Bayesian updating and decision making using correlated structural health monitoring observations

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Abstract:
A Bayesian approach is often applied when updating a deterioration model using observations from expected structural health monitoring or condition monitoring. Usually, observations are assumed to be independent conditioned on the damage size, but this assumption does not always hold, especially when using online monitoring systems. Frequent updating using correlated measurements can lead to an over-optimistic assessment of the value of information when the measurements are incorrectly modeled as independent. This paper presents a Bayesian network modeling approach for inclusion of the correlation between measurements through a location parameter and presents a generic monitoring model based on exceedance of thresholds for a damage indicator. Additionally, the model is implemented in a computational framework for risk-based maintenance planning, developed for maintenance planning for wind turbines. A generic example and a parameter study show that neglecting correlation in the decision model when the observations are in fact correlated, can lead to much higher costs than expected and to the selection of non-optimal strategies; much lower costs can be obtained when the correlation is properly modeled. In case of correlated observations, an advanced decision model using all past observations for decision making is needed to make monitoring feasible compared to only using inspections.

Keywords: Value of information, risk-informed decision support, condition monitoring, structural health monitoring, maintenance planning, wind turbines

1 Introduction
Condition monitoring or structural health monitoring is increasingly being applied in many industries such as the offshore wind industry [1] and for bridges [2,3]. A shift from a simple time-based inspection and maintenance regime to predictive maintenance based on an updated belief of the component health can lower the expected maintenance costs, especially if a risk-based approach is used to balance the expected costs of failures, preventive repairs, inspections, and monitoring [4]. The Bayesian pre-posterior decision theory [5] establishes the theoretical background for this, and Benjamin and Cornell [6] adapted it to civil engineering. For risk-based inspection planning (RBI) for offshore structures, efficient generic methods have been developed [7,8]. Here, inspection uncertainties are assumed independent given the crack size, and for inspections with a separation in time of several years as typically seen [9], this assumption seems fair. However, when frequent inspections or online monitoring is applied, the assumption of independence should be challenged.

Monitoring is defined as follows in COST 345 [10]: “Monitoring can be defined as any periodic or continuous operation where the behavior of a structure, or of its components – such as foundations, is quantified in some way so that its serviceability and stability can be evaluated.” The term “condition monitoring” (CM) is often used for mechanical components and “structural health monitoring” (SHM) is often used for structural components. Rytter [11] defined four levels for vibration-based inspection, which are widely used for CM and SHM: Detection, localization, assessment (quantification of size), and consequence (for safety). The second and third levels are related to diagnosis, and the last level is considered to also include the future performance of the structure and is therefore often called prognosis and is related to remaining useful life (RUL) estimation [12]. To include monitoring in a framework for predictive maintenance decisions, one must have the information of the diagnosis and prognosis.
There are many published studies on the use of monitoring for detection, localization, diagnosis, and prognosis. Monitoring can also be applied to assess the consequence of extreme events such as earthquake [13] or vehicle impact on a bridge [14], or it can be used to decrease uncertainties on loading [15] or to estimate the health of the structure or component. For modern wind turbines, CM systems based on vibration measurements are widely used for mechanical components in the drivetrain such as bearings and gears. Commonly used methods for diagnosis are developed based on envelope analysis utilizing the high-frequency resonance technique [16]. Methods for prognosis are being developed, for example, a data-driven failure prognostics mixture of Gaussian hidden Markov models for bearings [17], a mixture of Gaussian Bayesian belief networks [18], and neural network approaches [19].

Even for turbines without dedicated condition monitoring systems, SCADA (supervisory control and data acquisition) systems are always installed, and the data can be used for condition monitoring, although large uncertainties and false alarms are present [20]. For wind turbine main bearings, a non-linear model based on temperature measurements was developed [21], and in [22] an approach for stress monitoring was developed using neural networks and the proportional hazard model. For wind turbine blades there is a large potential for SHM [23]. Methods based on actuated vibrations [24], frequency response transmissibility analysis [25,26], and acoustic emission [27] have been proposed. Also for the wind turbine structure, vibration-based methods have been proposed [28].

Generally, monitoring does not provide direct measurements of damage size, as the observations are associated with uncertainty. To account for this uncertainty, Bayesian methods can be applied. Several studies apply Bayesian methods for prognosis based on vibration measurements [29–31], but the prognosis is made for the vibration measurements instead of for the underlying deterioration process, and the failure threshold for the signal is assumed to be known with certainty, thus giving a potential error. Qin et. al. [32] assumed that it was possible to monitor the annual damage increment, and the monitoring outcomes were used to update the mean value of the distribution for the annual damage increment. Straub [33] described how to estimate the value of information (VoI) of fatigue monitoring based on structural reliability methods, and did also consider monitoring outcomes to be independent.

To account for the uncertainty related to the monitoring signal, we can consider the damage size and the outcome of the monitoring system as two separate variables, where an indicator model relates the variables. This approach was applied in [4,34–39], and the monitoring observations were assumed to be independent given the damage size, as for inspections in RBI [7]. In classical RBI, rare inspections are applied, and the belief is only updated through the action repair/no repair based on the probability of detection (PoD) curve and repair threshold applied. When frequent inspections or online monitoring is applied for sequential Bayesian updating of a deterioration model, the assumption of independent observations might not be valid. A defect that is unusually hard to detect at one point in time due to e.g. the location would probably be equally hard to detect one month later if it has not grown significantly. When the observations are assumed independent, the probability of detection stays the same, and the more observations, the higher the total probability of detection, and the decision maker will find it less likely that a defect is present. If instead the observations were fully correlated, additional observations after the initial observation would not give new information and would not decrease decision makers’ belief of the likelihood of defects further. The reality would most likely be something in between these two boundaries, with higher correlation for inspections performed using the same method, the same equipment, and by the same person. If spatially distributed defects are considered and the decisions relate to the largest defect as proposed in [37], the location and type of defect would influence the probability of detection, and thus observations would be correlated.

The importance of unbiased measurements is also emphasized in COST TU1402 [40], where the principles for a theoretical framework for estimation of the VoI from SHM is presented, following the principles set by the Joint Committee on Structural Safety (JCSS) [41] and in ISO 2394 [42]. However, if the apparent bias
originates from the uncertainty of an underlying variable, this bias could be considered in the model, as the resulting correlation between observations can be accounted for.

Pozzi and Der Kiureghian [43] consider the assessment of VoI from monitoring and propose modeling the additive error terms as jointly normally distributed random variables, possibly correlated with a given covariance matrix. However, the estimation of such a covariance matrix or the implications for the results is not considered, and the example does not include the correlation.

Straub and Faber [44] considered the correlation between inspection outcomes of different but similar hot spots and studied how this correlation influenced the updated failure probability of the system. They concluded that normally the influence of inspection dependency would be limited compared to other contributors of uncertainty. They identified the following contributions to the uncertainty of inspection outcomes:

- Random noise (aleatoric)
- Statistical uncertainty due to limited data available for estimation of PoD model
- Model uncertainty due to the use of a parametric model
- Model uncertainty due to factors not included in the inspection model, e.g. human factors, environment, defect orientation, geometry, and location.

Thöns [45] and Thöns et. al. [15] use load monitoring of hot spots for a jacket structure and update the inspection plan according to the loading information obtained. The optimal inspection plan without and with monitoring is obtained using the generic approach for RBI presented in [7,46].

This paper considers monitoring methods, for which the observation is an indicator of the damage size or health state. This is the case for CM methods used for wind turbine drivetrains. Condition monitoring makes it possible to see an increase in the signal before failure, but the magnitude of the increase varies. Often the alarm threshold is set as the mean plus a factor times the standard deviation of the signal, and the signal is assumed to be associated with the damage size. If a probabilistic relation is assumed between such alarm levels and the damage size, and a risk-based decision framework is applied, where the probability of failure is updated sequentially based on non-exceeded alarm levels, the estimate of the reliability will increase each time the model is updated, if the observations are assumed to be independent. However, if the observations are in fact correlated due to joint model uncertainty not included in the monitoring model, it could lead to too optimistic reliability estimates, if the correlation is not accounted for. The correlation could be caused by e.g. joint model uncertainty related to the location of the defect. Before a defect is initiated, the model should reflect the overall uncertainty regarding the relationship between the defect sizes and monitoring outcome. However, once a defect is initiated in a location that makes it harder to detect than the average, it will continue to be hard to detect, and therefore the observations will be correlated.

In Section 2, this paper first presents a Bayesian network structure including the correlation between measurement for CM and SHM methods and include the approach in a Bayesian risk-based decision framework using Bayesian networks. Section 3 formulates a generic monitoring model for inclusion of correlation for use in the framework. Section 4 presents a generic example and parameter study, which examine the influence of correlation.

2 Bayesian network model

Bayesian networks provide an efficient way to model deterioration and perform Bayesian updating using measurements [47]. In the general case, where various types of distributions are applied, inference can be performed using sampling-based methods [48] or efficient exact inference methods can be used if all variables are discrete [49]. The latter approach can also be used for models with continuous variables if they are discretized, and thus an approximation is made. Bayesian networks consist of nodes representing
stochastic variables and links representing causal relationships. The dependencies are quantified through the conditional probability distributions specified for each node, conditioned on nodes with an arrow pointing toward the given node. An elaborate introduction can be found in [50].

Generally, deterioration can be modeled using dynamic Bayesian networks, which consist of identical time slices, each only connected to the following and previous time slice. Thereby a Markov chain is made, and if all variables in a present slice are observed, knowledge of previous time slices does not change the belief of the future. However, the deterioration process does not need to be Markovian in order to be modeled in this way; time-invariant model parameters can be included in each time step, causing the future damage size to be dependent of the past given the present. Fig. 1 shows a Bayesian network for a deterioration model with damage size $D$ and model parameter $M$. Additionally, a node for the monitoring outcome $I_M$ is shown for the case where observations are independent, as in [4]. This is modeled by a link pointing from the damage size to the monitoring observation, indicating that the monitoring outcome only depends on the damage size, and the monitoring models are specified as the conditional probability distribution $P(I_M|D)$. 

![Bayesian Network](image)

Fig. 1: Bayesian network for damage size $D$, model parameter $M$, and monitoring outcome $I_M$, when monitoring outcomes are not correlated.

One way to include correlation would be to add arrows between the nodes for the monitoring outcomes as shown in Fig. 2, so each observation would also depend on the previous observation, and the conditional probability distribution would be specified as $P(I_M,i|D_i,I_M,i-1)$. Another approach is to add a new variable to the network to model the ease of detection. In Fig. 3, a variable $L$ is used to model the ease of detection and will be referred to as a location parameter in the following, as the ease of detection could be affected by the location of the defect. For example, if a PoD curve was used to model the monitoring reliability, the variable $L$ could be a parameter in the PoD model, e.g. the expected size of detectable defects, and the models could be derived from trials if the location of detected defects is noted.

Here, the conditional probability distribution for the monitoring observation should be specified by $P(I_M,i|D_i,L_i)$. The location parameter could be time-invariant, representing that once a defect is initiated, the monitoring outcome will depend on this defect, or it could be time variant, representing that another defect can be initiated, and come to dominate the monitoring outcome. The model shown in Fig. 3 with the location parameter $L$ is used in the following, as it seems more intuitive to specify the model in this way, compared to the model shown in Fig. 2.

![Bayesian Network](image)
Fig. 2: Bayesian network for damage size $D$, model parameter $M$, and monitoring outcome $I_M$, when monitoring outcomes are correlated modeled by a link between them.

![Bayesian network](image1)

Fig. 3: Bayesian network for damage size $D$, model parameter $M$, monitoring outcome $I_M$, and location parameter $L$, when monitoring outcomes are correlated through the location parameter.

### 2.1 Updating

For the Bayesian network shown in Fig. 3, exact inference can be performed efficiently if all variables are discretized. The algorithms presented below are inspired by [47] and [49] and make use of conditional independencies between variables.

The network is defined in terms of the following probability distributions:

- $P(M_0)$: prior probability distribution of model parameter $M_0$
- $P(D_0)$: prior probability distribution of initial damage size $D_0$
- $P(L_0)$: prior probability distribution of location parameter $L_0$
- $P(M_i|M_{i-1})$: conditional probability distribution for the model parameter in time step $i$ given the model parameter in the previous time step
- $P(L_i|L_{i-1})$: conditional probability distribution for the location parameter in time step $i$ given the location parameter in the previous time step
- $P(D_i|D_{i-1}, M_i)$: conditional probability distribution for the damage size in time step $i$ given the damage size in the previous time step and the model parameter
- $P(I_{M,i}|D_i, L_i)$: conditional probability distribution for the monitoring outcome given damage size and location parameter

Prediction of future deterioration is performed sequentially using the following expression, where elementwise multiplication is performed for multidimensional distributions:

$$P(D_i, M_i, L_i) = \sum_{L_{i-1}} P(L_i|L_{i-1}) \sum_{D_{i-1}} P(D_i|D_{i-1}, M_i) \sum_{M_{i-1}} P(M_i|M_{i-1}) P(D_{i-1}, M_{i-1}, L_{i-1})$$

(1)

When monitoring observations are received, Bayesian updating is performed using Bayes rule, which is expressed as:

$$P(D_i, M_i, L_i|I_{M,i} = i_{M,i}) \propto P(D_i, M_i, L_i, I_{M,i} = i_{M,i}) = P(D_i, M_i, L_i) P(I_{M,i} = i_{M,i}|D_i, L_i)$$

(2)

When monitoring observations are received in all time steps, equations (1) and (2) are used alternating with increasing $i$, and the joint distributions $P(D_i, M_i, L_i)$ are conditioned on all observations from all past time steps, although for simplicity this is not indicated in the notation applied.
2.2 Inclusion in risk-based operation and maintenance framework

The model formulation used above corresponds to the formulation used in the computational framework for risk-based maintenance planning presented in [4], except that monitoring observations were assumed to be independent. The framework considers several types of decision rules for decisions on inspections and repairs, and two distinct methods are used for the estimation of probabilities of inspections, preventive repairs, and failures in each time step. For both methods, these probabilities are estimated for a range of candidate strategies and parameters in decision rules and are then multiplied by the specific costs of inspection, preventive repair, and failure respectively, to obtain the total expected lifetime costs. In the first method, Bayesian networks are used directly for the computation of probabilities, and only simple decision rules depending on constants or observed variables can be used. The second method uses simulations, and Bayesian networks are used within simulations to estimate the probability of failure, such that decision rules using the probability of failure can be used. For these decision rules, all past observations are considered when a decision is made, as they are used for the computation of the probability of failure. If the simulation-based approach is extended to include correlation between observations through the location parameter $L$, the Bayesian network shown in Fig. 4 can be applied for decision making. The network is updated each time new information from inspection or monitoring is received and each time decisions are made. All types of decision rules can be included in this approach, as decisions are not modelled within the Bayesian network, but instead entered as evidence. For this network, inference can be performed using the same principles as in (1) and (2) and in [4], and the effect of including or excluding the correlation can be assessed. The first method, using Bayesian networks directly for decision making, have been extended in a similar way to include correlation. The example in Section 4 will use both methods.

Fig. 4: Bayesian network for damage size $D$, damage size after potential repair $D_{R}$, model parameter $M$, monitoring outcome $I_{M}$, location parameter $L$, inspection outcome $I$, inspection decision $I_{D}$, and repair decision $R$.

3 Modelining correlated monitoring observation

This section presents a generic monitoring model including the effect of correlation through a location parameter. A monitoring model is here understood as a probabilistic model relating the outcome of the monitoring system to the health of the considered component. For most monitoring systems, e.g. vibration-based CM or SHM, the direct outcome will be a high-frequency signal, and it is necessary to apply methods to derive a single variable which is related to the component health, sometimes referred to as a damage indicator. For CM of the drivetrain of a wind turbine, a short time series is usually recorded e.g. once a day when the turbine is operating within specified conditions, for the outcomes to be less noisy. Methods such as envelope analysis are used to derive a single variable, which is related to the component health, e.g. the crest factor or a root mean square (RMS) value. In that case, a measurement could be available for updating of the model every day. However, for the purpose of a risk-based decision model, the computation time will be
large if decisions are included with time steps in the order of days. A more appropriate time step for decisions could be weeks or months, and therefore all monitoring observations received in a time step needs to be summarized into one variable, e.g. a mean or maximum value. Fig. 5 illustrates the concept.

For a simple monitoring model with damage indications that are independent given the damage size, the outcome of the damage indicator $X$ could be equal to the damage size $D$ plus a zero-mean error term $\varepsilon$:

$$X = D + \varepsilon$$  \hspace{1cm} (3)

This corresponds to a linear relationship between expected outcome and damage size. In a more general case, still with independent outcomes, the model could be written as follows, where $f$ is a function of damage size, relating the expected value of the outcome and the damage size:

$$X = f(D) + \varepsilon$$  \hspace{1cm} (4)

If we now introduce correlation between outcomes through the location parameter $L$, the general model can be written as:

$$X = f(D, L) + \varepsilon$$  \hspace{1cm} (5)

where $f$, representing the expected damage size, is now a function of both damage size and location parameter, and the outcomes are considered as independent given the damage size $D$ and the location parameter $L$. The location parameter is defined as the relative increase in $X$ for a defect in a given location compared to the increase for a defect in an optimal location. For an optimal location, it is one; for the worst possible location with no possibility of detection, it is zero.

If the error is normally distributed with mean zero and standard deviation $\sigma$, the outcomes $X$ will then follow a normal distribution with mean $f(D, L)$:

$$X \sim N(f(D, L), \sigma)$$  \hspace{1cm} (6)

The distribution function of $X$ given $D$ and $L$ is then:

$$F(x|D, L) = \Phi\left(\frac{x - f(D, L)}{\sigma}\right)$$  \hspace{1cm} (7)

If the node for the monitoring outcome $I_M$ is discretized into $k$ intervals with upper bounds $x_k$, the probability of obtaining an outcome in interval $k$ is found by:

$$P(I_{M,k}|D, L) = F(x_k|D, L) - F(x_{k-1}|D, L)$$  \hspace{1cm} (8)

If the model is updated every time a monitoring observation is received, Eqs. (7) and (8) define the conditional probability distribution $P(I_{M,k}|D, L)$. 

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**Fig. 5: Principle for obtaining the monitoring observation used for updating the model.**

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where $f$, representing the expected damage size, is now a function of both damage size and location parameter, and the outcomes are considered as independent given the damage size $D$ and the location parameter $L$. The location parameter is defined as the relative increase in $X$ for a defect in a given location compared to the increase for a defect in an optimal location. For an optimal location, it is one; for the worst possible location with no possibility of detection, it is zero.

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**Monitoring signal**
- Continuous time series
- High frequency (Hz)

**Damage indicator**
- Derived using e.g. envelope analysis
- Once a day

**Monitoring observation**
- Statistical value of damage indicator
- Once a month
However, as described above, for decision-making purposes, it is computationally inefficient to update the model many times during each time step. Instead, the model can be updated once in each time step using a statistical value of \( X \). If the sample mean of the monitoring observations, \( \bar{X} \), for each time step is used, and the standard deviation of the error is known, the distribution function for the mean value is:

\[
F(\bar{x}|D, L) = \Phi \left( \frac{\bar{x} - f(D, L)}{\sigma} \sqrt{n} \right)
\]  

(9)

where \( n \) is the number of measurements used to compute the mean \( \bar{X} \).

If instead the maximum value of the sample were used for updating, the appropriate statistical model would be an extreme value distribution. For the case with identically normal distributed and independent random variables, the distribution function for the maximum value \( x_{\text{max}} \) of \( n \) measurements is given as [51]:

\[
F(x_{\text{max}}|D, L) = \exp \left( -\exp \left( -\frac{\alpha_n}{\sigma} \left( x_{\text{max}} - f(D, L) - \sigma u_n \right) \right) \right)
\]  

(10)

\[
\alpha_n = (2 \ln n)^{0.5}
\]

(11)

\[
u_n = \alpha_n - \frac{\ln(\ln n) + \ln(4\pi)}{2\alpha_n}
\]

(12)

The deterministic function \( f(D, L) \) states how the expected value of the monitoring outcome depends on damage size \( D \) and location parameter \( L \). A simple case would be to have a linear dependency between the expected value of the monitoring outcome and the product \( DL \), where the model can be written as:

\[
f(D, L) = \mu_0 + kDL
\]

(13)

where \( \mu_0 \) and \( k \) are constants. The constant \( \mu_0 \) is the mean value of the damage indicator for a healthy component, and the constant \( k \) is the increase of the damage indicator per unit increase in damage size for a damage where the location parameter is equal to one.

### 3.1 Estimation of model based on knowledge and data

To use the monitoring model described above, knowledge of the function \( f(D, L) \), the standard deviation \( \sigma \) of the damage indicator, and the prior distribution of \( L \) is needed. The model assumes that the standard deviation of the damage indicator is independent of the damage size and location parameter. If this is the case, the standard deviation can be estimated from data after a run-in period with a healthy component. If this is not the case, this dependence should be included in the model and should be found based on experience. The mean of the signal for a healthy component \( \mu_0 \), can also be estimated after a run-in period with a healthy component. The parameter \( k \) for the increase per unit increases in damage size and the prior distribution of \( L \) must be estimated based on experience, experiments, or models. The prior distribution of \( L \) should be based on the probability of a defect initiating in each possible location and the relative increase that it will give the signal compared to a defect in an optimal location.

### 4 Generic example

To quantify the influence of the correlation between observations, a generic example is presented, where optimal risk-based strategies for inspections and preventive repairs are found when correlated or uncorrelated monitoring observations are available. The same overall probability of each monitoring outcome, given the damage size, is used. Additionally, the influence of neglecting an existing correlation is found. Finally, a parameter study is presented to assess the generality of the conclusions. The method is based on the
computational framework presented in [4], but the framework has been extended to include the location parameter \( L \), as described in Subsection 2.2.

4.1 Input models

The attempt in this example is to construct a generic case, where variables can be adjusted to assess how the conclusions will be affected. A rather simple deterioration model with discrete states has been applied, which can easily be adjusted to represent linear, concave and convex deterioration models. The example follows the assumptions used in the example in [4], except for the monitoring model. In [4], monitoring observations were assumed to be independent, and the present paper uses the monitoring model presented in Section 3. The time step used in the model is one month, and the planned lifetime is 20 years. For the base case, a linear Markov-based deterioration model with seven states is used; however, the transition probability is assumed to be uncertain and is modeled using a time-invariant parameter. The last state ‘fault’ is entered at a damage size of one, and the mean time to failure is set to 20 years for the expected value of the transition probability. The lower bound for the first state is zero, and the six non-faulty states are of equal size. For inspections, the probability of detection for the damage states are respectively 0, 0.4, 0.8, 0.9, 0.95, 0.98, and 1, and if a defect is detected, it is quantified correctly. For details on these models, see [4].

The monitoring model is defined by the probability distributions:

- \( P(L_0) \): prior probability distribution of location parameter \( L \)
- \( P(L_i|L_{i-1}, R_{i-1}) \): conditional probability distribution for location parameter \( L \) in time step \( i \) given the location parameter in the previous time step and the repair decision \( R \) in the previous time step
- \( P(I_{M,i}|D_i, L_i) \): conditional probability distribution for monitoring outcome given damage size and location parameter

Due to the definition of the location parameter, it can only take values from zero to one. It is zero if the defect is in a location where the defect does not change the expected monitoring outcome, and it is one if the defect is in an optimal location for detection. In the base case, \( L \) is assumed to take values 0.0, 0.2, 0.4, 0.6, 0.8, 1.0, and that the probability of each value is the same, such that the prior \( P(L_0) \) is a uniform distribution.

The location parameter is assumed time-invariant between repairs. This means that once a defect is initiated, no defects in other locations are considered. Then, the conditional probability distribution for no repair, \( P(L_i|L_{i-1}, R_{i-1} = 0) \), is equal to the identity matrix. In case of repair, the distribution for \( L \) is equal to the prior distribution, \( P(L_0) \), regardless of \( L_{i-1} \).

For the base case, linear dependency between the mean of the damage indicator and the damage size and location parameter is assumed, and the damage indicator is scaled to have the damage size as the expected value for \( L=1 \). Thereby, the expected value of the damage indicator is given by \( f(D, L) = DL \).

The updating is based on the mean of the outcomes received in a time step, and the distribution \( P(I_{M,i}|D_i, L_i) \) is evaluated using Eqs. 9 and 8. The standard deviation of the damage indicator is assumed to be 0.2, and the number of samples in each month (independent given \( D \) and \( L \)) is assumed to 30. With these assumptions and the interval boundaries in Table 1, the monitoring model for correlated observations, given by the conditional probability distribution \( P(I_{M,i}|D_i, L_i) \) illustrated in Fig. 6a is obtained. For the case of uncorrelated observations, the same overall variation of monitoring outcomes is assumed, and the distribution \( P(I_{M,i}|D_i) \) is obtained by:

\[
P(I_{M,i}|D_i) = \sum_{L_i} P(I_{M,i}|D_i, L_i)P(L_i)
\]

(14)

where \( P(L_i) = P(L_0) \). This leads to the distribution illustrated in Fig. 6b.
To quantify the resulting correlation coefficient between consecutive monitoring observations, 100 000 sets of samples of monitoring states were drawn from the distributions for each of damage state, and the sample correlation coefficient was found; the results are shown in Fig. 7. When using the models for uncorrelated observations, the correlation coefficient was very close to zero, as expected, and when the model for correlated measurements was used, the correlation coefficient increased with damage size from around 0.18 to 0.96.

\[
\begin{array}{cccccccccc}
I_M & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & F \\
\hline
x_{k,1} & - & 0.1 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 1.2 & - \\
\end{array}
\]

Table 1: Monitoring outcome, \(I_M\), and the associated lower threshold for \(\bar{x}\).

\(a)\) Conditional probability of each monitoring outcome given the state for the damage size \(D\) and location parameter \(L\).
b) Conditional probability of each monitoring outcome given the state for the damage size $D$.

Fig. 6: Monitoring models for correlated (a) and uncorrelated (b) observations.

Fig. 7: Sample correlation coefficient between two consecutive monitoring observations as a function of damage size for 100 000 samples for uncorrelated and correlated observations.

4.2 Strategies

The influence of correlation between monitoring observations on expected lifetime costs will depend on how they are used for decision making. As described in Subsection 2.2, the computational framework presented in [4] includes two distinct alternatives for this, based directly on the outcome, or based on a quantity summarizing all past information, such as the probability of failure. Additionally, a strategy without monitoring is used for comparison and quantification of the VoI of monitoring. For all strategies, pre-inspections are assumed necessary prior to repairs. For the decision on when to inspect, three strategies are considered:

1) Equidistant inspections
2) Based on a threshold for monitoring outcome
3) Based on a threshold probability of failure, considering past monitoring and inspections

For all strategies, preventive repairs are made when a threshold for the inspection outcome is exceeded. As the damage size is quantified correctly in case of detection, this is also a threshold for the state for the damage size. Therefore, for each strategy, there are two decision parameters: one for decisions on inspections (inspection interval, monitoring threshold, or probability of failure threshold), and one for repairs (damage state threshold). Corrective repairs are made after failures and are more expensive than preventive repairs.

The specific costs are normalized with respect to the inspection costs, such that the inspection costs are equal to 1. For the base case, the costs of a preventive repair are set to 37.5, and the costs of a corrective repair are 500. These relative specific costs correspond to the costs used in [4]. All costs shown in the following are therefore relative costs. The costs of purchasing, operating, and maintaining the monitoring system are not included but should be added to the costs obtained with strategies 2 and 3.
4.3 Results of base case

The expected lifetime costs are estimated for the following seven combinations of strategy and assumptions on the correlation between monitoring observations:

- 1: no monitoring
- 2-un: uncorrelated observations
- 2-co: correlated observations
- 2-co*: correlated observations, assumed uncorrelated when making decisions
- 3-un: uncorrelated observations
- 3-co: correlated observations
- 3-co*: correlated observations, assumed uncorrelated when making decisions

First, the expected lifetime costs are estimated for strategy 1, where equidistant inspections are made. The expected lifetime costs are estimated for 45 combinations of inspection interval and threshold for repairs as shown in Fig. 8. The costs are found directly using Bayesian networks using exact inference algorithms, and the computed expected values are therefore exact given the input. The lowest costs are obtained when inspections are performed every 12 months, and repairs are made when a defect in state 5 or above is detected. The costs are 64.39 times the cost of an inspection.

![Chart showing expected lifetime costs for strategy 1](image)

Fig. 8: Expected lifetime costs for strategy 1 for inspection intervals from 3 to 60 months. For each inspection interval, five bars are shown for damage state thresholds for repairs 2, 3, 4, 5, and 6.

For strategy 2, decisions on inspections are made based on the state of the monitoring outcome. As for strategy 1, the exact expected costs are found using Bayesian networks. Fig. 9 shows the relative expected lifetime costs for 30 combinations of thresholds for inspections and repairs for the cases when monitoring outcomes are uncorrelated (9a) and correlated (9b).

For uncorrelated observations, the lowest costs are 46.66 and are obtained at the monitoring threshold 4, and damage state threshold 5. Here, there are almost no failures. However, if the monitoring observations are correlated, much more failures are generally seen. The lowest costs are 154.12 and are obtained when the monitoring threshold is 2 and damage state threshold is 3. If the optimal strategy for uncorrelated observations was used, the costs were 181.73.
Fig. 9: Expected lifetime costs for strategy 2 for thresholds for states of monitoring observations 1 to 6. For each threshold, five bars are shown for damage state thresholds for repairs 2, 3, 4, 5, and 6.

For strategy 3, decisions on inspections are made based on the probability of failure within a time step (one month). The probability is estimated using all past monitoring and inspection observations. Therefore, the expected costs are estimated using simulations, and Bayesian networks are used within simulations to update the probability of failure in each time step; see Subsection 2.2 and [4] for details. Using 10 000 simulations, the relative expected lifetime costs shown in Fig. 10 were obtained. The costs are shown for 40 combinations of threshold for the probability of failure and thresholds for repairs for the cases, when monitoring observations are uncorrelated (10a), correlated (10b), and when they are correlated; however, the probability of failure used for decision making is estimated assuming that they are uncorrelated (10c).

For uncorrelated observations, the lowest costs are 46 and for correlated they are 47. For both observations, the optimal threshold for the probability of failure is $3 \cdot 10^{-4}$, and the optimal threshold for the damage state for repairs is 5. If the monitoring observations are correlated but are assumed uncorrelated when updating and the optimal thresholds for uncorrelated observations are used, the expected costs are 184. Although much higher than the costs obtained in the optimal preventive strategy, it is still less than the costs obtained when using only corrective maintenance, which is 285.
a) Uncorrelated

b) Correlated

c) Correlated, but assumed uncorrelated
Fig. 10: Expected lifetime costs for strategy 3 probability of failure thresholds for inspections $1 \cdot 10^{-5}$ to $3 \cdot 10^{-2}$. For each probability of failure threshold, five bars are shown for damage state thresholds for repairs 2, 3, 4, 5, and 6. The vertical black bars show 95% confidence intervals for the expected costs.

Fig. 11a summarizes the expected costs for optimal decision parameters for strategies 1, 2, and 3, for uncorrelated and correlated monitoring outcomes, and for the case with correlated monitoring observations that are assumed uncorrelated when making decisions. Fig. 11b shows the corresponding VoI found as the difference between the costs for strategy 1, and the costs for each of the other strategies. As the costs of the monitoring system are not included, these are also the maximum total lifetime costs of a monitoring system that would be feasible to install. Negative VoI implies that it is better not to use monitoring at all than to use that strategy.

![Graph a) relative expected costs](image)

a) Relative expected costs

![Graph b) value of information](image)

b) Value of information in percent of the costs of strategy 1 (without monitoring).

Fig. 11: Expected costs and VoI for strategy 1 and for strategies 2 and 3 for monitoring observations that are uncorrelated (un), correlated (co), and correlated but assumed uncorrelated (co*).

For uncorrelated observations, both strategies 2 and 3 give a decrease in costs and VoI around 18. For correlated observations, strategy 3 still performs very well, and a VoI of around 17 is obtained. However, using strategy 2 with correlated observations causes a substantial increase in the costs, and thereby a negative
VoI of around -90. When the observations are assumed uncorrelated but are in fact correlated, the VoI is around -120 for both strategies 2 and 3.

4.4 Parameter study
To investigate the generality of the tendencies observed in the base case, a parameter study is performed. Variables are changed, one at a time and the optimal strategies and expected costs are found for each combination of strategy and assumptions on correlation, in the same way as for the base case. To reduce computation time for strategy 3, only values of the probability of failure $10^{-8}$, $10^{-7}$, $10^{-6}$, $10^{-5}$, $10^{-4}$, $10^{-3}$, $10^{-2}$ and values for the damage state thresholds 4, 5, and 6 are used.

The following six parameters are considered, where 1 to 4 are changes to the monitoring model.

1) Exponential dependency on damage size  
2) Standard deviation of damage indicator  
3) Shape of prior distribution of location parameter  
4) Minimum value of location parameter  
5) Deterioration model – increasing transition probability  
6) Relative costs

4.4.1 Exponential dependency on damage size
The base case assumed linear dependency between damage size and the expected value of monitoring signal: $f(D, L) = DL$. If instead exponential dependency between those is assumed, the model could be formulated as:

$$f(D, L) = \frac{\exp(c_{exp} D) - 1}{\exp(c_{exp}) - 1} L$$  \hspace{1cm} (15)

Where $c_{exp}$ is a constant, determining the shape. Fig. 12 shows how the constant affects the shape of the function $f(D, L)$. Is can be seen that the relationship is closer to linear for small values $c_{exp}$, and for $c_{exp} = 0.1$ the relationship is practically linear.

Fig. 13 shows the costs obtained for each strategy and assumption on correlation for five values of $c_{exp}$. It is seen that increased exponential dependency makes the bad strategies less bad, although VoI would still be negative.
Fig. 12: Expected value of monitoring signal \( f(D,L) \) as a function of damage size \( D \) for \( L=1 \).

![Graph showing expected value of monitoring signal as a function of damage size]

Fig. 13: Expected lifetime costs as a function of \( c_{exp} \) for exponential dependency between expected monitoring outcome and damage size. Base case value: \( \sim 0.1 \).

**4.4.2 Standard deviation of damage indicator**

The base case assumed a standard deviation of the damage indicator \( \sigma \) equal of 0.2, and the number \( n \) of samples per time step was 30. Fig. 14 shows the influence of increasing or decreasing the standard deviation. Multiplying the standard deviation by a factor \( x \) gives the same effect as reducing the number of samples by a factor of \( x^2 \), as the sample distribution for the mean of normally distributed signals has the standard deviation \( \frac{\sigma}{\sqrt{n}} \).

For strategy 2, both cases with correlated observations benefit from increasing the standard deviation. Increasing the standard deviation leads to more inspections and preventive repairs, and fewer failures and an optimal strategy with lower costs is obtained for the assumptions used here.

![Graph showing relative expected lifetime costs as a function of standard deviation]

Fig. 14: Relative expected lifetime costs as a function of the standard deviation of the signal \( \sigma \). Base case value: 0.2.
4.4.3 Shape of prior distribution of location parameter

For the base case, the location parameter $L$ was assumed to have a uniform prior. If instead a triangular distribution is assumed, this would reduce or increase the probability of low or high values. Three cases, with an increasing expected value of $L$, are considered here:

- Linearly decreasing triangular distribution (zero probability of $L=1$)
- Uniform distribution (base case)
- Linearly increasing triangular distribution (zero probability of $L=0$)

Fig. 15 shows the costs obtained in the three cases. The uncorrelated cases are also affected, as the monitoring model for uncorrelated observations is computed using Eq. 14.

For most strategies, the costs decrease when the expected value of $L$ increase, and vice versa if it decreases. However, for strategy 3, when correlated observations are assumed to be uncorrelated, both triangular distributions give smaller costs than the uniform. This could be because the variance of the location parameter is smaller when a triangular distribution is used.

Fig. 15: Expected lifetime costs as a function of the shape of the prior distribution of $L$. Base case value: uniform.

4.4.4 Minimum value of location parameter

In the base case, the minimum value of the location parameter $L$ was assumed to be zero. If the minimum value is increased, and the prior is still assumed to be uniform, the costs are affected as shown in Fig. 16. Generally, the costs are seen to decrease when the minimum value of $L$ is increased; although, there is a local increase at a minimum value of 0.6 for strategy 2 with correlated outcomes, assumed to be uncorrelated. The reason for this is that here the optimal decision parameters for the uncorrelated case change from 4 to 5, which is unfavorable for the correlated case.
4.4.5 Deterioration model – increasing transition probability

For the base case, the transition probability was assumed constant, corresponding to linear damage growth and constant mean sojourn time. For deterioration models with increasing transition probability, the sojourn time is decreasing, and vice versa. For this sensitivity study, the mean sojourn time is assumed to increase or decrease linearly with damage size. The mean time to failure is kept constant, and the transition probabilities are fully defined when the increase in sojourn time per state is defined. For convenience, this will be referred to as the deterioration growth parameter. Values above one give a convex damage model, and values below one give a concave model. The general tendency is that costs decrease when the deterioration growth parameter increase; although for some strategies, it begins increasing again.
4.4.6 Changed specific costs

In the base case, the specific costs were defined relative to the costs of an inspection, such that the cost of a preventive repair was 37.5, and the cost of failure was 500. To assess the influence of changing the specific costs, each of the costs is in turn multiplied by a factor ranging from 0.1 to 5, and the optimal decision parameters are identified for each cost. Fig. 18 shows the influence on total expected costs, of changing the specific costs. Naturally, increasing any of the specific costs generally leads to increased costs. What is interesting is how the strategies perform in relation to each other. The two cases where the observations are assumed uncorrelated but are correlated, continue to perform the worst. However, decreasing the failure costs to values near the repair costs makes them perform similarly to the best strategies. For strategy 2, with correlated outcomes, reducing the inspection costs makes it perform more like the three good strategies. With uncorrelated outcomes, strategies 2 and 3 are similar, but strategy 3 is always slightly better. Strategy 3 for correlated outcomes is generally similar to the cases with uncorrelated observations.

Fig. 19 shows the influence on VoI of changing the specific costs. For the three strategy-correlation combinations with positive VoI, increasing any of the specific costs leads to an increase in VoI. For strategy 2 with correlated observations, increasing or decreasing the inspection or repair costs compared to the base case leads to increased VoI, whereas VoI always decreases with increasing failure costs.

![Graph showing the influence of changing specific costs on total expected costs](image1)

a) Changed inspection costs

![Graph showing the influence of changing specific costs on VoI](image2)

b) Changed repair costs
c) Changed failure costs

Fig. 18: Relative expected lifetime costs as a function of the factor on specific costs.

a) Changed inspection costs

b) Changed repair costs
c) Changed failure costs

Fig. 19: Value of information from monitoring as a function of the factor on specific costs.

5 Discussion

This section will explore the significance of the results from the generic example and parameter study. In the base case, for uncorrelated observations, using the advanced strategy 3 which include all past observations, is only marginally better than the simple strategy 2 which use only the most recent observation. However, the difference between the strategies is astonishing when observations are correlated. Where strategy 3 can provide almost the same VoI as for uncorrelated observations, strategy 2 is much worse than using no monitoring at all. Even with rather low monitoring thresholds for inspections, many failures will still happen, because of the influence of the location parameter, and the costs are much higher than for equidistant inspections. The reason that strategy 3 performs so well is that the probability distribution for the location parameter $L$ is also updated, each time observations are received. As such the decision model acknowledges the probability of defects, even if the monitoring system shows no increase in measurements.

When decisions are made assuming uncorrelated observations, but they are in fact correlated, both strategies lead to much larger costs than assumed. This clearly shows the importance of including an existing correlation in the model; neglecting it can cause monitoring to increase costs instead of decreasing costs, and the estimated VoI will be incorrect.

Generally, changing one of the parameters does not significantly change the overall performance of the strategies with monitoring in relation to each other. However, using strategy 2 for correlated monitoring observations will become feasible in some cases, as will be discussed.

An exponential relationship between damage size and monitoring observation increases the feasibility of the three infeasible strategy-correlation combinations. For the uncorrelated cases, the costs would also decrease slightly. Exponential behavior is beneficial because a lower threshold for inspections can be used, and still inspections are only carried out when there is a defect that should be repaired and still only a few failures occur. If preventive repair costs were assumed to increase with damage size, instead of being constant, it could be feasible to repair earlier, and exponential behavior could be less beneficial. In contrast to the other strategies, for correlated observations in strategy 3, the costs were found to increase slightly with behavior that is more exponential. This could be because the late increase (in terms of damage size) of monitoring observations makes it harder to update the location parameter, because an increase is not expected until quite late, leading to both slightly more inspections and slightly more failures.
The effect of the standard deviation of the signal is interesting. Intuitively, you might assume that less uncertainty is always better. This also seems to be true for the three feasible strategies. For the infeasible strategy 2 for correlated measurements, increasing the standard deviation actually leads to a decrease in costs. This suggests that combining this strategy with inspections occurring almost randomly actually makes it better. In case of correlated observations, an alternative to the advanced strategy using the probability of failure could be a strategy combining equidistant inspections with monitoring, such that some inspections are still made after a given period, even if the monitoring system does not indicate the presence of defects. This would reduce the number of defects with low values of the location parameter developing to failure.

Intuitively, having a low probability of low values of the location parameter should lead to decreased costs, as defects will generally be easier to detect. This effect can be observed both when changing the shape of prior distribution from uniform to triangular and when increasing the minimum value. When the minimum value of the location parameter is increased to 0.8, all strategies result in positive VoI. Strategy 3 with correlated monitoring observations – assumed uncorrelated – gives only slightly higher costs than the three best strategies and is better than using the simple strategy 2, considering the correlation. However, if the minimum value of the location parameter is 0.6, strategy 2 with correlated monitoring observations still gives positive VoI, unlike both of the strategies in which the correlation is present, but not accounted for.

Increasing the deterioration growth parameter gives a similar effect as when having an exponential relationship between damage sizes and monitoring observations; a higher deterioration growth parameter gives lower costs. The reason is that it is then easier to repair defects as late as possible before failure, as thresholds based on damage size are used. When increasing the deterioration growth parameter, most strategies will generate less preventive repairs and inspections (for strategies 2 and 3) but more failures for each set of decision parameters. This means that the optimal strategy can change, and in some cases, the expected costs for the new optimal strategy will be lower.

Changing the specific costs shows no difference in how the strategies perform in relation to each other. Smaller inspection costs make strategy 2 better for correlated observations, but it is still worse than equidistant inspections, thus monitoring is not feasible. When reducing the specific failure costs to 50 (0.1 of the base case value), only a third higher than the cost of preventive repairs, all strategies converge to the same costs. Here, a minimum preventive effort is optimal, and the costs are close to those obtained using corrective maintenance only, which is found to 28.5 for failure costs of 50.

5.1 Conclusions

In this paper, the influence of correlation between monitoring observations on the expected lifetime costs and the value of information is investigated for a generic case, and a parameter study is applied to assess the generality of the conclusions. Correlation between measurements is introduced using a defect specific location parameter, thus modeling whether defects in some locations gives less increase in monitoring observations than in other locations. The monitoring observations are assumed independent given the damage size and location parameter, and high variance on the defect specific location parameter gives high correlation between measurements.

Three strategies are applied for estimation of expected lifetime costs and VoI: Strategy 1 without monitoring but with equidistant inspections, Strategy 2 where inspections are made when a threshold for the monitoring observation is exceeded, and the advanced strategy 3 where inspections are made when a threshold for the probability of failure is exceeded. For strategies 1 and 2, Bayesian networks are used directly to estimate the costs. For strategy 3, all past monitoring and inspection outcomes are used to update the probability of failure in each time step using Bayesian networks, and Monte Carlo simulations are used to estimate the costs.

The advanced strategy 3 generally performs very well for both uncorrelated and correlated observations. In fact, the presence of correlation generally only gives a minor increase in the expected lifetime costs. The
simpler strategy 2 performs similarly to strategy 3 for uncorrelated observations. However, in the case of highly correlated observations, it leads to much higher costs than when no monitoring was used at all. If decisions are made assuming uncorrelated measurements when they are in fact correlated, it leads to much higher costs than when no monitoring is used at all for both strategies 2 and 3.

The parameter study shows that if the monitoring observations are less than 20% smaller for defects that are hard to detect, compared to the defects that are most easy to detect, strategy 2 can be used with good results, and assuming uncorrelated measurements in case of correlated measurements will be less critical, it will still give positive VoI.

It is also found that having an exponential relation between damage size and monitoring observations will make strategy 2 more beneficial and make it less critical to assume uncorrelated measurements, in case of correlated measurements.

In conclusion, it is crucial to assess whether or not correlation is present between monitoring observations and, if present, to include the correlation when estimating expected lifetime costs and VoI. Failing to do so can cause much higher costs of strategies using monitoring than when using equidistant inspections. In case of high correlation, the advanced strategy using the probability of failure can provide good results, but the simple strategy using a threshold for the monitoring outcome often cannot provide good results.

5.2 Future work

This paper presented a generic example and parameter study to draw general conclusions on the feasibility of using monitoring using two different strategies. The parameter study concerned the change of eight parameters that were assessed most relevant for the performance of strategies using monitoring. Future work can consider the influence of more parameters and combined cases where more parameters are changed simultaneously. Relevant parameters would be in relation to inspection reliability, the variance of time to failure, the possibility of new defects appearing after initiation of one defect, and the use of the maximum for the damage indicator for decision making. The strategies assume that inspections are possible, non-destructive, significantly cheaper than preventive repairs, and required prior to preventive repairs. The influence of changing these assumptions can also be assessed.

Another interesting option is to use methods for damage localization in combination with the strategies used in this paper. In the advanced strategy using the probability of failure, localization methods can be used to update the location parameter more directly, thus making the approach even more powerful. The simple strategy, using the outcome directly, could have a threshold that depends on the observed location parameter. For example, the decision parameter could be a threshold for the expected damage size, considering just the most recent monitoring observation, thus thresholds for the monitoring outcome could be extracted for each value of the location parameter. Localization methods could also make inspections cheaper, as the inspector will know where to look for the defect.

The uncertainties included in the model should reflect not only the aleatory uncertainty, but also epistemic uncertainties such as model uncertainties, including the uncertainty of input values, or sensitivity studies should be performed to make sure that the effect of changing a parameter would be low. The variance of the total cost predictions should be assessed in case financial risks are associated with excessive costs or in case of a risk-averse decision-maker.

For use in real applications, past experience can be used to formulate the probabilistic models. For example, for wind turbine drivetrains operators and manufacturers have extensive databases with CM and inspection data, which could be used to formulate models. Alternatively, in the case of less mature applications, such as structural health monitoring of structures or wind turbine blades, models of the response can be used to predict the response of defects in different locations and thereby estimate the location parameter.
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7 References


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