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A General Statistical Framework for the Analysis of Tendency Survey Data of the IFO Type

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The purpose of the Danish Tendency Survey for Manufacturing Industries[2] is to provide rapid and current data on the business cycle in manufacturing industries, comprising data on current judgments over the preceding period and data on future expectations. The tendency survey is a supplement to other short-term statistics. To this is added information on a number of factors considered difficult to extract from the traditional statistics.

The publicised results are mainly descriptive statistics, the key indicator being the so-called *balance number*, defined as the difference in percentage-points between the proportions of respondents with an (actual or expected) increase and a decrease in e.g. production. Several indicators are used in the survey (production, employment, incoming orders, financial results etc.)

Despite the long history of the Tendency Survey (in Denmark it started in 1963) as well as in other countries, including Germany, where the method originated, to our knowledge no genuine statistical model for the statistical analysis of this special kind of data has been proposed. An explanation of this may be that econometricians are not trained in analyzing the kind of data that are generated by Tendency Surveys, i.e.

- Panel data, i.e. the same respondents are interviewed in repeated surveys
- The measurements are qualitative, i.e. nominally or ordinally scaled

An early effort was made in (Poulsen 1983) in Danish, using the latent class model. In this paper some extensions of this approach are considered. Especially, the dynamics of the data will be explored using latent Markov modelling. All models will be estimated using Latent GOLD in its most recent version 5.0 that includes a very flexible modelling syntax language, (Vermunt and Magidson 2013b). The program provides access to a whole class of dynamic models, based on panel data.

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[2] http://www.dst.dk/HomeUK/Guide/documentation/Varedeklarationer/emnegruppe/emne.aspx?sysrid=000934

Our purpose is to present an individual response model, based on (latent) Markov models. Hence, we will *not* be analyzing aggregate time series, derived from the IFO survey, e.g. Ifo Business Climate for industry and trade, (Abberger and Nierhaus 2010). Our work may be seen as a precursor to further analysis, proving more reliable input to subsequent modelling.

The plan of the paper is as follows. First, the latent class (LC) model and its application to the Tendency Survey are reviewed. Then we go on to the main focus in the paper: the application of the latent Markov (LM) model to capture the dynamics in the Tendency Survey. The *classes* of the static LC model are now seen as *states* in a dynamic model that allow individual (firms) to change their state from one time to the next according to a Markov process. Stationary as well as non-stationary models are admissible. Finally, in the concluding section we point to a more general class of Markov models - the Mixed Latent Markov (MLM) with time-constant and time-varying covariates - that offers a framework for Markov modelling that can be utilised in the further analyses of the Tendency Survey.

All applications are based on the Business Survey for Northern Jutland that has been published quarterly since 4th quarter 1998 till 4th quarter 2006 with two quarters (3rd q. 2004 and 1st q. 2006) missing and again from 1st quarter of 2011. A description can be found at the web-site www.business.aau.dk/njk/ and in (Christensen et al. 2005). In this paper only the most recent data from 2011 and onwards consider is used. For estimation of the models we use Latent Gold 5.0, (Vermunt and Magidson 2013a; Vermunt and Magidson 2013b).

One time point/wave – multiple indicators: The Latent Class (LC) model

We shall start by demonstrating the use of latent class (LC) analysis to the Business Cycle data, as already proposed in (Poulsen 1983). The LC model explains the observed association (dependency) between a set of categorical variables A, B, C, and D by way of a latent or unobserved categorical variable ξ that represents classes of respondents with different sets of response probabilities. When this variable is ignored, i.e. we only consider the aggregated table [ABCD], the variables are associated (an hypothesis of independence [A][B][C][D] cannot not be accepted). However, if ξ is controlled for, the dependency disappears: the model with conditional independency, given ξ, [Aξ][Bξ][Cξ][Dξ], cannot be rejected.

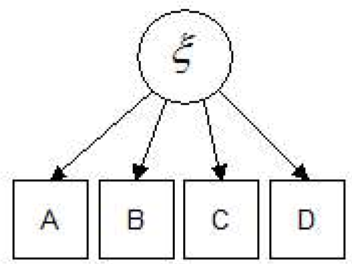
The LC model can be estimated, using the EM-algorithm, (Dempster et al. 1977). However, to speed things up, it is often combined with the Newton-Raphson which – in addition - gives asymptotic variance-covariances of the parameters as a by product.

To illustrate we consider the Tendency Survey for Northern Jutland for the 1st quarter of 2011. We focus on four variables (indicators) that compare the current with previous quarter (4^{th} quarter of 2010):

A: Production
B: Orders
C: Employment
D: Result

These are the four indicators used for describing the *short-term* development in the business cycle, cf. http://www.business.aau.dk/njk/konstruktion/Konstruktion.html. For each indicator three valid responses are possible: less, unchanged, or more. Although these may be treated in the model as ordinal, for simplicity we shall see them as nominal variables throughout this paper. Figure 1 gives a pictorial representation of the LC model.

Figure 1. The Latent Class model



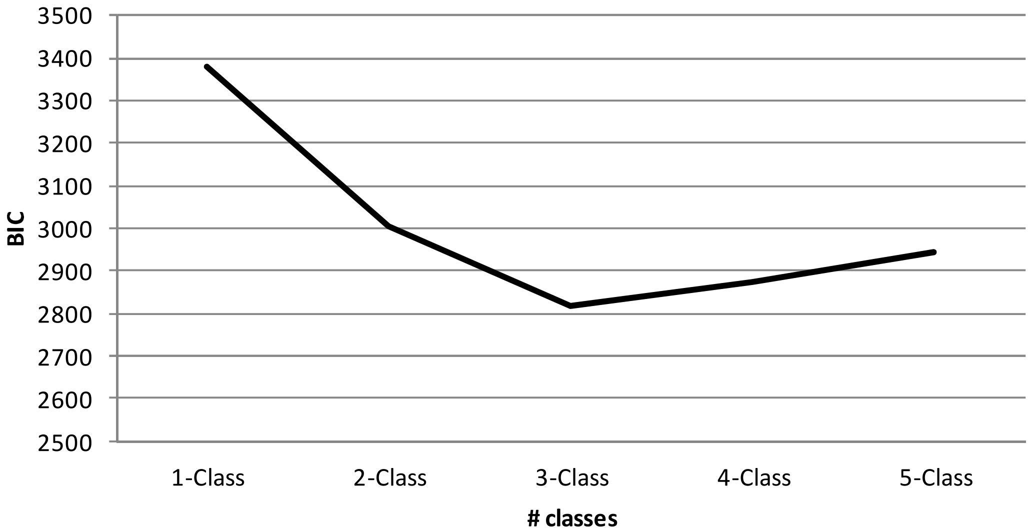
In terms of probabilities the LC model can be written:

$$
\begin{aligned}
\pi_{ABCD} &\equiv \sum_{\xi=1}^{\#\xi} \pi_{\xi} \cdot \pi_{ABCD|\xi} \\
&= \sum_{\xi=1}^{\#\xi} \pi_{\xi} \cdot \rho_{A|\xi} \cdot \rho_{B|\xi} \cdot \rho_{C|\xi} \cdot \rho_{D|\xi}
\end{aligned}
\tag{1}
$$

i.e. for a given number of classes $\#\xi$ the joint probabilities of the [ABCD] – table π_{ABCD} are given as the product-sum of the marginal probability π_{ξ} and the conditional probabilities $\rho_{A|\xi}, \rho_{B|\xi}, \rho_{C|\xi}$, and $\rho_{D|\xi}$. This property is called "local independence" in the literature on LC models.

Using LG 5.0 we start the analysis by exploring the 'best' number of levels (classes) of the latent variable ξ, using the BIC–criterion. The results point to a 3-class model and the estimated response probabilities are displayed in table 1.

Figur 2. The BIC-measure of 'goodness-of-fit' for #classes = 1, 2, 3, 4.



The overall pattern of response is a tendency to respond in a similar fashion across all four indicators, and the three clusters or classes correspond to the levels of responses on the indicators. Another way of looking at this structure is to see the LC model as a measurement model for the latent construct "short-term development" and interpret the probabilities on the main diagonal (from NW to SE) for each indicator as *reliabilities* for the indicators. Then, it is quite easy to interpret the three classes: In class 1 with 16% of the respondents, there is a tendency to answer 'less' on all four indicators. This class can be labelled 'truly less'. Similarly, class 2 with 51% of the respondents is 'truly unchanged' and class 3 with 33%, is 'truly more'.

The **bold** numbers on the main diagonal may be interpreted as loadings (as in factor analysis) or reliabilities (as in measurement theory). Their variation across indicators implies that latent variable ξ is related more closely to e.g." production" than to "results". Also, indicator "employment" is different from the other indicators by having the largest loading of any class associated with "unchanged". This may reflect the fact that hiring and laying-off people are more long-term decisions.

Table 1. Estimated response probabilities for the 3-class LC model.
Actual development 1st quarter 2011 compared with 4th quarter 2010.

	Class size		
	1	2	3
	0,159	0,511	0,331
	A: Production		
Class	less	unchanged	more
1	**0,789**	0,210	0,000
2	0,050	**0,885**	0,065
3	0,000	0,062	**0,938**
	B: Orders		
Class	less	unchanged	more
1	**0,984**	0,016	0,000
2	0,035	**0,835**	0,130
3	0,000	0,075	**0,925**
	C: Employment		
Class	less	unchanged	more
1	**0,455**	0,537	0,008
2	0,086	**0,814**	0,100
3	0,011	0,584	**0,405**
	D: Results		
Class	less	unchanged	more
1	**0,685**	0,303	0,013
2	0,179	**0,620**	0,201
3	0,009	0,268	**0,722**

One of the most useful features of the LC model is the computation of so-called "recruitment probabilities, i.e. the posterior probabilities of class memberships, given the observed responses (*abcd*):

$$\pi_{\xi|abcd} \equiv \frac{\pi_{\xi} \cdot \pi_{abcd|\xi}}{\pi_{abcd}} = \frac{\pi_{\xi} \cdot \rho_{a|\xi} \cdot \rho_{b|\xi} \cdot \rho_{c|\xi} \cdot \rho_{d|\xi}}{\sum_{\xi'}^{\#\xi} \pi_{\xi'} \cdot \rho_{a|\xi'} \cdot \rho_{b|\xi'} \cdot \rho_{c|\xi'} \cdot \rho_{d|\xi'}} \tag{2}$$

Based on the estimated model in table 1 we may now define an LC analogue to the balance number of actual development as the difference between the percentage of "truly more" and "truly less":

$$Balance\_Actual^{LC} = 33,1\% - 15,9\% = 17,2\%.$$

The focus of this paper, however, is not the latent structure at a given time point (panel wave), but rather the *dynamics*, i.e. how this latent structure develops over time. We want to formulate a dynamic model that reflects the latent structure, found in the static analysis, but at the same time allows for a dynamic development.

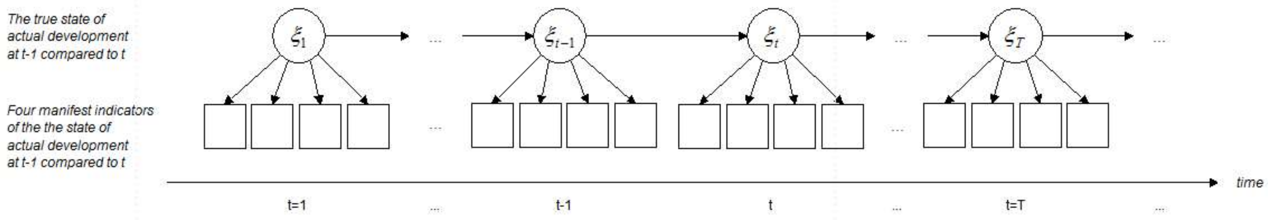
Multiple time points – multiple indicators: The latent Markov (LM) model

One possible model that fulfils these requirements is a (stationary) Markov model, defined on the latent table, and consequently called the latent Markov model. This was first proposed by (Wiggins 1973), while (Poulsen 1982) was the first to demonstrate how the model can be estimated by the EM-algorithm. The model may be seen as an extension of the traditional (manifest) Markov model to include measurement error in determining the states. The model can be applied to single or multiple indicators. We shall merely consider multiple indicators here.

Among the many possible specifications of Markov models, the most widely used is the first-order and stationary, i.e. the transition matrix is unchanged over time and transitions in the next period depends only on the states of the current one. Clearly, the stationary model is nested within a non-stationary one with time dependent transition probabilities. Further, the order restriction can be circumvented by redefining the state space and put restrictions on the transition matrix, see (Poulsen 1982) for how this can be done.

Markov models consist of two connected submodels: (i) a model for starting the process, i.e. the initial state probabilities and (ii) a model for the subsequent transitions. In Latent GOLD 5.0 different explanatory variables and/or covariates may be used for these submodels, i.e. variables that influence where the process starts can be different from the variables that govern the dynamics.

Figure 2. The Latent Markov model – one chain.



In terms of probabilities the LM model is defined by equation (3):

$$\pi_{\mathbf{X}_1\mathbf{X}_2\ldots\mathbf{X}_T} = \sum_{\xi_1}\sum_{\xi_2}\cdots\sum_{\xi_T}\pi_{\xi_1}\cdot\rho_{\mathbf{X}_1|\xi_1}\cdot\tau_{\xi_2|\xi_1}\cdot\rho_{\mathbf{X}_2|\xi_2}\cdot\ldots\cdot\tau_{\xi_T|\xi_{T-1}}\cdot\rho_{\mathbf{X}_T|\xi_T} \tag{3}$$

Here, $\mathbf{X}_t = \left[ABCD\right]_t$ is the table of observed responses at time point *t* and $\mathbf{X}_1\mathbf{X}_2\ldots\mathbf{X}_T$ is the entire response vector for *T* time points. Eq. (3) describes the probability of observing any pattern of responses as the product-sum of initial state probabilities π_{ξ_1}, conditional response probabilities $\rho_{\mathbf{X}_t|\xi_t}$, given the state, and transition probabilities $\tau_{\xi_t|\xi_{t-1}}$.

For each time point we can compute the estimated state proportions and define a dynamic version, *Balance_Actual* ^{LM}, of the balance number, corresponding to the static measure *Balance_Actual* ^{LC}. The results are displayed as part of in figure 3. Here we have included two additional aspects of the LM model: the possibility to include time-varying covariates, viz. a seasonal variation in the state proportions, and ability to project the development beyond the periods of data collection. The season factor is a qualitative variable with four levels, one for each quarter, which may enter the model using dummy-coded variables. Note that in this way we have introduced time-variation in the otherwise stationary Markov model.

Table 2. The estimated LM model with three latent states and four manifest indicators for each panel wave. Actual development 1st quarter 2011 compared with 4th quarter 2013.

Initial state probabilities $\hat{\pi}_{\xi_1}$

actual[=0]		
"less"	"unchanged"	"more"
0,174	0,453	0,374

Transition probabilities $\hat{\tau}_{\xi_t|\xi_{t-1}}$

	actual			
actual[-1]	"less"	"unchanged"	"more"	Total
"less"	0,443	0,312	0,245	1,000
"unchang	0,182	0,582	0,236	1,000
"more"	0,162	0,303	0,535	1,000

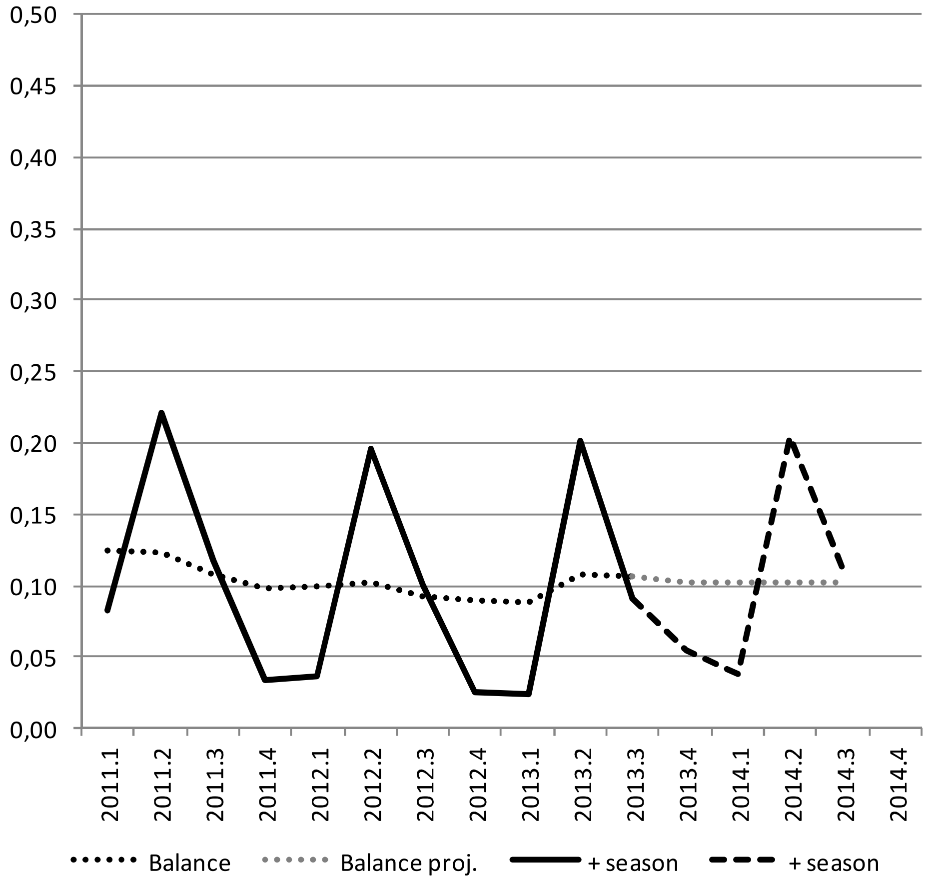
Conditional response probabilities

	A: Production		
State	less	unchanged	more
"less"	**0,832**	0,160	0,008
"unchanged"	0,009	**0,962**	0,029
"more"	0,007	0,103	**0,891**
	B: Orders		
State	less	unchanged	more
"less"	**0,899**	0,073	0,028
"unchanged"	0,048	**0,874**	0,079
"more"	0,018	0,097	**0,885**
	C: Employment		
State	less	unchanged	more
"less"	**0,516**	0,467	0,017
"unchanged"	0,088	**0,853**	0,059
"more"	0,028	0,475	**0,496**
	D: Results		
State	less	unchanged	more
"less"	**0,773**	0,179	0,048
"unchanged"	0,127	**0,750**	0,124
"more"	0,050	0,253	**0,698**

Figure 3. Development in *Balance_Actual* ^{LM} with and without seasonal factor.

1st quarter 2011 – 1st quarter 2014 is based on empirical data.

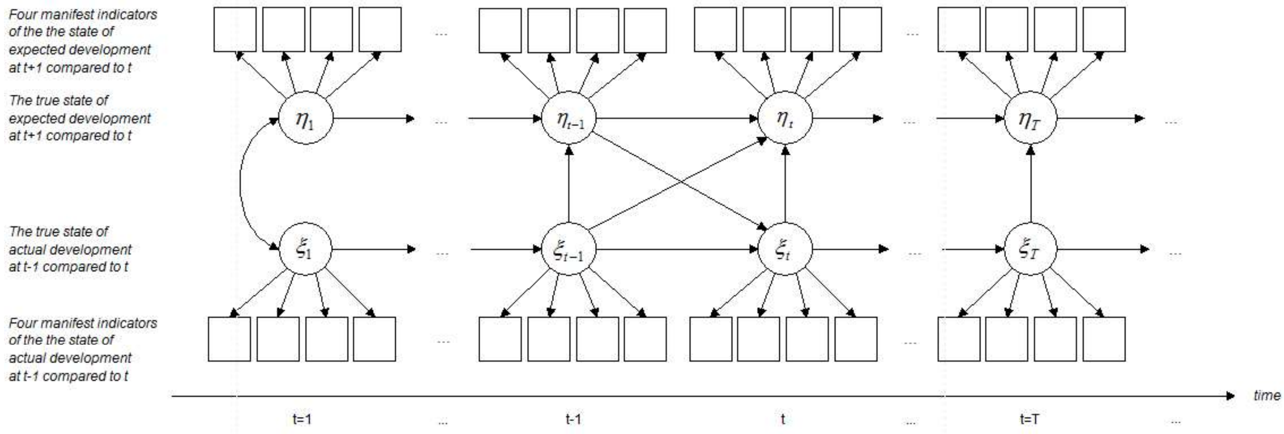
2nd quarter 2014 – 3rd quarter 2014 is projected.



Two recursively related Latent Markov chains

We now consider the case involving more than one Markov chain, see ??? for an early example. Consider the two chains representing the actual and expected development. An obvious question is: how closely are expectations and realisations over time related? How good are expected quarterly developments as predictors of the subsequently reported realised developments? To answer these questions we consider a model with two Markov chains that are recursively interdependent, cf. figure 4.

Figure 4. The Latent Markov model – two recursively related chains



Probabilistically the structure is expressed in equation (4):

$$\pi_{\xi_1\eta_1\mathbf{X}_1\mathbf{Y}_1\xi_2\eta_2\mathbf{X}_2\mathbf{Y}_2\ldots\xi_T\eta_T\mathbf{X}_T\mathbf{Y}_T} = \sum_{\xi_1,\eta_1}\sum_{\xi_2,\eta_2}\cdots\sum_{\xi_T,\eta_T}\pi_{\xi_1\eta_1}\cdot\rho_{\mathbf{X}_1\mathbf{Y}_1|\xi_1\eta_1}\cdot\tau_{\xi_2\eta_2|\xi_1\eta_1}\cdot\rho_{\mathbf{X}_2\mathbf{Y}_2|\xi_2\eta_2}\cdot\ldots\cdot\tau_{\xi_T\eta_T|\xi_{T-1}\eta_{T-1}}\cdot\rho_{\mathbf{X}_T\mathbf{Y}_T|\xi_T\eta_T} \quad (4)$$

Based on the prior information about the time ordering among the variables, the joint probability, $\pi_{\xi_1\eta_1\mathbf{x}_1\mathbf{y}_1\xi_2\eta_2\mathbf{x}_2\mathbf{y}_2\ldots\xi_T\eta_T\mathbf{x}_T\mathbf{y}_T}$ in (4) may be decomposed into a set of conditional probabilities. According to figure 3, we assume that the variable ξ_t is (in part) causing the states η_t, a further decomposition is natural:

$$\tau_{\xi_t\eta_t|\xi_{t-1}\eta_{t-1}} = \tau_{\eta_t|\xi_t\xi_{t-1}\eta_{t-1}}\cdot\tau_{\xi_t|\xi_{t-1}\eta_{t-1}};t=2,\ldots,T \quad (5)$$

Note, however, that the model (4) cannot in general determine cause and effects–relationships between concurrent variables/states. In addition, as conditional independence in the response probabilities, given the current state, is usually assumed, a further simplification is obtained by introducing

$$\rho_{\mathbf{X}_t\mathbf{Y}_t|\xi_t\eta_t} = \rho_{\mathbf{X}_t|\xi_t}\cdot\rho_{\mathbf{Y}_t|\eta_t};t=1,\ldots,T \quad (6)$$

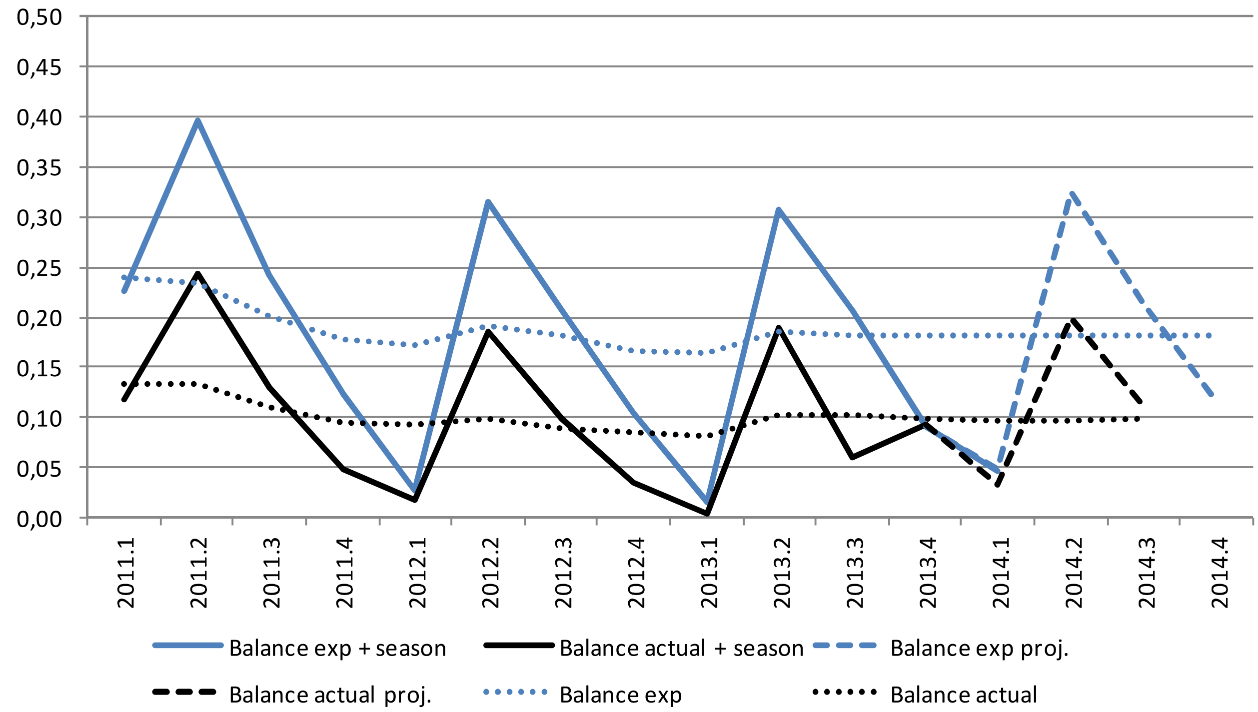
To save space we merely report the resulting profiles of the balance numbers for the two processes, cf. figure 5.

We note immediately that expectations are systematically above the realised results when the balance numbers are computed based on the latent states rather than the manifest variables. Interestingly, the smallest error between expected and actual balance figure is at the first quarter of the year. Whether this is related to the budgeting process around New Year remains to be confirmed.

Figure 5. Model based balance numbers. Expectations and actual result.

1st quarter 2011 – 1st quarter 2014 is based on empirical data.

2nd quarter 2014 – 3rd quarter 2014 is projected.



Several time points – mixed latent Markov model

So far, we have (implicitly) assumed that the parameters of LM model are shared by all firms in the panel. This may suffice for a first approximation when the purpose of the model is prediction of future aggregate development. Heterogeneity across firms should, however, be allowed for, when a more detailed analysis is called for. There are two ways to accommodate heterogeneity between respondent firms. One is to perform a group analysis, where an observed variable, like size, branch, geography, is available, and the LM model may be estimated in full or partly as conditional on this variable. Since the grouping variable is observed (without error) this procedure may be called manifest or observed heterogeneity. In contrast, latent or unobserved heterogeneity may be allowed for by introducing a latent class variable that accounts heterogeneity that cannot be referred to a single group variable. This is called the Latent Class Latent Markov or Mixed Latent Markov model, first proposed by (Langeheine 1990).

The probabilistic structure of the grouped LM model is:

$$
\begin{aligned}
\pi_{\xi_1\eta_1\mathbf{X}_1\mathbf{Y}_1\xi_2\eta_2\mathbf{X}_2\mathbf{Y}_2\ldots\xi_T\eta_T\mathbf{X}_T\mathbf{Y}_T} &= \sum_{g=1}^{G}\pi_g\cdot\pi_{\xi_1\eta_1\mathbf{X}_1\mathbf{Y}_1\xi_2\eta_2\mathbf{X}_2\mathbf{Y}_2\ldots\xi_T\eta_T\mathbf{X}_T\mathbf{Y}_T|g} \\
&= \sum_{g=1}^{G}\pi_g\cdot\sum_{\xi_1,\eta_1}\sum_{\xi_2,\eta_2}\cdots\sum_{\xi_T,\eta_T}\pi_{\xi_1\eta_1|g}\cdot\rho_{\mathbf{X}_1\mathbf{Y}_1|\xi_1\eta_1 g}\cdot\tau_{\xi_2\eta_2|\xi_1\eta_1 g}\cdot\rho_{\mathbf{X}_2\mathbf{Y}_2|\xi_2\eta_2 g}\cdot\ldots\cdot\tau_{\xi_T\eta_T|\xi_{T-1}\eta_{T-1}g}\cdot\rho_{\mathbf{X}_T\mathbf{Y}_T|\xi_T\eta_T g}
\end{aligned}
\tag{7}
$$

where the group variable has $g=1,2,\ldots,G$ levels. Usually, restrictions are put on the parameters, e.g. response probabilities are common to all groups, $\rho_{\mathbf{X}_t\mathbf{Y}_t|\xi_t\eta_t g}=\rho_{\mathbf{X}_t\mathbf{Y}_t|\xi_t\eta_t}, \forall t$, independent of g.

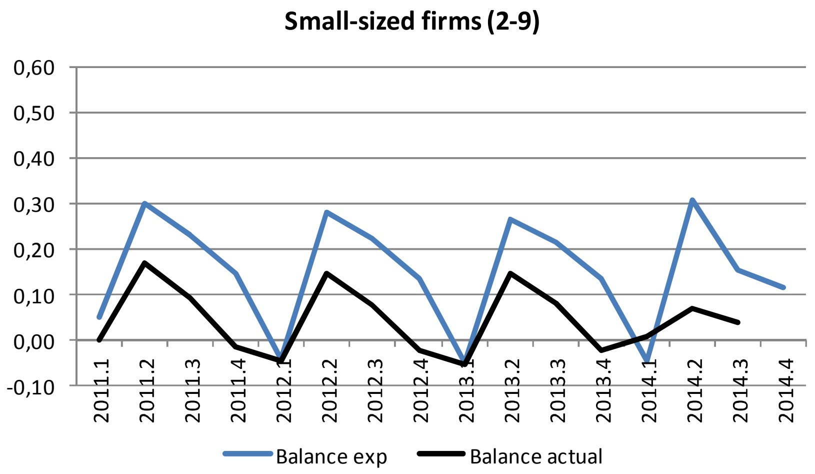
The Mixed LM model has formally the same structure. However, in this case class sizes have to be estimated in addition.

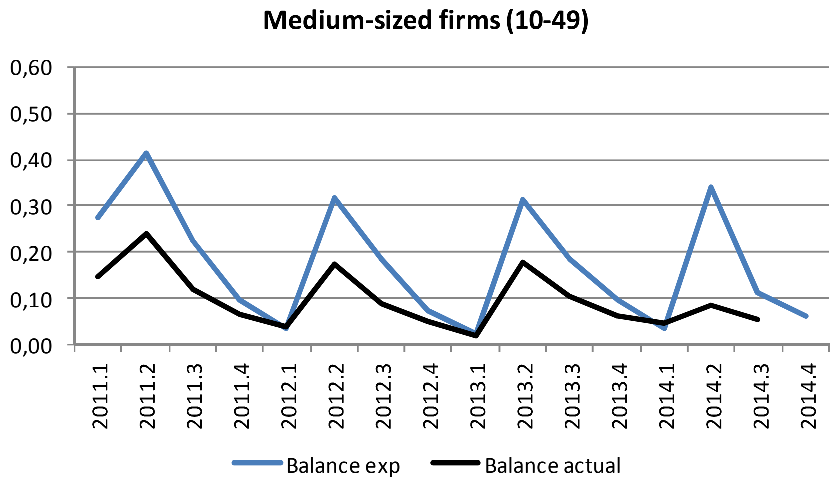
We illustrate the group analysis, using the variable firm size, defined by the number of employees with G=3 levels: small (2-9), medium (10-49), and large (50+) employees. Model based balance numbers are estimated for the tree groups, assuming common response error structure. The results are displayed in figure 6.

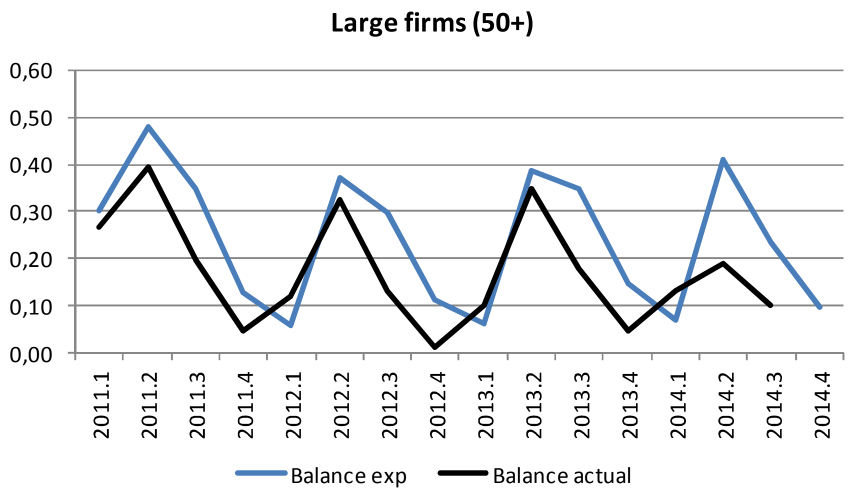
Although the overall patterns are the same we notice that the variation takes place at different levels with small-sized firms at the lowest, and large firms at the highest.

(A Mixed LM model with two latent groups was estimated, but not justifiable compared to the one-group solution according the BIC-criterion).

Figure 6. Model based balance numbers for three size groups. Expectations and actual result with seasonal variation.
1st quarter 2011 – 1st quarter 2014 is based on empirical data.
2nd quarter 2014 – 3rd quarter 2014 is projected.







Conclusion

As stated in the introduction our main purpose in this paper is to propose a statistical framework for analysing Business Cycle Surveys that take their special data qualities, i.e. panel data (repeated measurements at the individual level) and nominally/ordinally scaled response variables, into account. The framework seems rich enough to test a number of research questions that so far has gone unanswered. To name a few: How is predictions and actual development related? Does the precision change during the business cycle? Are small firms better or worse as forecasters of their future? Can turning points be predicted more reliably? Also, the external validity of Tendency Surveys may be tested by relating the data e.g. the Balance Number, to time series, available at a later date. We hope with this paper to have spurred some interest in working along these lines.

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