Containment and Consensus-Based Distributed Coordination Control to Achieve Bounded Voltage and Precise Reactive Power Sharing in Islanded AC Microgrids

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Abstract—This paper presents a novel distributed approach to achieve both bounded voltage and accurate reactive power sharing regulation in a microgrid. The coupling/trade-off effects between bus voltages and reactive power sharing regulation are first analyzed in detail to provide a guideline for coordinated control design. Furthermore, a containment and consensus-based distributed coordination controller is proposed, by which the bus voltage magnitudes can be bounded within a reasonable range, instead of only controlling average voltage value. Furthermore, the accurate reactive power sharing between distributed generators can be achieved simultaneously. Then, a detailed small-signal model is developed to analyze the stability of the system and the sensitivity of different parameters. Experimental results are presented and compared, where the controller performance, robust performance under communication failure, and plug-and-play operation are successfully verified.

Index Terms—Bounded voltage, consensus algorithm, containment algorithm, coordinated control, microgrid (MG), reactive power sharing.

I. INTRODUCTION

THE MICROGRID (MG) concept provides a promising mean of integrating large amount of distributed generators (DGs) into the power grid and improving reliability [1]. Based on the MG concept, hierarchical control architecture, which was commonly applied in power systems, has been adopted and modified to coordinate different control objectives and to standardize the MG operation [2], [3]. Typically, the primary level deals with the voltage/current regulation and power sharing local control. The secondary level is used to restore the system frequency and voltage to nominal values, regardless of load changes. The tertiary level deals with the energy management and optimization issues.

For the primary and secondary levels, one of main challenges is to achieve the coordination between reactive power sharing and output voltage magnitudes regulation. The reactive power-to-voltage (Q–V) droop control [4] is applied to achieve reactive power sharing in a decentralized manner. However, the Q–V droop control is very sensitive to line impedance differences thus producing inaccurate reactive power sharing and deviating the voltage excessively when researchers try to correct it by increasing the droop coefficient value [5]. In [6], the reactive power sharing problem is analyzed and a two-step estimation method is proposed to calculate the local reactive load, based on which the droop gain is adjusted accordingly to improve the reactive power sharing. Both the effects from local loads and line impedances are considered to improve the power sharing performance. However, the load information should be identified in advance. Then, with a centralized energy management system, a dynamic virtual impedance is proposed in order to satisfy the reactive power sharing requirements based on the different load conditions [7]. However, the load information is also required by the centralized controller, which limits the expandability of the proposed controller. In addition, the centralized controller can cause the single point of failure. In [8], according to the electrical topology of the MG, the relationship between active/reactive load changes and reactive power sharing error is analyzed, and a genetic algorithm is applied to optimize the virtual impedance, in order achieve reactive power sharing. However, when the topology of MG is changed, the whole optimization problem needs to be reformulated. Notice that all the aforementioned controllers are focusing just on the reactive power sharing, while the problem of voltage recovery is not considered at the same time.
Recently, distributed control algorithms [9]–[13] are coming up to stage finding their applications in the MG area [14]–[21]. A distributed method is proposed in [14] to achieve reactive power sharing through acquiring the average value of reactive power. Similarly, in [15], considering the converters parameters related to the power limit and maximum active power capability, a reactive power sharing algorithm is developed. However, for the above-mentioned two kinds of controllers, each distributed controller needs to know the output reactive power and output voltage magnitudes of all the other DGs in the MG. Thus, if one DG fails, the operating DG number should be updated for all the other controllers, which limits the flexibility and robustness of the system. Furthermore, in [15], the voltage regulation is not considered, which may result in considerable voltage deviations. On the other hand, based on the distributed leader-following tracking algorithm [9], another work [16] proposes a secondary voltage tracking strategy by using the feedback linearization method, achieving the output voltage magnitudes tracking the leader. However, the problem of reactive power sharing is not considered during the controller design process. In another work [17], in order to evaluate the issues between accurate reactive power sharing and voltage magnitude regulation, a simple trade-off analysis is proposed for the two control objectives within the secondary control level. Then, an averaging-based method [13] has been applied to achieve reactive power sharing and to keep the average voltage value equal to the nominal value. However, in this work only the average value of all output voltage magnitudes is regulated, being possible large local voltage deviations.

Alternatively, in [19], a droop-free distributed method is proposed to achieve power sharing and to fix the average output voltage to the nominal value. However, the system cannot be operated in practical applications without the use of droop control when the communication system is disabled or presents a global failure. Furthermore, only controlling the average voltage value is not enough for some operation conditions. For example, when one or several DGs are disconnected from the MG or the difference among output line impedances is large, the voltage magnitudes at some buses deviate out from the allowed limit, which can affect the power quality of the system, even though the average voltage value is kept at the nominal value [22]. Compared with the voltage deviation standard for electrical distribution networks, e.g., IEEE 1547, the voltage deviations in an islanded MG should be smaller to guarantee stable of the MG. Accordingly, the existing literature only focus on regulating the average value of voltage magnitudes rather than bounding all bus voltage magnitudes in a reasonable range.

Thus, instead of only controlling the average voltage value, a more flexible control strategy is required to bound all bus voltage magnitudes into a reasonable range, and to achieve accurate reactive power simultaneously. Under more serious conditions, to guarantee the bus voltages are bounded and to provide voltage support for the system stability, the performance of reactive power sharing should be compromised to some extent. In addition, the coupling and trade-off effects between critical system parameters, including droop gains, line impedances, voltage magnitude deviations, and reactive power sharing errors should be analyzed in detail by considering different conditions.

To solve the above-mentioned critical issues, a containment-based controller [23] is identified and considered as a reasonable and flexible approach. It can bound objects within a convex range, while maintaining the distributed fashion, which makes it highly suitable for DG-based MG applications.

In this paper, there are four main technical contributions. First, coupling and trade-off effects between different control parameters and objectives are analyzed within primary and secondary control level. Second, a fully distributed coordination control scheme including containment-based and consensus-based algorithms is proposed realizing a well coordination between bounded bus voltage and accurate reactive power sharing. Third, the small-signal model is derived for system stability analysis and control parameter design. Finally, experimental results including control and robust performance comparison, especially under communication links failure and plug-and-play conditions, are shown to prove high resiliency of the proposed controller.

The paper is organized as follows. In Section II, the coupling and trade-off effects within the hierarchical control structure are analyzed. In Section III, containment and consensus-based distributed coordination control strategy is introduced in detail. In Section IV, the small-signal model and its stability analysis are provided. In Section V, experimental results are presented to prove the effectiveness of proposed controller. Finally, the paper is concluded in Section VI.

II. COUPLING AND TRADE-OFF ANALYSIS WITHIN THE HIERARCHICAL CONTROL

This section gives the coupling analysis among $Q$–$V$ droop gains, line impedance differences, reactive power sharing errors, and relative deviations of voltage magnitudes. Then, the trade-off effects between the accurate reactive power sharing regulation and voltages regulation are analyzed.

A. Coupling Analysis in the Primary Control Level

The simplified islanded MG for analysis purposes is shown in Fig. 1, including two DGs, two local loads, and one common load.

During the islanded operation, DG units are operated under conventional $Q$–$V$ droop control, defined as

$$E_{DG_i} = E^* - n_i Q_i$$

where $E_{DG_i}$ is the reference voltage for the inner control loop, $E^*$ is the voltage magnitude reference of droop control, $n_i$ is the $Q$–$V$ droop gain, and $Q_i$ is the output reactive power.
In Fig. 1, according to the line impedances, the voltage drop can be calculated as [24]

$$E_{DGi} - V_{pcc} = \frac{R_i (P_i - P_{Li}) + X_i (Q_i - Q_{Li})}{V^*}$$  \hspace{1cm} (2)

where $V^*$ is the nominal voltage according to the system requirement, $R_i$ and $X_i$ are the line impedance, $P_i$ and $Q_i$ are the output active and reactive powers, and $P_{Li}$ and $Q_{Li}$ are the local active and reactive powers for $i$th DG.

From (1) and (2), the coupling effects of droop gains and line impedance ratios on reactive power sharing error and voltage magnitude deviations can be analyzed by using the control variate method, shown in Fig. 2. For convenience, same droop gains, $n_1 = n_2 = n$, are assumed in this analysis. Note that in this figure logarithm horizontal $x$-axis is used to show clearly the trends.

Fig. 2(a-1) shows the effect of absolute $Q$–$V$ droop gains and the ratio of line impedances on reactive power sharing error and voltage magnitude deviations can be analyzed by using the control variate method, shown in Fig. 2. For convenience, same droop gains, $n_1 = n_2 = n$, are assumed in this analysis. Note that in this figure logarithm horizontal $x$-axis is used to show clearly the trends.

Fig. 2(a-1) shows the effect of absolute $Q$–$V$ droop gains and the ratio of line impedances on reactive power sharing error. To be more specific, Fig. 2(a-2) shows the relationship between absolute $Q$–$V$ droop gains and reactive power sharing error. It is shown that the reactive power sharing error can be effectively reduced by increasing droop gains regardless of line impedance ratios. Fig. 2(a-3) shows the relationship between the line impedance ratios and the reactive power sharing error. It is shown that with the increasing of line impedance ratios, the reactive power sharing error is increased, while larger absolute $Q$–$V$ droop gain can reduce this error.

From the other standpoint, Fig. 2(b-1) shows the effect of absolute $Q$–$V$ droop gains and the line impedance ratios on relative deviations of output voltage magnitudes of DG units. As shown in Fig. 2(b-2), both smaller and larger droop gains can help to reduce the relative deviations of voltage magnitudes. Here, it is emphasized that due to the feature of $Q$–$V$ droop control, the voltage magnitudes are always deviated from the nominal value and in this section, the relative deviations between voltage magnitudes of only two DGs are discussed to make the explanation easier. The voltage restoration will be discussed in Section II-B. In addition, there exist several peak values for different line impedances conditions shown in Fig. 2(b-2), which is also proven by Fig. 2(b-3). Thus, by combining the results with those from Fig. 2(a-1)–(a-3), the relatively larger droop gains within the stability range can relieve the reactive power sharing error and voltage magnitude relative deviations effectively and simultaneously.

Based on the above-mentioned analyses, the following four main conclusions can be obtained.

1) Both larger and smaller droop gains can help decreasing the relative deviations of output voltage magnitudes to reduce the reactive circuit current.
2) Larger droop gains can weaken the effect of line impedance differences and decrease the reactive power sharing errors.
3) Based on 1) and 2), larger droop gains are more suitable to decrease two errors under the assumption that the droop gain can satisfy the stability requirements.
4) The absolute droop gains and line impedances have decisive influence over the relative voltage deviations and reactive power sharing errors rather than the relative ratios.

**B. Trade-Off Analysis Between Voltage Magnitude Regulation and Reactive Power Sharing in the Secondary Control Level**

Due to the features of above-mentioned analyses in the primary level, two control objectives should be achieved: 1) volt-
Fig. 3. Voltage magnitudes regulation control for two parallel DGs with smaller droop gain and larger line impedance difference.

Fig. 4. Voltage magnitudes regulation control for two parallel DGs with larger droop gain and smaller line impedance difference.

Fig. 5. Reactive power sharing regulation control for two parallel DGs with smaller droop gain and larger line impedance differences.

Fig. 6. Reactive power sharing regulation control for two parallel DGs with larger droop gain and smaller line impedance differences.

Fig. 3 shows the condition with smaller $Q-V$ droop gain and larger line impedance difference, after regulating the voltage magnitude to the nominal value, $Q_1$ becomes much smaller and $Q_2$ becomes much larger than before. If larger $Q-V$ droop gain and smaller line impedance difference are considered, as shown in Fig. 4, after voltage regulation, the deviation between $Q_1$ and $Q_2$ also becomes larger. Thus, no matter what kind of parameter conditions are chosen in the system, the trade-off effect is observable.

In Fig. 5, a smaller $Q-V$ droop gain and a larger line impedance difference condition are assumed, after reactive power sharing regulation, the voltage deviation between two DGs is enlarged. In Fig. 6, a relative larger $Q-V$ droop gain and a smaller line impedance difference are assumed as another condition, after the reactive power sharing regulation, the voltage deviation from two DGs is almost not changed.

Thus, regulating voltage magnitudes to nominal value can cause larger deviation of reactive power sharing. By comparison, reactive power sharing regulation can cause relative little effects on voltage magnitude deviations. Based on above-mentioned discussions, a reasonable operation strategy is to ensure accurate reactive power sharing regardless of relative voltage deviations between buses, but necessarily, bounding voltage magnitudes within a reasonable range defined by standards to guarantee the stability and power quality.

III. PROPOSED DISTRIBUTED COORDINATION CONTROL FOR BOUNDING VOLTAGE DEVIATIONS AND REACTIVE POWER SHARING

This section explains the proposed distributed coordination control in detail. Comparing with the algorithms presented in the literature [14], [17], [19], the proposed algorithm can achieve the controllability of all the bus voltages instead of only controlling the average value of them. A containment-based voltage controller is first proposed to bound all the bus voltages into allowed range, which can guarantee the power quality of the system even under worst conditions, especially for the system with the larger line impedance differences or the system with necessary plug-and-play operations. Compared with the proposed method, the conventional average value based controller may result in over/under-voltage ranges in some of the buses although the average value of all bus voltages can be kept at nominal value. At the same time, a consensus-based reactive power controller is also
involved in the proposed control algorithm to achieve proper reactive power sharing. Thus, the proposed two controllers can achieve the coordinated control including the global bounded bus voltages and reactive power sharing simultaneously. A hierarchical control structure can be formulated to integrate of multiple functions seamlessly.

A. Definitions and Notations

For the control system with \( n \) distributed controllers, a controller is called leader if it only provides information to its communication neighbors and does not receive information. A controller is called follower if it can receive information from one or more neighbors through communication topology. Let \( N_i \) denote the set of \( i \)th-controller neighbors chosen from followers, and \( R_i \) denote the set of leaders, which can send its information to \( i \)th-agent directly. The definition mentioned above is applied to containment-based voltage controller. At the same time, the consensus-based reactive power controller only uses the neighbors’ information without the leaders’ information.

Let \( C \) be a set in a real vector space \( V \subseteq \mathbb{R}^p \). The set \( C \) is called convex if, for any \( x \) and \( y \) in \( C \), the point \( (1 - z)x + zy \) is also in \( C \) for any \( z \in [0, 1] \). The convex hull for a set of points \( X = \{x_1, \ldots, x_q\} \) in \( V \) is the minimal convex set containing all points in \( X \). Let \( \text{Co}(X) \) denote the convex hull of \( X \). In particular, when \( V \subseteq \mathbb{R}, \text{Co}(X) = \{x | x \in [\min x_1, \max x_1]\} \) which will be used in following. In addition, define vector \( Z \in \mathbb{R}^n \), then diag\((Z) \in \mathbb{R}^{n \times n} \) as the diagonal matrix whose diagonal elements are the elements in vector \( Z \). \( I_n \) is the unit matrix and \( 0_n \) is the zero \( n \times n \) matrix.

For consensus-based control, an adjacency matrix is defined as \( A = [a_{ij}] \in \mathbb{R}^{n \times n} \) with \( a_{ij} > 0 \) if node \( i \) can receive information from node \( j \) otherwise \( a_{ij} = 0 \). The Laplacian matrix is defined as \( L = [l_{ij}] \in \mathbb{R}^{n \times n} \) with \( l_{ii} = \sum_{j=1}^{n} a_{ij} \) and \( l_{ij} = -a_{ij}, i \neq j \).

For containment-based control, the range is formed by two leaders which are called the lower and upper voltage boundaries. Thus, another adjacency matrix is defined as \( B = [b_{ij}] \in \mathbb{R}^{n \times 2} \) with \( b_{ij} = 1 \) if node \( i \) can receive information from one of the two leaders otherwise \( b_{ij} = 0 \), in which \( l \) represents the label of two leaders; another Laplacian matrix is defined as \( L_E = [l_{ij}] \in \mathbb{R}^{n \times (n+2)} \) with \( l_{ii} = \sum_{j=1}^{n} a_{ij} + \sum_{l=n+1}^{n+2} b_{il} \) for other rows, when \( j < n, l_{ij} = -a_{ij}, \) otherwise when \( j > n, l_{ij} = -b_{ij} \).

B. Proposed Coordination Controller

The containment-based controller generates a correction term \( e_{Ei} \) for each DG to keep the voltage within a range which is as convex hull. The controller expression is defined as

\[
\dot{e}_{Ei} = -\sum_{j \in N_i} a_{ij} (E_{Di} - E_{Dj}) - \sum_{l \in R_i} b_{il} (E_{Di} - E_l)
\]  

(3)

where \( E_{Di} \) and \( E_{Dj} \) are the voltage magnitudes of \( i \)th DG and \( j \)th DG, respectively, and \( E_l \) is the voltage leader which can be either upper boundary \( E_{Ubou} \) or lower boundary \( E_{Lbou} \).

Equation (3) can be written into matrix form as

\[
\dot{e}_E = -L_E \mathbf{E}
\]

(4)

where \( E_{DG} = [E_{DG1}, \ldots, E_{DGn}]^T \), \( E_{\text{leader}} = [E_{Ubou} \ E_{Lbou}]^T \), \( E = [E_{DG} \ E_{\text{leader}}]^T \), and \( e_E = [e_{E1} \ldots e_{En}]^T \).

Then the error \( \dot{e}_E \) is fed into a proportional-integral (PI) controller.

Consensus-based reactive power controller is defined as

\[
\dot{e}_{nQ_i} = -\sum_{j \in N_i} a_{ij} (n_i Q_i - n_j Q_j)
\]

(5)

where \( n_i \) and \( n_j \) are the reactive power droop gains, and \( Q_i \) and \( Q_j \) are the output reactive for \( i \)th DG and \( j \)th DG.

Equation (5) can be written into matrix form as

\[
\dot{e}_{nQ} = -L_Q N\mathbf{Q}
\]

(6)

where \( N = \text{diag}([n_1 \ldots n_n]^T) \) and \( e_{nQ} = [e_{nQ1} \ldots e_{nQn}]^T \).

Then, the error \( \dot{e}_{nQ} \) is fed into another PI controller.

The configuration of the proposed controller is shown in Fig. 7, including the containment-based voltage controller and the consensus-based reactive power controller. The information format from DGs (followers) is defined as \( Y_{fj} = [n_j Q_j, V_j] \), and the information format from the leader is defined as \( Y_l = [0, E_l] \).
C. Different Work Conditions

The proposed algorithms are based on the hierarchical structure, which consists of primary, secondary, and tertiary levels. In the primary level, the stability can be guaranteed by the conventional droop controller. The proposed two controllers are implemented in the secondary level, corresponding to two critical operation conditions.

Condition 1: Under this condition, DGs have enough power capacity to support voltage and reactive power regulations. The containment-based controller guarantees that all the bus voltages are kept within a prescribed range defined by standards, while consensus-based controller realizes proportional reactive power sharing by all the DGs at the same time.

Condition 2: Under this condition, all the bus voltages are controlled to be maintained within a dynamic boundary which could be adapted based on the command from the higher control level. Even though the voltage boundary may change, the total reactive power is proportional shared among DGs. It is worth saying that the principle about how to change the voltage boundary is out of the scope of this paper, which should be designed in the tertiary level to achieve power management operation.

Notice that, under both work conditions, the two controllers are activated at the same time. The bounded voltage deviations and precise reactive power sharing can be achieved simultaneously. Both the work conditions have been tested in Section V-A.

IV. SMALL-SIGNAL STABILITY ANALYSIS

This section develops the small-signal model for stability analysis and parameter designs purposes. The model includes the proposed containment-based voltage controller, consensus-based reactive power controller, active and reactive power calculation, low-pass filter, and droop controller. The whole model is based on the synchronous reference frame.

A. Small-Signal Model for Proposed Controllers

For the containment-based voltage controller shown in (4), the small signal model is expressed as

$$\dot{\Delta x}_E = -L'_E \Delta E_{DG}$$

(7)

where $L'_E$ is the matrix which deletes the last two columns of matrix $L_E$ neglecting the dynamic of leaders $\Delta x_E = [\Delta e_{E1} \ldots \Delta e_{En}]^T$ and $\Delta E_{DG} = [\Delta E_{DG1} \ldots \Delta E_{DGn}]^T$.

For the consensus-based reactive power controller in (6), the small signal model is expressed as

$$\dot{\Delta x}_nQ = -L_{Qn} \Delta Q$$

(8)

where $\Delta x_{nQ} = [\Delta e_{nQ1} \ldots \Delta e_{nQn}]^T$ and $\Delta Q = [\Delta Q_1 \ldots \Delta Q_n]^T$.

Considering the dynamic of voltage changes, by adding a voltage disturbance term $\Delta E_{DG}$ in left part of (1), then it can be rewritten as

$$\dot{E}_{DG} = E^* - E_{DG} - NQ$$

(9)

which can be seen as the dynamic of the primary level control. As explained above, the two proposed controllers should provide control signals added into (9) through PI controllers. Thus, the system can be written as

$$\dot{\Delta E}_{DG} = -\Delta E_{DG} - N\Delta Q - K_{pQ}L_{Qn}N\Delta Q - K_{PE}'L'_E \Delta E_{DG} + K_{IQ}\Delta e_{nQ} + K_{IE}\Delta e_{E}$$

(10)

where $K_{pQ} = \text{diag}([k_{pQ1} \ldots k_{pQn}]^T)$ corresponds to the proportional parameters, $K_{IQ} = \text{diag}([k_{IQ1} \ldots k_{IQn}]^T)$ to the integral parameters of the PI controllers for the consensus-based reactive power controller, $K_{PE}' = \text{diag}([k_{pE1} \ldots k_{pEn}]^T)$ to the proportional parameters, and $K_{IE} = \text{diag}([k_{IE1} \ldots k_{IEn}]^T)$ to the integral parameters of the PI controllers for the containment-based voltage controllers.

Due to the low-pass filter effect, the small-signal model of output reactive power $Q_i$ can be written as

$$\dot{\Delta q} = -\omega_i \Delta Q + \omega_i \Delta q$$

(11)

where $\omega_i$ is the cut-off frequency of low-pass filter and the instant output reactive power is $\Delta q = [\Delta q_1 \ldots \Delta q_n]^T$.

Considering synchronous reference frame for the $i$th DG, the vector voltage $\hat{E}_{DGi}$ can be written as

$$\hat{E}_{DGi} = E_{di} + jE_{qi}$$

(12)

where $E_{di} = E_{DGi} \cos \delta_i$, $E_{qi} = E_{DGi} \sin \delta_i$, and $\delta_i = \arctan(E_{di}/E_{qi})$.

Linearizing (12) of $\delta_i$, we can get

$$\delta_i = (\partial \delta_i/\partial E_{di}) \Delta E_{di} + (\partial \delta_i/\partial E_{qi}) \Delta E_{qi}$$

$$= m_{di} \Delta E_{di} + m_{qi} \Delta E_{qi}$$

(13)

where $m_{di} = -E_{qi}/(E_{di}^2 + E_{qi}^2)$ and $m_{qi} = E_{di}/(E_{di}^2 + E_{qi}^2)$. Since $\Delta \omega_i(s) = s \Delta \delta_i(s)$, (13) can be rewritten as

$$\Delta \omega_i = m_{di} \Delta E_{di} + m_{qi} \Delta E_{qi}$$

(14)

Considering that $E_{DGi} = |\hat{E}_{DGi}| = \sqrt{E_{di}^2 + E_{qi}^2}$, it can be linearized as

$$\Delta E_{DGi} = n_{di} \Delta E_{di} + n_{qi} \Delta E_{qi}$$

(15)

where $n_{di} = E_{di}/\sqrt{E_{di}^2 + E_{qi}^2}$ and $n_{qi} = E_{qi}/\sqrt{E_{di}^2 + E_{qi}^2}$.

It follows:

$$\Delta E_{DGi} = n_{di} \Delta E_{di} + n_{qi} \Delta E_{qi}$$

(16)

Thus, from the equation set consisting of (14) and (16) for variables $\Delta E_{di}$ and $\Delta E_{qi}$, we have

$$\begin{cases} 
\Delta \hat{E}_{di} = m_{1i} \Delta \omega + m_{2i} \Delta \hat{E}_{DGi} \\
\Delta \hat{E}_{qi} = m_{3i} \Delta \omega + m_{4i} \Delta \hat{E}_{DGi}
\end{cases}$$

(17)

where $m_{1i} = n_{qi}/(m_{di} n_{di} - m_{qi} n_{qi})$, $m_{2i} = -n_{qi}/(m_{di} n_{di} - m_{qi} n_{qi})$, $m_{3i} = n_{di}/(m_{di} n_{di} - m_{qi} n_{qi})$, and $m_{4i} = -n_{di}/(m_{di} n_{di} - m_{qi} n_{qi})$. 
Substituting (10) and (15) into (17) and writing into matrix form as

\[
\begin{align*}
\Delta \dot{E}_q &= M_1 \Delta \omega + A_1 N_3 \Delta E_d + A_1 N_q \Delta E_q \\
&\quad + A_2 \Delta Q + M_2 K_{1E} \Delta e_E + M_2 K_{1Q} \Delta e_{nQ} \\
\Delta \dot{E}_q &= M_3 \Delta \omega + B_1 N_3 \Delta E_d + B_1 N_q \Delta E_q \\
&\quad + B_2 \Delta Q + M_4 K_{1E} \Delta e_E + M_4 K_{1Q} \Delta e_{nQ}
\end{align*}
\]

where

\[
M_1 = \text{diag}([m_{11} \ldots m_{1n}]^T),
M_3 = \text{diag}([m_{31} \ldots m_{3n}]^T),
N_d = \text{diag}([n_{d1} \ldots n_{dn}]^T),
N_q = \text{diag}([n_{q1} \ldots n_{qn}]^T),
A_1 = -M_2(I_n + K_{pE} L'_E),
A_2 = -M_2(I_n + K_{pQ} L'_Q)N, B_1 = -M_3
\]

Substituting (10) and (15) into (17) and writing into matrix form as

\[
\Delta \dot{E}_q = M_1 \Delta \omega + A_1 N_3 \Delta E_d + A_1 N_q \Delta E_q \\
+ A_2 \Delta Q + M_2 K_{1E} \Delta e_E + M_2 K_{1Q} \Delta e_{nQ}
\]

By combining (21) and (23), the small signal model of the instant active and reactive powers can be expressed as

\[
\begin{align*}
\Delta p &= S_1 \Delta E_d + S_2 \Delta E_q \\
\Delta q &= S_3 \Delta E_d + S_4 \Delta e_q
\end{align*}
\]

where

\[
S_1 = 3/2(I_q + E_q G + E_q B),
S_2 = 3/2(-I_q + E_q G - E_q B),
S_3 = 3/2(I_q - E_q B - E_q G).
\]

By substituting (24) into (11) and (19), and by substituting (15) into (7) and combining with (8) and (18), we can obtain the whole system model as follows:

\[
\dot{X} = FX
\]

where

\[
F =
\begin{bmatrix}
-\omega_c I_n & -\omega_c M S_1 & 0_n & 0_n & 0_n \\
M_1 & A_1 N_3 & A_1 N_q & A_2 & M_2 K_{1E} & M_2 K_{1Q} \\
M_3 & B_1 N_3 & B_1 N_q & B_2 & M_4 K_{1E} & M_4 K_{1Q} \\
0_n & \omega_c S_3 & \omega_c S_4 & -\omega_c I_n & 0_n & 0_n \\
0_n & -L'_P N_d & -L'_Q N_q & 0_n & 0_n & 0_n \\
0_n & 0_n & 0_n & -LQ_N & 0_n & 0_n \\
\end{bmatrix}
\text{and } X = [\Delta \omega \Delta E_d \Delta E_q \Delta Q \Delta e_E \Delta e_{nQ}]^T.
\]

In order to make the modeling process more clear, Fig. 8 shows the equivalent small signal model diagram for the proposed controller and the ac MG system.

C. Stability Analysis

In order to analyze the model quantitatively, an MG including four parallel connected DGs, a local load, and a common load are considered as a study case. In this section, root locus plots are shown to reflect the dynamical behavior of the system by considering different control parameters.

Fig. 9 shows the root locus movement considering the proportional coefficient $K_{pE}$ of PI controller for containment-based control changing from 7 to 15. From the enlarged part in Fig. 9, it is shown that the two dominating poles located near the imaginary axis are moving toward the real axis and away from the imaginary axis, which indicate that the system is becoming more damped. Six complex poles which are also affected by $K_{pE}$ are moving away from the imaginary axis, thus improving the response speed.

Fig. 10 shows root locus considering integral coefficient $K_{iE}$ of the PI controller for containment-based control changing from 1 to 100. From the enlarged part in Fig. 10, it is shown that two dominating poles are moving away from the real axis, which means that the system is becoming less damped. Simultaneously, two poles on the real axis are moving away from original point. The rest six complex poles are less affected than the proportional coefficient $K_{pE}$.

Fig. 11(a) shows root locus considering proportional coefficients $K_{pQ}$ for a PI controller for consensus-based control changing from 7 to 15. It can be observed that two dominating poles in the blue circle are barely affected. In addition, one
pole on the real axis moves toward origin point, which can slow down the response speed. Six complex poles are moving away from real axis, which means the system is becoming less damped. Fig. 11(b) shows root locus considering integral coefficients $K_{iQ}$ of PI controller for consensus-based control changing from 30 to 120. The two dominating poles in the blue circle are also not affected. One dominating pole on the real axis...
TABLE II
ELECTRICAL DATA COMPARISON

<table>
<thead>
<tr>
<th>Category</th>
<th>In this paper</th>
<th>In [17]</th>
<th>In [19]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line Impedance</td>
<td>( Z_{12} = 1.2\Omega + 5.4 \text{mH}; )</td>
<td>( Z_{12} = 0.8\Omega + 3.6 \text{mH}; )</td>
<td>( Z_{12} = 0.8\Omega + 3.6 \text{mH}; )</td>
</tr>
<tr>
<td></td>
<td>( Z_{23} = 0.4\Omega + 1.8 \text{mH}; )</td>
<td>( Z_{23} = 0.4\Omega + 1.8 \text{mH}; )</td>
<td>( Z_{23} = 0.4\Omega + 1.8 \text{mH}; )</td>
</tr>
<tr>
<td></td>
<td>( Z_{34} = 0.8\Omega + 3.6 \text{mH}; )</td>
<td>( Z_{34} = 0.7\Omega + 1.9 \text{mH}; )</td>
<td>( Z_{34} = 0.7\Omega + 1.5 \text{mH}; )</td>
</tr>
<tr>
<td>Nominal Voltage</td>
<td>325 V</td>
<td>325 V</td>
<td>120 V</td>
</tr>
<tr>
<td>Total Reactive Power</td>
<td>3000 W</td>
<td>1260 W</td>
<td>750 W</td>
</tr>
</tbody>
</table>

Fig. 13. Root locus: (a) Changing from 0.8\( \times Z_{12} \) to 1.2\( \times Z_{12} \); (b) changing from 0.8\( \times Z_{23} \) to 1.2\( \times Z_{23} \); and (c) changing from 0.8\( \times Z_{34} \) to 1.2\( \times Z_{34} \).

values of the line impedances for this analysis are presented in Table II.

V. EXPERIMENTAL RESULTS

The proposed control scheme is implemented and tested in an experimental MG setup operating in islanded mode, shown in Fig. 14 at the AAU-Microgrid Research Laboratory. The setup consists of four parallel-configured power electronics inverters, a real-time control and monitor platform, LCL filters, and RL loads. Communication link is only built between neighboring units shown in the top left corner of Fig. 13. Converters rated active and reactive power are 2:2:1:1 for DG1 – DG4. The nominal voltage magnitude is set to 325 V with 1% voltage boundary (325 ± 1%). The experimental results are shown in Figs. 15–19. At \( t = T_0 \), four DGs are controlled by the conventional droop control, and at \( t = T_1 \) the proposed controller is enabled.

A. Case 1: Performance Assessment and Comparison

Fig. 15 shows the performance comparison between the conventional droop controller and the proposed one. Fig. 15(a) shows the voltage performance and Fig. 15(b) shows the reactive power sharing characteristic. Before \( t = T_1 \), the system is controlled by the conventional droop controller. The bus voltage magnitudes are dropped more than 18 V, which exceed the lower voltage boundary 320 V. The reactive power sharing among four DGs does not follow the proportionality 2:2:1:1. At \( t = T_1 \), the proposed controller is activated. Then, the bus volt-
age magnitudes can be bounded within prescribed range and the reactive power is proportionally shared to 2:2:1:1. Furthermore, between $t = T2$ and $t = T4$, the boundary is changed and the bus voltage magnitudes are followed the changed boundary into the new range which is between 320 and 310 V. In addition, the performance of reactive power sharing can also be guaranteed, being still equal to 2:2:1:1.

In addition, at $t = T3$, the load is changed and the performance of reactive power sharing is also kept accurate. After $t = T4$, the voltage boundary is changed back to the nominal range and both the voltage and reactive power sharing performance are kept well. It is shown that after activating the proposed controller, the bus voltage magnitudes can be bounded within the dynamic range. Meanwhile the output reactive power can be proportionally shared to 2:2:1:1 during the whole process.

**B. Case 2: Communication Failure Resiliency**

Resiliency to a single communication link failure is studied in Fig. 16.

The communication link between DG$_2$ and DG$_3$ has been disabled at $t = T6$. As shown in the zoomed in part of Fig. 16(a) and (b), after small oscillations occur around ±0.5 V, it does not have any impact on the performance of bounded bus voltage and reactive power sharing. After that, the load is switched at $t = T7$ and $T8$. The performance is also kept well. It is concluded that both the dynamic and steady-state performance of the proposed controller cannot be affected as long as the communication network remains connected from the perspective of graph theory.

**C. Case 3: Plug-and-Play Comparison Study**

Fig. 17 shows the plug-and-play capability of the proposed controller. DG$_4$ is unplugged at $t = T9$. Thus, the bus voltage and reactive power from DG$_4$ decay to zero. Notice that a source failure also means loss of communication links connected to other DGs. As shown in Fig. 17(a), the bus voltages of DG$_1$-DG$_3$ are kept inside the acceptable range. In addition, Fig. 17(b) shows the per-unit value of output reactive power to decrease the range of $y$-axis, indicating that the per-unit values of output reactive power among DG$_1$-DG$_3$ are all equal to
0.55 p.u. At $t = T10$, DG$_4$ begins to synchronize the frequency and phase with the MG. After successful synchronization, at $t = T11$, DG$_4$ is connected without activating the proposed controller. At $t = T12$, the proposed controller and communication are activated for DG$_4$. Fig. 17 shows that bus voltage magnitudes can be bounded within the range and the per-unit value of reactive power sharing among four DGs are equal to 0.47 p.u. after activating the proposed controller for DG$_4$.

Furthermore, to make the proposed containment-based voltage controller more convincing, the control performance by
using controllers of [17] and [19], which are very popular in the literature about voltage and reactive power sharing regulation, is shown in Figs. 18 and 19 by using the same electrical topology, respectively.

Fig. 18 shows the plug-and-play performance of the controller proposed in [17]. When DG4 is disconnected from the MG, the bus voltage of DG2 exceeds the voltage boundary which can affect the power quality of the system. In addition, as shown in Fig. 19, by using the controller proposed in [19], when DG4 is plugged out of the system, even though the average voltage value can be kept at 325 V, shown in Fig. 19(c), the bus voltage of DG2 also exceeds the voltage boundary shown in Fig. 19(a).

Notice that the plug-and-play tests are also shown in [17] and [19] without observing this problem. The reason is that by comparing the electrical parameters of this paper with the ones in [17] and [19], the line impedance deviations that we use in this paper are much larger, and thus the total reactive power as well. The electrical data comparison is shown in Table II in detail. Thus, it is proved that the proposed containment-based controller is more suitable for larger systems with relatively larger line impedance deviations and higher reactive power sharing requirements.

Remark 1: If the line impedance deviations in the system are much higher and some DGs are disconnected of the system, it is not possible to guarantee both the performance of bus voltages bound and reactive power sharing simultaneously due to the electrical limitations. Under these conditions, the advantage of proposed controller is that it can decide either to guarantee the bus voltage bound through setting error saturation of reactive power sharing performance, or to guarantee the reactive power sharing performance through enlarging the voltage boundary of the system according to the MGs or electrical distribution standards. Previously proposed controllers cannot perform this high level of freedom to achieve global bus voltage bound, so that the control performance of all global voltages in the system cannot be compromised among DGs, especially under extreme conditions.

VI. CONCLUSION

This paper presents the coupling/trade-off effects between voltage regulation and reactive power sharing in different levels of a hierarchical control structure. The coupling effects among Q–V droop gains, line impedance differences, reactive power sharing errors, and voltage magnitudes deviations are analyzed in the primary control level. The trade-off effects between the accurate reactive power sharing and voltage magnitudes regulations are further analyzed in the secondary level. A fully distributed coordination controller including both containment-based and consensus-based controllers is proposed to offer a highly flexible and reliable operation of DGs, thus achieving both bound the voltage magnitudes within a reasonable range and achieving accurate reactive power sharing. A detailed small signal model is derived and the effects of different parameters change for the proposed controller are analyzed. Experimental results are presented to demonstrate the effectiveness of proposed controller including performance assessment and comparison, resiliency for communication failure, and plug-and-play study.

REFERENCES

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