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Plug-and-Play Model Predictive Control for Water Supply Networks with Storage

Carsten Skovmose Kallesøe *, ** Tom Nørgaard Jensen * Jan Dimon Bendtsen *

* Aalborg University, Fredrik Bajers Vej 7c, DK-9220 Aalborg, Denmark (e-mail: {cst,kc,jimon}@es.aau.dk)
** Grundfos, Poul Due Jensens Vej 7, DK-8850 Bjerringbro, Denmark (e-mail: ckallesoe@grundfos.com)

Abstract: This paper addresses cost optimal control of pumping stations in water distribution networks with elevated reservoirs. We consider a topology in which a single pumping station and a single elevated reservoir is present in the network. This configuration is often seen at smaller water utilities. Typically, advanced network models and staff with control experience are not available at such utilities, therefore we pursue a plug-and-play approach that identifies a reduced network model from measurements and use the obtained model in an economic model predictive control (economic MPC) scheme.

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Keywords: Water Distribution, Model Predictive Control, Parameter Identification, Model Reduction, Smart Grid.

1. INTRODUCTION

This paper deals with optimization of reservoir filling via pump control in water distribution networks containing one pumping station and one elevated reservoir. Optimality is here measured as the minimum cost for operating the pump station. This cost is a function of the power consumption of the pumping station and fluctuating energy prices. The aim is to develop a plug-and-play solution enabling the use of the method at small water utilities where the development of complex algorithms and detailed network models are economically out of reach. The configuration with one pumping station and one elevated reservoir is often used at small utilities, as pump control becomes particularly simple in this case. The pumping station is connected to the reservoir through a main pipeline, and, in addition to the reservoir, a number of small town districts are also connected to the main pipeline, see Fig. 1.

Optimal control of pumping stations in water distribution is investigated in many papers. Ocampo-Martinez et al. (2013) describes a Model Predictive Control (MPC) framework for controlling both water distribution and wastewater networks. Distributed MPC control of water distribution networks is considered in Leirens et al. (2010), and a decentralised MPC approach is proposed in Ocampo-Martinez et al. (2011). Sun et al. (2014) considers the non-linearities of the hydraulic network using a constraint satisfaction problem formulation. The MPC control scheme proposed in the current exposition utilizes a reduced network model, which is also the main contribution. Reduced network models are also treated in Kallesøe et al. (2015), Maschler and Savic (1999), Paluszczykszyn et al. (2013). In Kallesøe et al. (2015) a reduced network model that is identified from measurements is used for pumping station control. While the networks investigated in Kallesøe et al. (2015) are without elevated reservoirs, we here consider networks including reservoirs. Maschler and Savic (1999) proposes model reduction for water distribution network models by using knowledge about the underlying graph. Paluszczykszyn et al. (2013) proposes a fast algorithm for reducing network models for various control and network analysis purposes. This algorithm is also based on the underlying graph of the network. While we in the current exposition assume the underlying graph of the network to have a tree structure as the system shown in Fig. 1, we have no assumptions about the distribution of the demand nodes in the tree. Nevertheless, the reduced model, which we propose, is able to capture the relation between flow and pressure to a degree which is sufficient for the purpose of our control scheme, while having the benefit of low complexity and few parameters. Methods for online identification of user demands as well as identification of the parameters of the reduced network

Fig. 1. Sketch of the type of water distribution network considered in this paper. The control objective is to control the flow d1 to minimize the operating cost of the pump with variable energy prices taken into consideration.
model are proposed in the paper. After identification of the parameters of the reduced model, it is subsequently utilized in a plug-and-play control approach Stoustrup (2009) for economic Model Predictive Control (economic MPC). Referring to the control structure presented in Ocampo-Martinez et al. (2013), the proposed economic MPC is placed on the Control Level, while a local pump controller is assumed at the Local Control Level. The resulting control scheme is self commissioning and adaptive (plug-and-play). Thereby the approach presented here distinguishes from Baunsgaard et al. (2016) by the structure of the network under consideration and the automatic identification of the user demands and the network model parameters.

The paper starts by presenting a reduced network model of the network sketched in Fig. 1. This is described in Section 2. In Section 2, the developed model is utilized in a parameter identification scheme that can identify the parameters of the model in Section 3. In Section 4, the model is utilized in a nonlinear MPC scheme. Section 5 presents the results of a numerical experiment which exemplifies the operation and benefits of the proposed control scheme. The paper ends with some concluding remarks.

Nomenclature: We use boldface low case letters for vectors and boldface capital letters for matrices such that \( \mathbf{x} \in \mathbb{R}^n \) and \( \mathbf{A} \in \mathbb{R}^{n \times m} \). 1 indicates a vector with one on all entries. By \( \mathbf{A} > (\geq) 0 \) we denote positive (semi-) definiteness of the matrix \( \mathbf{A} \). For a vector \( \mathbf{x} \in \mathbb{R}^n \), we use \( \mathbf{x} > (\geq) 0 \) to denote \( x_i > (\geq) 0 \) for every \( i \in \{1, 2, ..., n\} \). \( \mathbf{1}_{x>0} \) is an indicator function such that \( \mathbf{1}_{x>0} \) equals 1 when \( x > 0 \) otherwise it is 0.

2. REDUCED SYSTEM MODEL

The network in Fig. 1 is divided into several districts, collectively referred to as “Pressure Zone” (PZ). The flow from the pumping station into PZ1 is denoted \( d_1 \) and is the control variable in this work. The flow from the reservoir into PZ1 is denoted \( d_n \) and can be both positive and negative. Finally, the reservoir also supplies other parts (PZ2) of the network. The collective flow to these parts is denoted \( d_{n+1} \), which is always positive.

The pressure at node \( n \) that connects the reservoir to PZ1 is a function of the level \( h \) in the reservoir. This level is a function of the flow leaving the reservoir \( (d_n \) and \( d_{n+1} \)) and is described by

\[
A \mathbf{h} = -d_n - d_{n+1}
\]

where \( A \) is the constant cross sectional area of the reservoir and \( h \) belong to an interval restricted by the height of the reservoir. The pressure at the connecting node is found from the level by

\[
p_n = \alpha h + \alpha h_0
\]

where \( p_n \) is the pressure at node \( n \), \( \alpha \) is the scaling between the water level and the pressure unit and \( h_0 \) is the geodesic offset between node \( n \) and the geodesic level where \( h = 0 \) (zero point of the level sensor). Typically, the level is measured in meters and the pressure is measured in bar, hence \( \alpha \) scales from meter to bar.

The algorithm developed in this paper is restricted to networks with the structure shown in Fig. 1. This means that PZ1 is well modelled by a simple tree graph where each district is described as demand flow of the corresponding connection node (Maschler and Savic (1999)), see Fig. 2.

![Fig. 2. Simple tree graph that models the relation between the pressure at node 1 and the pressure at node n as a function of the demand flows d1 to dn.](Image)

Because of mass conservation in the network, the relation between the supply flow \( d_1 \), the reservoir flow \( d_n \) and the remaining demand flows is \( d_1 + d_n = -\sum_{i=2}^{n-1} d_i \). The sign convention of the demand flow is such that \( d_i \leq 0 \), meaning that water is always consumed inside the district. Under the assumption that the consumers of the districts has similar behaviours in average, then the demand flow \( d_i \) to the individual districts is

\[
d_i = -v_i d \quad \forall i = 2, \ldots, n - 1
\]

where \( d = d_1 + d_n \) is the sum of demand flows, \( v_i \) describes the distribution of the district demands and is constant for all \( i \), and \( \sum_{i=2}^{n-1} v_i = 1 \) due to mass conservation. The assumption that \( v_i \) is constant for all \( i \) implies that all district demands have the same daily profile. The study in Aquacraft (2011) implies that the assumption holds between districts with an equal distribution between consumer types (i.e., the distribution between residential-, industrial-, and other types of end-users is the same in all districts composing the PZ).

Now, define \( v_1 = 0 \), then the evaluation of the conservation of mass at the nodes of the graph in Fig. 2 implies that the pipe flows \( q_i \) are

\[
q_i = d_1 - \bar{d} \sum_{j=1}^{i} v_j \quad \forall i = 1, \ldots, m
\]

where \( m \) is the number of pipe elements, see Fig. 2. Note that \( m = n - 1 \) since the underlying graph of the network is a tree. The pressure \( \Delta p_i \) across each of the pipes in the graph is described by \( \Delta p_i = r_i |q_i|/q_i + a_i h_i \), where \( q_i \) is the flow through the pipe, \( r_i \) is the pipe resistance, and \( h_i \) is the level change from the inlet to the outlet node of the pipe.

Using (4), and the fact the the sum of the pressure drops between \( p_l \) to \( p_n \) equals \( \sum_{i=1}^{m} \Delta p_i \) we have

\[
p_1 = \sum_{i=1}^{m} r_i |d_1 - \bar{d} \sum_{j=1}^{i} v_j d| + a_i \sum_{i=1}^{m} h_i + p_n
\]

This model shows that the pressure at the supply node 1 is a function of the network structure (number of demand nodes), the distribution of the demands between the individual districts \( v_i \), and the pipe resistances \( r_i \). The plug-and-play approach we pursue in this paper requires automatic identification of the model (5). We do this by defining a heuristic reduced network model that approximates (5) for control purposes. The structure of the reduced network model is

\[
p_1 = \theta_1 |d_1| + \theta_2 |d_1 - \bar{d}| |d_1 - \bar{d}| + \theta_3 |d_1| |d_1 - \bar{d}| + \theta_4 |d_1 - \bar{d}| d_1 + \theta_5 + p_n
\]

where \( \theta_1 \) to \( \theta_5 \) are chosen to minimize the distance between the reduced model (6) and the real model (5). The reduced model fits the real model perfectly when \( m = 2 \) with \( \theta_1 = r_1, \theta_2 = r_2, \theta_3 = \theta_4 = 0, \) and \( \theta_5 = \alpha \sum_{i=1}^{m} h_i \). In
Section 2 presented a reduced network model (6) that predicts the pressure at node 1 given the pressure at node n. This model is a good approximation under the assumptions that the network of PZ1 has a structure similar to the network shown in Fig. 2 and that the distribution between the node demands of the district is fixed under varying loads \(d\) \((\nu_i\) constant for all \(i\) in (3)).

The dynamics of the level in the reservoir, shown in Fig. 1, is described by (1), and the level relates to the node pressure at node \(n\) in accordance with (2). This means that

\[
p_n = \alpha \frac{A}{d_1} - \frac{\alpha}{A} (\bar{d} + d_{n+1})
\]

where the relation \(d_{n} = -d_1 + \bar{d}\) is used. The total user demand \(\bar{d}\) of the zone is a stochastic process with an inherent periodicity of one day (Alcocer-Yamanaka et al., 2012). The reservoir acts on the system as a low-pass filter filtering out fast variations in the flow, meaning that only the mean value of the daily demand is important when dealing with reservoir control. The daily demand variations differ between work days and weekends. In the following, we only consider a model for workdays but the model for the weekends and holidays is obtained in a similar way. Let \(\frac{1}{\alpha} \bar{d}(t) = g_\lambda(t)\) where \(g_\lambda(t)\) is a periodic function given by

\[
g_\lambda(t) = \lambda_1 + \sum_{i=1}^{(k-1)/2} (\lambda_{2i} \cos(\omega t) + \lambda_{2i+1} \sin(\omega t))
\]

where \(\omega\) is the angular frequency corresponding a period of one day and \(k\) is the number of frequency terms in the truncated Fourier Series. Other methods as for example a season model could have been used as well. However, the Fourier Series requires less data history, hence is well suited for implementation in distributed controllers.

Using a backward Euler approximation of the derivative \(p_n\) in (7) and the consumption demand model (8) the following relation is obtained

\[
p_n(t) - p_n(t - \delta t) = \delta t \alpha_0 (d_1(t) - d_{n+1}(t)) - \delta t \alpha g_\lambda(t)
\]

where \(\lambda_0 = \frac{1}{\alpha}\). The expression (9) is linear in the parameters \((\lambda_0, \lambda)\) and therefore a good estimate of the parameters can be calculated, provided that \(g_\lambda\) can capture the shape of the consumption demand. Note that this can be controlled by increasing \(k\) in (8). Thus \(k\) should be chosen as a trade-off between model precision and model complexity.

We now assume that samples at times \(t_i\), \(i = 1, \cdots, N\) of the pressure \(p_n\) and the flows \(d_1\) and \(d_{n+1}\) are available for parameter identification, and let \(t_i - t_{i-1} = \delta t_i\). The parameters of (9) are then found by solving the following optimization problem

\[
\arg\min_{\lambda_0, \lambda} \sum_{i=1}^{N} (p_n(t_i) - p_n(t_{i-1}) -
\delta t_i \alpha_0 (\lambda_0 (d_1(t_i) - d_{n+1}(t_i)) - g_\lambda(t_i))^2
\]

(10)

The optimization problem is linear in the parameters \((\lambda_0, \lambda)\) meaning that standard recursive methods exist for the problem. Here, we use a recursive algorithm with a forgetting factor to enable tracking of changes in the system, Madsen (2008) and Ljung (1999). The recursive update is done once a day, meaning that a data-batch is collected over one day and used in the update.

To model the demand \(d_{n+1}\) of PZ2, we use a periodic function \(g_\mu(t)\). The arguments for \(g_\mu(t)\) being periodic are the same as for the demand in PZ1. The demand flow \(d_{n+1}\) is measured, therefore the optimization problem to identify the model parameters \(\mu\) of \(g_\mu\) is

\[
\arg\min_{\mu} \sum_{i=1}^{N} (d_{n+1}(t_i) - g_\mu(t_i))^2.
\]

Equation (6) describes the relation between the supply pressure and the flow \(p_1\), \(d_1\), the reservoir pressure \(p_{r2}\) and the total user demand \(\bar{d}\) in PZ1. The user demand \(\bar{d}(t)\) is modelled by \(Ag_\lambda(t)\). Using the definition of \(\lambda_0\) this means that \(\bar{d}(t) = \frac{1}{\lambda_0} g_\lambda(t)\). Define the function \(f_\theta(d_1, t)\) such that

\[
f_\theta(d_1, t) = \theta_1 |d_1| d_1 + \theta_2 \left| d_1 - \frac{g_\lambda(t)}{\lambda_0} \right| \left( d_1 - \frac{g_\lambda(t)}{\lambda_0} \right)
\]

\[
+ \theta_3 |d_1 - \frac{g_\lambda(t)}{\lambda_0}| + \theta_4 |d_1 - \frac{g_\lambda(t)}{\lambda_0}| d_1 + \theta_5
\]

(12)

then \(p_1(t) - p_{r2}(t) \approx f_\theta(d_1, t)\). Assuming that samples at times \(t_i, i = 1, \cdots, N\) of the pressures \(p_1\) and \(p_{r2}\) and the flow \(d_1\) are available for parameter identification, then the
parameters of \( f_\theta \) are found by solving the optimization problem
\[
\arg\min_\theta \sum_{i=1}^N (p_1(t_i) - p_\theta(t_i) - f_\theta(d_1(t_i), t_i))^2
\]  
(13)
Again, as (13) is linear in the parameters \((\theta)\), recursive methods can be used to estimate the parameters.

4. PREDICTIVE CONTROL

Here, we propose a controller which minimizes the cost of operating the pumping station. The controller utilizes the elevated reservoir to move the power consumption in time, such that variable energy prices can be utilized to minimize the cost of operating the system. We assume that the energy price \( c(t) \) is known in advance and is a function of time.

We further assume that the pumping station consists of a number of parallel connected pumps of equal size, and that a low level controller ensures optimal operation of the pumping station. That is, at a given flow and pressure, the number of pumps in operation is the number that results in the lowest power consumption. Under these operating conditions, the efficiency of the pumping station supplying a network can be assumed constant (Kallesøe et al., 2011) and the power consumption \( P \) is then
\[
P = \frac{(p_1-p_0)d_1}{\eta}
\]
with \( \eta \) the efficiency, \( p_0 \) the inlet pressure to the pumping station which is assumed constant, \( p_1 \) the outlet pressure which equals the pressure at node 1, and \( d_1 \) the supply flow which equals the demand flow into node 1.

The goal is to minimize the operation cost over a day by controlling the flow \( d_1 \). That is, we want to solve the minimization problem
\[
\min_{d_1} \int_{t_0}^{t_0+T} \frac{2c(\tau)}{\eta} \left( \frac{p_1(d_1(\tau), h(\tau), \tau) - p_0}{\eta} - d_1(\tau) \right) d\tau 
\]
\[+ c(h(t_0 + T) - h(t_0))^2 \]  
(14a)
where \( c \) is the cost and the term \((h(t_0 + T) - h(t_0))^2\) is introduced because the system is periodic and therefore the start and end reservoir level over the period \( T \) should be the same. This minimization problem (14a) is constrained by the system dynamics
\[
Ah(t) = d_1(t) - \frac{1}{\lambda_0}g_\lambda(t) - g_\mu(t) \]  
(14b)
\[
p_1(d_1, h, t) = f_\theta(d_1(t), t) + \alpha h \]  
(14c)
where \( f_\theta \) is the reduced network model described in (12) and \( g_\lambda \) and \( g_\mu \) are the prediction models for the user demands in PZ1 and 2. The constants \( \lambda_0 \) and \( \alpha \) describing the size of reservoir and the scaling between level and pressure respectively. Constraints on the reservoir level and the supply flow are
\[
0 \leq h < h < \bar{h}, \quad 0 \leq d_1 < d_1 < \bar{d}_1 \]  
(14d)
Water quality is an important parameter when controlling water networks with reservoirs. The average water age is a measure for the water quality, and as such must be kept below a certain value to ensure the quality. The daily water change controls the average water age. Therefore constraints are put on the water exchange rate in the reservoir.

The time integral of the inflow to the reservoir is a measure of the exchanged water volume. With the sign convention chosen here, the inflow to the reservoir equals \( -d_n \mathbf{1}_{d_n<0} = (d_1 - \frac{1}{\lambda_0}g_\lambda(t))\mathbf{1}_{d_1-\frac{1}{\lambda_0}g_\lambda(t)>0} \). The exchanged water volume is therefore given by
\[
V = \int_T \left( d_1 - \frac{1}{\lambda_0}g_\lambda(\tau) \right) \mathbf{1}_{d_1-\frac{1}{\lambda_0}g_\lambda(\tau)>0} \, d\tau 
\]  
(15)
The volume can as well be calculated from the water flowing out of the reservoir. The water flow out of the reservoir is \( d_n \mathbf{1}_{d_n>0} + d_{n+1} = (-d_1 + \frac{1}{\lambda_0}g_\lambda(t))\mathbf{1}_{d_1-\frac{1}{\lambda_0}g_\lambda(t)<0} + g_\mu(t) \), where \( d_{n+1} \) is the water flow to PZ2, which is predicted by \( g_\mu \), see Fig. 1. We assume that \( d_{n+1}(t) \geq 0 \) for all times, which is always true in practice. Therefore, the exchanged water volume over the time horizon \( T \) is
\[
V = \int_T \left( d_1 - \frac{1}{\lambda_0}g_\lambda(\tau) \right) \mathbf{1}_{d_1-\frac{1}{\lambda_0}g_\lambda(\tau)<0} + g_\mu(\tau) \, d\tau 
\]  
(16)

To ensure the water quality, the water exchange must be larger than \( V \). Summing (15) and (16), the following inequality constraint on the water volume is obtained
\[
V < \frac{1}{2} \int_T \left| \frac{1}{\lambda_0}g_\lambda(\tau) + g_\mu(\tau) \right| \, d\tau 
\]  
(17)

4.1 Economic MPC

We will use standard economic MPC to solve the optimization problem (14). This requires a discrete system model. The only dynamic component in the water distribution network is the elevated reservoir. On integral form the reservoir dynamic is
\[
h(t) = h(t-\delta t) + \frac{1}{M} \int_{t-\delta t}^{t} d_1(\tau) - \frac{1}{\lambda_0}g_\lambda(\tau) - g_\mu(\tau) \, d\tau 
\]
where it is used that \( \frac{1}{\lambda_0}g_\lambda(t) \) predicts the demand \( d \) of PZ1 and \( g_\mu(t) \) predicts demand \( d_{n+1}(t) \) of PZ2. The economic MPC finds an optimal set of piece-wise constant flow values \( d_1(t_0), d_1(t_0+\delta t), \ldots, d_1(t_0+M\delta t) \), where \( M \) is the prediction horizon and \( M\delta t = T \). Therefore the reservoir model (14b) rewrites to
\[
h(t) = h(t-\delta t) + \frac{\delta t}{M} d_1(\tau) + \left( \frac{1}{M}(v_\lambda(t) - v_\mu(t)) \right) \]  
(18)
where \( d_1(\tau) \) is constant on \( \tau \in [t-\delta t; t] \) and
\[
v_\lambda(t) = \int_{t-\delta t}^{t} \frac{1}{\lambda_0}g_\lambda(\tau) \, d\tau \]  
\[v_\mu(t) = \int_{t-\delta t}^{t} g_\mu(\tau) \, d\tau \]  
(17)

Now, define vectors \( h \in \mathbb{R}^M \), \( d_1 \in \mathbb{R}^M \), \( v_\lambda \in \mathbb{R}^M \), and \( v_\mu \in \mathbb{R}^M \) where \( [v_\lambda]_i = v_\lambda(t_i) \) and \( [v_\mu]_i = v_\mu(t_i) \) are the demand flow predictions for the sample times \( t_i \); \( i = 1, \ldots, M \). The dynamic system in (18) is a pure integrator, which means that
\[
h = 1h(t_0) + \lambda_0 G(\delta t d_1 - v_\lambda - v_\mu) 
\]  
(19)
where \( G \in \mathbb{R}^{M \times M} \) is a lower triangular matrix with 1 in the lower triangle and 1 on the diagonal.

The supply pressure \( p_1 \) is described by (14c). To put the expression on vector form we stack the pressure for times \( t_1, t_2, \ldots, t_M \) such that \( [p_1]_i = p_1(t_i) \). Thus (14c) implies
\[
p_1 = f_\theta(d_1(t), t_i) \]  
(20)
where \( [f_\theta(d_1)]_i = f_\theta(d_1(t_i), t_i) \) and \( f_\theta(d, t) \) is given by (12).

Let \( e_M \in \mathbb{R}^M \) be a unit vector with 1 at the last entry and
zero on all other entries then $h(t_M) = e_{M}^{T} h$, which implies that
\[
h(t_0 + T) = e_{M}^{T} (1h(t_0) + \lambda_0 G(\delta t d_1 - \nu_\lambda - \nu_\mu))
\]
which again implies
\[
h(t_0 + T) - h(t_0) = \lambda_0 T (\delta t d_1 - \nu_\lambda - \nu_\mu)\tag{21}
\]
since $e_{M}^{T} G = 1^T$.
This shows that in the case where $h(t_0 + T) - h(t_0) = 0$ the sum of the flows into the reservoir $\delta t d_1 - \nu_\lambda - \nu_\mu$ must equal zero, which makes perfectly sense from a physical point of view.

Using (21), the discrete optimization problem becomes
\[
\min d_1^{T} C (p_1 - p_0) + (p_1 - p_0)^{T} T d_1 + \kappa_0^2 (\delta t d_1 - \nu_\lambda - \nu_\mu) 11^T (\delta t d_1 - \nu_\lambda - \nu_\mu)\tag{22}
\]
where $C$ is a diagonal cost matrix with the costs $c(t_i)$ on the diagonal. Inserting (20) and (19) in (22) the optimization problem simplifies to
\[
\min (d_1^{T} C f_0 (d_1) + f_0 (d_1)^{T} T d_1 + d_1^{T} Q d_1 + b^{T} d_1)\tag{23a}
\]
where constant terms are left out and
\[
Q = \alpha \lambda_0 (CG + C^{T} C) + \kappa_0^2 \delta t^2 11^T
\]
\[
b = 2(\alpha h(t_0) - p_0) C 1 - 2\lambda_0 (\alpha CG + \kappa t \lambda_0 11^T) (\nu_\lambda + \nu_\mu)
\]
from which it is evident that $Q = Q^{T} > 0$. The optimization problem (23a) is subject to the following constraints
\[
\frac{1}{2} < h < \frac{1}{2}\quad d_1 < d_1 < d_1\tag{23b}
\]
and the nonlinear water quality constraint
\[
\frac{1}{2} < \frac{1}{2} d_1 (|\delta t d_1 - \nu_\lambda| + \nu_\mu)\tag{23c}
\]
where $|x|$ denotes the vector consisting of absolute values of the entries in $x$. Furthermore, in (23c) we have used the approximation $\delta t \lambda_0 a \lambda \approx v_\lambda$.

5. NUMERICAL EXPERIMENT
The MPC control developed in Section 4 is tested by a numerical experiment where a model of a pipe network simulating PZ1 in Fig. 1 is used. Fig. 4 presents a sketch of this network, which has 10 nodes n2 to n11 with a demand flow, one supply node n1, and a node n12 that connects the elevated reservoir to the network. A geodetic level $h_i$ is related to each of the node. The edges of the network is denoted $e1$ to $e11$. Each of the edges has a flow resistance $r_i$. The elevated reservoir has a cross-sectional area $A = 400$ m² and is lifted 30 meter above node n12 (since n = 12 in this case) hence $h_0 = 30$ m. In addition to the consumer demands of PZ1, there is a second consumer demand to PZ2 of Fig. 1. This demand is not affecting the network of PZ1 only the reservoir.
All the consumer demands are modelled by a nominal demand pattern added with individual stochastic variation in the form of a noise source. The nominal demand pattern is the same for all the consumer demands at nodes 2 to 11 in PZ1. The demand flow out of each of these nodes is shown in Fig. 5. The red curve is the nominal demand flow and the blue curve is the nominal flow added with noise which models natural variation in the demand. The demand flow to PZ2 is similar to the flow pattern in Fig. 5 but scaled to have a maximum flow around 18 m³/h.

The model identification and the control are tested by simulating the network over period of 10 days. The first two days, the flow $d_1$ is controlled by standard on/off control based on the reservoir level. The parameters $\lambda$, $\lambda_0$, $\mu$, and $\theta$ of the reduced network model are identified based on data from these two days. After the reduced model is identified, the predictive controller uses the model for calculating the optimal flow profile using the interior point algorithm in Matlab $fmincon$ solver. The algorithm uses measurements of the reservoir level $h$, the pressure at the supply node $p_1$ the flow at the supply node $d_1$, and the flow into PZ2 $d_{n+1}$.

The results of the parameter identification described in Section 3 are presented in Fig. 6. The top plot shows the real and estimated demand flows of PZ1. The middle plot shows the real and estimated demand flow to PZ2, and finally the lower plot shows the real and estimated pressure at the supply node (node 1 on the network in Fig. 4). The first day, there are no estimates as the data needed is not available before the end of the day. After the first day the real and estimated signals align closely, which exemplifies the usability of the parameter identification algorithm described in Section 3.

The numerical results obtained with the predictive control is presented in Fig. 7. The top plot shows the level change in the elevated reservoir together with the maximum and minimum level requirements, the middle plot shows the price signal, which here is a repeating signal that is high during daytime and low at night. The repeating behaviour is not necessary for the algorithm to work, but it enables a better understanding of the behaviour of the algorithm. Finally, the lower plot shows the demand flows of PZ1 (blue curve), the demand flow of PZ2 (green curve) and the supply flow calculated by the predictive control algorithm.
area noise which models natural variation in the demand. The flow and the blue curve is the nominal flow added with shown in Fig. 5. The red curve is the nominal demand network of PZ1 only the reservoir.

Fig. 4. Network diagram over the piping in PZ1. Node 1 is connected to pumping station and node 12 connects to the consumer demands of PZ1, there is a second consumer network is denoted of this network, which has 10 nodes simulating PZ1 in Fig. 1 is used. Fig. 4 presents a sketch approximation δt of the entries in and the nonlinear water quality constraint

\[ \begin{align*}
e_1: r &= 2.3 \times 10^{-4} \\
e_2: r &= 2.3 \times 10^{-4} \\
e_3: r &= 2.2 \times 10^{-4} \\
e_4: r &= 2.2 \times 10^{-4} \\
e_5: r &= 2.2 \times 10^{-4} \\
e_6: r &= 2.2 \times 10^{-4} \\
e_7: r &= 3.5 \times 10^{-4} \\
e_8: r &= \ldots \\
1: h &= 3.33 \\
n2: h &= 5.00 \\
n3: h &= 6.67 \\
n4: h &= 8.33 \\
n5: h &= 7.50 \\
n6: h &= 1.50 \\
n7: h &= 3.17 \\
n8: h &= 4.75 \\
n9: h &= 6.33 \\
n10: h &= 10.00 \\
11: h &= 10.00 \\
12: h &= 10.00 \\
\end{align*} \]

5. NUMERICAL EXPERIMENT

The results of the parameter identification described in Fig. 7. The top plot shows the level change exemplifies the usability of the parameter identification that the flow d1 is increasing during the emptying phase. This is because the power consumption is a function of the pressure at node 1, which is a function of the flow and the reservoir level. Therefore, the flow is increased with decreased reservoir level, which results in almost constant supply pressure p1 (see the lower plot of Fig. 6).

6. CONCLUSION

This paper studies Model Predictive Control (MPC) of a network configuration found in many European water distribution networks. The paper utilizes a reduced network model that enables automatic identification of the network behaviour. An economical MPC control approach is developed based on the model which takes non-linearities in the hydraulic network into account. The approach is tested on a numerical simulation of a small network. The tests show that the system is able to optimize the pump operation after two days, without the need for any pre-knowledge about the network and reservoir, making the control system plug-and-play commissionable.

Future work include methods for controlling multiple pumping stations by utilizing distributed MPC and for smoothing the supply flow d1 by the use of continuous control methods.

REFERENCES


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