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Semi-analytical approach to modelling the dynamic behaviour of soil excited by embedded foundations

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Abstract

The underlying soil has a significant effect on the dynamic behaviour of structures. The paper proposes a semi-analytical approach based on a Green's function solution in frequency–wavenumber domain. The procedure allows calculating the dynamic stiffness for points on the soil surface as well as for points inside the soil body. Different cases of soil stratification can be considered, with soil layers with different properties overlying a half-space of soil or bedrock. In this paper, the soil is coupled with piles and surface foundations. The effects of different foundation modelling configurations are analysed. It is determined how simplification of the numerical model affects the overall dynamic behaviour.

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Keywords: Ground vibration; Green's function; foundation modelling.

1. Introduction

Proper evaluation of vibrations is a complex problem, which is especially hard to reproduce numerically. Vibration propagation through soil can be modelled using the Finite Element Method (FEM), but this approach requires modelling of large soil domains and special boundary conditions, and this leads to high computation times and cumbersome calculation procedures. For computational tools to be useful in practice—especially in the early design phase—when a large number of different cases need to be analysed, they have to be relatively fast. Therefore, more computationally efficient approaches are needed.

One way of modelling the response of the soil is by using a semi-analytical approach, based on the Green's function. The semi-analytical solution provides an analytical solution for the Green's function if frequency–wavenumber

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domain and afterwards a numerical inverse Fourier transform is performed. For the formulation used in this paper, a layer transfer matrix is used. The transfer matrix describes the displacements and traction relation between the top and the bottom of a single layer. The layer transfer matrix was developed by Thomson [1] and further expanded by Haskell [2]. The Green's function approach has been commonly used for various problems concerning the vibrations of soil. Sheng and Jones [3] used it to model the vibrations propagating from a railway track placed on the soil surface. Further solutions for rigid surface footings were provided by Andersen and Clausen [4] and Lin *et al.* [5]. An alternative formulation to the Green's function approach is the stiffness matrix approach by Kausel and Roesset [6]. This method still uses the same layer transfer matrix, however the solution is formulated in terms of stiffness not flexibility.

While the Green's function based model is well known, it is most commonly used to model structure–interaction through the soil surface. This work aims to provide an approach to finding the Green's function between points not only on the soil surface, but also embedded inside the soil body. The obtained system can be modified in order to couple the soil with structures modelled using the FEM. The formulation of the solution is given in Section 2. Further, in Section 3, to illustrate the described approach, some analyses are carried out to evaluate how different foundation-modelling approaches affect the surrounding soil behaviour. Finally, Section 4 list the main conclusions.

Semi-analytical soil model

1.1. Transfer matrix for layered soil based on Green's function

Consider two points: in or on a horizontally stratified half-space Point 1 placed at the coordinates (x_1, y_1, z_1) and Point 2 placed at (x_2, y_2, z_2). The traction p_1 is applied at Point 1 and at time t_1, while the displacement resulting from the load are investigated at Point 2 at time t_2. The relation between the traction and displacement can be expressed using Green's function, provided in time–space domain:

$$\mathbf{u}_2(x_2, y_2, z_2, t_2) = \int_{-\infty}^{t_2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{0} \mathbf{g}_{12}(x_2 - x_1, y_2 - y_1, z_1, z_2, t_2 - t_1)\mathbf{p}_1(x_1, y_1, z_1, t_1)\mathrm{d}z_1\, \mathrm{d}y_1\, \mathrm{d}x_1\, \mathrm{d}t_1. \tag{1}$$

Unfortunately, an analytical expression for $\mathbf{g}_{12}$ cannot be found. To overcome this problem, a triple Fourier transformation is performed over two spatial coordinates (x, y) and time. The resulting equation is:

$$\mathbf{U}_2(k_x, k_y, z_2, \omega) = \mathbf{G}_{12}(k_x, k_y, z_1, z_2, \omega)\mathbf{P}_1(k_x, k_y, z_1, \omega), \tag{2}$$

where $\mathbf{U}_2, \mathbf{P}_1$, $\mathbf{G}_{12}$ are the triple Fourier transforms of displacement, traction and Green's function, respectively. Further, k_x and k_y are the wavenumbers in the respective horizontal coordinate directions and ω is the circular frequency. The displacement and traction acting on the interface of a layer are combined in the state vector $\mathbf{S}^{j,i}$:

$$\mathbf{S}^{j,i} = [(\mathbf{U}^{j,i})^{\mathrm{T}} \quad (\mathbf{P}^{j,i})^{\mathrm{T}}]^{\mathrm{T}}. \tag{3}$$

The layer number is denoted by the first superscript, j, while the second superscript, i, denotes the top interface of the layer, when equal to 0, or the bottom interface, when equal to 1.

The expression for the Green's function is obtained by solving the Navier equations in frequency–wavenumber domain. After some manipulation, the following expression is obtained, as described by Thomson [1] and Haskell [2]:

$$\mathbf{S}^{j,1} = \mathbf{B}^j \mathbf{S}^{j,0} = \mathbf{S}^{j-1,0}. \tag{4}$$

Matrix $\mathbf{B}^j$ is the transfer matrix and describes the relationship between displacements and tractions at the top and bottom of layer j. A detailed description of the procedure to obtain the transfer matrix can also be found in the work by Sheng *et al.* [3] as well as Andersen and Clausen [4]. The second reformulation in Eq. (4) prescribes continuity of displacements and equilibrium of traction, between adjacent layers at interfaces.

Following the same procedure, a relation between the soil surface $\mathbf{S}^{1,0}$ and the bottommost interface $\mathbf{S}^{J,1}$ can be found. Starting from Layer J and going upwards through the layers, the following is obtained:

$$\mathbf{S}^{J,1} = \mathbf{B}^J \mathbf{B}^{J-1} \mathbf{B}^{J-2} \cdots\, \mathbf{B}^1 \mathbf{S}^{1,0}. \tag{5}$$

Now assume that a forced discontinuity in the displacement and/or traction occurs at an interface via the incremental state vector $\Delta\mathbf{S}^{n,0}$ as illustrated in Fig. 1. This results in the following state at the bottom of Layer J:

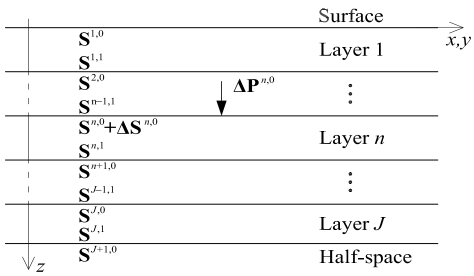


Fig. 1. Assembly of multiple soil layers over a half-space.

$$\mathbf{S}^{J,1} = \mathbf{B}^{J}\mathbf{B}^{J-1}\mathbf{B}^{J-2}\cdots\ \mathbf{B}^{n}(\mathbf{S}^{n,0} + \Delta\mathbf{S}^{n,0}). \tag{6}$$

Including all the layers up to the soil surface, Eq. (7) can be reformulated into

$$\mathbf{S}^{J,1} = \mathbf{T}^{1}\mathbf{S}^{1,0} + \mathbf{T}^{n}\Delta\mathbf{S}^{n,0}; \qquad \mathbf{T}^{n} = \mathbf{B}^{J}\mathbf{B}^{J-1}\mathbf{B}^{J-2}\cdots\ \mathbf{B}^{n+1}\mathbf{B}^{n}. \tag{7}$$

The stratum is assumed to overly a homogeneous half-space of soil. The relationship between the soil displacement and traction acting at the top of the half-space can be defined as:

$$\mathbf{U}^{J+1,0} = \mathbf{G}_{\mathrm{hh}}\mathbf{P}^{J+1,0}. \tag{8}$$

The half-space is denoted as Layer $J+1$, and $\mathbf{G}_{\mathrm{hh}}$ is the Green's function for the half-space defining a relation between displacement and traction acting on the surface. The derivation of this expression can be found in Ref. [4].

The traction and displacements at the bottom of Layer J have to be equal to the traction and displacement at the top of Layer $J+1$, as shown by Eq. (5). Combining Eq. (8) and Eq. (9), and further assuming that the traction applied to the soil surface and displacement discontinuity applied at Layer n are both equal to zero, the following is obtained:

$$\begin{bmatrix}\mathbf{U}^{J+1,0}\\ \mathbf{P}^{J+1,0}\end{bmatrix} = \begin{bmatrix}\mathbf{U}^{J,1}\\ \mathbf{P}^{J,1}\end{bmatrix} = \begin{bmatrix}\mathbf{G}_{\mathrm{hh}}\mathbf{P}^{J,1}\\ \mathbf{P}^{J,1}\end{bmatrix} = \begin{bmatrix}\mathbf{T}_{11}^{1} & \mathbf{T}_{12}^{1}\\ \mathbf{T}_{21}^{1} & \mathbf{T}_{22}^{1}\end{bmatrix}\begin{bmatrix}\mathbf{U}^{1,0}\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{T}_{11}^{n} & \mathbf{T}_{12}^{n}\\ \mathbf{T}_{21}^{n} & \mathbf{T}_{22}^{n}\end{bmatrix}\begin{bmatrix}\mathbf{0}\\ \Delta\mathbf{P}^{n,0}\end{bmatrix}. \tag{9}$$

This results in a relationship between the traction applied at the top of Layer n and the displacement on the soil surface:

$$\mathbf{U}^{1,0} = \mathbf{G}_{\mathrm{lh}}\Delta\mathbf{P}^{n,1}; \qquad \mathbf{G}_{\mathrm{lh}} = (\mathbf{G}_{\mathrm{hh}}\mathbf{T}_{21}^{1} - \mathbf{T}_{11}^{1})^{-1}(\mathbf{T}_{12}^{n} - \mathbf{G}_{\mathrm{hh}}\mathbf{T}_{22}^{n}). \tag{10}$$

The matrix $\mathbf{G}_{\mathrm{lh}}$ is the Green's function for displacements on the soil surface caused by a traction applied at the top of the Layer n. To calculate the displacements at lower layers, the original Eq. (4) is used. Further, similar relations can be established for a layered stratum over rigid bedrock.

1.2. Coupling of the layered soil model to a finite element model

The obtained Green's function describes a relation between two points for traction and displacements in three directions (k_x, k_y, z). Calculating a dynamic stiffness matrix for the soil that can be coupled with an FEM model to analyse soil–structure interaction (SSI) involves a number of steps:

1. The Green's function needs to be established for a sufficient number of wavenumbers k_x and k_y to ensure good coverage of the analysed soil body. This can be achieved by evaluating the Green's function along a single axis in the wavenumber domain and rotating the result according to the needed combinations of k_x and k_y.
2. The displacement at a point caused by a distributed load of unit magnitude centred on another point is found in frequency–wavenumber domain and converted to frequency–space domain by inverse Fourier transformation. This is carried out for every combination of two nodal points in which the FEM model interacts with the soil.
3. The obtained 3×3 matrices, describing the relations between two points in three directions, are placed in a single matrix $\mathbf{G}$ with dimensions $3m \times 3m$, when m is the total number of SSI nodes in the system.
4. Unit displacements are prescribed for each degree of freedom in the system and stored in matrix $\mathbf{U_0}$. If n SSI nodes move together as a rigid body, the number of degrees of freedom associated with these points is reduced from $3n$ to 6. The unit displacement matrix size is $3m \times l$, where l is the number of degrees of freedom in the system, and $l \leq 3m$.
5. The Green's function describes the displacements due to the applied loads. Thus:

$$\mathbf{G}\mathbf{P_0} = \mathbf{U_0}. \tag{11}$$

The matrix $\mathbf{P_0}$ containing the acting forces from unit displacement can then be found. It is further integrated over the contact area between rigid bodies and soil (if any rigid bodies are present) to obtain the dynamic stiffness matrix $\mathbf{D}_{\mathrm{soil}}$. This can be achieved by pre-multiplying matrix $\mathbf{P_0}$ with the transposed unit displacement matrix:

$$\mathbf{D}_{\mathrm{soil}} = \mathbf{U_0}^{\mathrm{T}}\mathbf{P_0}. \tag{12}$$

The final obtained dynamic stiffness matrix has the dimensions $l \times l$. To couple the soil with finite elements using the standard stiffness matrix $\mathbf{K}_{\mathrm{FE}}$, damping matrix $\mathbf{C}_{\mathrm{FE}}$ and mass matrix $\mathbf{M}_{\mathrm{FE}}$, the dynamic stiffness matrix for SSI is defined as:

$$\mathbf{D}_{\mathrm{full}} = \mathbf{D}_{\mathrm{FE}} + \mathbf{D}_{\mathrm{soil}}; \qquad \mathbf{D}_{\mathrm{FE}} = \mathbf{K}_{\mathrm{FE}} + \mathrm{i}\omega\mathbf{C}_{\mathrm{FE}} - \omega^2\mathbf{M}_{\mathrm{FE}}. \tag{13}$$

Further coupling with a structure above ground level can be obtained by classical FEM assembly. The final model is a computationally efficient solution that still considers fully coupled structure–soil system.

2. Foundation modelling using the semi-analytical soil model

2.1. Rigid and flexible surface footings

A soil body excited by a surface footing is analysed. The surface footing is square, with a length of 2 m. The height of the footing is 0.6 m. It is constructed from concrete, for which the material properties are given in Table 1. The surface footing is modelled using two approaches.

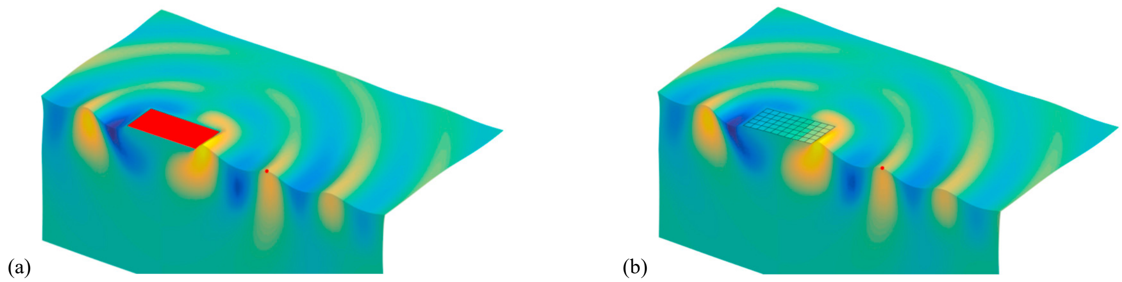
The first approach is to model the footing as a rigid body. In this case, the local deformations of the footing are not considered. Therefore, only the mass density is utilized, whereas the remaining material properties (Young's modulus, Poisson's ratio and damping ratio) are not used. In this case, the footing contact area is discretized into a number of points on the soil surface and, by using the procedure described in Section 2.2, an impedance matrix for 6 degrees of freedom is obtained. The six degrees of freedom include three translational and three rotational degrees of freedom. The footing mass and rotational mass moment of inertia are added to the corresponding degrees of freedom.

The second approach is to model the footing using Mindlin shell finite elements with quadratic interpolation and selective integration. A description of the elements can be found in Ref. [7]. The interface between the soil and the footing is once again discretized into the same number of points. In this case each point has only three translational (or displacement) degrees of freedom, which are coupled with the nodes of the shell elements.

Two different soil cases are tested. The first case considers a half-space of clay material (all the material properties are given in Table 1), while in the second case a half-space of sand is overlaid by a 3 m thick layer of clay. The

Table 1. Materials used in the calculations.

Material	Young's modulus (MPa)	Poisson's ratio (-)	Mass density (kg/m^3)	Damping ratio (-)
Clay	100	0.48	2000	0.045
Sand	250	0.30	2000	0.050
Concrete	34000	0.15	2400	0.010



(a) (b)

Fig. 2. Surface footing modelled as rigid (Case a) and flexible body (Case b), excited at 35 Hz. Soil stratification: 3 m layer of clay overlaying half-space of sand. Blue/yellow shades indicate positive/negative displacements in the *z*-direction. The red dot is the observation point for soil displacement analysis.

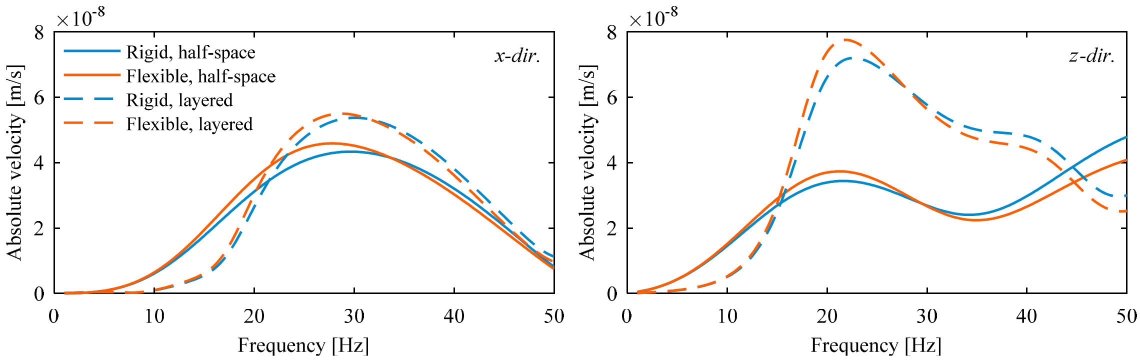


Fig. 3. Velocity dependency on frequency for a point placed 4m from the footing. The footing is modelled as a rigid or flexible plate.

footings are excited by applying a unit moment around the *y*-axis, at every frequency. The behaviour of the footings and soil for one frequency is illustrated in Fig. 2. Further, the soil displacements are analysed at a point placed 4 m from the edge of the footing, on the *x*-axis (position shown in Fig. 2). The results are illustrated in Fig. 3.

From Fig. 2 and Fig. 3 it can be seen that the two approaches provide very similar results. The flexible foundation produces somewhat higher excitation for lower frequencies, but the difference is not significant. Representing the surface footing as a completely stiff plate can be concluded to be a fair assumption in the considered frequency range. However, care should be taken as thinner footings might provide different results.

2.2. Pile foundations

A single pile foundation is analysed. A 5 m long pile is embedded 3 m into the soil body. The pile is modelled using three-dimensional beam elements with six degrees of freedom in each node. A detailed description of the elements is available in Ref. [8]. To couple with soil, the embedded part of the pile is discretized into SSI nodes for which the soil dynamic stiffness matrix is found. The connecting SSI nodes are modelled in three different ways:

1. Each connecting node is a single point in the soil. It has three degrees of freedom which are later coupled to the pile. Therefore, rotational degrees of freedom of the pile are not coupled to the soil.
2. For each connecting node, a horizontal rigid disc with the same diameter as the pile is created, illustrated in Fig. 4a. The disc is discretized into a number of points and the dynamic stiffness is obtained. In this case, the rigid disc has 6 degrees of freedom that are coupled to the translational and rotational degrees of freedom of the pile.
3. Instead of the disk, a ring with same the diameter as the pile is created, see Fig. 4b. The ring is assumed rigid and later coupled to the six degrees of freedom of the pile.

The same two soil-stratification cases as in previous subsection are used. The system is excited by placing a unit moment around the *y*-axis at the very top of the pile. The behaviour of the system with rigid discs and rings, excited at 45 Hz, can be seen in Fig. 4. Further, the displacements of the soil surface 3 metres from the pile are investigated (see Fig. 5). It is observed that different approaches produce significant differences in the behaviour of the system. The peaks, corresponding to the first eigenmode of the pile, are at 5.4 Hz, 10.8 Hz and 11.4 Hz, respectively. This could lead to dramatically different behaviour if used for a bigger system. The first approach, while computationally

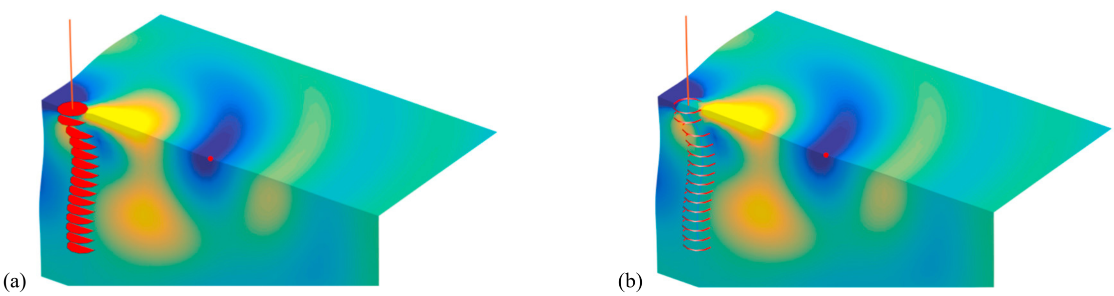


Fig. 4. Pile modelled with rigid disks (Case a) and rings (Case b), excited at 45 Hz. Soil stratification: half-space of clay. Blue/yellow shades indicate positive/negative displacements in the *z*-direction. The red dot is the observation point for soil displacement analysis.

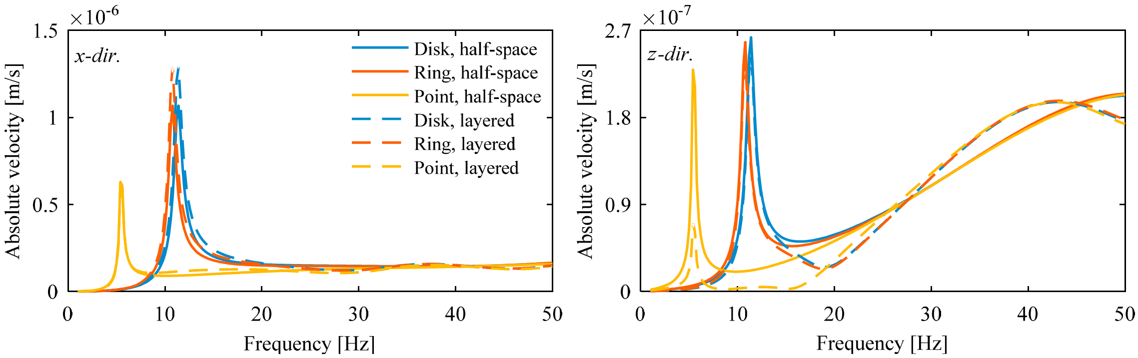


Fig. 5. Velocity dependency on frequency for a point placed 3m from the pile. The interaction points between the soil and pile modelled with three different approaches.

faster compared the other two, underestimates the stiffness added to the pile by the soil. This is caused by the lack of rotational stiffness added from the soil. The second and third approaches produce results much closer to each other. However, to determine the most suitable approach, further validation should be carried out. Most likely, the exact choice of pile modelling approach should be determined depending on the particular modelling case.

3. Conclusions

A semi-analytical soil model is described in this work. A formulation for obtaining Green's function between two points embedded at different depths in the stratum is provided. The semi-analytical model has some advantages when compared to modelling the same problem using an FEM based approach. This approach does not require complicated boundary conditions as infinite boundaries are already assumed in the formulation. Therefore, when optimized correctly, the semi-analytical approach is faster for a wide range of analyses. This formulation could be useful when considering various embedded foundation types or underground structures.

Further, some cases of modelling foundations using the described numerical approach are presented. Surface footings are modelled assuming the footing to be rigid or flexible, modelling the footing using shell finite elements. It was found that the difference between the two approaches is not significant. However, caution should be taken when considering thinner and more flexible surface footings, as investigated by Iguchi and Luco [9].

Finally, pile foundations were analysed. Three different approaches of coupling the pile to the soil were considered. It was found that the coupling between the pile and the soil has a significant effect and that a proper model of the pile cross section interacting with the soil must be used.

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References

[1] W. T. Thomson, "Transmission of elastic waves through a stratified solid medium", Journal of applied Physics 21.2 (1950) 89–93.
[2] N. A. Haskell, "The dispersion of surface waves on multilayered media", Bulletin of the seismological Society of America 43.1 (1953) 17–34.
[3] X. Sheng, C. J. C. Jones, and m. Petyt, "Ground vibration generated by a harmonic load acting on a railway track", Journal of sound and vibration 225.1 (1999) 3–28.
[4] L. Andersen and J. Clausen, "Impedance of surface footings on layered ground", Computers & structures 86.1 (2008) 72–87.
[5] G. Lin, Z. Han, and J. Li, "An efficient approach for dynamic impedance of surface footing on layered half-space", Soil Dynamics and Earthquake Engineering 49 (2013) 39–51.
[6] E. Kausel and J. M. Roësset. "Stiffness matrices for layered soils", Bulletin of the Seismological Society of America 71.6 (1981) 1743-1761.
[7] O. C. Zienkiewicz and R. L. Taylor, "The finite element method for solid and structural mechanics", Butterworth-heinemann, 2005.
[8] M. Paz, "Structural dynamics: theory and computation", Springer Science & Business Media, 2012.
[9] M. Iguchi and J. E. Luco, "Dynamic response of flexible rectangular foundations on an elastic half-space.", Earthquake Engineering & Structural Dynamics 9.3 (1981) 239-249.