A Two-layer Distributed Cooperative Control Method for Islanded Networked Microgrid Systems

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Abstract—This paper presents a two-layer distributed cooperative control method for networked microgrid (NMG) systems, taking into account the proprietary nature of microgrid (MG) owners. The proposed control architecture consists of an MG-control layer and an NMG-control layer. In the MG layer, the primary and distributed secondary controls realize accurate power sharing among distributed generators (DGs) and the frequency/voltage reference following within each MG. In the NMG layer, the tertiary control enables regulation of the power flowing through the point of common coupling (PCC) of each MG in a decentralized manner. Furthermore, distributed quaternary control restores the system frequency and critical bus voltage to their nominal values and ensures accurate power sharing among MGs. A small-signal dynamic model is developed to evaluate the dynamic performance of NMG systems with the proposed control method. Time-domain simulations and experiments on NMG test systems are performed to validate the effectiveness of the proposed method.

Index Terms—networked microgrids, hierarchical control, distributed cooperative control, resiliency, small-signal stability.

I. INTRODUCTION

Resiliency against major disasters, such as major hurricanes or earthquakes, is considered by the U.S. Department of Energy (DOE) as the most essential characteristic of future smart distribution systems [1]. Concerning the enhancement of system resiliency, interconnecting microgrids (MGs) to form a networked microgrid (NMG) system after a major outage has been proved to be an effective option [2].

Three types of NMG systems (or multi-microgrid systems) have been reported in the literature: low-voltage (LV) MGs interconnected through LV tie lines [3], medium-voltage (MV) MGs networked via MV feeders [4], and LV MGs interconnected through an MV feeder and distribution transformers [5], [6]. Disregarding the way in which MGs are networked, NMG systems have some features in common. First, MGs within an NMG system may belong to different entities, and limited information may be shared with others because of their proprietary nature. Second, multiple control objectives need to be realized by effective coordination among DGs and MGs.

A. Literature Review

1) Control architectures of NMG systems

A proper control architecture for NMG systems considering the above features is needed. The classic three-level control architecture [7] is widely applied to a single MG [8]-[14]. The existing control philosophies for NMG systems are usually based on this architecture and fall into two categories, i.e., one-layer architecture and two-layer architecture. The one-layer architecture ignores the boundary of each MG and considers the NMG system as a large MG. Thus, the three-level control architecture of MGs can be modified to fit the NMG system [15]-[17]. The two-layer architecture adds an extra NMG-control layer and considers the three-level control of MGs as the MG-control layer [18]-[31]. The two-layer architecture is inspired by the multi-layer and multi-area control concept in the bulk power system [30]. The NMG layer regards each MG as a control entity and thus avoids directly controlling each DG unit. Therefore, the two-layer architecture is preferred in this study.

However, the existing two-layer architectures have the following limitations: i) they cannot realize load sharing among MGs automatically and enable the plug-and-play capability of each MG, and ii) the NMG-control layer requires too much information of DGs within MGs, which may be inaccessible, e.g., DG capacity information and load consumption data.

2) Control methods of NMG systems

Centralized or distributed methods can both be applied to the control of NMG systems. The centralized control methods for NMG systems have been reported in [19]-[26]. In [19], the NMG layer is responsible for calculating power and voltage reference values and then sending them to the MG layer to realize power sharing and voltage control objectives. In [20],
the frequency control issue is addressed in the NMG layer by coordinating each MG. In [21-26], an interface converter is assumed to be deployed with each MG to realize power sharing among MGs. Thus, the NMG layer is actually responsible for the control of the interfaced converter.

Compared with centralized methods, the distributed control methods employing consensus-based cooperative control theory [33] have some advantages, e.g., communication failure robustness and good re-configurability due to their neighboring communication features. Therefore, they have attracted much attention in recent years.

Among the existing publications with distributed control methods [15], [16], [27]-[29], both one-layer architecture [15], [16] and two-layer architecture have been adopted [27]-[29]. In [15], [16], a distributed communication network including all DG units of the NMG system exists under a one-layer architecture. Thus, there will be a large and complex communication network, which may result in a slow convergence speed of the distributed control algorithm. In [27], [28], the NMG layer is distributed, while the MG layer is still centralized. Thus, the DG units cannot flexibly plug in or out because of the centralized MG layer. In [29], both the MG and NMG layer use distributed control methods. However, MG output power sharing and critical bus voltage control objectives cannot be realized.

In sum, the above methods cannot simultaneously realize the control objectives of both the NMG layer (frequency/voltage regulation and power sharing among MGs) and the MG layer (load sharing among DGs), especially under the two-layer distributed control architecture.

3) Stability modeling and analysis of NMG systems

Compared with a single MG, the dynamic interactions among MGs and among multiple control layers in an NMG system may introduce new low-damping oscillation modes that may even destabilize the system. Therefore, a small-signal dynamic model of the NMG system and its corresponding stability analysis are of significant importance. However, only several publications about the small-signal stability issues of NMG systems have been reported [25], [26], [32]. In [25], [26], a small-signal dynamic modeling method for the NMG system is proposed in which each MG is simplified as a DG unit without considering its internal dynamics. This simplification will inevitably lead to analysis errors. In [32], a detailed small-signal dynamic model of a PV-based NMG is proposed, and the analysis results indicate that the coupling among MGs will weaken the system stability margin. However, only decentralized primary control is employed with each DG in [32], which means the impacts of distributed control methods and other control layers are not studied in [32].

Based on the above analysis, to the best of the authors’ knowledge, a detailed small-signal dynamic model and corresponding stability analysis of the NMG system considering distributed control methods and multiple control layers have not been previously reported.

B. Contribution

Compared with the state of the art, the major contributions of this paper are threefold:

1) A two-layer, four-level distributed cooperative control architecture is proposed. In this architecture, each MG is represented by an agent, and only the total spare power capacity information is provided to the NMG layer. Thus, proprietary information of the MG entities is well protected. In addition, an interface level is designed in the NMG layer, which can realize load sharing among MGs automatically as well as enable the plug-and-play capability of each MG.

2) In the NMG-control layer, a control method for the interface level is proposed, and a distributed cooperative control strategy based on it is developed. In the MG-control layer, the classic MG distributed control is adjusted to accommodate the NMG layer. The proposed control strategy is capable of simultaneously i) regulating system frequency and critical bus voltage to desired values and ii) achieving accurate active and reactive power sharing among MGs as well as among DGs within each MG.

3) A unified small-signal dynamic model of the NMG system with the proposed control strategy is constructed. The model is sufficiently detailed, which means the dynamics of every line and load, especially the two-layer distributed cooperative controllers, are included. Moreover, the stability analysis based on the proposed model reveals the newly introduced low-damping oscillation modes and their impact factors. Then, general guidelines are provided for controller parameter tuning based on the stability assessment results.

C. Paper Organization

The remainder of this paper is organized as follows. The proposed control architecture is introduced in Section II. In Section III, the design of controllers and corresponding coordination principles are presented. Section IV develops a unified small-signal dynamic model of a test NMG system. Stability analysis and numerical simulation results are discussed in Section V. Section VI provides the experimental results. Conclusions are summarized in Section VII.

II. THE HIERARCHICAL CONTROL ARCHITECTURE

In this section, the control objectives and overall control architecture for NMG systems are described.

A. Control Objectives

This paper presents a hierarchical architecture to perform frequency and voltage regulation and power sharing control of the NMG system. The control objectives include the following:

(i) To maintain the system frequency $f_{\text{sys}}$ at its rated value $f_{\text{sys}}^\ast$.

(ii) To restore the critical bus voltage $V_C$ to the desired value $V_C^\ast$. Note that only one critical bus is assumed, and it can be selected according to the operation requirement.

(iii) To share active and reactive power among MGs in proportion to the reference values, i.e.,

$$\frac{1}{r_{\text{PCC1}}} P_{\text{PCC1}} = \frac{1}{r_{\text{PCC2}}} P_{\text{PCC2}} = \cdots = \frac{1}{r_{\text{PCCm}}} P_{\text{PCCm}}$$  \hspace{1cm} (1)
where $P_{k}^{PCC}$, $Q_{k}^{PCC}$, $P_{k}^{G}$, and $Q_{k}^{G}$ are the active and reactive power references for the point of common coupling (PCC) of MG, and the output active and reactive power of PCC, respectively, with $k \in \mathcal{M}$, $\mathcal{M} = \{1,2,\ldots,m\}$. In this paper, $P_{k}^{PCC}$ and $Q_{k}^{PCC}$ are equal to the active and reactive capacity of MG, denoted by $P_{\max MG}$ and $Q_{\max MG}$, respectively. Note that $P_{k}^{PCC}$ and $Q_{k}^{PCC}$ can also be determined according to optimal power flow results, which makes the control objectives flexible. Equations (1) and (2) are denoted as objective (iii)-(1) and objective (iii)-(2), respectively.

(iv) Within each MG, active and reactive power can be shared among DGs based on their power capacities, i.e.,

$$\frac{1}{P_{\max k}}P_{k1}^{1} = \frac{1}{P_{\max k}}P_{k2}^{1} = \cdots = \frac{1}{P_{\max k}}P_{knk}^{1}$$

$$\frac{1}{Q_{\max k}}Q_{k1} = \frac{1}{Q_{\max k}}Q_{k2} = \cdots = \frac{1}{Q_{\max k}}Q_{knk}$$

where $P_{\max k}$, $Q_{\max k}$, $P_{k}$, and $Q_{k}$ are active and reactive power capacities and active and reactive power outputs of DG, in MG, respectively, with $i \in N_{k}$, $N_{k} = \{1,2,\ldots,n_{k}\}$. Equations (3) and (4) are denoted as objective (iv)-(1) and objective (iv)-(2), respectively.

In our previous work [6], an NMG power flow model considering the above objectives is proposed, and only one solution exists, which demonstrates that the objectives (i)-(iv) can be met simultaneously.

B. The Proposed Two-Layer Control Architecture

To realize the aforementioned control objectives, a two-layer control architecture is proposed, as shown in Fig. 1. There are $m$ MGs in the system, marked as MG$_{1}$, MG$_{2}$, ..., MG$_{m}$. In the NMG-control layer, each MG is modeled as an agent to form the upper distributed communication network $G$. Each MG agent includes a distributed quaternary controller (DQC) and a tertiary controller (TC). The DQC exchanges information with its neighbors to generate control variables and sends them to the corresponding TC to realize NMG layer control objectives. In the MG-control layer, MG$_{k}$ is selected and magnified to present the MG-layer control. In MG$_{k}$, all the DG units are assumed to be communication nodes to form a lower communication network $G_{k}$. Each DG unit deploys a distributed secondary controller (DSC) and a primary controller (PC). The DSC communicates with its neighbors to generate control variables and sends them to the corresponding PC to realize MG layer control objectives.

Note that the dotted lines with arrows represent directed communication links. Each TC sends commands to a DSC in the corresponding MG to realize interactions between upper and lower communication networks, as shown by the red dotted link.

1) MG-control layer: This layer aims at meeting the frequency, voltage and power sharing control objectives of the MG layer as well as supporting the NMG layer control.

a) The primary controller (PC) level is responsible for regulating the output power of DGs via the droop method [11].

b) The distributed secondary controller (DSC) level is responsible for regulating the MG frequency and PCC voltage according to the reference values received from the tertiary controller level. The control actions are taken by sending compensation signals to the primary controllers.

2) NMG-control layer: This layer handles the control of system frequency and critical bus voltage as well as power sharing among MGs.

c) The tertiary controller (TC) level is responsible for sharing power among MGs by controlling the PCC power flow according to the droop characteristics. By adjusting the frequency and PCC voltage reference values, which are sent to the distributed secondary level, the PCC power flow is controlled. Note that only the total spare capacity information of MGs is needed at this level.
d) The distributed quaternary controller (DQC) level regulates the system frequency and critical bus voltage to their desired values by coordinating MGs in a distributed manner.

In sum, the proposed architecture has the following advantages: i) only the spare capacity information of each MG is needed by the NMG layer, which well respects the proprietary nature of MG entities; ii) the TC level can realize load sharing among MGs automatically; iii) the two-layer distributed feature enables the plug-and-play of both DG and MG unit.

III. CONTROLLER DESIGN

Based on the proposed architecture, the corresponding controllers and coordination strategy are described in this section. Fig. 2 shows the control block diagram of an NMG system. To simplify the description, MGs and its inside DGs are selected to present the control principles. DGs is employed with LCL filter and a local load and then connects with the PCC bus of MGs through an LV line. For MGs, the system connects with the MV critical bus through the circuit breaker CBs. LV/MV distribution transformer and an MV line.

In Fig. 2, the red and yellow blocks on the right present control strategies of DQC and TC in the NMG-control layer for MGs. Detailed control principles will be described in the following subsections. Note that the proposed control is also suitable for other types of NMG systems introduced in Section I.

A. PC Level

At this level, the droop-based control is adopted, which consists of the power controller, inner voltage controller and current controller, as shown in Fig. 2. The power controller allows DGs to share active and reactive power demand based on their power capacities by setting droop coefficients, i.e.,

\[
\omega_{ki} = \omega_n - D_{Prk} P_{ki}
\]

(5)

\[
V_{fki} = V_n - D_{Qrk} Q_{ki}
\]

(6)

where \(\omega_{ki}\) is the angular frequency of DGk in MGk, \(\omega_n\) is the rated angular frequency, \(V_n\) is the rated voltage of the LV network, and \(V_{fki}\) is the inverter AC-side voltage reference provided to the inner voltage controller. \(D_{Prk}\) and \(D_{Qrk}\) are the active and reactive power droop coefficients, given by

\[
D_{Prk} = \frac{\omega_{max} - \omega_{min}}{f_{maxk}}\quad D_{Qrk} = \frac{V_{max} - V_{min}}{q_{maxk}}
\]

(7)

where \(\omega_{max}\) and \(\omega_{min}\) are the upper and lower limits of the angular frequency, respectively. \(V_{max}\) and \(V_{min}\) are the upper and lower limits of the DG output voltage, respectively.

B. DSC Level

The DSC level is responsible for realizing the power-sharing objectives of DGs within each MG as well as tracking the voltage and frequency reference values from the tertiary level. The consensus-based distributed cooperative control theory [33] is used to design the DSC. The term “distributed” means that each agent only needs its own information and that of its neighbors in a distributed communication network to update its state. The term “cooperative” means that all agents work as a group to realize a common synchronization goal. Based on the continuous consensus algorithm, the states of all the agents will synchronize to a common value in the steady state, i.e., they reach a “consensus”.

The communication network in this level contains \(m\) di-
graphs, $G_1, G_2, \ldots, G_m$, corresponding to $m$ MGs, respectively. For MG$i$, the set of neighbors of DG node $i$ is denoted as $L^k_i$. Each node requires its own information and that of its neighbor $j \in L^k_i$ on the digraph to update its states. The associated adjacency matrix is $A^k = [a^k_{ij}]$. Details of the communication network at this level are provided in the Appendix.

1) Distributed secondary frequency control: With this control, objectives (iv)-(iii) can be achieved. In addition, the frequency reference value $\omega_{MGk}$ from TC$k$ can be tracked. The reference value $\omega_{MGk}$ for different MGs can be different during transient events to adjust the PCC power flow but will converge to the system rated angular frequency $\omega_{sys}$ when the system reaches a steady state. The controller design is a combination of the regulator synchronization problem [33] and the tracking synchronization problem [34], given by

$$\frac{d\Omega_k}{dt} = -c_{\Omega k}\omega_k + \alpha_k - D_{PK}P_{PK} + \Omega_k$$

where $\omega_k$ is the angular frequency of MG$k$, $\alpha_k$ is the positive control gain, and the pinning gain $\gamma_k \geq 0$ is the weight of the edge connected to the reference. It is non-zero only for a few nodes (at least one node). Equation (8) is transformed from (5) with an additional DSC variable $\Omega_k$.

2) Distributed secondary PCC voltage control: This controller is responsible for controlling each MG’s PCC voltage to the reference $V_{PCCk}$ from TC$k$. The correction term $\lambda_{k}$ is added in the reactive power droop control (6), i.e.,

$$V_{k} = V_n - D_{Qk}Q_k + \lambda_k$$

where $\lambda_k$ is the local neighbor tracking error of MG$k$ to its reference (9). The correction term $\lambda_k$ is generated through a PI controller such that $V_{PCCk}$ converges to its reference $V_{PCCk}^*$, which is received from TC$k$, i.e.,

$$V_{PCCk}^* = V_n + k_{P}V_{PCCk} - V_{PCCk}$$

where $k_{P}$ and $k_{I}$ are the gains of the PI controller.

3) Distributed secondary reactive power control: This controller addresses the inaccuracy of the reactive power sharing problem due to the unbalanced line impedance [10]. Thus, the voltage correction term $h_k$ is added to the right-hand side of (10) to realize accurate reactive power sharing among DGs within MG$k$, namely, objective (iv)-(4), by regulating the voltage reference, i.e.,

$$E_{odk} = V_n - D_{Qk}Q_k + \lambda_k + h_k$$

where $E_{odk}$ and $E_{oqk}$ are provided to the inner voltage controller. $h_k$ is selected such that $D_{Qk}Q_k$ of each DG in MG$k$ converges to a common value, which is a regulator synchronization problem [33] given by

$$\frac{dh_k}{dt} = -c_{Qk}\sum_{j \in L_k^k} a^k_{ij}(D_{Qk}Q_{kj} - D_{Qk}Q_k)$$

where $c_{Qk}$ is a positive control gain, and $\sum_{j \in L_k^k} a^k_{ij}(D_{Qk}Q_{kj} - D_{Qk}Q_k)$ is the local neighbor tracking error that enables accurate reactive power sharing.

C. TC Level

This level is an interface level that can realize load sharing among MGs automatically as well as enable the plug-and-play capability of each MG. In addition, the proprietary information of each MG can be well protected by introducing this level. The droop control is modified to control the output power through the PCC of MGs, i.e.,

$$\omega_{MGk} = \alpha_n - D_{PK}P_{PCCk}$$

$$V_{PCCk} = V_n - D_{Qk}Q_{PCCk}$$

where $D_{PK}$ and $D_{Qk}$ are the active and reactive droop coefficients of MG$k$, respectively, determined by

$$D_{PK} = \frac{\omega_{max} - \omega_{min}}{P_{maxMGk}}, \quad D_{Qk} = \frac{\nu_{max} - \nu_{min}}{Q_{maxMGk}}$$

At this level, each MG is considered a droop-controlled node, and only the spare power capacity of each MG is needed.

D. DQC Level

The DQC level is responsible for regulating system frequency and critical bus voltage to desired values, as well as realizing accurate power sharing among MGs. The controller at this level is also designed based on consensus-based distributed cooperative control. The communication network in this level is denoted as $G$ with the associated adjacency matrix $A = [a_{ij}]$. For MG node $k$, the set of neighbors of node $k$ is denoted as $H_k$. Three controllers are included in this level. Details of the communication network at this level are given in the Appendix.

1) Distributed quaternary frequency control: With this control, objective (i) and objective (iii)-(1) can be achieved. The controller design is as follows.

$$\omega_{MGk} = \alpha_n - D_{PK}P_{PCCk} + \lambda_k$$

$$\frac{d\Omega_k}{dt} = -c_{\Omega k}\omega_k + \alpha_k - D_{PK}P_{PK} + \Omega_k$$

where $\omega_k$ is a positive control gain, $\sum_{j \in H_k} a^k_{ij}(V_{PCCk} - V_{PCCk}) + g^k(V_{PCCk} - V_{PCCk})$ is the local neighbor tracking error of MG$k$, which enables voltage regulation. $V_{PCCk}$ is generated through a PI controller such that $V_{PCCk}$ converges to its reference $V_{PCCk}^*$, which is received from TC$k$, i.e.,

$$V_{PCCk}^* = V_n + k_{P}V_{PCCk} - V_{PCCk}$$

where $k_{P}$ and $k_{I}$ are the gains of the PI controller.
3) Distributed quaternary reactive power control: The inaccurate reactive power sharing among MGs is managed by this controller. The voltage correction term \( h_k \) is added to the right side of (21) to achieve accurate reactive power sharing by regulating the voltage reference, i.e.,

\[
V_{\text{PCCk}}^* = V_k - D_{Qk} Q_{\text{PCCk}} + \lambda_k + h_k \tag{24}
\]

where the voltage reference \( V_{\text{PCCk}}^* \) is generated as the reference of (12). \( h_k \) is selected such that \( q_k Q_{\text{PCCk}} \) of each MG converges to a common value given by

\[
\frac{dh_k}{dt} = -c_{kq} \sum_{i \in L_k} a_{ki} (D_{Qk} Q_{\text{PCCi}} - D_{Qi} Q_{\text{PCCi}}) \tag{25}
\]

where \( c_{kq} \) is a positive control gain. Thus, objective (iii)-(2) can be realized.

E. Flow chart and implementation steps of the proposed method

To illustrate the execution process of the proposed method, a flow chart diagram is shown in Fig. 3. In addition, the corresponding implementation steps are given as follows:

Step 1 (DQC level): The rated system frequency \( \omega_{\text{sys}} \) and desired critical bus voltage \( V_k^* \) are transmitted to agent MG; in the upper communication network \( \mathcal{G} \). Through the distributed consensus algorithm (20), (22), (23) and (25), the control variables \( \Omega_k, \lambda_k \) and \( h_k \) are obtained and sent to TC level;

Step 2 (TC level): The TC is a droop-based controller that is used to adjust the PCC power flow of each MG. \( \Omega_k \) is applied to shift the frequency-active power droop curve to realize objective (i) and objective (iii)-(1). \( \lambda_k \) and \( h_k \) are applied to shift the voltage-reactive power droop curve to realize objective (ii) and objective (iii)-(2). The output variables \( \omega_{\text{MGk}} \) and \( V_{\text{PCCk}}^* \) are sent to DSC level;

Step 3 (DSC level): \( \omega_{\text{MGk}} \) and \( V_{\text{PCCk}}^* \) are received by agent MG in the lower communication network \( \mathcal{G}_k \). Through the distributed consensus algorithms (9), (11), (12) and (15), the control variables \( \Omega_k, \lambda_k \) and \( h_k \) are obtained and sent to PC level;

Step 4 (PC level): PC is droop-based for each DG unit. \( \Omega_k \) regulates the angular frequency of \( \Omega_{\text{MGk}} \) to \( \omega_{\text{MGk}} \) and realizes objective (iv)-(1) by shifting the frequency-active power droop curve of PC. \( \lambda_k \) and \( h_k \) regulate the output voltage of \( \Omega_{\text{MGk}} \) to \( V_k^* \) and realize objective (iv)-(2). The output frequency reference \( \omega_{\text{MGk}} \) and voltage reference \( E_{\text{odk}} \) are sent to the voltage and current controller, and the switching signals through the PWM module are finally generated to control the inverter of \( \text{DG}_{ki} \).

In sum, with the proposed two-layer distributed cooperative control method, the multiple control objectives summarized in section II-A can be coordinated and simultaneously realized.

IV. SMALL-SIGNAL DYNAMIC MODEL OF NMG SYSTEMS

The existing works on small-signal dynamic modeling of NMG systems have the limitations of i) oversimplification of MG’s internal dynamics [25], [26] and ii) omission of multiple control levels and distributed controllers in modeling [32]. Thus, the stability analysis based on these models will inevitably introduce errors. To accurately reveal the dynamic interaction mechanism and evaluate the dynamic performance of the proposed method, this section develops a detailed small-signal dynamic model of the NMG system in Fig. 2.

A. MG Layer Modeling

The MG layer model represents the dynamics of the PC and DSC controllers as well as the lines and loads within MGs. Note that each MG is modeled separately and will be combined in Section IV-C.

1) DG unit model: In this paper, the local dq-frame of \( \text{DG}_{ij} \), namely, \( \text{DG}_1 \) in \( \text{MG}_1 \), is selected as the common DQ-frame. The symbol \( \omega_g \) denotes the rotating frequency of DQ-frame and \( \omega_{11}, \delta_{ij} \) is the angle between the local dq-frame of \( \text{DG}_{ij} \) and the common DQ-frame, and then

\[
\delta_{ij} = \omega_{11} - \omega_g \tag{26}
\]

This paper focuses on the dynamics of the power controller. Therefore, the relatively fast dynamics of voltage and current controllers can be neglected by assuming

\[
v_{\text{odk}} = E_{\text{odk}}, \quad v_{\text{oqk}} = E_{\text{oqk}} \tag{27}
\]

where \( v_{\text{odk}} \) and \( v_{\text{oqk}} \) are the d-axis and q-axis component of DG output voltage \( v_{\text{odk}} \), respectively, as shown in Fig. 2.

By linearizing (8)-(11), (13)-(15) and (26) around an operating point, the model of \( \text{DG}_{ij} \) can be derived as

\[
[\Delta X_{\text{DGk}}] = A_{\text{DGk}}[\Delta \omega_k] + B_{\text{DGk}}[\Delta v_{\text{odk}}] + C_{\text{DGk}}[\Delta \omega_k] + \sum_{j \in L_k} F_{\text{DGk}}[\Delta X_{\text{DGj}}] + H_{\text{DGk}}[\Delta V_k] \tag{28}
\]

where \( \Delta v_{\text{odk}} \) is the deviation of \( v_{\text{odk}} \) (bus voltage as shown in Fig. 2) in the common DQ-frame, and \( A_{\text{DGk}}, B_{\text{DGk}}, C_{\text{DGk}}, F_{\text{DGk}} \) and \( H_{\text{DGk}} \) are parameter matrices. Note that \( F_{\text{DGk}} \) reflects the correlation between \( \text{DG}_{ij} \) and its neighbors \( \text{DG}_{ij}, j \in L_k \). The state variables of each DG unit are

\[
[\Delta X_{\text{DGk}}] = [\Delta \delta_{ij}, \Delta P_{ij}, \Delta Q_{ij}, \Delta \Omega_{ij}, \Delta \lambda_{ij}, \Delta h_{ij}, \Delta v_{\text{odk}}, \Delta v_{\text{oqk}}]^T \tag{29}
\]
2) PCC voltage controller model: Introduce $\psi_k$ as the state variable to describe the dynamics of (12), i.e.,

$$\dot{\psi}_k = V_{\text{PCC}} - V_{\text{FCC}}$$  \hspace{1cm} (30)

where $V_{\text{PCC}} = \sqrt{V_{\text{PCCD}}^2 + V_{\text{PCCQ}}^2}$. Then, the small-signal dynamic model of the PCC voltage controller is obtained by linearizing (30) and (12), i.e.,

$$\Delta \dot{\psi}_k = -A_{\text{PCC}}[\Delta V_{\text{PCCD}}] + \Delta V_{\text{PCC}}$$  \hspace{1cm} (31)

where $A_{\text{PCC}}$ is the parameter matrix.

3) Network and load models within MGk: The network and load models [9] are developed based on the lumped, series RL feeder lines and the RL-type constant-impedance loads, respectively, i.e.,

$$\Delta i_{\text{lineDQ}} = A_{\text{net}}[\Delta i_{\text{lineDQ}}] + B_{\text{net}}[\Delta \psi_{\text{DQ}}] + C_{\text{net}} \Delta \omega_k$$  \hspace{1cm} (33)

$$\Delta i_{\text{loadDQ}} = A_{\text{load}}[\Delta i_{\text{loadDQ}}] + B_{\text{load}}[\Delta \psi_{\text{DQ}}] + C_{\text{load}} \Delta \omega_k$$  \hspace{1cm} (34)

where $\Delta i_{\text{lineDQ}}$, $\Delta i_{\text{loadDQ}}$ and $\Delta \psi_{\text{DQ}}$ are variables of all lines, loads and buses within MG, respectively. The deviation of $i_{\text{DG}}$ of all the DG units and $i_{\text{PCC}}$ of MGk, shown in Fig. 2, is denoted as $\Delta i_{\text{DG}}$ and $\Delta i_{\text{PCC}}$ in the common DQ-frame, respectively. Then, $\Delta \psi_{\text{DQ}}$ is represented as [9]

$$\Delta \psi_{\text{DQ}} = R_N[M_{\text{DG}} \Delta i_{\text{DG}}] + M_{\text{net}}[\Delta i_{\text{lineDQ}}] + M_{\text{load}}[\Delta i_{\text{loadDQ}}] + M_{\text{PCC}}[\Delta \psi_{\text{DQ}}]$$  \hspace{1cm} (35)

Since PCCk is also a bus within MG, $\Delta V_{\text{PCCDQ}}$ can be expressed in terms of $\Delta i_{\text{DG}}$, $\Delta i_{\text{PCCDQ}}$, $\Delta i_{\text{loadDQ}}$, and $\Delta \psi_{\text{DQ}}$ and named $\Delta V_{\text{PCCDQ}}$ expression.

4) The complete model of MGk: Use the state variables of all DG units within MG as $\Delta x_{\text{DGk}}$, then combine (28), (31), (33), (34) and replace $\Delta x_f$, $\Delta \psi_f$, $\Delta x_{\text{PCCDQ}}$ with (32), (35) and $\Delta x_{\text{PCCDQ}}$ expression, respectively. The small-signal dynamic model of MGk is obtained, i.e.,

$$[\Delta s_{\text{MGk}}] = A_{\text{MGk}}[\Delta x_{\text{MGk}}] + B_{\text{MGk}}[\Delta \psi_{\text{DQ}}] + C_{\text{MGk}}[\Delta \psi_{\text{DQ}}]$$  \hspace{1cm} (36)

where $\Delta s_{\text{MGk}} = [\Delta x_{\text{DGk}} \Delta i_{\text{lineDQ}} \Delta i_{\text{loadDQ}} \Delta \psi_{\text{DQ}}]$, $A_{\text{MGk}}$, $B_{\text{MGk}}$, and $C_{\text{MGk}}$ are parameter matrices.

B. NMG Layer Modeling

The NMG layer modeling covers the dynamics of TC and DQC controllers as well as MV lines and loads. Note that each MG will be viewed as a black box and referred to as an MG unit.

1) MG unit model: By linearizing (19)-(22), (24)-(25) around an operating point, the model of MGk becomes

$$\Delta x_{\text{MGk}} = A_{\text{MGk}}[\Delta x_{\text{MGk}}] + B_{\text{MGk}}[\Delta \psi_{\text{DQ}}] + C_{\text{MGk}} \Delta \omega_k + \sum_{i \in \mathcal{H}_k} F_{\text{MGk}} \Delta x_i + H_{\text{MGk}} \Delta v_{\text{br}} + l_{\text{MGk}} \Delta \psi_{\text{DQ}}$$  \hspace{1cm} (37)

where $\Delta \psi_{\text{DQ}}$ is the deviation of MV bus voltage $v_{\text{br}}$ in the common DQ-frame, and $A_{\text{MGk}}$, $B_{\text{MGk}}$, $C_{\text{MGk}}$, $F_{\text{MGk}}$, and $H_{\text{MGk}}$ are parameter matrices. Note that $R_{\text{MGk}}$ reflects the correlation between unit MGk and its neighbors MGi, $i \in \mathcal{H}_k$. The state variables of each MG unit are

$$[\Delta x_{\text{MGk}}] = \begin{bmatrix} \Delta x_{\text{k}} \Delta P_{\text{PCCk}} \Delta Q_{\text{PCCk}} \Delta \omega_k \Delta \lambda_k \Delta h_k \Delta i_{\text{PCCDQk}} \Delta i_{\text{PCCQk}} \end{bmatrix}^T$$  \hspace{1cm} (38)

2) Critical bus voltage controller model: Denote $\psi$ as the state of (23), i.e.,

$$\dot{\psi} = \psi^* - V_c$$  \hspace{1cm} (39)

where $V_c = \sqrt{V_{\text{cd}}^2 + V_{\text{cQ}}^2}$. By linearizing (23) and (39), the model of the critical bus voltage controller can be obtained:

$$\Delta \dot{\psi} = -A_c[\Delta \psi_{\text{DQ}}] + \Delta \psi$$  \hspace{1cm} (40)

where $\Delta \psi_{\text{DQ}} = \begin{bmatrix} \Delta \psi_{\text{cD}} \Delta \psi_{\text{cQ}} \end{bmatrix}^T$, and $A_c$ is the parameter matrix.

3) MV network and load models: The modeling of the MV network and load is the same as that in MG layer modeling and can be expressed as

$$\Delta i_{\text{lineDQ}} = A_{\text{net}}[\Delta i_{\text{lineDQ}}] + B_{\text{net}}[\Delta \psi_{\text{DQ}}] + C_{\text{net}} \Delta \omega_k$$  \hspace{1cm} (42)

$$\Delta i_{\text{loadDQ}} = A_{\text{load}}[\Delta i_{\text{loadDQ}}] + B_{\text{load}}[\Delta \psi_{\text{DQ}}] + C_{\text{load}} \Delta \omega_k$$  \hspace{1cm} (43)

where $\Delta i_{\text{lineDQ}}$, $\Delta i_{\text{loadDQ}}$, and $\Delta \psi_{\text{DQ}}$ are variables of all MG units, lines, buses and loads, respectively. $\Delta \psi_{\text{DQ}}$ denotes $\Delta \psi_{\text{DQ}}$, all the MG units. Then, $\Delta \psi_{\text{DQ}}$ is represented as

$$\Delta \psi_{\text{DQ}} = R_N[M_{\text{MG}} \Delta \psi_{\text{DQ}}] + M_{\text{net}}[\Delta i_{\text{lineDQ}}] + M_{\text{load}}[\Delta i_{\text{loadDQ}}]$$  \hspace{1cm} (44)

Since the critical bus is also an MV bus, $\Delta \psi_{\text{DQ}}$ can be expressed in terms of $\Delta \psi_{\text{DQ}}$ and $\Delta \psi_{\text{DQ}}$ and named $\Delta \psi_{\text{DQ}}$ expression.

4) Complete the NMG layer model: Denote the state variables of all MG units in the NMG system as $\Delta x_{\text{MG}}$. Combine (37), (40), (42), (43) and replace $\Delta V_f$, $\Delta \psi_f$, $\Delta x_{\text{PCCDQ}}$, $\Delta V_{\text{PCCDQ}}$ with (41), (44) and $\Delta \psi_{\text{DQ}}$ expression. Then, the small-signal dynamic model of the NMG layer is obtained. That is,

$$[\Delta s_{\text{NMG}}] = A_{\text{NMG}}[\Delta x_{\text{NMG}}] + B_{\text{NMG}}[\Delta \psi_{\text{DQ}}]$$  \hspace{1cm} (45)

where $\Delta s_{\text{NMG}} = [\Delta x_{\text{MG}} \Delta i_{\text{lineDQ}} \Delta i_{\text{loadDQ}} \Delta \psi_{\text{DQ}}]^T$, $A_{\text{NMG}}$ and $B_{\text{NMG}}$ are parameter matrices.

C. Complete NMG System Model

In (36) and (45), the coupling states $\Delta v_{\text{PCCk}}$, $\Delta \psi_{\text{DQ}}$ and $\Delta \psi_{\text{DQ}}$ can be dealt with as follows: i) linearize (24), and then $\Delta v_{\text{PCCk}}$ in (36) can be represented by $\Delta x_{\text{MGk}}$ in (38), which is part of $\Delta s_{\text{NMG}}$; ii) represent $\Delta \psi_{\text{DQ}}$, $\Delta \psi_{\text{DQ}}$, which is part of $\Delta s_{\text{NMG}}$, and iii) replace $\Delta \psi_{\text{DQ}}$ by $\Delta \psi_{\text{DQ}}$ expression, which is then represented by $\Delta s_{\text{NMG}}$. By combining all MG layer models (36) and the NMG layer model (45), the complete NMG system model can be obtained as
\[ [\Delta \mathbf{S}_{\text{sys}}] = A_{\text{sys}} [\Delta \mathbf{S}_{\text{sys}}] \]  
(46)

where \( \Delta \mathbf{S}_{\text{sys}} = [\Delta S_{\text{MG1}}, \ldots, \Delta S_{\text{MGm}}, \Delta S_{\text{NMG}}]^T \). \( A_{\text{sys}} \) is the state matrix of the NMG system.

V. NUMERICAL STUDY

To validate the effectiveness of the proposed two-layer distributed control method, stability analyses and time-domain simulation studies in the PSCAD/EMTDC platform are carried out in this section based on a test NMG system.

A. Test System

The test NMG system consisting of 3 MGs is shown in Fig. 4. The circuit breakers (CB) 1, 2, and 3 are closed. The rated voltages of the MV and LV networks are 10 kV and 0.38 kV, respectively. MG1 and MG3 include 3 DGs, 3 LV lines and 3 loads. MG2 includes 3 DGs, 3 LV lines and 2 loads. \( L_c \) is the coupling inductance. Each MG connects with the MV feeder through a 10 kV/0.38 kV \( \Delta/Y \) transformer. The MV feeder is regarded as the critical bus. Tables I, II and III provide the system and controller parameters.

<table>
<thead>
<tr>
<th>Line</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1: 0.08 + j0.12 Ω, Line 2: 0.05 + j0.07 Ω, Line 3: 0.07 + j0.11 Ω, Lines 4,8,10: 0.15 + j0.05 Ω, Lines 6,7,12: 0.11 + j0.07 Ω, Lines 5,9: 0.11 + j0.11 Ω,</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load 9 = 100 kw + 30 kvar, Load 10 = 20 kw + 5 kvar, Loads 1,5,8 = 15 kw + 7.5 kvar, Loads 3,6,9 = 12 kw + 5 kvar, Loads 2,4 = 40 kw + 15 kvar, Loads 7,11,12 = 50 kw + 20 kvar,</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transformer</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1/T2/T3: 1MVA, ( u_s = 4% ), ( r_s = 1% ), 10/0.38 kV(( \Delta/Y ))</td>
<td></td>
</tr>
</tbody>
</table>
proposed control method are presented in this subsection.

1) Participation factors: Fig. 6 compares the low-frequency eigenvalue spectra of single MG1, MG2, MG3 and the NMG system. Note that a single MG only employs the MG layer controllers in Section III, with \( \omega_{MG} \) and \( V_{SRC} \) set as desired values. The typical dominant modes of the NMG system are labeled as mode \( i \) (\( i = 1, 2, \ldots, 8 \)).

Fig. 6 indicates that interconnecting MGs i) significantly change the shaping of the eigenvalues on the complex plane, ii) retain the single MG modes, i.e., MG modes 1-3, and iii) introduce four pairs of low-damping modes (modes 4-7), leading to many more oscillatory system responses compared with single MGs.

![Fig. 6 Low-frequency eigenvalue spectra of three single MGs and NMG system](image)

To identify the correlation between system states and dominant oscillatory modes, a participation factor analysis is conducted. The participation factor is calculated by multiplying corresponding elements in the right and left eigenvectors of the state matrix \( A_{sys} \). This analysis can be used to measure the association between the state variables and the modes.

Fig. 7 illustrates the participation factors of MG layer states \( (\Delta \delta_{ki}, \Delta P_{ki}, \Delta Q_{ki}, \Delta \Omega_{ki}, \Delta \lambda_{ki}, \Delta h_{ki}) \). NMG layer states \( (\Delta \delta_{ki}, \Delta P_{ki}, \Delta Q_{ki}, \Delta \Omega_{ki}, \Delta \lambda_{ki}, \Delta h_{ki}) \) and the critical bus voltage controller state \( (\Delta \psi) \). As indicated by Fig. 7, mode 1 is a typical MG inner mode that is almost solely affected by states of DG_{11}~DG_{13} units within MG. Modes 5 and 7 are intercoupling modes mainly affected by states of both MG and NMG layers. For simplicity, the participation factors of modes 2 and 3 (MG inner modes of MG2 and MG3) and modes 4 and 6 (intercoupling modes) are not presented. In addition, mode 8 is mainly affected by the critical bus voltage controller. The strongly associated states, controllers and parameters with modes 1-8 are summarized in Table IV. Table IV indicates that the most dominant modes 4-7 (with damping less than 10%) are affected by the control parameters of DSCs and DQCs. Therefore, the impact of these parameters on system stability should be carefully analyzed.

![Fig. 7 Participation factors of modes 1, 5, 7 and 8](image)

2) Sensitivity analysis of DSC and DQC parameters: Fig. 8 shows the traces of modes 4-7 as a function of \( c_{pki} \) and \( c_{qki} \). The impacts of other DSC and DQC parameters are summarized in Table IV. Fig. 8 shows that the variation of parameters may bring instability risk to the system. The summary in Table IV indicates that a dominant mode may be affected by multiple controllers and their parameters. In addition, a control parameter may affect different modes.

![Fig. 8 Traces of the most dominant modes 4-7](image)

3) Summary: The above results reveal that i) interconnecting MGs introduces new low-frequency oscillatory modes and therefore complicates the system dynamic behavior; ii) the new low-damping modes (modes 4-7) reduce the stability margin due to the coupling among neighboring MGs and between the
TABLE IV. RESULTS OF PARTICIPATION FACTORS ANALYSIS

<table>
<thead>
<tr>
<th>Modes</th>
<th>Strongly associated control layer</th>
<th>Strongly associated states</th>
<th>Associated controllers</th>
<th>Parameters of the associated controller</th>
<th>Impact of parameters on mode damping (MD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>MG layer-DSClevel</td>
<td>$\Delta q_{ki}$, $\Delta k_{qi}$, $\Delta h_{ki}$</td>
<td>(15), (25)</td>
<td>$c_{vkl}, c_{qkl}$</td>
<td>$c_{vkl}, c_{qkl}$ (MD ↓)</td>
</tr>
<tr>
<td>4-5</td>
<td>MG layer-DSClevel</td>
<td>$\Delta q_{ki}$, $\Delta k_{qi}$, $\Delta h_{ki}$</td>
<td>(11), (15)</td>
<td>$c_{vkl}, c_{qkl}$</td>
<td>Mode 4: $c_{vkl}$, $c_{qkl}$, $c_{vkl}$ (MD ↓); Mode 5: $c_{vkl}$, $c_{qkl}$ (MD ↓); $c_{vkl}$, $c_{qkl}$ (MD ↑)</td>
</tr>
<tr>
<td>6-7</td>
<td>NMG layer-DQlevel</td>
<td>$\Delta h_{ki}$</td>
<td>(22), (25)</td>
<td>$c_{vkl}, c_{qkl}$</td>
<td>Mode 6: $c_{vkl}$ (MD ↓); $c_{vkl}$, $c_{qkl}$ (MD ↑); Mode 7: $c_{vkl}$ (MD ↓); $c_{vkl}$, $c_{qkl}$ (MD ↑)</td>
</tr>
<tr>
<td></td>
<td>NMG layer-DQlevel</td>
<td>$\Delta h_{ki}$</td>
<td>(25)</td>
<td>$c_{vkl}, c_{qkl}$</td>
<td>$k_{q}, k_{i}$ (MD ↓); $k_{q}, k_{i}$ (MD ↓)</td>
</tr>
</tbody>
</table>

Two control layers; and iii) the dominant oscillatory modes are affected by multiple controllers. Note that the parameters in Table III are carefully tuned based on the guidelines in Table IV.

C. Time-Domain Simulation Results in PSCAD/EMTDC

Three cases are designed for the simulation. Case 1 demonstrates the steady-state performance of the proposed control strategy, namely, the capability to meet control objectives (i)-(iv) simultaneously under normal conditions. Case 2 verifies the system dynamic performance under communication failures as well as sudden load changes. Case 3 verifies the plug-and-plug functionality of DG and MG units.

1) Case 1 – steady-state performance: The PCs are initially engaged, the DSCs and TCs are activated at t=1.5 s, and DQCs are employed at t=3 s.

![Fig. 9 Steady-state performance of the proposed method.](image)

Fig. 9 (a) indicates that a 0.25 Hz frequency deviation is introduced by the TCs, while the DQCs restore the system frequency to 50 Hz (objective i). Fig. 9 (b) indicates that the critical point voltage is restored to 1 p.u. by the DQC (objective ii). Fig. 9 (d) indicates that after t=3 s, the DQCs achieve accurate reactive power sharing among MGs with ratios of $Q_1$ to $Q_3$ being 2:3:2 (objective iii). Fig. 9 (f) indicates that after t=1.5 s, the DSCs achieve accurate reactive power sharing among DGs within each MG, with ratios of $Q_1$ to $Q_3$ being 1:1:2 (objective iv). Fig. 9 (c) and (e) indicate that the output active powers of MGs and DGs are always accurately shared. Note that the power sharing ratios are presented in Table II.

2) Case 2 – communication link failures: In this case, all the controllers are activated at t=0.8 s, and then the system reaches a steady state. It is worth noting that, based on the proof in [34], for the tracking synchronization problem and regulator synchronization problem, if a spanning tree exists in the corresponding distributed communication network and $g_k \neq 0$ for at least one root node, the proposed controllers can reach a steady state, and objectives (i)–(iv) can still be realized.

![Fig. 10 System behaviors when communication failures occur in $G$ and $G_3$.](image)

Stage 1 (1.5–3 s): in the upper communication network $G$, the communication link between MG2 and MG3, as shown in Fig. 5, fails at t = 2 s. Subsequently, 25% of load 9 is switched...
off at \( t = 2.5 \) s. The results in Fig. 10 show that the steady-state objectives (i)-(iv) can still be achieved after the communication link failure since the remaining communication network still contains a spanning tree.

Stage 2 (3–4 s): During this stage, a worse scenario, which refers to communication failures occurring in both upper network \( \mathcal{G} \) and lower network \( \mathcal{G}_k \), is set up. After one communication link fails at \( t = 2 \) s in upper communication network \( \mathcal{G} \), for lower communication network \( \mathcal{G}_3 \) in MG3, the communication link between DG3 and DG33, as shown in Fig. 10, fails at \( t = 3 \) s. Subsequently, at \( t = 3.5 \) s, 80% of load 6, which is the internal load of MG3, is switched on. The results in Fig. 10 show that the steady-state objectives (i)-(iv) can be realized since both the network \( \mathcal{G} \) and \( \mathcal{G}_3 \) still contain a spanning tree after communication failure at this stage. In addition, Fig. 10 also indicates that the NMG system reaches a steady state within 0.5 s after a disturbance, and no significant overshoot is observed, even under communication failure events.

3) Case 3 – plug-and-play operation: In this study, all the controllers are activated at \( t = 0.8 \) s, and then the system reaches a steady state.

![Fig. 11 System behaviors under plug-and-play operation of MG3 and DG33.](image)

Stage 1 (2–4 s): MG3 is disconnected at \( t = 2 \) s and reconnected at \( t = 4 \) s to evaluate the plug-and-play capability of MGs. Note that (i) the MG will lose all the communication links with its neighboring units when it disconnects with the NMG system, then these links will recover after its reconnection; (ii) the synchronization process is necessary for MG3 before its reconnection (specifically, in this study, the synchronization of MG3 starts at \( t = 3 \) s during its islanded state); (iii) the MGs will transfer to the islanded operation state with only MG layer controllers employed after the disconnection event at \( t = 2 \) s, and the reference angular frequency \( \omega_{MGk} \) and reference PCC voltage \( V_{PCC} \) will be set as the rated value \( 2 * \pi * 50 \) rad/s and 1.0 p.u., respectively, to maintain a stable operation of the islanded MG3. The active and reactive power through PCC of each MG are presented in Fig. 11 (a) and (b), respectively.

Stage 2 (5–6 s): DG33 in MG3 disconnects at \( t = 5 \) s and reconnects at \( t = 6 \) s to evaluate the plug-and-play capability of the DGs. Similarly, DG33 will lose all the communication links with its neighboring units when it disconnects, and the communication links will recover after its reconnection. The synchronization process of DG33 starts immediately after it disconnects at \( t = 5 \) s. The active and reactive power of DG units in MG3 are presented in Fig. 11 (c) and (d), respectively.

The above results show that after the disconnection of MG1 in the NMG layer and DG33 in the MG layer, objectives (i)-(iv) can still be realized (for the sake of simplicity, only the results of active and reactive power are given). This is because the remaining communication network \( \mathcal{G} \) of the NMG-control layer and \( \mathcal{G}_3 \) of the MG-control layer still contain a spanning tree. After MG3 and DG33 reconnect, their power sharing objectives can be realized. In addition, during plug-and-play operation, no significant overshoots are observed, and the time of recovering to a steady state is within \( 0.4 \) s.

VI. EXPERIMENTAL VALIDATION

This section provides experimental results to validate the practical implementation feasibility of the proposed methods.

A. Experimental setup

Fig. 12 shows the experimental NMG system setup, which includes a real-time dSPACE 1006 platform, four Danfoss inverters, resistive loads, inductive loads, line impedances, switches, and a control desk. The control strategy is programmed and executed in the dSPACE 1006 platform to switch the inverters. The switching frequency is 10 kHz.

![Fig. 12 Experimental setup in the laboratory](image)

The physical configuration of the experimental NMG system is shown in Fig. 13. There are two microgrids in the system, and each microgrid consists of two DG units. The circuit breakers CB1 and CB2 are closed. MG1 and MG2 are connected to the critical bus through line impedances. The system rated frequency is 50 Hz, and the rated rms voltage is 200 V.

Table V provides the electrical parameters of the experimental NMG system. The parameters of the four level controllers are shown in Tables VI and VII. From Table VI, it can be seen that (i) the active and reactive power capacities between MG1 and MG2 are equal, i.e., \( P_{SMG1} = P_{SMG2} \) and \( Q_{SMG1} = Q_{SMG2} \), and (ii) the ratios of the active and reactive power capacities between DG1 and DG2 are both 4:3, i.e., \( P_{max1}:P_{max2} = 4:3 \) and \( Q_{max1}:Q_{max2} = 4:3, k = 1, 2 \).
Fig. 14 shows the topology of two-layer distributed communication networks for the experimental NMG system. In the upper network $G$, MG$_1$ receives reference values $\omega_{\text{sys}}$ and $V_c^*$.

In the lower network $G_1$ and $G_2$, DG$_{11}$ and DG$_{21}$ receive the references from $\hat{G}$.

![Fig. 13 Physical configuration of the experimental NMG system](image)

![Fig. 14 Topology of the two-layer distributed communication network for the experimental NMG system](image)

**TABLE V. ELECTRICAL PARAMETERS OF EXPERIMENTAL NMG SYSTEM**

<table>
<thead>
<tr>
<th>Inverter</th>
<th>Filter inductance: 1.8 mH</th>
<th>Filter capacitance: 27 $\mu$F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line 1</td>
<td>1.8 mH</td>
<td></td>
</tr>
<tr>
<td>Line 2</td>
<td>1.8 mH</td>
<td></td>
</tr>
<tr>
<td>Line 3</td>
<td>1.8 mH</td>
<td></td>
</tr>
<tr>
<td>Line 4</td>
<td>1.8 mH</td>
<td></td>
</tr>
<tr>
<td>Line 5</td>
<td>1.6 $\Omega$ + 2.1 mH</td>
<td></td>
</tr>
<tr>
<td>Line 6</td>
<td>2.1 $\Omega$</td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load 1</td>
<td>92 $\Omega$</td>
<td></td>
</tr>
<tr>
<td>Load 2</td>
<td>153.3 $\Omega$</td>
<td></td>
</tr>
<tr>
<td>Load 3</td>
<td>38.1 + j32.9 $\Omega$</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE VI. PARAMETERS OF PCs AND TCs FOR THE EXPERIMENTAL SYSTEM**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DG$_{11}$</th>
<th>DG$_{12}$</th>
<th>DG$_{22}$</th>
<th>MG$_1$</th>
<th>MG$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{PC}}/D_{\text{PG}}$</td>
<td>0.625</td>
<td>0.833</td>
<td>0.625</td>
<td>0.833</td>
<td>0.357</td>
</tr>
<tr>
<td>$D_{\text{TC}}/D_{\text{PG}}$</td>
<td>6.479</td>
<td>8.639</td>
<td>6.479</td>
<td>8.639</td>
<td>3.702</td>
</tr>
<tr>
<td>$P_{\text{max}}/P_{\text{PG}}$ (kW)</td>
<td>1.8</td>
<td>1.35</td>
<td>1.8</td>
<td>1.35</td>
<td>3.15</td>
</tr>
<tr>
<td>$Q_{\text{max}}/Q_{\text{PG}}$ (kVar)</td>
<td>1.2</td>
<td>0.9</td>
<td>1.2</td>
<td>0.9</td>
<td>2.1</td>
</tr>
</tbody>
</table>

**B. Experimental results**

The system is initially operated with PC. At $t=6.9$ s, the DSC and TC are activated, and at $t=8.55$ s, the DQC is activated. Fig. 15 shows the corresponding experimental results. Fig. 15(a) and (b) indicate that after $t=8.55$ s, the DQC can restore the system frequency and critical bus voltage to their rated values of 50 Hz and 1 p.u., i.e., objectives (i) and (ii) are realized. Fig. 15(c) indicates that the active powers through the PCC of each MG are equal after applying TC at $t=6.9$ s (objective (iii)-(1)). Fig. 15(d) indicates that the reactive powers through PCC of each MG are equal after applying DQC at $t=8.55$ s (objective (iii)-(2)). Fig. 15(e) and (f) indicate that after applying DSC at $t=6.9$ s, the output active power and reactive power of DG units in each MG can realize accurate sharing with $P_{k1} : P_{k2}$ and $Q_{k1} : Q_{k2}$ both being 4:3 (objective (iv)).

![Fig. 15 Experimental results](image)

**TABLE IV. PARAMETERS OF DSCS AND DQCS FOR EXPERIMENTAL SYSTEM**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DSC level</th>
<th>DQC level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{CHS}}/C_{\text{CHF}}$</td>
<td>400</td>
<td>80</td>
</tr>
<tr>
<td>$C_{\text{CHS}}/C_{\text{CHF}}$</td>
<td>400</td>
<td>80</td>
</tr>
<tr>
<td>$C_{\text{CSH}}/C_{\text{CFH}}$</td>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>$C_{\text{SHH}}/C_{\text{FHH}}$</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>$k_{\text{PB}}/k_{\text{P}}$</td>
<td>1.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$k_{\text{PB}}/k_{\text{P}}$</td>
<td>42</td>
<td>10</td>
</tr>
</tbody>
</table>
This paper also develops a detailed small signal model and power sharing analysis for islanding process, and the associated adjacency matrix $\mathcal{A}_k = [a_{ij}]$. Note that the DG units are considered nodes of the communication digraph. An edge from node $j$ to node $i$ is denoted by $(\mathcal{V}^k_j, \mathcal{V}^k_i)$, which indicates that node $i$ receives information from node $j$. $a_{ij}$ is the weight of edge $(\mathcal{V}^k_j, \mathcal{V}^k_i)$, and $a_{ij} > 0$ if $(\mathcal{V}^k_j, \mathcal{V}^k_i) \in \mathcal{E}^k$; otherwise, $a_{ij} = 0$. Node $i$ is a neighbor of node $j$ if $(\mathcal{V}^k_j, \mathcal{V}^k_i) \in \mathcal{E}^k$. The set of neighbors of node $i$ is denoted as $\mathcal{N}_k(i) = \{j | (\mathcal{V}^k_j, \mathcal{V}^k_i) \in \mathcal{E}^k\}$.

The upper communication network is deployed with the NMG layer, which has only one digraph $\mathcal{G}$. Each MG unit is considered a node of this digraph. Similarly, this digraph is expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with nodes set $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, ..., \mathcal{V}_M\}$, edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$, and the associated adjacency matrix is $\mathcal{A} = [a_{ij}]$. Node $l$ is a neighbor of node $k$ if $(\mathcal{V}_l, \mathcal{V}_k) \in \mathcal{E}$. The set of neighbors of node $k$ is denoted as $\mathcal{N}_k = \{l | (\mathcal{V}_l, \mathcal{V}_k) \in \mathcal{E}\}$.

A directed path from node $i(k)$ to node $j(l)$ is a sequence of edges. A digraph has a spanning tree if there is a node $i_r$ (called the root node), such that there is a directed path from the root node to every other node in the graph.

APPENDIX

The communication networks of the NMG system can be modeled by several directed graphs (digraphs), where the DG units (or MG units) are considered as the nodes of the communication digraphs.

The lower communication networks are deployed with the MG layer, which contains $m$ digraphs, $G_1, G_2, ..., G_m$, corresponding to $m$ MGs. The digraph for the $k$th MG is expressed as $G_k = (\mathcal{V}^k, \mathcal{E}^k, \mathcal{A}^k)$, with a non-empty finite set of $n_k$ DG nodes $\mathcal{V}^k = \{\mathcal{V}^k_1, \mathcal{V}^k_2, ..., \mathcal{V}^k_{n_k}\}$, a set of edges $\mathcal{E}^k \in \mathcal{V}^k \times \mathcal{V}^k$, and the associated adjacency matrix $\mathcal{A}^k = [a_{ij}]$. Note that the DG units are considered nodes of the communication digraph.

![Graph](image_url)  
**Fig. 15** Experimental results (DSC and TC are activated at t=6.9 s, and DQC is activated at t=8.55 s)

VII. CONCLUSION

REFERENCES


VIII. BIOGRAPHIES

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