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Control of District Heating System with Flow-dependent Delays

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Abstract: All flow systems are subject to transport delays, which are governed by the flow rates in the system. When the flow rates themselves are control inputs, the system becomes subject to input-dependent state delays, which poses significant theoretical problems. In an earlier paper, we proposed a guaranteed stable control design for a system of this type; in this paper, we provide experimental evidence of the usefulness of the design.

Keywords: Transport delays, Lyapunov-Krasovskii stability, flow systems

1. INTRODUCTION

District heating is a common method for distributing heat generated at a centralized location to residential and commercial buildings. It is widely used in urban areas in northern Europe, see for instance Gabrielaitiene et al. [2007], Verda and Colella [2011], or Dotzauer [2002]. In this paper, we consider control of a district heating system with non-negligible transport delay between a central district heating provider and a number of consumers that require heat in order to maintain comfort.

A significant challenge in flow systems such as district heating grids is that the transport delay is dependent on the control input; in particular, the delay is inversely proportional to the flow rate in the system, which is one of the controlled inputs. Systems with delays in states and/or inputs have received considerable attention in the literature (see for instance Gu et al. [2003] and the references therein). However, computing an input signal to compensate for a delay based on feedback of a delayed state, where the delay depends on the same input in the first place, is naturally a difficult problem. Results on input-dependent delays in literature are consequently relatively few. The references Bresch-Pietri and Krstic [2010], Krstic [2008], Krstic [2010] and Fischer et al. [2012] all considered time-varying input delays in various settings. More recently, Bekiaris-Liberis and Krstic [2013] presented a predictor-based methodology for compensating state-dependent input delays for both linear and nonlinear systems. A class of nonlinear systems with input-dependent parameters and delays was considered in Dieulot and Richard [2001], in which an open-loop motion planning problem was solved using an explicit parametrization of trajectories. A hot/cold-water mixing loop was considered in Bresch-Pietri and Krstic [2010], Bresch-Pietri et al. [2012a] and Bresch-Pietri et al. [2012b], in which the authors transformed the time axis of the problem in such a way that the delay could be considered constant.

In a previous paper, Bendtsen and Krstic [2013], a simple control design for a thermodynamical flow system was proposed, which overcame the input-dependent delay difficulties by fixing the flow rate at heat input/output equilibrium and proving that the temperature states could be stabilized using Lyapunov-Krasovskii stability theory. In this paper we provide a more detailed simulation of the flow system in question – reformulated as a district heating system – along with experimental evidence that the design indeed works as intended.

The structure of the remainder of the paper is as follows. Section 2 first provides an overview of the system under consideration whereupon Section 3 reviews the control design from Bendtsen and Krstic [2013]. Section 4 then presents a detailed flow simulation of the system, Section 5 presents the experimental results along with a brief discussion, and finally Section 6 sums up the contributions of the work. Note that, throughout the paper, all quantities mentioned are real and scalar.

2. DISTRICT HEATING SYSTEM

2.1 System overview

The layout of the district heating system under consideration is outlined in Figure 1.

The system consists of a heat supplier (typically a combined heat and power plant in case of large-scale/industrial consumers or a district heating substation in case of residential consumers) and a number of consumers that draw district heating water from the supplier. Each consumer is equipped with a heat exchanger that enables the required heat to be transferred from the district heating system to the consumer. Each consumer is situated in parallel, but at different distances from the supplier, [1]

[1] In case of residential buildings, this is obviously an abstraction since consumers will also be placed in series; however, by considering groups of consumers in series as single consumers, we can maintain the simple structure in Figure 1.

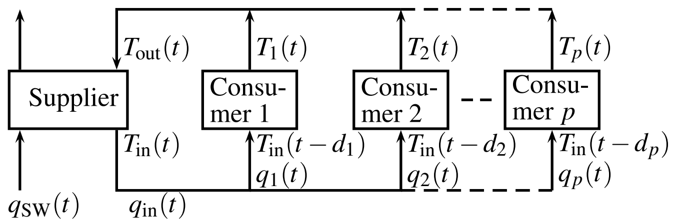

Fig. 1. District heating topology; $q_i(t)$ are heated-water flows to the consumers at time t, indexed by $i = 1,\dots,p$; $T_{\text{in}}(t)$ is the forward flow temperature at time t, T_i are the corresponding temperatures of water exiting the consumers' heat exchangers, and d_i are flow-dependent transport delays.

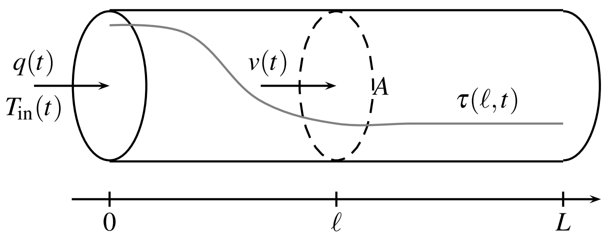

Fig. 2. Flow of heated water through a pipe section; $\tau(\ell,t)$ is the temperature distribution in the pipe at time t and $v(t)$ is the average speed of the water through a cross-section A.

which implies that the heated water has to travel different distances to reach each consumer. The transport of heated water is facilitated by a variable-speed pump at the supplier, which is equipped with a local controller ensuring fast flow control compared to the temperature dynamics. Each consumer in turn is equipped with a valve regulating the flow into the consumer's heat exchanger. Thus, we can assume that the volumetric flows $q_1(t), q_2(t), \dots, q_p(t)$ are control inputs, and $q_{\text{in}}(t)$ is controlled to enforce $q_{\text{in}}(t) = \sum_{i=1}^{p} q_i(t)$ at all times.

2.2 Transport from supplier to consumers

Next, we look at the flow from the supplier to a consumer; see Figure 2. Let $\tau : [0,L] \times \mathbb{R}_+ \to \mathbb{R}_+$ denote the temperature of the water in the pipe at a distance $\ell \in (0,L)$ from the end of the pipe at time $t \in \mathbb{R}_+$, assume the pipe is completely filled with fluid, and the fluid is incompressible. The transport equation representing the distribution of temperature along the pipe is then

$$A\rho c_p \frac{\partial \tau(\ell,t)}{\partial t} + v(t) A\rho c_p \frac{\partial \tau(\ell,t)}{\partial \ell} = 2\gamma\pi r (T_{\text{amb}} - \tau(\ell,t)) \quad (1)$$

$$\tau(0,t) = T_{\text{in}}(t) \quad (2)$$

where A and r are resp. the cross-section area and radius of the pipe, ρ is the density and c_p the heat capacity of water, and γ is a heat transfer coefficient. Furthermore, $v(t) = q(t)/A$ is the speed of the water through the pipe and T_{amb} is the ambient temperature.

Assuming heat losses through the pipe walls to be negligible (since district heating pipes are usually well insulated), (1) reduces to

$$\frac{\partial \tau(\ell,t)}{\partial t} + \frac{q(t)}{A} \frac{\partial \tau(\ell,t)}{\partial \ell} = 0$$

Integrating the transport equation over the pipe length L then leads to the following temperature at the inlet valve of the i'th consumer:

$$\tau(L_i,t) = T_{\text{in}}(t - d_i) \text{ where } d_i = d_i(q_i(t)) = \frac{\alpha_i}{q_i(t)} \quad (3)$$

That is, d_i are *input-dependent transport delays* between the supplier and each individual consumer, and α_i are consumer-specific constants that depend on pipe length and diameter of the piping leading to the consumer in question; simply put, the slower the water flows, the longer the delay becomes.

2.3 Thermodynamics

The thermodynamics of the system cover heat transfer between the heat source (power plant boiler, substation, etc.), the heated water in the distribution grid (hereinafter DH water for short), and the consumers. For simplicity, it is assumed that heat transfer only takes place within the heat exchangers; consequently, a single model with consumer-specific parameters can be used to describe the thermodynamics of each consumer. Indeed, the heat balance for the heat exchanger in each consumer is modeled as a simple first-order ordinary differential equation:

$$\dot{T}_i(t) = \frac{1}{V_i} q_i(t)(T_{\text{in}}(t-d_i) - T_i(t)) - w_i(t) \quad (4)$$

where $T_i(t)$ is the temperature of the DH water when it leaves the i'th consumer, V_i is the effective volume of the consumer's heat exchanger (typically radiators and local hot-water storage tanks in case of residential housing) and $w_i(t) \in [\underline{w}_i; \overline{w}_i]$ is a slowly varying disturbance (the consumer load). $T_{\text{in}}(t-d_i)$ is the temperature of the DH water as it enters the i'th consumer, as explained above.

The heat supplier is modeled in the same way as the consumers, except that in this case the supply side is providing the heat flow (power) $Q(t)$ to the DH water:

$$\dot{T}_{\text{in}}(t) = \frac{1}{V_{\text{S}}} q_{\text{in}}(t)(T_{\text{out}}(t) - T_{\text{in}}(t)) + Q(t) \quad (5)$$

where V_{S} is the effective volume of the supplier's heat exchanger, and T_{out} is the return flow from the consumers, which is modeled as simple mixing of the return flow from each consumer:

$$\begin{aligned} T_{\text{out}}(t) &= \frac{1}{\sum_{i=1}^{p} q_i(t)} \left(q_1(t)T_1(t) + \cdots + q_p(t)T_p(t)\right) \\ &= \frac{1}{q_{\text{in}}(t)} \left(q_1(t)T_1(t) + \cdots + q_p(t)T_p(t)\right) \end{aligned}$$

For simplicity, the return flow delay is ignored here.

The goal is to stabilize the temperatures $T_i(t)$ at some desired values $\overline{T}_i$ in the face of strictly positive loads w_i, i.e., positive heat *demands* at each consumer. The reference and load values will be considered constant; in practice, they will vary with the environment conditions, such as ambient temperature, weather etc., but we shall assume these variations to be slow.

3. CONTROL DESIGN

We first rewrite the physical model above in a form more amenable to control design.

3.1 Bilinear model for control

With p consumers, we have the set of model equations

$$\dot{T}_1(t) = \frac{q_1(t)}{V_1}(T_{\text{in}}(t-d_1) - T_1(t)) - w_1 \tag{6}$$

$$\vdots$$

$$\dot{T}_p(t) = \frac{q_p(t)}{V_p}(T_{\text{in}}(t-d_p) - T_p(t)) - w_p \tag{7}$$

$$\dot{T}_{\text{in}}(t) = \frac{1}{V_S}(q_1(t)T_1(t) + \cdots + q_p(t)T_p(t)) - \frac{1}{V_S}\sum_{i=1}^{p} q_i(t)T_{\text{in}}(t) + Q(t) \tag{8}$$

where $Q(t)$ is considered a control input, as mentioned above.

In steady-state operation, for given fixed w_i and with the consumer outlet temperatures equal to their respective reference temperatures $\overline{T}_i$, we have the static relations

$$w_1 = \frac{\overline{q}_1}{V_1}(\overline{T}_{\text{in}} - \overline{T}_1) \tag{9}$$

$$\vdots$$

$$w_p = \frac{\overline{q}_p}{V_p}(\overline{T}_{\text{in}} - \overline{T}_p) \tag{10}$$

$$\overline{Q} = \sum_{i=1}^{p} w_i \tag{11}$$

where $\overline{T}_{\text{in}}$ is the steady-state temperature of the forward flow of DH water leaving the supplier and

$$\overline{q}_i = \frac{V_i w_i}{\overline{T}_i - \overline{T}_{\text{in}}}, \quad i = 1, \ldots, p \tag{12}$$

are the corresponding steady-state flows through the i consumers.

Let $n = p + 1$ and define the new coordinates

$$x_i(t) = T_i(t) - \overline{T}_i, u_i(t) = \frac{q_i(t) - \overline{q}_i}{V_i}, \; i = 1, \ldots, n-1$$

$$x_n(t) = T_{\text{in}}(t) - \overline{T}_{\text{in}}, u_n(t) = Q(t) - \overline{Q}$$

We can then write (6)–(8) on the bilinear form

$$\dot{x}_1 = a_1(-x_1(t) + x_n(t-d_1)) + (-b_1 - x_1(t) + x_n(t-d_1))u_1(t) \tag{13}$$

$$\vdots$$

$$\dot{x}_{n-1} = a_{n-1}(-x_{n-1}(t) + x_n(t-d_{n-1})) + (-b_{n-1} - x_{n-1}(t) + x_n(t-d_{n-1}))u_{n-1}(t) \tag{14}$$

$$\dot{x}_n = \sum_{i=1}^{n-1} a_i(x_i(t) - x_n(t)) + \sum_{i=1}^{n-1}(b_i + x_i(t) - x_n(t))u_i(t) + u_n(t) \tag{15}$$

where $a_i = \overline{q}_i/V_i$ and $b_i = \overline{T}_i - \overline{T}_{\text{in}}$ are positive constants.

Furthermore, in order to have a well-posed system of delay-differential equations, it is necessary to know initial conditions for the states and inputs leading up to time $t = 0$. These functions (sometimes referred to as history functions) are here assumed to be given by

$$x_i(\theta) = \phi_i(\tau), u_i(\theta) = \varphi_i(\tau), \; i = 1, \ldots, n \tag{16}$$

where $\phi_i, \varphi_i : [-\max_i(\alpha_i/\bar{q}_i), 0] \to \mathbb{R}$ are continuously differentiable functions.

3.2 Control design

The main contribution of the previously mentioned paper, Bendtsen and Krstic [2013], was the following simple control design, which we briefly re-iterate here.

Theorem 1. Consider the system (13)–(15) with history functions $\phi_1, \ldots, \phi_n, \varphi_1, \ldots, \varphi_n : [-\max(\alpha_i/\bar{q}_i), 0) \to \mathbb{R}$; the control law

$$u_1(t) = u_2(t) = \cdots = u_{n-1}(t) \equiv 0 \; \forall t \geq 0 \tag{17}$$

$$u_n(t) = -kx_n(t) \tag{18}$$

with

$$k > \sum_{i=1}^{n-1} \frac{1}{3} a_i e^{\delta_i \alpha_i/\bar{q}_i}, \; \delta_i > 0$$

renders the origin globally exponentially stable.

The proof involves the positive semi-definite Lyapunov-Krasovskii function candidate (weighted L_2-norm):

$$V(x) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n-1} \varepsilon_i \int_{-d_i}^{0} e^{\delta_i \zeta_i} x_n(t+\zeta_i)^2 d\zeta_i \tag{19}$$

with $\varepsilon_i, \delta_i \in \mathbb{R}_+$ constants to be determined and fixed $d_i = \alpha_i/\bar{q}_i, i = 1, \ldots, n-1$. Inserting the control law stated in the theorem, integrating by substitution and completing squares then reveals that the time derivative of V is negative definite for any k and δ_i that satisfy the stated conditions. See Bendtsen and Krstic [2013] for the details.

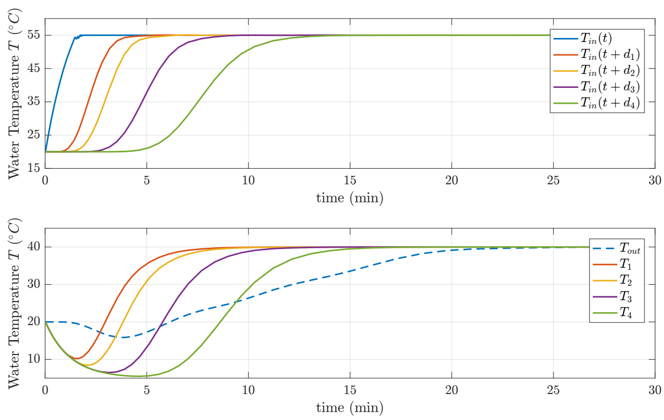

Fig. 3. Simulation with the stabilizing control law from Theorem 2. Top: forward flow leaving the supplier (blue) and reaching each consumer after delays d_i. Bottom: Outlet temperatures from each consumer. The dashed line shows T_{out}, the temperature of the return flow at the supplier

4. SIMULATION

To verify the usefulness of the proposed control law, we first simulate the system depicted in Figure 1. Four identical consumers were modeled using the heat balances (6)–(7), but with

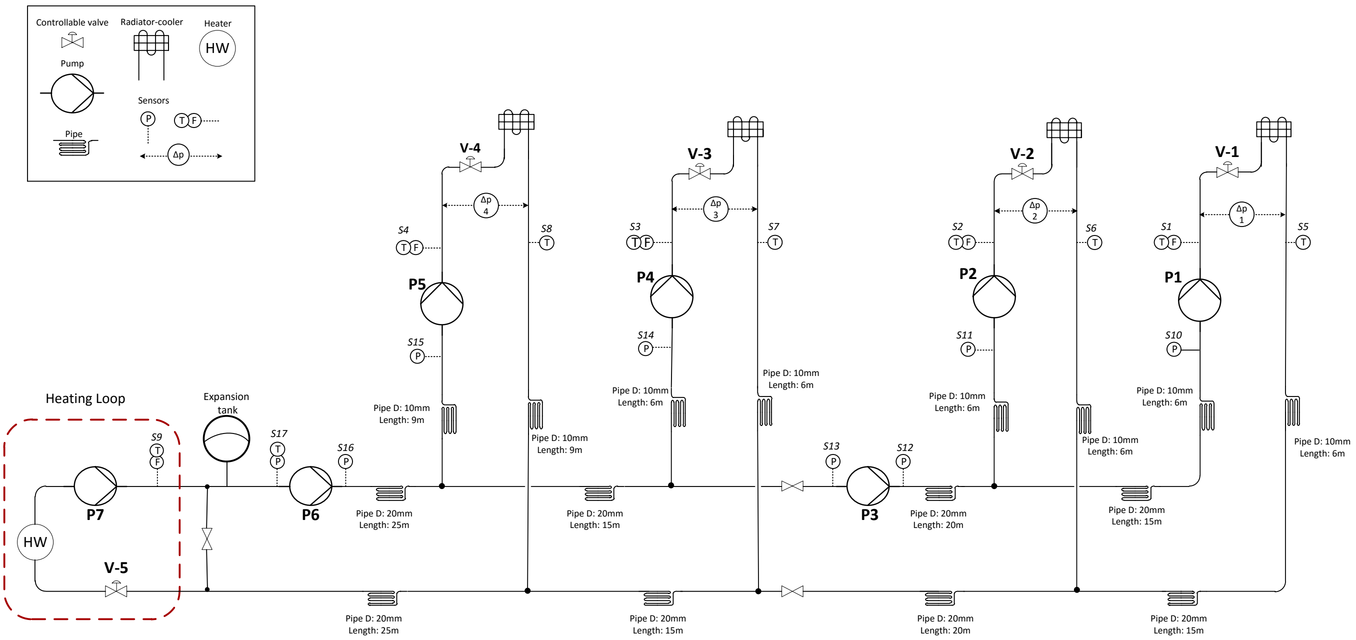

Fig. 4. Detailed topology of laboratory setup.

$T_{\text{in}}(t-d_i)$ given as the output of four separate versions of the transport equation generating (3) with different values of α_i. The transport equations were discretized using the celebrated Backward-in-Time, Centered-in-Space scheme, i.e., we divide each pipe into small sections of length $\Delta\ell$ and for each consumer $i \in \{0,\ldots,4\}$, pipe segment $m \in \{0,\ldots,L_i/\Delta\ell\}$ and timestep t_k solve for $\tau_i^m(t_k)$ in the expression

$$\frac{\tau_i^m(t_k)-\tau_i^m(t_k-\Delta t)}{\Delta t} = -\frac{q(t_k)}{\alpha_i}\frac{\tau_i^{m-1}(t_k)+\tau_i^{m+1}(t_k)}{2\Delta\ell}$$

where superscript m denotes the m'th segment. The flows $q_i(t_k)$ were governed by simple PI controllers to satisfy (9)–(10) in steady state.

Finally, the substation was simulated using (8) with $Q(t_k)$ given by (18). Note that, although the delay in return flow was ignored in the previous section, in the present simulation the return flow was actually simulated using the transport equation as well. The control gain k was chosen arbitrarily to $k = \sum_{i=1}^{4} a_i$.

The flows and water temperatures for a simulation with control are shown in Figure 3. The reference outlet temperatures were set to 40 °C. Furthermore, the consumers were subjected to a significant heat demand, causing the temperatures to actually drop below their initial values until the supplier/substation was able to increase the heat input.

As can be seen from the figure, the flows quickly stabilize at the target values, while the substation increases the temperature from the rather low initial value of 20 °C to 55 °C over a period of about 90 seconds. The transport delays are clearly evident from the graph. The bottom graph shows how the temperature at the outlet of each consumer eventually increases and converges to the setpoint, but not before both the transport delay and the consumer's own temperature dynamics permit.

5. EXPERIMENTAL RESULTS

The proposed control law was then tested using the laboratory setup depicted in Figure 4; Figure 5 shows a physical view of the laboratory setup. Pumps P1, P2, P4 and P5 are consumer pumps, while P6 is the supplier's feedforward pump (delivering the flow q_{in}). In principle, the flow should be delivered by the supplier pump only; however, since P6 in our laboratory setup is not powerful enough to deliver the entire required flow by itself, the consumer pumps were used to ensure the correct flow to the consumers. The controllable valves V1-V4 are serially connected with radiators to emulate thermal loads. Note that the white wall shown on the photo in fact hides most of the physical piping; as indicated on Figure 4, the consumers are separated from the substation by between approximately 35 m and 80 m of piping. The heating loop at the left end of the diagram represents the DH supplier/substation. V5 is a throttle valve, which can be used to limit the flow through the heater (denoted HW in the diagram).

Note that, throughout the experiments, all signals were sampled at 1 Hz. Before plotting, they have been filtered through a 3rd order Butterworth filter with a cutoff frequency of 0.05 Hz.

Figures 6 to 8 show the result of a step in heat demand on all four consumers at time 50 s. The controller reacts immediately by closing the mixing valve, shutting off the flow through the heating circuit (bottom plot of Figure 8) and thereby rapidly reducing T_{in}, as can be seen from the temperature graph in Figure 6. This control action results in the inlet temperatures of each of the consumers to decrease approximately 30, 40,

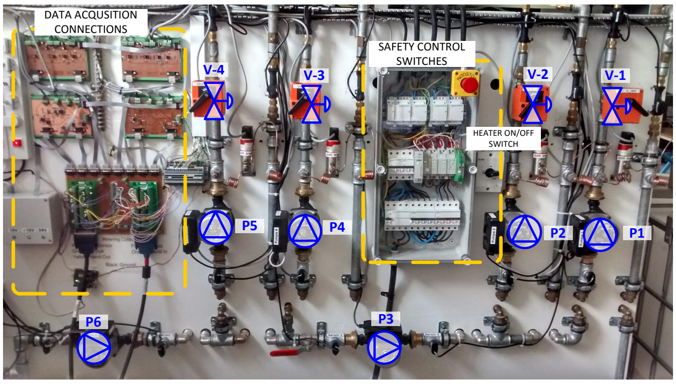

Fig. 5. Photo of the laboratory setup with indication of instrumentation.

70 and 120 seconds later, respectively. Figure 8 also shows how the supply temperature is maintained at the desired steady-state inlet temperature of 36 °C. Subsequently, the mixing valve opens gradually, as the temperature of the flow through the heating loop approaches the reference.

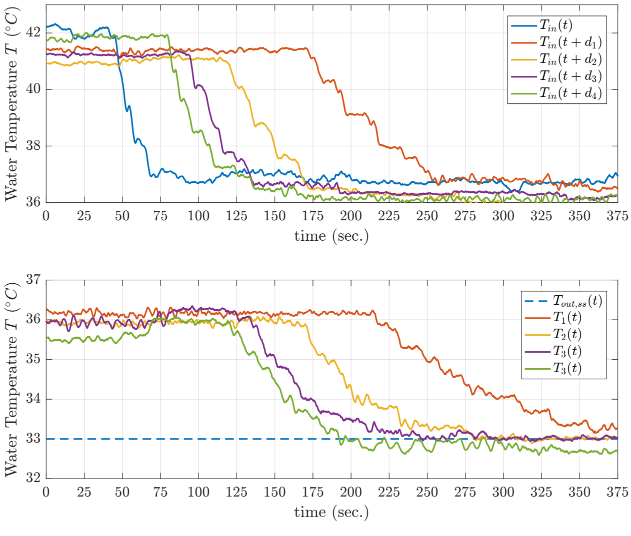

Fig. 6. Top: forward flow temperature measurements. The supply temperature is plotted in blue. Bottom: Outlet temperatures from each consumer; the dashed line shows the reference temperature.

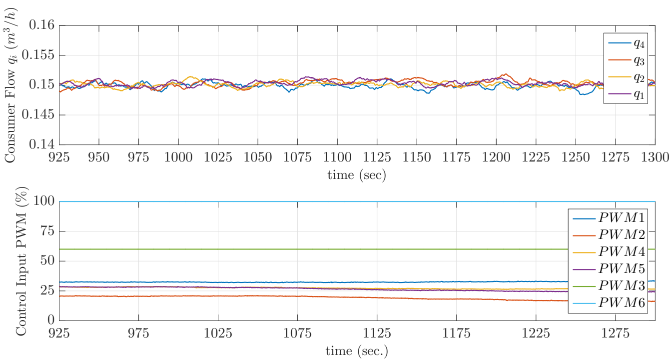

Fig. 7. Top: flow measurements at the inlets of each consumer. Bottom: PWM signals to pumps (see Figure 5).

Even though the simulation in Figure 3 shows a different temperature range than Figure 6, the simulated and measured

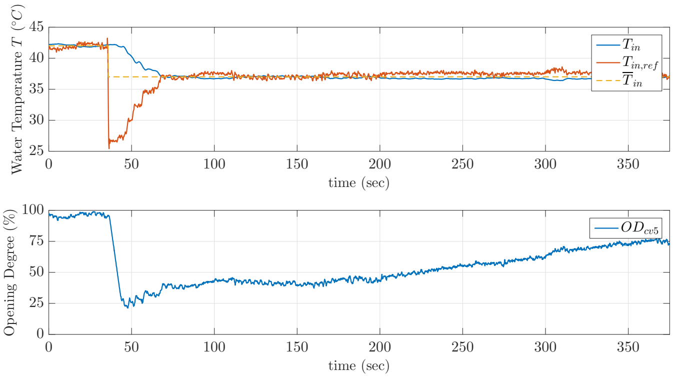

Fig. 8. Top: control signal (red), supply temperature (blue) and reference (dashed). Bottom: Valve opening

temperatures are qualitatively very much alike. This indicates that the simulation model is indeed an acceptable description of the actual system, as long as the actuators do not saturate. It can also be verified that the control law stabilizes the system at the desired steady-state value, and it can be seen from Figure 8 that it aggressively tries to eliminate the control error (i.e., drive $x_n(t)$ to zero).

6. CONCLUSION

This paper has presented a practical test of a control design based on Lyapunov-Krasovskii theory proposed in an earlier paper. We considered a simplified model of a district heating system with non-negligible transport delays between a supplier (substation) and a number of consumers that require heat to be supplied in real time. We presented a model in which the transport delay was simulated using partial differential equations, and verified through simulations as well as practical measurements that the control law behaved as predicted and stabilized the temperatures at the outlets of the consumers in spite of significant delays.

Even though the control design as such is very simple – fixing the flows in the system and using a proportional gain to regulate the forward temperature – we find the concept (along with the theoretical analysis in Bendtsen and Krstic [2013]) to be quite useful, considering how often flow systems transporting heating or cooling fluids are encountered in engineering applications.

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