf{Q} &= \begin{bmatrix} \frac{1}{ML} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \frac{1}{ML} \\ & & & \epsilon \end{bmatrix}, \\ \mathbf{A}_m &= \left[\begin{array}{ccc|c} 1 & & \mathbf{0} & -My_m(t-L+1) \\ & \ddots & & \vdots \\ \mathbf{0} & & 1 & -My_m(t) \\ \hline 0 & \cdots & 0 & P_{X_m,X_{\mathrm{ref}}} \end{array}\right], \\ \bar{\mathbf{b}} &= \Big[0, \ldots, 0, \frac{1}{M}\Big]^T. \end{aligned}\tag{19}$$

Equation (18) is in a distributed form, yet it contains node constraints instead of edge constraints. In order to obtain the desirable form, we formulate the dual problem as follows. The Lagrangian for the optimization problem in (18) is

$$\begin{aligned}\mathcal{L}(\bar{\boldsymbol{\omega}}_m, \bar{\boldsymbol{\mu}}) = \sum_{m\in\mathcal{V}} \Big(& \bar{\boldsymbol{\omega}}_m^H \mathbf{Q} \bar{\boldsymbol{\omega}}_m \\ & - 2\Re\Big\{ \bar{\boldsymbol{\mu}}^H (\mathbf{A}_m \bar{\boldsymbol{\omega}}_m - \bar{\mathbf{b}})\Big\}\Big), \end{aligned}\tag{20}$$

where $\Re\{\cdot\}$ denotes the real part operator. By equating the gradient of (20) to zero,

$$\vec{\nabla}_{\bar{\boldsymbol{\omega}}_m^H} \mathcal{L}(\bar{\boldsymbol{\omega}}_m, \bar{\boldsymbol{\mu}}) = \mathbf{Q}\bar{\boldsymbol{\omega}}_m - \mathbf{A}_m^H \bar{\boldsymbol{\mu}} = 0, \tag{21}$$

the optimum solution at node m will be reached,

$$\bar{\boldsymbol{\omega}}_m = \mathbf{Q}^{-1}\mathbf{A}_m^H \bar{\boldsymbol{\mu}}. \tag{22}$$

By putting the optimal primal variable, $\bar{\boldsymbol{\omega}}_m$ from (22), into the Lagrangian function in (20), the dual function is obtained,

$$\begin{aligned} g(\bar{\boldsymbol{\mu}}) &= \mathcal{L}(\bar{\boldsymbol{\omega}}_m, \bar{\boldsymbol{\mu}}) \\ &= \sum_{m\in\mathcal{V}} \Big(- \bar{\boldsymbol{\mu}}^H \boldsymbol{\Omega}_m \bar{\boldsymbol{\mu}} + 2\Re\Big\{ \bar{\mathbf{b}}^H \bar{\boldsymbol{\mu}} \Big\}\Big), \end{aligned}\tag{23}$$

where

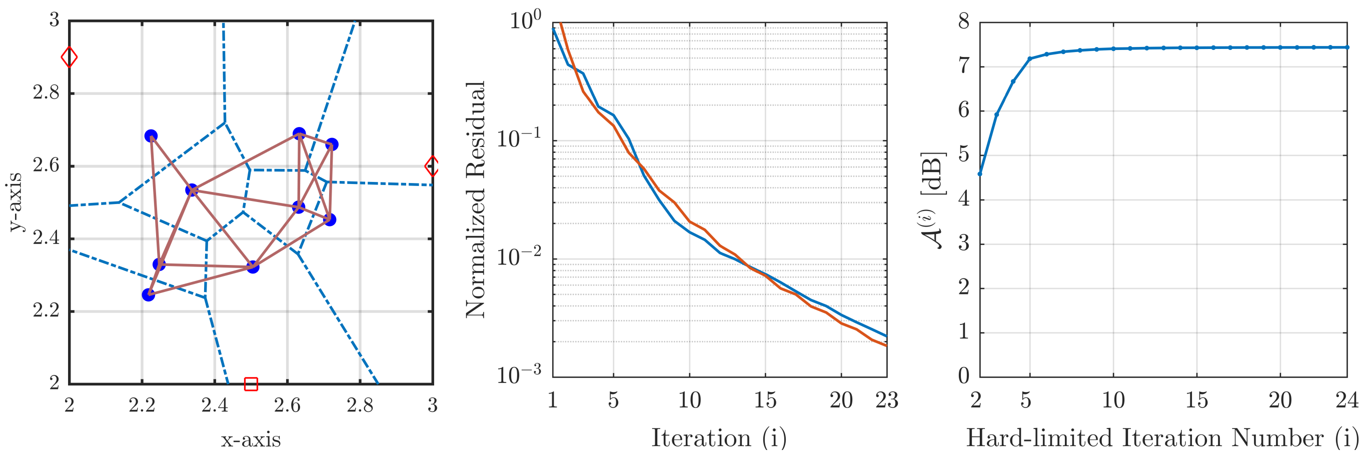
$$\boldsymbol{\Omega}_m = \mathbf{A}_m \mathbf{Q}^{-1} \mathbf{A}_m^H. \tag{24}$$

The dual problem of (18), which maximizes the dual function in (23), can be solved in different ways. For the PDMM method to be applicable, the dual problem is decoupled into

$$\begin{aligned}\min_{\bar{\boldsymbol{\mu}}_m} \quad & \sum_{m\in\mathcal{V}} \Big(\bar{\boldsymbol{\mu}}_m^H \boldsymbol{\Omega}_m \bar{\boldsymbol{\mu}}_m - 2\Re\Big\{ \bar{\mathbf{b}}^H \bar{\boldsymbol{\mu}}_m \Big\}\Big) \\ \text{s.t.} \quad & \bar{\boldsymbol{\mu}}_m = \bar{\boldsymbol{\mu}}_n \qquad \forall (m,n) \in \mathcal{E}, \end{aligned}\tag{25}$$

where $n \in \mathcal{N}(m)$ is connected to node m with an edge, and $\bar{\boldsymbol{\mu}}_m$ is the local dual parameter at node m.

Now that the optimization problem in (25) is formulated in accordance to the form in (11), the PDMM method can be used to solve it iteratively. Algorithm 1 shows the procedure, briefly. Depending on the choice of synchronous or asynchronous regimes, all or an active subset of nodes, $\mathcal{I} \subseteq \mathcal{V}$, are being updated at each iteration, respectively. For the contributing nodes,



(a) (b) (c)

Fig. 1. Experimental Results.

Algorithm 1 The proposed enhancement algorithm

set up the graph, form $\mathbf{Q}$ and $\bar{\mathbf{b}}$
while new time-frames are available **do**
  buffer L time-frames
  initialize $i := 0$, $\bar{\boldsymbol{\mu}}_n^{(0)} \leftarrow \mu_0$, $\bar{\boldsymbol{\nu}}_n^{(0)} \leftarrow \nu_0$
  update $\mathbf{A}_m$ and $\boldsymbol{\Omega}_m$
  repeat {PDMM updates}
    $i \leftarrow i+1$
    for $\forall m \in \mathcal{I}$ **do**
      forward edge: $r=-1$, backward edge: $r=+1$
      $\bar{\boldsymbol{\mu}}_m^{(i)} \leftarrow \left(\boldsymbol{\Omega}_m + \sum_{n\in\mathcal{N}(m)} \mathbf{R}_{mn}\right)^{-1} \left(\bar{\mathbf{b}} + \sum_{n\in\mathcal{N}(m)} \left(\mathbf{R}_{mn}\bar{\boldsymbol{\mu}}_n^{(i-1)} + r\bar{\boldsymbol{\nu}}_{n|m}^{(i-1)}\right)\right)$
      $\bar{\boldsymbol{\nu}}_{m|n}^{(i)} \leftarrow \bar{\boldsymbol{\nu}}_{n|m}^{(i-1)} - r\mathbf{R}_{mn}\left(\bar{\boldsymbol{\mu}}_m^{(i)} - \bar{\boldsymbol{\mu}}_n^{(i-1)}\right)$
    end for
  until convergence
  $\bar{\boldsymbol{\omega}}_m \leftarrow \mathbf{Q}^{-1}\mathbf{A}_m^H\bar{\boldsymbol{\mu}}_m$
end while

the set of dual parameters, $\{\bar{\boldsymbol{\mu}}_m^{(i)} | m \in \mathcal{I}\}$, are being calculated and then broadcasted to neighboring nodes, at the i-th iteration. The set of dual-dual parameters, $\{\bar{\boldsymbol{\nu}}_{m|n}^{(i)} | (m,n) \in \mathcal{E}\}$, are also get updated for the subset of active nodes; however, it is possible to either unicast them along the edges, or to calculate the dual-dual parameters of each edge, locally. Unicasting the dual-dual parameter has the advantage of making the algorithm robust against packet losses at the cost of increased communication overheads. Alternating signs along each edge, denoted by term r, is intrinsic to PDMM algorithm, but is only an assignment. The dual and dual-dual parameters can be initialized with zero vectors; however, it is possible to initialize them with the final values of the latest time-frame. With proper selection of the positive definite penalty terms, $\mathbf{R}_{mn}$, the algorithm will converge in acceptable number of iterations. To check convergence, one can look into the gradients.

III. Experiments

In order to clarify the theories in Section II, and to understand the behavior of the proposed algorithm, pilot experiments are performed in this section. In our experiments, speech signals from the TSP speech database are used for one desired and two interfering speakers. A sampling frequency of 8000 Hz is used with time-frames of 512 samples, with 50% overlapping Hanning window. A $5\times5\times3$ m room is simulated with the image method [30] with 150 ms reverberation time. The applicability of the algorithm was also verified using ray tracing method. Fig.1a shows an instance of such a geometry. Nine nodes (blue circles) in this random geographic network are connected only to the nodes in their neighborhood through edges (brown lines). The desired speaker (red square) and the interfering speakers (red diamonds) are placed at $(2.5, 2, 1.5)$ m, $(2, 2.9, 1.5)$ m, and $(3, 2.6, 1.5)$ m, respectively. Pseudo-coherence vectors are calculated using recursive time averaging with forgetting factor equal to 0.2. Here, $L=1$. Spatially white noise is added to microphones with variance equal to the variance of the desired speech at the reference microphone.

In the first experiment, we study the convergence of parameters of the proposed algorithm for the randomly positioned array with synchronous updates. Convergence curves for the dual and the dual-dual parameters are obtained at one arbitrary microphone by time and frequency averaging the normalized residuals. The normalized residuals for every element of the dual and dual-dual parameters are defined as

$$\xi_\mu^{(i)} = \frac{\left|\mu^{(i+1)} - \mu^{(i-1)}\right|}{2\mu^{(i)}}, \quad \xi_\nu^{(i)} = \frac{\left|\nu^{(i+1)} - \nu^{(i-1)}\right|}{2\nu^{(i)}}. \tag{26}$$

This definition allows us to average over different time-frequency bins without concern regarding the scale of dual and dual-dual parameters. As Fig.1b shows, the normalized residuals are decreasing very fast. Sufficient reduction in the residuals is achieved only within a few cycles. After 20 iterations, the residual of both the dual and the dual-dual parameters is decreased to about 0.2 percent.

In the second experiment the performance of the proposed algorithm w.r.t. noise/interference suppression is studied. The amount of interference plus noise is measured by time averaging the power of error signals at the input and the output of the beamformer as

$$e_{\rm in}(m) = \sum_t \sum_k \left|X_{\rm ref}(k,t) - Y_m(k,t)\right|^2,$$
$$e_{\rm rn}^{(i)}(m) = \sum_t \sum_k \left|X_{\rm ref}(k,t) - Z_m^{(i)}(k,t)\right|^2, \qquad (27)$$

where $e_{\rm in}(m)$ is the received error signal (considering both noise and interferences), and $Z_m^{(i)}(k,t)$ and $e_{\rm rn}^{(i)}(m)$ are the beamformer output signal and the remaining error signal at node m after i iterations, respectively. Then, the average array gain after i iterations is

$$\mathcal{A}^{(i)} = \frac{\sum_{m\in\mathcal{V}} \left|e_{\rm in}(m)\right|^2}{\sum_{m\in\mathcal{V}} \left|e_{\rm rn}^{(i)}(m)\right|^2}. \qquad (28)$$

Fig.1c shows how a hard-limit on the number of iterations affects the defined array gain. As can be seen, the array gain approaches its optimal value just after a few iterations.

IV. Conclusions

In this paper, we have proposed a distributed optimization algorithm for the minimum variance distortionless response beamformer using the primal-dual method of moments. The pseudo-coherence-based signal model is used in derivations. Experimental results show fast convergence for random graphic networks. Moreover, it is studied how the array gain for noise and interference suppression is affected by imposing hard-limits on the number of iterations. The results show that the overall performance of the microphone array just drops few dBs, even when only a few iterations are allowed.

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