# Modelling in-room radio channels using point processes

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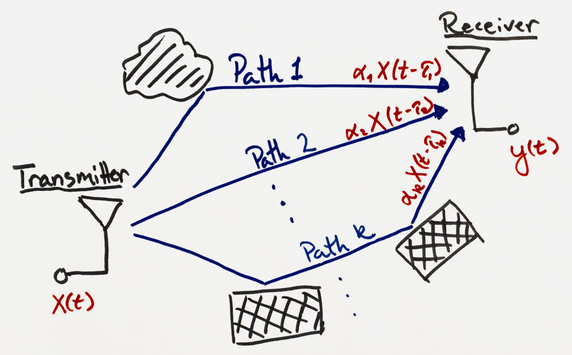
Point processes in space, time, and beyond, Skagen, 2019.

# Multipath Propagation in Radio Channels

A radio channel is measured by transmitting a known signal $x(t)$ and recording another signal $y(t)$ at the receiver. The signal propagates via multiple paths:




The received signal is a superposition of the multipath components

$$y(t) = \sum_k \alpha_k x(t - \tau_k),$$

For simplicity, we ignore additive noise.

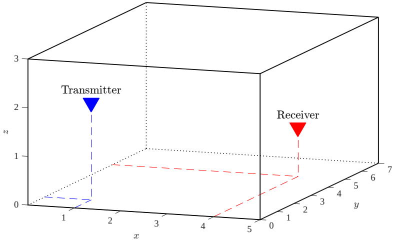
- The number of paths, delays $\mathcal{T} = \{\tau_k\}$, and gains $\{\alpha_k\}$ are unobserved.
- The delay and gain pairs form a marked point process $\{(\tau_k, \alpha_k)\}$.
- The signal $y(t)$ is a shot noise driven by $\{(\tau_k, \alpha_k)\}$.

# Multipath Models

- Multipath propagation induces delay dispersion, signal fading etc. which should be accounted for in radio system.
- Radio channel models are used in system design, analysis and simulation.
- Defining a stochastic multipath model (in the simplest setting shown before) amounts to defining a marked point process.
- Numerous such multipath models have been proposed, with delays generated from various point processes and gain distributions.
- Example: Turin's model [Turin et al., 1972], $\{(\tau_k, \alpha_k)\}$ is a marked Poisson point process fully specified by an intensity function $\lambda(\tau)$ or *path arrival rate*, and a mark density, or *conditional gain distribution* $p(\alpha|\tau)$.
- Since Turin, many other models for various scenarios have been studied in the literature.
- Here we consider a stochastic model for in-room radio channels.

# Rectangular room channel

Transmit and receive antennas located in a rectangular room of volume $V$:




The gain of the transmit antenna in direction $\Omega \in \mathbb{S}_2$ is denoted $G_T(\Omega) \geq 0$.

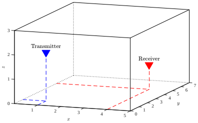
Let *beam supports*, $\mathcal{O}_T$ is the support of the transmit antenna gain function and define the *beam coverage fraction* as

$$\omega_T = \frac{1}{4\pi} \int_{\mathcal{O}_T} d\Omega$$

This is the fraction of the sphere which the antenna radiates power.

For the receiver, we define $G_R(\Omega)$ and $\omega_R$ similarly.

# Mirror sources for a rectangular room



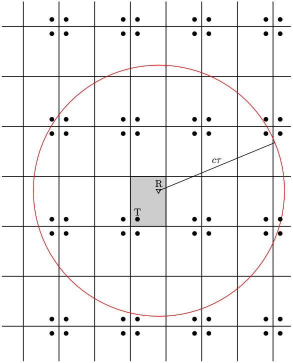

Mirror source $k$ corresponds to a path $k$ with

- propagation delay $\tau_k$ given by the propagation distance,
- gain $\alpha_k$

$$|\alpha_k|^2 = g^{|k|} \cdot \frac{G_T(\Omega_{Tk})G_R(\Omega_{Rk})}{(4\pi c\tau_k/l_c)^2}$$

where $g$ is the wall reflection gain, $|k|$ is the reflection order for source $k$, $l_c$ is carrier wavelength and $c$ is speed of light.

Iteratively "mirroring" the transmitter position in boundaries, gives an infinte set of mirror source positions in 3D. 2D cut of mirror source process:




The pattern continues similarly in the direction perpendicular to the drawing plane.

# Mirror Source Positions as Spatial Point Process

Randomize mirror source process: uniformly distributed transmit antenna locations and directions are uniformly distributed on the sphere.

Then mirror source positions form a spatial (here 3D) homogeneous point process $\mathcal{M}$ with constant intensity $\varrho(r) = 1/V$. The antennas filter/thin out, some mirror sources. The remaining set of 'visible' is

$$\mathcal{V} = \left\{ r \in \mathcal{M} : \frac{r - r_T}{\|r - r_T\|} \in \mathcal{O}_T, \frac{r - r_R}{\|r - r_R\|} \in \mathcal{O}_R \right\}$$

with intensity function (transmitter acts as a thinning)

$$\varrho_{\mathrm{v}}(r) = \mathbb{1}\left( \frac{r - r_R}{\|r - r_R\|} \in \mathcal{O}_R \right) \frac{\omega_T}{V}, \quad r \in \mathbb{R}^3.$$

Mapping points in $\mathcal{V}$ onto the delay axis gives the delay process:

$$\mathcal{T} = \{\|r - r_R\|/c : r \in \mathcal{V}\}$$

with arrival rate which is easy to compute

$$\lambda(\tau) = \frac{4\pi c^3 \tau^2}{V} \omega_T \omega_R \mathbb{1}(\tau > 0).$$

*Higher moment intensities are intractable.*

# Poisson Approximation of Mirror Source Process

The point processes $\mathcal{M}, \mathcal{V}$, and $\mathcal{T}$ give only access to the first moments.

We approximate the point process $\mathcal{M}$ and $\mathcal{V}$ as spatial Poisson processes as

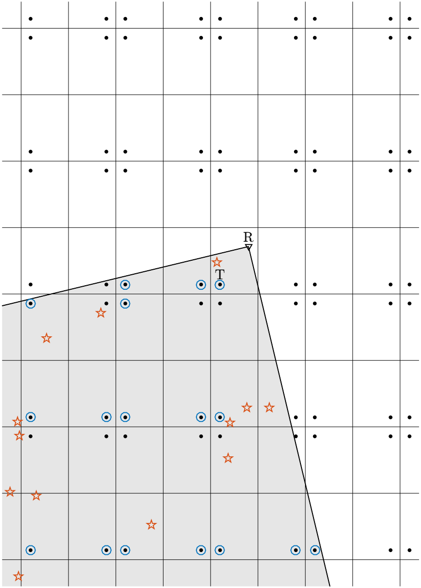
$$\mathcal{M} \approx \mathcal{M}_{\mathrm{PPP}} \sim \mathrm{PPP}(\mathbb{R}^3, \varrho_{\mathrm{m}}) \qquad \text{and} \qquad \mathcal{V} \approx \mathcal{V}_{\mathrm{PPP}} \sim \mathrm{PPP}(\mathbb{R}^3, \varrho_{\mathrm{v}}).$$

Mapping the process $\mathcal{V}_{\mathrm{PPP}}$ to the delay axis yields a Poisson point process with intensity function $\lambda(t)$

$$\mathcal{T} \approx \mathcal{T}_{\mathrm{PPP}} = \{\|r - r_R\|/c : r \in \mathcal{V}_{\mathrm{PPP}}\} \sim \mathrm{PPP}(\mathbb{R}, \lambda).$$

By construction, this Poisson approximation of the mirror source process has the correct intensity function, but *higher moments are disregarded*.

# Realizations of the Point Processes




- For readability, only sources at the same height as the transmitter are shown projected to the horizontal plane.
- Gray area: beam coverage of receiver (R).
- Transmitter (T) has a hemisphere antenna pointing at the receiver.

- • Mirror source positions ($\mathcal{M}$).
- ○ The visible mirror sources ($\mathcal{V}$).
- ⋆ Poisson approximation ($\mathcal{V}_{\mathrm{PPP}}$).

# Power delay spectrum — Variance of Mark Density

For zero-mean and conditionally uncorrelated gains $\{\alpha_k\}$, the second moment of the received signal reads

$$\mathbb{E}[|y(t)|^2] = \int_{-\infty}^{\infty} P(\tau - t)|x(t)|^2 dt,$$

with a *power-delay spectrum* $P(\tau)$ that factorizes as

$$P(\tau) = \sigma_\alpha^2(\tau) \times \lambda(\tau),$$

where $\sigma_\alpha^2(\tau)$ is the variance of $p(\alpha|\tau)$.

Choosing $p(\alpha|\tau)$ is a *circular complex Gaussian* with variance $\sigma_\alpha^2(\tau)$ according to (a good) approximation of the mirror source theory [Pedersen, 2018]

$$\sigma_\alpha^2(\tau) = \frac{e^{-\tau/T}}{(4\pi c\tau/l_c)^2} \cdot \frac{1}{\omega_T \omega_R} \qquad \text{gives} \qquad P(\tau) = \frac{e^{-\tau/T}}{4\pi V/l_c^2 c},$$

where the decay rate $T$ is called the reverberation time. Antenna dependencies cancel in the power delay spectrum (but affects other channel characteristics).

# Simulation Setup

We compare three models with identical power delay spectra:

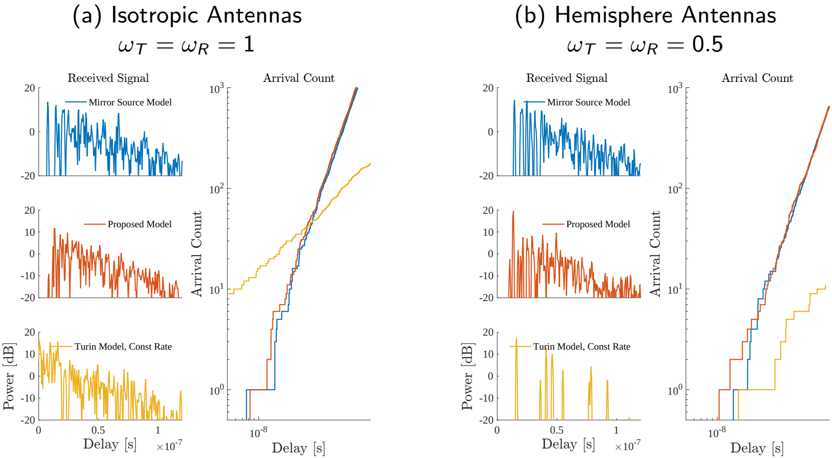
- **MS:** Mirror source model with uniformly distributed antenna positions and orientations.
- **Proposed:** Proposed inhomogeneous Poisson approximation with complex Gaussian gains.
- **Constant Rate:** Poisson model with constant arrival rate $\varrho_0 = \omega_T \omega_R \cdot 150/\tau_{\mathsf{max}}$ and complex Gaussian gains.

Simulation settings:

| | |
|---|---|
| Room dim., $L_x \times L_y \times L_z$ | $5 \times 5 \times 3\,\mathrm{m}^3$ |
| Reflection gain, $g$ | 0.6 |
| Center Frequency | $60\,\mathrm{GHz}$ |
| Bandwidth, $B$ | $2\,\mathrm{GHz}$ |
| Transmitted signal, $s(t)$ | Hamming pulse |
| Antennas | Isotropic or hemisphere. |

# Realizations of Channel Responses and Arrival Counts

Example realizations of the received signal and corresponding arrival counts.

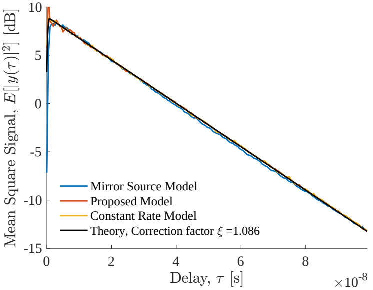



The constant rate model clearly does not capture the diffusion process.

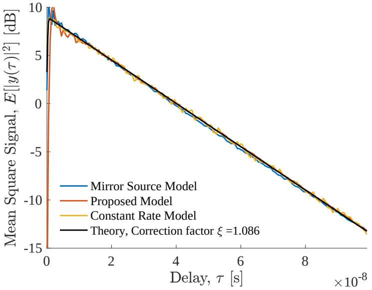
# Simulated Second Moment Of Received Signal

Recieved power delay spectra simulated using $10^4$ Monte Carlo Runs.

(a) Isotropic Antennas $\omega_T = \omega_R = 1$




(b) Hemisphere Antennas $\omega_T = \omega_R = 0.5$



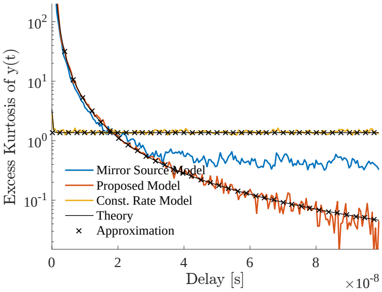

As they should by construction, all three models match the theory.

# Kurtosis-Delay Profile

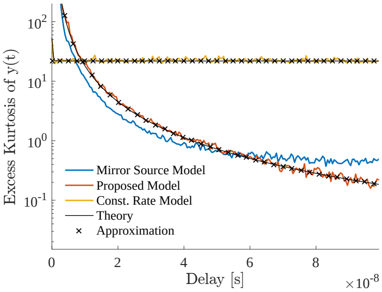
The kurtosis delay profile is the excess kurtosis of the received signal $y(t)$.

The Poisson assumption permits derivation of all cumulants and higher moments of the received signal [Pedersen, 2019].

(a) Isotropic Antennas $\omega_T = \omega_R = 1$




(b) Hemisphere Ants. $\omega_T = \omega_R = 0.5$

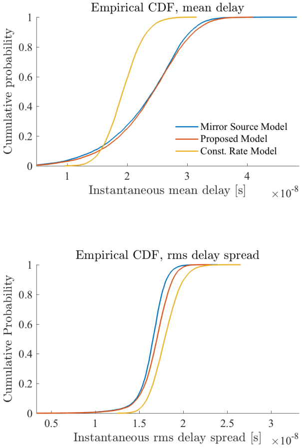



- Antennas directivity affects the kurtosis (inversely proportional to $\omega_T \omega_R$).
- The proposed model fits (with some discrepancy) the decay in kurtosis-delay profile of the mirror source model.
- The constant rate models does not replicate the behaviour.

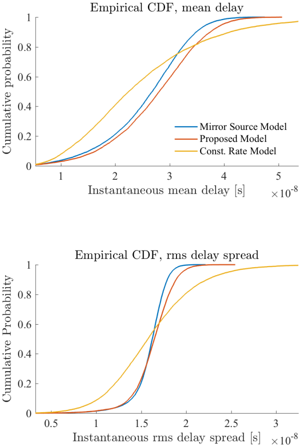
# Distributions of Instantaneous Mean Delay and RMS Delay Spread

The (standardized) first and second temporal moments of the received signal (Mean delay and RMS delay spread) are an important summary statistics.

(a) Isotropic Antennas $\omega_T = \omega_R = 1$




(b) Hemisphere Antennas $\omega_T = \omega_R = 0.5$




The proposed model follow the trends of the mirror source model on mean and rms delay. The constant rate model does not capture the antenna effects.

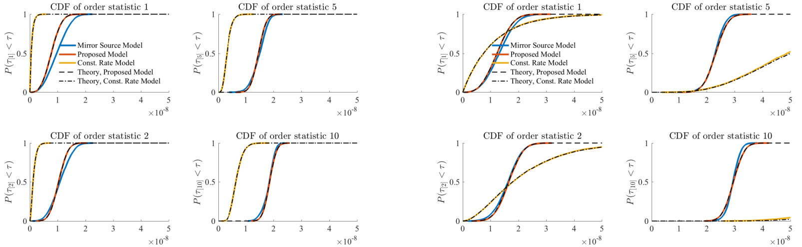
# Order Statistics

The $n$th order statistic $\tau_{[n]}$ is the delay of the $n$th arrival are obtained by sorting the delays:

$$\tau_{[1]} \leq \tau_{[2]} \leq \tau_{[3]} \leq \dots \tag{1}$$

The order statistics for the proposed model are generalized gamma distributed.

(a) Isotropic Antennas $\omega_T = \omega_R = 1$ (b) Hemisphere Antennas $\omega_T = \omega_R = 0.5$




The proposed model closely matches the order statistics of the mirror source model.

# Conclusion

- For the inroom scenario, the positions of mirror sources is described by 3D point proces (homogenous, but not Poisson).
- The path arrival rate is quadratic:

$$\lambda(\tau) = \frac{4\pi c^3 \tau^2}{V} \omega_T \omega_R \mathbb{1}(\tau > 0)$$

- Compared to the constant rate model, the proposed model captures better several other characteristics of the mirror source model, including:
  - gradual diffusion of the received signal
  - mean delay and rms delay spread
  - kurtosis delay profile
  - order statistics of path delays.
- Thus, to accurately model entities important for design of radio systems, the model should account for the arrival rate.
- The quadratic model is conservative – in real where clutter is present, the rate may be higher. This, motivates a more general model

$$\lambda(\tau) = a\tau^b \mathbb{1}(\tau > 0).$$

The two parameters $a, b$ should be estimated from observations of $y(t)$.

# Beyond: Other Topics With Point Processes within Telecommunications

Result of a 5 min brainstorm of where point processes and related inference techniques are/could be considered in telecommunications:

- Spatial radio channel modelling (directional models, point cloud models)
- Spatio-temporal traffic models (when and where does traffic occur)
- Spatio-temporal modeling of user behaviour (movement)
- Modeling of user behaviour (movement)
- Spatio-temporal interference modeling
- Design of communication protocols (packet collision modeling)
- Spatial models for quality of service QoS (e.g. spatio-temporal models for dropped calls)

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