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Published in:
IEEE Transactions on Industrial Electronics

DOI (link to publication from Publisher):
10.1109/TIE.2018.2883261

Publication date:
2019

Document Version
Accepted author manuscript, peer reviewed version

Link to publication from Aalborg University

Citation for published version (APA):
Passivation of Current-Controlled Grid-Connected VSCs Using Passivity Indices

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Abstract—Passivity-based analysis and controller design offer a promising approach to guarantee the stability of power systems. If all grid-connected components act strictly passive, critical oscillations cannot arise. Recent research established design guidelines for current-controlled voltage-source converters (VSCs) that allow obtaining a non-negative real part of the converter input admittance for a wide frequency range. In this paper, the findings are extended and generalized from a control engineering point of view. Utilizing passivity indices to quantify the degree of the input admittance’s passivity, the current controller design is reviewed and assessed. Further, generic and necessary passivation design criteria are proposed and carried out exemplarily for passive as well as for active damping using voltage feed-forward filters. Finally, the theoretical findings are validated by experiments.

Index Terms—Converter control, passivity, resonances, stabilization, voltage-source converter (VSC).

I. INTRODUCTION

W ITH the rapid increase of sustainable and distributed renewable energy sources, grid-connected voltage source-source converters (VSCs) have become a key component in present power systems. However, the interactions between one VSC or multiple VSCs, with poorly damped grid resonances are known to cause oscillations or even to destabilize the power system [1], [2]. In fact, the current control as well as the total time delay introduced by the computation plus pulsewidth modulation (PWM) were identified as one of the main reasons for exciting high-frequency harmonic resonances or resonances near to the fundamental grid frequency [2]–[4].

Interpreting the small-signal VSC dynamics as a frequency-dependent converter input admittance that is interconnected to a grid impedance, harmonic stability is most frequently analyzed by applying the Nyquist stability criterion [2], [5]–[8]. This impedance-based method can also be extended to networks that consist of multiple power converters [8]. Nevertheless, since each VSC contributes to the power system stability, every network has to be analyzed individually, and thus, it is not possible to deduce general stability statements.

On the other hand, passivity-based control [9], [10] represents a powerful approach allowing to apply passivity theorems which give sufficient conditions for power system stability, despite of the configuration and number of grid-connected VSCs. Assuming that the power network only consists of passive components, stability is guaranteed by design, if all grid-connected converters are passive, i.e., the real part of each converter input admittance is non-negative.

Among others, especially Harnefors et al. investigated the frequency-domain passivity-based controller design and stability assessment of grid-connected VSCs, see e.g., [3], [4], [11], [12]. If any computation and PWM time delay is neglected, it is easy to achieve a non-negative real part of the input admittance for all frequencies and various controllers [11]. However, considering non-idealized, practical conditions, a passive input admittance for all frequencies is hard to obtain and additional damping has to be introduced [3], [4], [12], [13]. Regarding present standards, e.g., the railway standard EN 50388-2 [14], typical requirements enforce dissipative active-front-ends up to the Nyquist frequency, while latest works motivate a non-negative real part of the input admittance even above the Nyquist frequency, see e.g., [12].

Nevertheless, even though the authors of [3] proposed an active damping control scheme, that is based on a stationary-frame proportional-resonant (PR) current controller with point of common coupling (PCC) voltage feed-forward, the feed-forward filter design still lacks a thorough scientific investigation. In particular, the question has to be answered, which filter types are able to render the real part of the input admittance non-negative, and moreover, generic filter design guidelines would be desirable. In addition, common literature, e.g., [3], [4], [11]–[13], concentrates on methods that achieve a passive VSC input admittance. From a practical point of view, this design may be sufficient to prevent critical oscillations, but can also be extended to obtain strictly passive components, allowing to guarantee asymptotic stability of the power system.

Hence, this paper aims to extend the findings of frequency-domain passivity-based controller design and stability assessment of grid-connected VSCs. Using passivity indices, which allow to quantify the degree of a system’s passivity, the topic is rounded off from a more system-theoretic point of view. The main contributions are twofold. After giving the background on passivity theory and introducing the VSC system under study, in Section IV-A, the current controller design is reviewed. Besides introducing an analytical method for passive damping dimensioning, it is shown that damped
are derived and carried out exemplarily.

Secondly, in Section IV-B, generic and necessary passivation design criteria for VSCs with PCC voltage feed-forward filters are necessary to render the input admittance strictly passive. PR controllers are beneficial to obtain a passive system and feedback interconnection of passive subsystems.

II. FREQUENCY-DOMAIN PASSIVITY THEORY

A. Passivity of Linear Systems

A linear single-input single-output (SISO) system with transfer function \( G(s) \) is said to be passive, if 1) \( G(s) \) is stable, i.e., \( \text{Re} \{ p_i \} \leq 0, i = 1, \ldots, n \), were \( p_i \) are the poles of \( G(s) \) and 2) the real part of the frequency response is always nonnegative, i.e., \( \text{Re} \{ G(j\omega) \} \geq 0, \forall \omega \in \mathbb{R} \). Moreover, \( G(s) \) is said to be strictly passive, if 1) \( G(s) \) is Hurwitz, i.e., \( \text{Re} \{ p_i \} < 0, i = 1, \ldots, n \), 2) \( \text{Re} \{ G(j\omega) \} > 0, \forall \omega \in \mathbb{R} \), 3) \( G(\infty) \geq 0 \), and 4) \( \lim_{\omega \to \pm \infty} \omega^2 \text{Re} \{ G(j\omega) \} > 0 \) [9], [10].

Equivalently, the phase response of each stable SISO passive system lies always within \([-90^\circ, 90^\circ]\) and the phase response of a SISO strictly passive system is always within \((-90^\circ, 90^\circ)\).

B. Passivity Indices and Passivation

If passive or nonpassive subsystems are combined, passivity indices can be utilized to quantify the degree of passivity of the resulting system, and thus, help to render a system passive.

The input feed-forward passivity (IFP) index for a stable linear SISO system \( G(s) \) is defined by the minimum real part of the frequency response, i.e., \( \text{IFP}(G) = \nu = \min_{\omega \in \mathbb{R}} \text{Re} \{ G(j\omega) \} \). Similarly, the output feedback passivity (OFP) index for a minimum phase system is defined by \( \text{OFP}(G) = \rho = \min_{\omega \in \mathbb{R}} \text{Re} \{ G^{-1}(j\omega) \} \) [9]. As illustrated in Fig. 1a, if \( \nu \) is negative, then the system shows a shortage of passivity and can be rendered passive by employing a minimum positive feed-forward of \( \nu \). Graphically examined, the passivation by input feed-forward shifts the Nyquist plot of \( G(j\omega) \) by \( \nu \) in the right direction, such that \( \text{Re} \{ G(j\omega) \} \geq 0, \forall \omega \in \mathbb{R} \). Equally, as shown in Fig. 1b, if \( \rho \) is negative, the system lacks passivity and can be rendered passive by employing a minimum negative feedback of \( \rho \), but does not have a direct graphically interpretation.

C. Passivity Theorems

Since passivity implies stability, each passive system is stable, that is, all poles satisfy \( \{ p_i \} \leq 0 \). Further, each strictly passive system is asymptotically stable, i.e, \( \{ p_i \} < 0 \).

Considering interconnected systems, two linear passive subsystems that are either connected in parallel or in a feedback-loop both result in a passive system again. Regarding Fig. 1c, the closed-loop system always results in an asymptotically stable system, if the subsystem in the forward path is strictly passive and the subsystem in the feedback path is passive [9], [10]. This can directly be seen from the open-loop Bode plot, where the total phase response never reaches \(-180^\circ\), and thus, always satisfies the Nyquist stability criterion.

Remark 1: It should be emphasized that contrary to passivity theory, common literature, e.g., [3], [4], [11], [12], requires a passive VSC input admittance to assess (asymptotic) stability. Even though only strict passivity guarantees asymptotic stability, passive components are often sufficient to prevent the power system from unwanted oscillations.

III. SYSTEM MODEL

The contributions of this paper can be applied to single-phase as well as three-phase systems. If three-phase systems are considered, the variables should be interpreted as complex space vectors in stationary \((\alpha, \beta)\)-coordinates, e.g., \( v = v_\alpha + jv_\beta \). For convenience, linear, continuous-time SISO systems are assumed throughout the paper. Moreover, although digitally implemented controllers are supposed, the analysis is performed in the continuous \( s \)-domain.

A. Grid-Connected VSCs

Fig. 2a shows the basic current-control structure of a grid-connected VSC. Here, it is assumed that the grid impedance

\( G_{el}(s) \).

Moreover, it is possible to specify frequency-dependent passivity indices, i.e., \( \nu(\omega) = \text{Re} \{ G(j\omega) \} \) and \( \rho(\omega) = \text{Re} \{ G^{-1}(j\omega) \} \), which allow for a frequency-domain filter design in Section IV.
$Z_g(s)$ represents a resistive-inductive-capacitive (RLC) network that interconnects the VSC to a stiff grid with voltage $v_g$ and fundamental frequency $f_s$, e.g., 50 Hz or 60 Hz. The converter is equipped with a symmetric resistive-inductive (RL) output filter $Z_{fc}(s) = R_{fc} + sL_{fc}$, where $R_{fc}$ and $L_{fc}$ are the output filter resistance and inductance, respectively. Hence, the dynamics of the controlled grid current $i$ in the Laplace domain are given by

$$i(s) = Y_{fc}(s) \left( E(s) - v_c(s) \right) = \frac{1}{L_{fc} / R_{fc} s + 1} \left( E(s) - v_c(s) \right) \quad (1)$$

where $E$ represents the voltage at the PCC and $v_c$ is the VSC output voltage.

Supposing that the converter always operates within its permissible voltage range and overmodulation does not occur, the converter plus PWM unit is typically approximated by the average model $v_c(s) = e^{-sT_{pl}} v_{ref}(s)$, where $v_{ref}$ is the desired converter voltage and $T_{pl}$ is the associated computation plus PWM time delay [3], [4], [6], [8], [13], [15]–[17]. Due to the synchronous sampling with the PWM cycle, switching harmonics are effectively damped and antialiasing analog-to-digital converter prefiltering can be avoided [3], [4]. For typical two-level VSCs that implement synchronous current sampling with sampling time $T_s$ and computation time $T_c$, the (maximum) total time delay is $T_d = T_c + T_s / 2$. Commonly, the update instant of $v_{ref}$ is delayed by the sampling period, yielding a delay of $T_d = 1.5 T_s$. The latter assumption can be considered as a worst-case approximation, which, in many cases, gives too conservative results [4], [12], [15].

Alternatively to a pure time delay, a zero-order-hold (ZOH) element can be used as a more accurate PWM model [12], [15]. In doing so, the converter model, consisting of the PWM and an associated computation time is modeled by

$$v_c(s) = G_d(s) v_{ref}(s) = e^{-s T_c} \frac{1 - e^{-s T_s}}{s T_s} v_{ref}(s), \quad (2)$$

where the first term takes the processing delay $T_c = T_s$ into account. While the phase shift of $e^{-s T_s}$ is identical to the phase shift of (2) for frequencies below the sampling frequency $\omega_s = 2 \pi / T_s$, the magnitude of (2) decreases from 1 at $\omega = 0$ rad/s to 0 at $\omega = \omega_s$. This property relaxes the worst-case PWM approximation, especially close and above the Nyquist frequency $\omega_N = \omega_s / 2$ [12].

Then, using an $(\alpha, \beta)$-frame PR current controller with a transfer function $G_{PR}(s)$ to regulate the grid current $i$ and implementing an additional active damping feed-forward filter $H(s)$, the VSC output voltage is calculated by

$$v_c(s) = G_d(s) \left[ -G_{PR}(s) \left( i_{ref}(s) - i(s) \right) + H(s) E(s) \right] \quad (3)$$

where $i_{ref}$ is the reference current. To be consistent with [3], [4], [11], the current $i$ is defined to flow in the direction of the converter, which results in the negative sign of the first term in (3).

### B. Impedance-Based Equivalent Circuit

Interpreting the VSC in terms of electrical circuit components, the system of Fig. 2b can be understood as a controlled current source that is connected to the grid [2]–[5], [11], [13], as shown in Fig. 2b. Here, $G_{cl}(s)$ represents the closed-loop transfer function between the reference current $i_{ref}$ and the injected grid current $i$, while $Y_i(s)$ is the (inner) input admittance, which describes the disturbance behavior from $E$ to $i$. Given (1) and (3), the dynamic behavior of the current source in the Laplace domain can be described by the single-frequency model

$$i(s) = G_{cl}(s) i_{ref}(s) + Y_i(s) E(s) \quad (4)$$

with

$$G_{cl}(s) = \frac{G_{PR}(s) G_d(s) Y_{fc}(s)}{1 + G_{PR}(s) G_d(s) Y_{fc}(s)} \quad (5)$$

$$Y_i(s) = \frac{[1 - H(s) G_d(s)] Y_{fc}(s)}{1 + G_{PR}(s) G_d(s) Y_{fc}(s)} \quad (6)$$

where aliasing effects are disregarded [12]. Referring to Fig. 2b, the PCC voltage can also be expressed in terms of the grid voltage $v_g$ and the grid current $i$, i.e., $E = v_g - Z_g(s) i$. Hence, after substituting, the closed-loop stability of (4) in combination with the grid can be checked by analyzing

$$i(s) = \frac{G_{cl}(s)}{1 + Y_i(s) Z_g(s)} i_{ref}(s) + \frac{Y_i(s)}{1 + Y_i(s) Z_g(s)} v_g(s) \quad (7)$$

Since the reference transfer function $G_{cl}(s)$ is designed to be stable, the overall system stability is assured if the open-loop plot of the grid impedance multiplied with the input admittance, also referred to as the minor-loop gain, satisfies the Nyquist stability criterion [5]–[8]. However, assuming that the connected RLC network only consists of (strictly) passive elements, $Z_g(s)$ is also (strictly) passive. Hence, according to Section II-C and Fig. 1c, stability can be obtained if the VSC also represents a passive system, i.e., the input admittance $Y_i(s)$ shows a non-negative real part [3], [4], [11], [13], while asymptotic stability is guaranteed, if $Y_i(s)$ is strictly passive.

**Remark 2:** Although the definition of passivity requires that the criteria of Section II-A are fulfilled for all $\omega \in \mathbb{R}$, standards like the EN50388-2 [14] enforce dissipative active front-ends up to the Nyquist frequency $\omega_N$. Hence, throughout this paper, the passivity analysis is performed with respect to the domain $\mathbb{D} = [0, \omega_N]$ [3], [4], [11], [13], which implies that condition (3) and (4) of Section II-A, which define whether a system is strictly passive or not, can be omitted.

**Remark 3:** Because of the discrete implementation of the converter control system, the resulting feedback-loop consists of a discrete and a continuous part, and thus, represents a hybrid sampled-data system [18]. However, since this paper aims to present the theoretical passivity background to previous research and to propose a new controller and filter design idea, sampling effects are disregarded and shall be addressed in future contributions. In this context, [12] provides an initial analysis of sampling effects that suggests to implement an input admittance, which also has a non-negative real part above the Nyquist frequency. In doing so, the critical excitation of above-Nyquist-frequency grid resonances can be prevented and a power system destabilization is unlikely to occur. Even though the effects of sampling and aliasing require further
Further, supposing that \((it can be observed that OFP)\) a series connection of the PR controller plus converter model, sufficient damping by the resistance \(R_{fe}\) system in Fig. 2a can be rendered passive by introducing passive in general, the input admittance \(Y_{fe}(s)\) of the RL filter admittance \(Y\) in the forward path and the (\(a\) and \(b\)) \(\nu_{cc}\) in the feedback path. Given (1), it can be observed that \(\text{OFP}(Y_{fe}) = \rho_{fe} = R_{fe} = \text{const.}\). Further, supposing that \(G_{cc}(s)\) shows a frequency-dependent IFP index of IFP \((G_{cc}) = \nu_{cc}(\omega)\), \(G_{cc}(s)\) can be decomposed into a passive system with IFP \((G'_{cc}) = \nu'_{cc}(\omega) = 0\) plus an input feed-forward element with IFP index \(\nu_{cc}(\omega)\). Then, with respect to Section II-B and similar to the idea of [13], where the input admittance has been decomposed as a passive filter and an active admittance, \(Y_{i}(s)\) can be represented as a feedback interconnection as shown in Fig. 3. Since both, the integrator as well as \(G''_{cc}(s)\) are passive elements with Re \(\{G(j\omega)\} = 0\), \(\forall \omega \in \mathbb{D}\), the system in Fig. 3 will be (strictly) output passive, if

\[
\text{OFP}(Y_{i}) = \text{OFP}(Y_{fe}) + \text{IFP}(G_{cc}) \Leftrightarrow R_{fe} + \nu_{cc}(\omega) \geq 0, \forall \omega \in \mathbb{D}. \tag{8}
\]

Remark 4: While (8) is derived by means of passivity indices, [12] also presents the condition as a necessary requirement for stability, using the Nyquist stability criterion. In this context, the system decomposition of Section IV-A gives the system-theoretic interpretation of the findings from [12].

### IV. Controller Design

Even though a purely current-controlled converter is not passive in general, the input admittance \(Y_{i}(s)\) of the VSC system in Fig. 2a can be rendered passive by introducing sufficient damping by the resistance \(R_{fe}\) or implementing an additional feed-forward filter \(H(s)\). In this section, necessary passivation design guidelines are proposed using passivity indices. Moreover, the findings are extended to converters that are equipped with inductive-capacitive-inductive (LCL) output filters.

#### A. Current Controller

Starting with a purely current-controlled converter without any additional feed-forward filter, i.e., \(H(s) = 0\), the input admittance (6) can be interpreted as a feedback interconnection of the RL filter admittance \(Y_{fe}(s)\) in the forward path and the series connection of the PR controller plus converter model, \(G_{cc}(s) = G_{PR}(s)G_{d}(s)\), in the feedback path. Given (1),

\[
G_{cc}(s) = G_{PR}(s)G_{d}(s),
\]

where \(\omega_r = 2\pi f_i\) is the resonant frequency, \(K_P\) is the proportional gain in \(\Omega\), \(K_I\) is the integral gain in \(\Omega/s\), and \(\phi\) represents the compensation angle allowing to introduce an additional phase lead at \(\omega_r\). The proportional gain can be selected by common design guidelines, e.g., \(K_P = \alpha_c L_{fe}\), where \(\alpha_c\) is the desired closed-loop current-control bandwidth, for which \(\alpha_c \leq \omega_s/10\) is recommended [3], [4], [11], [16], [19]. Neglecting the influence of the compensation angle, i.e., \(\phi \approx 0\), the phase margin \(\Phi_m\) of the open-loop transfer function \(G_{o}(s) = G_{PR}(s)G_{d}(s)Y_{fe}(s)\) can be approximated by

\[
\Phi_m \approx \pi - \tan^{-1} \left( \frac{K_1}{\omega_r K_P} \right) - \alpha_c T_d - \frac{\pi}{2}.
\tag{10}
\]

Due to its resonant character, (9) provides an infinite gain at \(\omega_r\), while the phase response at frequencies infinitely close to \(\omega_r\) shows a relative phase change of \(180^\circ\) [20]. Using the compensation angle \(\phi\) as an additional degree of freedom, the minimum phase value at \(\omega_r\) can be increased by \(\phi\), where \(\phi = 0^\circ\) yields the ideal PR controller with a phase jump from \(-90^\circ\) to \(90^\circ\).

Regarding the series connection of \(G_{PR}(s)\) and \(G_{d}(s)\), the converter model introduces a phase lag of \(\arg \{G_{d}(j\omega)\} = -1.5\omega T_s\). Hence, the passivity of \(G_{cc}(s)\) close to the resonant frequency can be obtained by choosing [3]

\[
\phi = \omega_r T_d, \quad T_d = 1.5 T_s. \tag{12}
\]

However, even though the phase lag of \(G_{d}(s)\) at \(\omega_r\) is exactly compensated, \(\text{Re} \{G_{cc}(j\omega)\} = \nu_{cc}(\omega)\) will become negative for higher frequencies. Thus, condition (8) can only be fulfilled, if the resistance of the RL output filter satisfies

\[
R_{fe} \geq \inf_{\omega \in \mathbb{D}} \nu_{cc}(\omega) \geq \inf_{\omega \in \mathbb{D}} \left| \text{Re} \{G_{PR}\} \text{Re} \{G_{d}\} - \text{Im} \{G_{PR}\} \text{Im} \{G_{d}\} \right|. \tag{13}
\]
Example 1: The parameters of a grid-connected VSC are given by Table I. Adopting the suggested design guidelines and choosing a closed-loop current-control bandwidth of \( \alpha_c = 6000 \text{ rad/s} \) and a phase margin of \( \Phi_{\text{in}} \approx 38^\circ \), the gains are calculated as \( K_P = \alpha_c L_{\text{ic}} = 18 \Omega, \ K_I = 0.0185 \alpha_c^2 L_{\text{ic}} = 2000 \Omega/\text{s} \). Further, using (12) to compensate for the total time delay, i.e., \( \phi = 2.7^\circ \), the red evolution in Fig. 4 shows the IFP index of \( G_{\text{cc}}(j\omega) \). As can be observed, there exists a negative region for frequencies in \( [\omega_c/6, \omega_N] \), which also results in an input admittance phase response with \( |\nu_I(j\omega)| \leq 90^\circ \), the red Bode plot in Fig. 5. Then, referring to Fig. 4 and condition (13), and increasing the filter resistance to e.g., \( R_{\text{tc}} = 15.1 \Omega \), the input admittance \( Y_I(s) \) is passivated. This is verified by Fig. 5, which shows that the phase response always lies within \( [-90^\circ, 90^\circ] \), \( \forall \omega \in \mathbb{D} \).

As demonstrated in Fig. 4 from Example 1, the IFP index of \( G_{\text{cc}}(j\omega) \) for high frequencies above the fundamental frequency \( \omega \geq \omega_c/6 \) can be approximated by

\[
\nu_{\text{cc}}(\omega) \approx K_P \text{Re} \{G_d(j\omega)\} = K_P \frac{2 \sin(\omega T_s/2)}{\omega T_s} \cos(\omega T_d).
\]

(14)

In this context, the resonant part of the PR controller (9) has negligible influence and \( G_{\text{cc}}(s) \) can be regarded as a series connection of a pure proportional (P) controller and the converter model \( G_d(s) \). Observing that, regardless of the switching frequency \( \omega_s \), (14) always has a local minimum at \( \omega_{\min} \approx 2.014/T_r \), the passivation criterion (13) yields the design guideline

\[
R_{\text{fc}} \approx K_P \text{Re} \{G_d(j\omega_{\min})\} \approx 0.84 K_P.
\]

(15)

With respect to (13), this simplifies the design process, but may require some empirical fine-tuning. For instance, using the parameters of Example 1, (15) gives a resistance of \( R_{\text{fc}} = 0.84 \cdot 18 \Omega = 15.12 \Omega \), where Fig. 5 illustrates that a lower resistance renders the input admittance passive as well. The deviation increases for lower switching frequencies, since the resonant part of the PR controller in (13) increasingly influences the complex part of the converter model \( G_I(s) \).

At this time it becomes also clear that the specified current-control bandwidth is directly related to passivity by choosing the proportional gain \( K_P \). In general, if the bandwidth is increased, more damping has to be introduced, and thus, the passivity of \( Y_I(s) \) is harder to obtain. Therefore, a trade-off between fast reference current tracking and disturbance rejection has to be found during the design process.

Remark 5: The identified frequency close to \( \omega_c/6 \), where the real part of the input admittance becomes negative, was also observed as critical frequency in previous works, e.g., [3], [13]. Given the IFP index of \( G_{\text{cc}}(j\omega) \) and its approximation (14), this finding can be verified, since \( K_P \text{Re} \{G_d(j\omega_c/6)\} = 0 \) and \( \nu_{\text{cc}}(\omega) \approx K_P \text{Re} \{G_d(j\omega)\} < 0 \) for \( \omega_c/6 < \omega < \omega_N \).

Remark 6: Although the input admittance can be rendered strictly output passive, i.e. OFP (\( Y_I > 0 \), \( \forall \omega \in \mathbb{D} \)), this does not imply strict passivity as defined in Section II-A. In general, strict passivity for linear systems requires strict IFP [9]. Observing that \( \text{Re} \{Y_I(j\omega_c)\} = 0 \) or \( \arg \{Y_I(j\omega)\} \in [-90^\circ, 90^\circ] \) for frequencies close to \( \omega_c \), despite of the chosen resistance and PR controller gains, the ideal PR controller of (9) is not able to transform the input admittance into a strictly passive system. If strict passivity, and thus, asymptotic stability is required, additional damping at \( \omega_r \) has to be introduced. This motivates to use a damped PR controller instead.

2) Damped PR Controller: Again focusing on reference current tracking and utilizing \( \omega_r \) as the resonant cut-off frequency, a damped PR current controller of the form

\[
G\_{PR}(s) = K_P + K_I \frac{s \cos(\phi) - \omega_r \sin(\phi)}{s^2 + \omega_r s + \omega_r^2}
\]

(16)

is proposed. Here, the same controller gain design can be performed as for the standard ideal PR controller, assuming a small resonant cut-off frequency [16]. The converter phase lag at \( \omega_c \) can again be compensated by (12). Given (16), \( \omega_r \) represents an additional degree of freedom, which reduces the gain at the resonance frequency to \( |G\_{PR}(j\omega_c)| = K_P + K_I/\omega_c \), but also compresses the phase response close to \( \omega_c \), yielding \( \arg \{G\_{PR}(j\omega)\} \in [-90^\circ, 90^\circ], \forall \omega \in \mathbb{D} \).

Therefore, if \( \omega_r > 0 \) and the RL filter resistance \( R_{\text{fc}} \) is chosen according to (13), where the \( > \) sign is replaced with a \( \geq \) sign, the input admittance \( Y_I(s) \) becomes strictly passive.

Example 2: Again supposing the converter system parameters of Table I and the PR controller parameters to be \( K_P = 18 \Omega, K_I = 2000 \Omega/\text{s}, \phi = 2.7^\circ \). Then, the blue evolution in Fig. 4 shows the IFP index of \( G_{\text{cc}}(j\omega) \) using a cut-off frequency of \( \omega_c = 0.2 \text{ rad/s} \). A positive peak close
to $\omega_i$ can be observed, while $\nu_{cc}(\omega)$ matches the red evolution of Example 1 for higher frequencies. Hence, again adopting a resistance of $R_{fc} = 15.1 \Omega$, the input admittance can be passivated, i.e., $\arg \{Y_i(j\omega)\} \in (-90^\circ, 90^\circ), \forall \omega \in \mathbb{D}$.

**Remark 7:** In [21], Zmood et al. claimed that a damped PR controller does not have any benefit compared to the ideal implementation. This might be true, if only reference current tracking is considered, but does not apply to disturbance rejection. As demonstrated, the additional damping at $\omega_i$ is necessary to render the input admittance strictly passive, and thus, is advantageous to guarantee asymptotic stability of the closed-loop system (7). As it will be shown in the following section, a damped PR controller is also preferable to passivate $Y_i(s)$, if a PCC voltage feed-forward filter is implemented.

### B. Feed-Forward Filter Design

In order to avoid a high passive filter resistance, and thus, high losses, an additional feed-forward filter $H(s)$ can be implemented. Observing that (6) consists of a series connection of the system shown in Fig. 3 and a system $N(s) = 1 - H(s)G_d(s)$, after rearranging, $Y_i(s)$ can be represented as a feedback interconnection as shown in Fig. 6. Then, similar to Section IV-A, $Y_i(s)$ will be passive if

$$\text{OFP}(Y_i) = \text{OFP}(G_i) + \text{IFP}(G_{fb1}) + \text{IFP}(G_{fb2}) \geq 0, \forall \omega \in \mathbb{D}$$

(17)

where $G_i(s) = \frac{N(s)}{T_m}, G_{fb1}(s) = R_{fc}N^{-1}(s), G_{fb2}(s) = N^{-1}G_{cc}(s)$ are the feedback systems. Regarding the IFP and OFP definitions of Section II-B, (17) can be rewritten as

$$\text{OFP}(Y_i) = \text{Re} \left\{ j L_{fc} \omega N^{-1}(j\omega) \right\} + \text{Re} \left\{ R_{fc} N^{-1}(j\omega) \right\} + \text{Re} \left\{ N^{-1}(j\omega) G_{cc}(j\omega) \right\}$$

$$= -L_{fc} \omega \text{Im} \left\{ N^{-1}(j\omega) \right\} + R_{fc} \text{Re} \left\{ N^{-1}(j\omega) \right\} + \text{Re} \left\{ N^{-1}(j\omega) \right\} \text{Re} \left\{ G_{cc}(j\omega) \right\}$$

$$- \text{Im} \left\{ N^{-1}(j\omega) \right\} \text{Im} \left\{ G_{cc}(j\omega) \right\} \geq 0, \forall \omega \in \mathbb{D}.$$  

(18)

Here, it should be noticed that (18) represents a set of generic inequalities, which do not depend on a certain PR controller, PWM model nor feed-forward filter. Moreover, (18) does not impose any restrictions on the converter’s power rating or sampling frequency. Thus, (18) can be understood as a set of general criteria that have to be fulfilled to passivate the VSC input admittance $Y_i(s)$ in the specified frequency region.

#### 1) Criteria in the Complex s-Plane

In order to interpret the criteria that are imposed by (18), it is reasonable to illustrate the inequalities in the complex s-plane. Given the converter model (2), expanding and rearranging (18) yields

$$\text{Im} \left\{ H(j\omega) \right\} \leq \frac{m_1(\omega)}{m_2(\omega)} \text{Re} \left\{ H(j\omega) \right\} + \frac{b(\omega)}{m_2(\omega)}, \forall \omega \in \mathbb{D}$$

(19) with

$$m_1(\omega) = \omega L_{fc} \sin(\omega T_d) - R_{fc} \cos(\omega T_d)$$

$$m_2(\omega) = \omega L_{fc} \cos(\omega T_d) + R_{fc} \sin(\omega T_d)$$

$$b(\omega) = R_{fc} G_{cc}(j\omega) + \text{Re} \left\{ G_{PR}(j\omega) \right\} \cos(\omega T_d)$$

$$+ \text{Im} \left\{ G_{PR}(j\omega) \right\} \sin(\omega T_d).$$

As can be seen, (19) represents a straight line in the complex s-plane for each frequency $\omega_i \in \mathbb{D}$. Using the damped PR controller of Example 2, Fig. 7 exemplarily illustrates filter criteria for distinct frequencies $\omega_i$. Depending on the sign of $m_2(\omega)$, which changes to $\geq$ in (19), the filter’s Nyquist plot $H(j\omega)$ has to lie below or above the plotted lines. In this context, the gray areas define regions, where the filter

Fig. 6: Block diagram of the input admittance $Y_i(s)$, where the effects of the feed-forward filter are considered by $N(s) = 1 - H(s)G_d(s)$.

Fig. 7: Exemplary filter criteria for varying frequencies $\omega_i$, where $\text{Im} \left\{ H(j\omega_i) \right\}$ has to lie below the respective line in (a) and (c) and above in (b) and (d).
This filter design procedure is proposed, i.e.,
more critical conditions for frequencies locked loops (PLLs) affect the low frequency behavior of the tracking, but compress the system's phase response close to damped PR controllers, which may impair reference current

Remark 9: These findings coincide with the observations of [3], but generalize the considerations concerning the PCC

Remark 8: In general, once a PCC feed-forward part to the system, considers the influence of the RL filter resistance vanishes, which results in particularly simple criteria. If the filter, the resulting D filter introduces an additional phase lag.

Example 3: Suppose Example 2, where a damped PR controller with phase compensation has been designed for a VSC with the parameters of Table I. First, a PD filter of the form \( H(s) = K_{H}(c_{1}s + c_{0}) \) is considered. Corresponding to Fig. 7c and 7d, Fig. 8 shows exemplary selected filter conditions. Following the proposed procedure, a single point in the complex s-plane is specified, e.g., \( H(j\omega /5) = 0.004 + j0.6 \). Setting \( K_{H} = 1 \), the coefficients are calculated as \( c_{0} = 0.004, c_{1} = 0.6 · 5/\omega_{s} \), which yields the filter \( H(s) = 0.004 + 4.77 · 10^{-5}s \). As can be observed from the phase response of Fig. 9, the adapted feed-forward filter \( H(s) \) renders the input admittance \( Y_i(j\omega) \) strictly passive, i.e., \( \arg \{ Y_i(j\omega) \} \in (-90^\circ, 90^\circ), \forall \omega \in \mathbb{D} \).

Example 4: Again suppose the parameters of Example 2. This time, the proposed design procedure shall be applied to a D filter, \( H(s) = c_{1}s \). In this case, the first term of (19) vanishes, which results in particularly simple criteria. If the influence of the RL filter resistance \( R_{L_c} \) and the imaginary part of the PR controller \( \text{Im}\{G_{PR}(j\omega)\} \) are neglected, (19) can be approximated by \( \text{Im}\{H(j\omega)\} \leq K_{P}(\omega L_{L_c}) \) for \( \omega \geq \omega_{s}/6 \). Then, choosing a frequency, e.g., \( \omega_{s} = \omega_{s}/6 \), yields the coefficient \( c_{1} = 36 K_{P}(\omega_{s}^2 L_{L_c}) = 36 \alpha_{c}/\omega_{s}^2 \). Similar to a PD filter, the resulting D filter \( H(s) = 5.4 · 10^{-5}s \) also passivates the input admittance, see Fig. 9.

Remark 10: As can be imagined, there exist system configurations that may result in very stringent passivation design criteria (18), which are hard to satisfy. Regarding sampling effects and the recommendation of [12], similarly harsh con-
In this paper, the passivation of current-controlled grid-connected VSCs has been discussed from a system-theoretical point of view. Introducing the concept of passivity indices, it was shown how the converter input admittance can be interpreted as interconnected subsystems. This allows to derive generic and necessary passivation design criteria, which were carried out exemplarily for passive as well as active damping of an VSC with RL output filter. It was discussed, how the results can be adapted to converters that implement LCL filters. Further, it can be stated that, if the input admittance is to be rendered passive, the feed-forward filter design must not be limited to high frequency components, but also has to consider frequencies less than the critical frequency.

VI. CONCLUSION

In this paper, the passivation of current-controlled grid-connected VSCs has been discussed from a system-theoretic point of view. Introducing the concept of passivity indices, it was shown how the converter input admittance can be interpreted as interconnected subsystems. This allows to derive generic and necessary passivation design criteria, which were carried out exemplarily for passive as well as active damping of an VSC with RL output filter. It was discussed, how the results can be adapted to converters that implement LCL filters. Further, it can be stated that, if the input admittance is to be rendered passive, the feed-forward filter design must not be limited to high frequency components, but also has to consider frequencies less than the critical frequency. Finally, experimental results validated the theoretical findings.
Fig. 11: Measured current $i$ as a result of a stepwise change of the current reference at $t = 20$ ms (a) without and (b) with feed-forward filter $H(z)$.

Fig. 12: VSC current response at nominal converter current, if the feed-forward filter $H(z)$ is enabled at the beginning and disabled at $t = 160$ ms.

showing that the proposed passivation process stabilizes a current-controlled VSC, which is connected to an undamped grid resonance.

REFERENCES

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