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Voltage Modulated Direct Power Control for a Weak Grid-Connected Voltage Source Inverters

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Abstract—In this paper, we design a voltage modulated direct power control (VM-DPC) for a three-phase voltage source inverter (VSI) connected to a weak grid, where the PLL system may make the system unstable if the conventional vector current control (VCC) method is applied. Compared with the conventional VCC method, the main advantage of the proposed VM-DPC method is that the PLL system is eliminated. Moreover, in order to inject the rated real power to the weak grid, the VSI system should generate some certain amount of reactive power as well. An eigenvalues based analysis shows the system with the proposed method tracks its desired dynamics in the certain operating range. Both simulation and experimental results match the theoretical expectations closely.

Index Terms—Voltage source inverter, voltage modulated direct power control (VM-DPC), vector current controller, weak grid, stable system.

I. INTRODUCTION

Voltage source converters (VSCs) are widely used in the application of smart grid, flexible AC transmission systems, and renewable energy sources (e.g., wind and solar) in the modern power grids [1]–[7]. One of the key devices in VSCs is grid-connected voltage source inverter (VSI), which is normally controlled as a current source injecting current into the grid. For grid-connected VSIs, the conventional vector current control strategy is typically used to provide satisfactory control performance [8]. However, it has been reported that the weak grid-connected VSI with the standard vector current control strategy suffers from stability and performance issues [9]–[12]. In addition, with the increasing penetration of renewable energy resources in modern power grids, it becomes more and more important to sustain stability and high power quality induced by grid-connected VSIs [13].

A widely used control scheme for VSIs is the vector current control, where the phase-locked loop (PLL) is used for the purpose of grid synchronization [14]. In recent years, the adverse impact of the PLL on the small-signal stability of VSIs have been reported. It is found out that the PLL may deteriorate the stability of VSIs by introducing the negative incremental resistance at low frequencies [15]–[17]. The frequency coupling dynamics of VSIs introduced by the PLL have also been explicitly revealed in [18]. The frequency range of the negative resistance is determined by the bandwidth of the PLL. Therefore, the low bandwidth PLL is usually adopted in order to improve the stability robustness of VSIs, which jeopardizes the dynamic performance of the system significantly. Moreover, even though the PLL is designed with a very low bandwidth, it is still very difficult for VSIs to remain stable under the very weak grid condition, in which the grid impedance is approaching 1.3 pu [19]. Recently, Wang and Blaabjerg summarized the harmonic stability caused by the grid-connected VSIs in modern power grids [20], where the small-signal dynamics of VSIs tend to introduce a negative damping, which may be in different frequency ranges, depending on both the specific controllers of the converters and power system conditions [19]–[23]. Therefore, in order to guarantee stable operation of VSIs under the weak grid condition, the control strategy without the PLL is needed.

Another control method, direct power control (DPC), has been researched for grid-connected VSIs to control the instantaneous real and reactive powers directly without using neither inner-loop current regulator nor PLL system [24], [25]. However, these methods have a main disadvantage as a variable switching frequency based on the switching state, which results in an unexpected broadband harmonic spectrum, i.e., it is not easy to design a line filter properly. To achieve a constant switching frequency, various DPC strategies have been proposed. Some of them are using space vector modulation [25], [26], or calculating the required converter voltage vector in each switching period [27], [28]. Moreover, with the consideration of the robustness, a sliding mode control is applied to the DPC method in order to guarantee a fast tracking performance of the real and reactive powers [29], and a passivity-based control via DPC is proposed by considering the system’s intrinsic dissipative nature [30]. However, there are still undesirable ripples in both real and reactive powers. One of the optimal control algorithms, model predictive control (MPC)-DPC, has been designed by considering the multivariable case, nonlinearities, and system constraints in an intuitive way [31]–[35]. In every sampling period, MPC-DPC selects voltage vector sequence and calculates the duty cycles. MPC-DPC provides a constant switching frequency as well. However, it may incur additional computational burden.

Recently, Gui et al. introduced a grid voltage modulated-DPC (GVM-DPC), which solves the main disadvantage of the DPC method, the steady-state performance [36], [37]. It may be easily designed and analyzed for the grid-connected VSIs through various linear control techniques since a linear time-invariant (LTI) system is obtained based on the GVM-DPC.
In this paper, we design a voltage modulated direct power control (VM-DPC) strategy for the three-phase VSI connected to a weak grid, where the PLL system may make the system unstable as discussed before. The main advantage of the proposed method is that the PLL system is eliminated. In order to use the concept of the GVM-DPC, we use a band-pass-filter (BPF) for a weak grid connected VSI system to apply the similar concept. In addition, in order to inject the rated real power to the weak grid, the system should generate some certain amount of reactive power to support the voltages at the PCC as well. Finally, a comprehensive analysis is presented to show the improvement of the stability of the system with the proposed method.

The rest of the paper is organized as follows. Section II presents the system modeling of the grid-connected VSI based on the DPC model and the GVM-DPC based on BPF. In Section III, we show a stability analysis for the whole system including the BPF with consideration of the parameter variations. Section IV shows the simulation results using MATLAB/Simulink, Simscape Power Systems and experimental test using a 15-kW-inverter system. Finally, the conclusions of this work are given in Section V.

II. MODELING OF GRID-CONNECTED VOLTAGE SOURCE INVERTERS

In this Section, at first, a DPC modeling of VSC is briefly introduced. Then, a VM-DPC is proposed for the VSI system to make it to become an LTI system.

A. Modeling

Fig. 1 shows a simplified single-line diagram of a two-level VSI connected to a weak grid through an L-filter. In this study, we assume that a stiff dc source \( V_{dc} \) is connected to the dc-side of the inverter, e.g., a dc-dc converter in a PV application or a rectifier in wind application. Hence, the dynamic from the dc input is not considered in this paper. In addition, a grid impedance \( L_g \) is considered at the grid-side. Normally, the voltages at the PCC, \( v_{pcc} \), are measured to synchronize the VSI with the grid through the PLL. For the current control, either the proportional-integral (PI) controller in the \( dq \)-frame or the proportional-resonant (PR) controller in the \( \alpha\beta \)-frame could be applied to generate the voltage references for the PWM. In this study, we only compare the PI controller in the \( dq \)-frame with the proposed method.

The dynamic equations consisting of the output voltages of the VSI, the voltages at the PCC, and the output currents can be expressed as follows:

\[
\begin{align*}
L \frac{di_{L,a}}{dt} &= -Ri_{L,a} + v_{inv,a} - v_{pcc,a}, \\
L \frac{di_{L,b}}{dt} &= -Ri_{L,b} + v_{inv,b} - v_{pcc,b}, \\
L \frac{di_{L,c}}{dt} &= -Ri_{L,c} + v_{inv,c} - v_{pcc,c},
\end{align*}
\]

where

\[
\begin{align*}
v_{pcc,a} &= L_g \frac{di_{L,a}}{dt} + v_{g,a}, \\
v_{pcc,b} &= L_g \frac{di_{L,b}}{dt} + v_{g,b}, \\
v_{pcc,c} &= L_g \frac{di_{L,c}}{dt} + v_{g,c},
\end{align*}
\]

where \( i_{L,a,b,c} \), \( v_{g,a,b,c} \), and \( v_{inv,a,b,c} \) are the output current, the grid voltage, and the output voltage of the VSI in the \( abc \) frame, respectively. \( L \) and \( R \) are the filter inductance and resistance, respectively. Based on a balanced grid voltage condition, the dynamic equations in (1) can be transformed into the stationary reference frame by using Clark transformation as follows:

\[
\begin{align*}
L \frac{di_{L,\alpha}}{dt} &= -Ri_{L,\alpha} + v_{inv,\alpha} - v_{pcc,\alpha}, \\
L \frac{di_{L,\beta}}{dt} &= -Ri_{L,\beta} + v_{inv,\beta} - v_{pcc,\beta},
\end{align*}
\]

where \( i_{L,\alpha,\beta} \), \( v_{pcc,\alpha,\beta} \), and \( v_{inv,\alpha,\beta} \) indicate the output currents, the voltages at the PCC, and the inverter output voltages in the \( \alpha\beta \)-frame, respectively.

In (2), we observe that the voltages at the PCC are affected by the injected currents. However, the GVM-DPC proposed in [37] starts from a non-distorted voltage. Consequently, if we only consider a fundamental frequency of the voltage at PCC, then the injected currents will be the fundamental ones as well. It is acceptable since the fundamental of real and reactive powers are expected to be injected in the grid from the grid-side. Hence, we will use a band-pass-filter (BPF) to obtain the fundamental component of the measured PCC voltages.

\[
v_{pcc,\alpha,\beta} = G_{bpf} * v_{pcc,\alpha,\beta},
\]

where \( G_{bpf} \) is the transfer function of the BPF and \( v_{pcc,\alpha,\beta} \) is the fundamental component of the measured voltages at the PCC. Consequently, we can obtain the instantaneous fundamental real and reactive powers injected from VSI to the grid in the stationary reference frame as follows [40], [41]:

\[
\begin{align*}
P_f &= \frac{3}{2} (v_{pcc,\alpha} i_{L,\alpha} + v_{pcc,\beta} i_{L,\beta}), \\
Q_f &= \frac{3}{2} (v_{pcc,\alpha} i_{L,\alpha} - v_{pcc,\alpha} i_{L,\beta}),
\end{align*}
\]
respectively. Differentiating (5) with respect to time, we can obtain the dynamics of the instantaneous real and reactive powers as follows:

\[
\begin{align*}
\frac{dP_f}{dt} &= \frac{3}{2} \left( i_{L,\alpha} \frac{dv_{pcc,\alpha_f}}{dt} + v_{pcc,\alpha_f} \frac{di_{L,\alpha}}{dt} + i_{L,\beta} \frac{dv_{pcc,\beta_f}}{dt} + v_{pcc,\beta_f} \frac{di_{L,\beta}}{dt} \right), \\
\frac{dQ_f}{dt} &= \frac{3}{2} \left( i_{L,\alpha} \frac{dv_{pcc,\beta_f}}{dt} + v_{pcc,\beta_f} \frac{di_{L,\alpha}}{dt} - i_{L,\beta} \frac{dv_{pcc,\alpha_f}}{dt} - v_{pcc,\alpha_f} \frac{di_{L,\beta}}{dt} \right).
\end{align*}
\]  

(6)

Since \( v_{pcc} \) is a fundamental part of the PCC voltages, we can obtain the following relationship such as

\[
\begin{align*}
v_{pcc,\alpha_f} &= V_{pcc} \cos(\omega f t), \\
v_{pcc,\beta_f} &= V_{pcc} \sin(\omega f t),
\end{align*}
\]  

(7)

where \( V_{pcc} \) is the magnitude of the fundamental PCC voltages, \( V_{pcc} = \sqrt{v_{pcc,\alpha_f}^2 + v_{pcc,\beta_f}^2} \). \( \omega f \) is the angular frequency of the fundamental PCC voltages and \( \omega f = 2\pi f \) and \( f \) is the fundamental frequency of the grid voltage. Differentiating (7) with respect to time, we can obtain instantaneous fundamental PCC voltage dynamics as follows:

\[
\begin{align*}
\frac{dv_{pcc,\alpha_f}}{dt} &= -\omega f v_{pcc} \sin(\omega f t) = -\omega f v_{pcc,\beta_f}, \\
\frac{dv_{pcc,\beta_f}}{dt} &= \omega f v_{pcc} \cos(\omega f t) = \omega f v_{pcc,\alpha_f}.
\end{align*}
\]  

(8)

Substituting (3) and (8) into (6), the state-space models of the fundamental real and reactive powers are generated as follows [29]:

\[
\begin{align*}
\frac{dP_f}{dt} &= -\frac{R}{L} P_f - \omega f Q_f + \frac{3}{2L} \left( v_{pcc,\alpha_f} v_{inv,\alpha} + v_{pcc,\beta_f} v_{inv,\beta} - V_{pcc}^2 \right), \\
\frac{dQ_f}{dt} &= \omega f P_f - \frac{R}{L} Q_f + \frac{3}{2L} \left( v_{pcc,\beta_f} v_{inv,\alpha} - v_{pcc,\alpha_f} v_{inv,\beta} \right).
\end{align*}
\]  

(9)

Note that the dynamics of instantaneous real and reactive powers in (9) are a multi-input-multi-output (MIMO) system, where \( v_{inv,\alpha} \) and \( v_{inv,\beta} \) are the control inputs, and \( P_f \) and \( Q_f \) are the outputs. Moreover, notice that the system is a time-varying one since both control inputs are multiplied by the grid voltages.

B. Controller Design

To simplify the dynamics in (9), we define a VM-DPC input as follows:

\[
\begin{align*}
u_P &= v_{pcc,\alpha_f} v_{inv,\alpha} + v_{pcc,\beta_f} v_{inv,\beta} - V_{pcc}^2, \\
u_Q &= v_{pcc,\beta_f} v_{inv,\alpha} - v_{pcc,\alpha_f} v_{inv,\beta},
\end{align*}
\]  

(10)

where \( u_P \) and \( u_Q \) are the new control inputs, which will be designed. With the new control inputs defined in (10), the dynamics of the real and reactive powers in (9) can be rewritten as follows:

\[
\begin{align*}
\frac{dP_f}{dt} &= -\frac{R}{L} P_f - \omega Q_f + \frac{3}{2L} u_P, \\
\frac{dQ_f}{dt} &= \omega P_f - \frac{R}{L} Q_f + \frac{3}{2L} u_Q.
\end{align*}
\]  

(11)

Note that the dynamics of the real and reactive powers in (11) are changed into a linear time-invariant (LTI) MIMO system with the coupling states, which has a simple structure like the model of \( d-q \) axes currents of VSI.

Let’s define the errors of the real and reactive powers as follows:

\[
\begin{align*}
e_P &:= P^* - P_f, \\
e_Q &:= Q^* - Q_f,
\end{align*}
\]  

(12)

where \( P^* \) and \( Q^* \) are the references of the real and reactive powers, respectively. In order to cancel the coupling terms in (11), a simple controller consisting of feedforward and PI feedback is designed as follows:

\[
\begin{align*}
u_P &= \frac{2L\omega_f}{3} Q_f + K_{P,p} e_P + K_{P,i} \int_0^t e_P(\tau)d\tau, \\
u_Q &= -\frac{2L\omega_f}{3} P_f + K_{Q,p} e_Q + K_{Q,i} \int_0^t e_Q(\tau)d\tau,
\end{align*}
\]  

(13)

where \( K_{P,p}, K_{Q,p}, K_{P,i}, \) and \( K_{Q,i} \) are the PI controller gains. Substituting (13) into (11), the error dynamics of real and reactive powers could be obtained as

\[
\begin{align*}
\dot{e}_P &= -(K_{P,p} + \frac{R}{L}) e_P - K_{P,i} \int_0^t e_P(\tau)d\tau, \\
\dot{e}_Q &= -(K_{Q,p} + \frac{R}{L}) e_Q - K_{Q,i} \int_0^t e_Q(\tau)d\tau.
\end{align*}
\]  

(14)

Roughly, the closed-loop system with the proposed method is exponentially stable in the operating range if the PI controller gains are positive. Finally, the original control inputs, \( v_{inv,\alpha} \) and \( v_{inv,\beta} \), could be calculated by means of (10) as follows:

\[
\begin{align*}
v_{inv,\alpha} &= v_{pcc,\alpha_f} u_P + v_{pcc,\beta_f} u_Q + V_{pcc}^2 v_{pcc,\alpha_f}, \\
v_{inv,\beta} &= v_{pcc,\beta_f} u_P - v_{pcc,\alpha_f} u_Q + V_{pcc}^2 v_{pcc,\beta_f}.
\end{align*}
\]  

(15)

The block diagram of the proposed method is shown in Fig. 2.

III. Stability Analysis

In this section, we investigate the eigenvalues of the error dynamics with the proposed method. Based on such eigenvalues, we analyze the stability of the weak grid-connected VSI.

At first, let us define the transfer function of the BPF used in this study as follows:

\[
G_{bpf} = \frac{2\omega_c s}{s^2 + 2\omega_c s + \omega_0^2},
\]  

(16)

where \( \omega_c = \zeta \omega_0 \) is the resonance bandwidth, \( \omega_0 \) is the resonance frequency, and \( \zeta \) is damping ratio. To obtain the state-space model of the BPF, we define the new state \( x_{bpf} \in \mathbb{R}^4 \),

\[
x_{bpf} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} v_{pcc,\alpha} \\ v_{pcc,\beta} \\ P_f \\ Q_f \end{pmatrix},
\]  

where \( x_1, x_2, x_3, \) and \( x_4 \) are the states of the BPF, are the PCC voltages, the real power, and the reactive power, respectively.

The state-space model of the BPF is obtained as follows:

\[
\dot{x}_{bpf} = \begin{pmatrix} \frac{1}{L} \frac{dP_f}{dt} \\ \frac{1}{L} \frac{dQ_f}{dt} \\ \frac{1}{L} \frac{dP_f}{dt} - \omega f Q_f + \frac{3}{2L} u_P \\ \frac{1}{L} \frac{dQ_f}{dt} + \omega P_f - \frac{3}{2L} u_Q \end{pmatrix} = G_{bpf} x_{bpf}.
\]
the control inputs \( u_{bpf} = [v_{pcc,α}, v_{pcc,β}]^T \), and the output \( y_{bpf} = [v_{pcc,α}, v_{pcc,β}]^T \), then the state-space model of the BPF can be obtained as follows:

\[
\dot{x}_{bpf} = \begin{bmatrix}
-2\omega_c & -\omega_0^2 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & -2\omega_c & -\omega_0^2 \\
0 & 0 & 1 & 0 \\
\end{bmatrix} x_{bpf} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} u_{bpf},
\]

\[
y_{bpf} = \begin{bmatrix}
2\omega_c \\
0 \\
0 \\
2\omega_c \\
\end{bmatrix} x_{bpf}.
\]

(17)

To simplify the analysis, we use the proportional controller instead of the PI controller in (13). In order to obtain the closed-loop system, we substitute (15) and (17) into (3). Finally, we can obtain the closed-loop system as

\[
\dot{x} = A(x)x + Bu,
\]

where \( x = [i_{α}, i_{β}, x_{bpf}]^T \in \mathbb{R}^6 \) and \( u = [v_{pcc,α}, v_{pcc,β}]^T \in \mathbb{R}^2 \). Moreover, \( A \) and \( B \) are listed in (19) at the bottom of the paper. Since the state variables are ac signals, the error dynamics is used to consider its tracking behavior, which has only one equilibrium point at the origin. We assume that there exists signal \( x^d \) to satisfy the following relationship:

\[
\dot{x}^d = A(x^d)x^d + Bu,
\]

where \( x^d = [i_{α}^d, i_{β}^d, x_{bpf}^d]^T \in \mathbb{R}^6 \). The superscript “\( d \)” indicates the desired value. The assumption in (20) is acceptable in this study, since we consider that the system dynamics are sufficiently smooth in an open connected set.

It should be noted that the proposed method stabilizes the system exponentially based on (14), i.e., \( P_f \) and \( Q_f \) converge to their references exponentially. Hence, in this study, we do not consider the dynamics of \( V_{pcc}^2 \) in (19), i.e., \( V_{pcc}^2 \approx V_{pcc}^2 \).

It is acceptable since \( V_{pcc}^2 \) is a dc value and has a slow dynamics compared to the currents. If we define an error as follows:

\[
e = x^d - x,
\]

then, the error dynamics could be obtained as

\[
\dot{e} = \dot{x}^d - \dot{x} = A(x^d)e
\]

(22)
where \( A(x^d) \) is listed in (23) at the bottom of the paper.

Consequently, we could consider \( V_{pcc,f} \) based on a phasor diagram, as shown in Fig. 3. It should be noted that upper-case letters are used for magnitude and lower-case letters are used for instantaneous variables in this study. We define \( V_{pcc}, V_g \), and \( I_L \), being the magnitude of \( V_{pcc}, v_g \), and \( i_L \), respectively. From Fig. 3, \( V_{pcc}^d \) has a relationship between \( V_g \) and \( I_L^d \) such as

\[
V_{pcc}^d = V_g^2 - (\omega_f L_g I_L^d)^2.
\]  
\[
(24)
\]

If we consider the only real power, then we can obtain the following relationship such as

\[
I_L^d = \frac{2 P^d}{3 V_{pcc}},
\]  
\[
(25)
\]

Substituting (25) into (24), \( V_{pcc} \) can be obtained as

\[
V_{pcc}^d = \frac{V_g^2}{2} \pm \sqrt{\frac{V_g^4}{4} - \left(\frac{2}{3} \omega_f L_g P^d\right)^2}.
\]  
\[
(26)
\]

When \( P^d = 0 \), \( V_{pcc}^d \) should be equal to \( V_g \). Hence, it should be \('+\' in (26). Notice that, \( \frac{V_g^2}{2} - \left(\frac{2}{3} \omega_f L_g P^d\right)^2 \) should be larger than zero since \( V_{pcc}^d \) should have real value. That means the inverter has a maximum real power injecting to the weak grid, which has been discussed in [42].

To analyze the weak-grid connected VSI, we assume that the capacity of the VSI is 3.5 kW. Consequently, we obtain \( L_g = 22 \text{ mH} \) when \( SCR = 1.5 \) and the root mean square (RMS) of the grid voltage, \( V_{g_{rms}} \), is 110 V. In addition, the BPF is designed as follows: \( \omega_0 = 2\pi f \) and \( \zeta = 0.707 \).

Fig. 4 shows the eigenvalues of the closed-loop system in (22) when the real power increases from \( P^d = 0 \) to \( P^d = 2.45 \text{ kW} \). In this case, we fix \( K_{P,p} = K_{Q,p} = 20 \). We can observe that the eigenvalues of the closed-loop system move to the right-half-plane when we increase the real power.

To inject the rated real power to the weak grid, the reactive power should generate to compensate the voltage at the PCC [42]. Consequently, (26) is changed into relationship in (27) based on the phasor diagram, as shown in Fig. 5.

\[
V_{pcc}^d = \frac{V_g^2}{2} + \frac{2}{3} \omega_f L_g Q^d
\]  
\[
+ \sqrt{\left(\frac{V_g^2}{2} + \frac{2}{3} \omega_f L_g Q^d\right)^2 - \left(\frac{2}{3} \omega_f L_g\right)^2 (P^d + Q^d^2)}.
\]  
\[
(27)
\]

Fig. 3. Phasor diagram of the voltages at PCC and grid when operating at unity power factor.

Fig. 4. Eigenvalues of the error dynamics when the real power increases from \( P^* = 0 \) to \( P^* = 2.45 \text{ kW} \) and \( L_g = 22 \text{ mH} \).

Fig. 5. Phasor diagram of the voltages at PCC and grid with consideration of reactive power [42].
operation could be calculated as
\[ Q^d = \frac{4}{3} \omega f L_g P^d (P^d + Q^d) \geq 0, \]
From (27), it should be noted that the following constraint should be satisfied.
\[ \frac{(V_g^2 + \frac{4}{3} \omega f L_g Q^d)^2}{4} - \frac{2}{3} (\frac{\omega f L_g}{3})(P^d + Q^d) \geq 0, \] (28)
Thus, the amount of reactive power to be injected for stable operation could be calculated as
\[ Q^d \geq \frac{(\frac{2}{3} \omega f L_g P^d)^2 - V_g^4}{\frac{2}{3} \omega f L_g V_g^2}. \] (29)
Fig. 6 shows the eigenvalues of the closed-loop system in (22) when the real power increases from \( P^d = 0 \) to \( P^d = 3.5 \text{ kW} \), and \( Q^d = 2 \text{ kvar} \). In this case, the inverter could inject its rated power with the compensation of the reactive power. The eigenvalues of the closed-loop system move close to imaginary axis when the more real power is injected into the weak grid. However, all the eigenvalues are located in the left-half-plane. When \( Q^d = P^d \), all the eigenvalues of the closed-loop system are also in the left-half-plane even if the VSI injects the rated power, as shown in Fig. 7. In addition, the eigenvalues move further from imaginary axis compared to \( Q^d = 2 \text{ kvar} \) when the VSI injects its rated power. We can conclude that the VSI could inject its rated real power to the weak grid when it also injects reactive power to support the voltages at PCC.
To check the robustness to the frequency variation, we change \( f \) from 49 Hz to 51 Hz, and \( P^d = 3.5 \text{ kW} \) and \( Q^d = P^d \). Fig. 8 shows the performance of the VSI with the proposed control method when the reference of \( P \) is changed from 0.5 kW to 2 kW at 0.8 s. However, when we increase more the real power reference, the system can not be stabilized, as shown in Fig. 11. Hence, the VSI injects some certain amount of reactive power into support the PCC voltages, and it can inject its rated power 3.5 kW real power into the grid, as shown in Fig. 12.

IV. PERFORMANCE VALIDATION

In this section, to validate the proposed control method, we use the MATLAB/Simulink, Simscape Power Systems and a prototype experimental setup at Aalborg University. The parameters of the system used in the simulation are listed in Table I.

A. Simulation Results

Fig. 10 shows the performance of the VSI with the proposed method when the reference of \( P \) is changed from 0.5 kW to 2 kW at 0.8 s. However, when we increase more the real power reference, the system can not be stabilized, as shown in Fig. 11. Hence, the VSI injects some certain amount of reactive power into support the PCC voltages, and it can inject its rated power 3.5 kW real power into the grid, as shown in Fig. 12.
Moreover, at 0.8 s, a converter load is connected at PCC and consumes 1.0 kW real power when the VSI regulates 3.5 kW real power and 2.0 kvar reactive power, as shown in Fig. 13. The proposed method could handle the nonlinear load as well. Once again, it is worth mentioning that the presented work is focused on the stability of grid-converter interaction under high impedance grid conditions. The steady-state harmonic (load) disturbance rejection is not the core contribution of this paper.

Fig. 14 shows the low voltage ride through capability of the proposed control method when the VSI regulates real and reactive powers to 0.5 kW and 2.0 kvar, respectively. In this case, the grid voltage has 20% sag at 0.8 s and recovers to its nominal value after 0.05 s. The proposed control method could regulate its real and reactive powers well with a small overshoot, as shown in Fig. 14(c). In this study, we did not consider the current limit to protect the VSI in the large voltage disturbance. However, the current limitation strategy could be easily implemented into the proposed control method, which has been discussed in other paper. Moreover, the reactive power requirement in the grid code is not considered as well. However, the power references could be modified based on the requirement, and it will be studied in the future.

We also test the effect of the variation of the grid frequency. As shown in Fig. 15(a), the frequency is changed from 49.5 Hz to 50.5 Hz at 0.8 s, and goes back to 49.5 Hz at 0.85 s. From Fig. 15(b), the VSI synchronizes the new frequency of

### TABLE I

**System Parameters Used in Simulations and Experiments**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal grid voltage</td>
<td>$V_{g,a,rms}$</td>
<td>110</td>
<td>V</td>
</tr>
<tr>
<td>Nominal grid frequency</td>
<td>$f$</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Dc-link voltage</td>
<td>$V_{dc}$</td>
<td>730</td>
<td>V</td>
</tr>
<tr>
<td>Filter inductance</td>
<td>$L_f$</td>
<td>6</td>
<td>mH</td>
</tr>
<tr>
<td>Grid inductance</td>
<td>$L_g$</td>
<td>22</td>
<td>mH</td>
</tr>
<tr>
<td>Grid capacitance</td>
<td>$C_g$</td>
<td>15</td>
<td>μF</td>
</tr>
<tr>
<td>SCR</td>
<td>$S_{cr}$</td>
<td>1.5</td>
<td>pu</td>
</tr>
<tr>
<td>Resonance bandwidth of BPF</td>
<td>$\omega_c$</td>
<td>222</td>
<td>rad/s</td>
</tr>
<tr>
<td>Resonance frequency of BPF</td>
<td>$\omega_0$</td>
<td>314</td>
<td>rad/s</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>$f_{sw}$</td>
<td>10</td>
<td>kHz</td>
</tr>
</tbody>
</table>
Fig. 11. Performance of the VSI with the proposed control method when the reference of $P$ is changed from 0.5 kW to 3.5 kW at 0.8 s. (a) PCC voltages, (b) currents, (c) real and reactive powers.

Fig. 12. Performance of the VSI with the proposed control method when the reference of $P$ is changed from 0.5 to 3.5 kW at 0.8 s and $Q$ is regulated to 2 kvar. (a) PCC voltages, (b) currents, (c) real and reactive powers.

the grid quickly. Hence, we can conclude that the proposed control method is robust to the variation of the grid frequency.

In this case, we also use the different BPF parameter (i.e., $\zeta = 0.3$.) It can be observed that the real and reactive powers have offset when the grid frequency changes, as shown in Fig. 16. However, the steady-state power ripples are smaller than the previous ones. That means if the bandwidth of BPF is too narrow, it is not easy to handle a wide variation in grid frequency. Otherwise, it is not easy to give sufficient attenuation in higher harmonic components presented in the measured PCC voltage. It is a trade-off. In addition, we inject 5th and 7th harmonics to grid side at 0.9 s, where the THD of the grid voltage is 5.8%. When the grid voltage is distorted, the proposed method slightly increases the THD of the current from 1.2% to 2.2% in the case of $P = 3.5$ kW and $Q = 2$ kvar, as shown in Fig. 17.

B. Experimental Results

The effectiveness of the proposed method is also validated by using a prototype experimental setup. Fig. 18(a) shows a three-phase 15 kW inverter system with a 6 mH $L$–filter connected to a weak grid, which consists of 22 mH–$L$ and 15 $\mu$F–$C$ in parallel, and a grid simulator generating 110 V RMS grid voltage, as shown in Fig. 18(b). The PCC voltages and the line currents are measured by using a DS2004 A/D board, and the proposed control strategy is implemented in the DS1007 dSPACE system, where the switching pulses of the inverter are generated by using the DS5101 digital waveform output board. In addition, a dc power supply supports a constant voltage at the dc-link. The parameters of the system used in the case study are summarized in Table I.

Fig. 19 shows the time response of the conventional VCC method when the reference of real power is changed from 0.5 kW to 2 kW. Notice that the settling time of the PLL is set to 0.05 s. As discussed in [20], the unstable phenomenon is observed when the inverter injects more real power into the grid. However, the proposed method can stabilize the system even if it injects more real power to the grid, as shown in Fig. 20. In addition, when the inverter injects more real power to the weak grid, the system becomes unstable and the protection of the system is activated, as shown in Fig. 21. This is also explaining the injected real power will affect the voltages at PCC.

Fig. 22 shows the time response of the system when the reference of real power is changed from 1.5 kW to 3 kW and the reactive power is regulated to 3 kvar. We can observe that the system is stable since the voltage is supported by the injected reactive power. Moreover, we increase the real power to its rated power 3.5 kW, as shown in Fig. 23. The inverter system is operating well with the proposed method. Consequently, we can conclude that the reactive power should also be injected when the rated real power is desired to inject into the weak grid by the inverter.
Fig. 13. Performance of the VSI when the converter load is connected at PCC. (a) PCC voltages, (b) currents, (c) real and reactive powers, (d) real and reactive powers of load.

V. CONCLUSIONS

In this paper, we have introduced a VM-DPC strategy for the three-phase VSI connected to a weak grid, where the PLL system may make the system unstable. We use a BPF for the weak grid connected VSI system to apply the concept of the GVM-DPC. From the comprehensive analysis based on the eigenvalues, the system is always stable in this operating range. In addition, in order to inject the rated real power to the weak grid, the system should generate some certain amount of reactive power to support the voltages at PCC as well. Finally, simulation and experimental results show that the proposed method is working well in the weak grid.

REFERENCES


Fig. 14. Performance of the VSI with the proposed control method when the grid voltage has 20% sag at 0.8 s and recovers to its nominal value after 0.05 s. (a) PCC voltages, (b) currents, (c) real and reactive powers.

Fig. 15. Performance of the VSI when the grid frequency is changed from 49.5 Hz to 50.5 Hz. (a) PCC voltages, (b) currents, (c) real and reactive powers.
Fig. 16. Performance of the VSI with different BPF (\(\zeta = 0.3\)) when the grid frequency is changed from 49.5 Hz to 50.5 Hz. (a) PCC voltages, (b) currents, (c) real and reactive powers.

Fig. 17. Performance of the VSI when the grid voltage is distorted. (a) PCC voltages, (b) currents, (c) real and reactive powers, (d) grid voltages (THD=5.8\%), (e) current spectrum.
Fig. 18. (a) Experimental setup at Aalborg University; (b) configuration of experimental setup.

Fig. 19. Measured performance of the VSI with the conventional VCC method when the real power is changed from 0.5 kW to 2 kW. Blue: $V_{pcc,ab}$ [250 V/div]; sky blue, bubble pink, and green: $i_{L,abc}$ [10 A/div].

Fig. 20. Measured performance of the VSI with the proposed control method when the real power is changed from 0.5 kW to 2 kW. Blue: $V_{pcc,ab}$ [250 V/div]; sky blue, bubble pink, and green: $i_{L,abc}$ [10 A/div].

Fig. 21. Measured performance of the VSI with the proposed control method when the real power is changed from 0.5 kW to 3 kW. Blue: $V_{pcc,ab}$ [250 V/div]; sky blue, bubble pink, and green: $i_{L,abc}$ [10 A/div].


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Fig. 23. Measured performance of the VSI with the proposed control method when it injects 3.5 kW and 3.5 kvar to the grid. Blue: \( V_{\text{pcc,ab}} \) [250 V/div]; sky blue: \( i_{L,a} \) [10 A/div]; bubble pink: \( P \) [3 kW/div]; green: \( Q \) [3 kvar/div].
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