Thermophysical Properties of Building Materials
Lecture Notes
Johra, Hicham

Publication date: 2019

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain

Take down policy
If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.
Thermophysical Properties of Building Materials: Lecture Notes

Hicham Johra

ISSN 1901-7286
DCE Lecture Notes No. 76
Thermophysical Properties of Building Materials: Lecture Notes

by

Hicham Johra

December 2019

© Aalborg University
Scientific Publications at the Department of Civil Engineering

*Technical Reports* are published for timely dissemination of research results and scientific work carried out at the Department of Civil Engineering (DCE) at Aalborg University. This medium allows publication of more detailed explanations and results than typically allowed in scientific journals.

*Technical Memoranda* are produced to enable the preliminary dissemination of scientific work by the personnel of the DCE where such release is deemed to be appropriate. Documents of this kind may be incomplete or temporary versions of papers—or part of continuing work. This should be kept in mind when references are given to publications of this kind.

*Contract Reports* are produced to report scientific work carried out under contract. Publications of this kind contain confidential matter and are reserved for the sponsors and the DCE. Therefore, Contract Reports are generally not available for public circulation.

*Lecture Notes* contain material produced by the lecturers at the DCE for educational purposes. This may be scientific notes, lecture books, example problems or manuals for laboratory work, or computer programs developed at the DCE.

*Theses* are monograms or collections of papers published to report the scientific work carried out at the DCE to obtain a degree as either PhD or Doctor of Technology. The thesis is publicly available after the defence of the degree.

*Latest News* is published to enable rapid communication of information about scientific work carried out at the DCE. This includes the status of research projects, developments in the laboratories, information about collaborative work and recent research results.
## Contents

1. Foreword ................................................................................................................................................... 6
2. Motivations................................................................................................................................................ 7
3. Density....................................................................................................................................................... 9
   3.1. Definitions ......................................................................................................................................... 9
   3.2. Measuring Density........................................................................................................................... 10
4. Specific Heat Capacity ............................................................................................................................. 12
   4.1. Definitions ....................................................................................................................................... 12
   4.2. Measuring Specific Heat Capacity ................................................................................................... 13
5. Volumetric Heat Capacity........................................................................................................................ 15
6. Thermal Capacity ..................................................................................................................................... 16
   6.1. Definitions ....................................................................................................................................... 16
   6.2. Thermal Capacity in Simple (Thermodynamic) Terms...................................................................... 17
   6.3. Effects of Thermal Capacity and Thermal Inertia in Buildings ......................................................... 21
7. Thermal Conductivity .............................................................................................................................. 23
   7.1. Definitions ....................................................................................................................................... 23
   7.2. Effects of Thermal Conductivity in Buildings ................................................................................... 26
   7.3. Calculation of Heat Transfer and Thermal Resistance ...................................................................... 26
   7.4. Measuring Thermal Conductivity .................................................................................................... 30
8. Thermal Diffusivity................................................................................................................................... 34
   8.1. Definitions ....................................................................................................................................... 34
   8.2. Measuring Thermal Diffusivity ........................................................................................................ 36
9. Some Other Interesting Material Properties for the Building Energy Efficiency and Indoor Environment Quality. ............................................................................................................................ 37

References ....................................................................................................................................................... 38
1. Foreword

The aim of this lecture note is to introduce the motivations for knowing and measuring the thermophysical properties of materials, and especially construction materials. The main material characteristics regarding thermodynamics are detailed together with some of their respective measurement methods and their implications in building physics. Those thermophysical properties of building materials can be measured at the Building Material Characterization Laboratory of Aalborg University - Department of Civil Engineering [1].
2. Motivations

Good knowledge of the thermophysical properties of materials in general, and construction materials in particular, is crucial for any calculation, modeling and simulation of heat transfer, thermodynamics and temperature distribution in solids, fluids or all sorts, mechanical systems and buildings.

In thermodynamics, the 3 main properties of the materials in which heat transfer occurs are “Density”, “Specific Heat Capacity” and “Thermal Conductivity”. They are the parameters of the fundamental differential equation of thermodynamics: “The Heat Equation”:

\[ \rho \times C_p \times \frac{\partial \theta}{\partial t} = \nabla \cdot (\lambda \times \nabla \theta) + q_V \]  

(1)

Where,
\( \theta \): Temperature [K or °C]
\( \rho \): Density of the material [kg/m³]
\( C_p \): Specific heat capacity of the material [J/kg.K]
\( \lambda \): Thermal conductivity [W/m.K]
\( q_V \): Volumetric heat source [W/m³]
\( t \): Time [sec]
This heat equation can be written in a simpler form by considering only one-dimensional heat transfer and only conductive term (no internal heat source and only heat transfer by conduction):

$$\rho \times C_p \times \frac{\partial \theta}{\partial t} = \lambda \frac{\partial^2 \theta}{\partial x^2}$$

(2)

Where,

$x$: Space coordinate in the 1-dimensional domain [m]

Because this *heat equation* is the basis of all heat transfer, heat losses, temperature distribution and thermodynamics numerical models commonly used in science and engineering to calculate the temperature at a specific point of the space domain at a given time, one can understand the importance of knowing the material properties with accuracy.
3. Density

3.1. Definitions

The density (or volumetric mass density) of a material is the amount of mass per unit volume of material. In simple words, for a given volume of material, the higher is the density of material in this volume and the higher is the mass of this volume of material.

The density is commonly denoted “\( \rho \)” (the lower case Greek letter “rho”), but it is also sometimes denoted “\( D \)”. The SI unit of kg/m\(^3\) is commonly used for material density. One can see in Figure 2 the density ranges of different construction and building materials (normal conditions of pressure and temperature). One can notice the very large range of densities of those different materials: from 10 kg/m\(^3\) for the lightest porous materials used as insulation (or even down to 1 kg/m\(^3\) for aerogels), and up to 22000 kg/m\(^3\) for the super-dense metals and alloys.

Figure 2: Density of common construction and building materials (normal conditions of pressure and temperature).
3.2. Measuring Density

The density of material at room temperature and atmospheric pressure can easily be assessed by simply measuring the volume of a sample (the measurement of its dimensions with a precision ruler, a caliper or a micrometer), and the mass of that sample with a precision scale (see Figure 3). The density can then easily be calculated as follows:

\[ \rho = \frac{m}{V} \]  (3)

Where,
\( \rho \): Density of the material \([\text{kg/m}^3]\)
\( m \): Mass of the sample \([\text{kg}]\)
\( V \): Volume of the sample \([\text{m}^3]\)

*Figure 3: Simple measurement of the material density by measuring the volume (dimensions) and the mass of a sample.*
To measure the density of smaller material samples at room temperature and atmospheric pressure with higher accuracy, it is recommended to use other methods such as gravimetric buoyancy (Archimedes’ principle of buoyancy, see Figure 4), gravimetric displacement or pycnometer [2].

**Figure 4:** Simple and accurate measurement of the density of a small sample with the Archimedes’ principle of buoyancy on a precision scale [2].

For a practical and accurate measurement of density as a function of temperature, one can use a digital density meter for liquid and gas samples (oscillating tube principle) [2], or a dilatometer for the measurement of the thermal expansion/dilatation of small solid samples at different temperatures [3].

**Figure 5:** Close up view on a horizontal dilatometer sample tube [3].
4. Specific Heat Capacity

4.1. Definitions

The specific heat capacity of a material is the amount of energy (in Joule) stored per unit mass of material for a temperature increase of 1K. The higher is the specific heat capacity and the more thermal energy can be stored in a given mass of material. Consequently, the higher is the specific heat capacity and the more energy is needed to change the temperature (heat up or cool down) of this mass of material.

The specific heat capacity is commonly denoted “Cp” (more precisely, $C_p$ denotes the specific heat capacity at constant pressure). The SI unit of J/kg.K is commonly used for the material specific heat capacity. One can see in Figure 6 the specific heat capacity of different construction and building materials. One can notice that, apart from metals, most of the common construction and building materials have a specific heat capacity comprised in the range of 800 – 2000 J/kg.K.

![Figure 6: Specific heat capacity of common construction and building materials [7].](image-url)
4.2. Measuring Specific Heat Capacity

The measurement of the specific heat capacity of a material consists in the precise monitoring of a uniform temperature change in the test sample as a function of the total amount of heat which is injected or extracted from the sample:

\[ C_p = \frac{1}{m} \cdot \frac{dQ}{d\theta} \]  \hspace{1cm} (4)

Where,

- \( C_p \): Specific heat capacity of the material \([\text{J/kg.K}]\)
- \( m \): Mass of the sample \([\text{kg}]\)
- \( dQ \): Amount of heat energy needed to uniformly raise or decrease the temperature of the sample \([\text{J}]\)
- \( d\theta \): Small uniform temperature change of the test sample \([\text{K}]\)

Differential Scanning Calorimetry (DSC) apparatus are commonly used to measure the specific heat capacity of all kinds of solid and liquid materials as a function of temperature (see Figure 7). DSC measurements are fairly fast to conduct and allow a detailed thermal analysis of the tested material (used a lot for pharmaceutical, food and polymer products). The test temperature can range from -180 °C up to 725 °C. However, DSC measurements are usually restricted to very small samples (3-20 mg).

*Figure 7: Differential Scanning Calorimetry (DSC) apparatus DSC Q2000 (TA Instruments, Inc.) [3].*
The specific heat capacity of larger samples can be measured with the Laser Flash Analysis (see Figure 8). Although the Laser Flash Analysis is slower than the DSC and not directly intended for specific heat capacity measurements, it allows the measurement of local (spatial) properties of large samples of inhomogeneous materials or layered samples. The common LFA apparatus can test samples at temperatures ranging from 10 °C up to 300 °C, but special LFA system can reach temperatures ranging from -125 °C to 2000 °C.

Figure 8: Laser Flash Analysis (LFA) Apparatus LFA 447 (Netzsch Gerätebau GmbH) [5].
5. Volumetric Heat Capacity

Similarly to the specific heat capacity, the volumetric heat capacity of a material is the amount of energy (in Joule) stored per unit volume of material for a temperature increase of 1K. The higher is the volumetric heat capacity and the more thermal energy can be stored in a given volume of material. Consequently, the higher is the volumetric heat capacity and the more energy is needed to change the temperature (heat up or cool down) of this volume of material.

The volumetric heat capacity is commonly denoted “\(C_V\)”. It should not be mistaken with the specific heat capacity at constant volume (isochoric), which is also denoted \(C_V\). The volumetric heat capacity can be directly calculated as the product of the density by the specific heat capacity of a given material:

\[
C_V = \rho \times C_p
\]  

(5)

Where,

- \(C_V\): Volumetric heat capacity of the material [J/m³.K]
- \(C_p\): Specific heat capacity of the material [J/kg.K]
- \(\rho\): Density of the material [kg/m³]

The SI unit of J/m³.K is commonly used for the material volumetric heat capacity. One can see in Figure 9 the volumetric heat capacity of different construction and building materials. One can notice that, since the volumetric heat capacity is the product of density and specific heat capacity, and since the specific heat capacities of building materials are all very similar in between each other, the volumetric heat capacity of common construction materials is very linearly proportional to their respective density.

![Figure 9: Volumetric heat capacity of common construction and building materials [7].](image-url)
6. Thermal Capacity

6.1. Definitions

The thermal capacity of a thermodynamic system is the amount of energy (in Joule) stored in the entire system (a wall, a room, a water tank, an entire building, etc) for a homogeneous temperature increase of 1K. The SI units of J/K, Wh/K or kWh/K are commonly used for the thermal capacity of a system. The higher is the thermal capacity and the more thermal energy can be stored in the entire system. Consequently, the higher is the thermal capacity and the more energy is needed to change the temperature (heat up or cool down) of this system. The thermal capacity (or total thermal capacity) of a system is calculated by multiplying the volumetric heat capacity of the system by the volume of the latter:

\[ C_m = V \times C_V = V \times \rho \times C_p \]  

Where,
- \( C_m \): Thermal capacity of the system [J/K]
- \( V \): Volume of the system [m³]
- \( C_V \): Volumetric heat capacity of the system’s material [J/m³.K]
- \( C_p \): Specific heat capacity of the system’s material [J/kg.K]
- \( \rho \): Density of the system’s material [kg/m³]

If the system is composed of sub-systems with different volumetric heat capacities, the total thermal capacity of the system is the sum of the thermal capacity of each sub-system:

\[ C_m = \sum_i^n C_m^i = \sum_i^n V^i \times C_V^i = \sum_i^n V^i \times \rho^i \times C_p^i \]  

Where,
- \( C_m^i \): Thermal capacity of the sub-system “i” among the “n” sub-systems [J/K]
- \( V^i \): Volume of the sub-system “i” among the “n” sub-systems [m³]
- \( C_V^i \): Volumetric heat capacity of the material sub-system “i” among the “n” sub-systems [J/m³.K]
- \( C_p^i \): Specific heat capacity of the material sub-system “i” among the “n” sub-systems [J/kg.K]
- \( \rho^i \): Density of the material sub-system “i” among the “n” sub-systems [kg/m³]

The thermal capacity of a system is associated with (or sometimes interchangeably named) the thermal inertia or thermal mass of a system. However, those terms do not necessarily have the same meanings, depending on the situations. If the other system’s parameters do not change, a larger thermal capacity will produce higher thermal inertia (thermal mass), which induces a longer time for the system to change temperature or reach thermal equilibrium.

However, the thermal inertia of a system only corresponds to its thermal capacity if the system is isothermal (homogeneous temperature at every point in the system) at all times. This situation is rarely encountered but can be assumed correct in the case of small/thin systems with very high thermal diffusivity (low thermal capacity and high thermal conductivity). In most of the cases, the thermal inertia of a system is better
described by its “effective” thermal capacity. This effective thermal capacity is a fraction of the total thermal
capacity of the system depending on the thermal conductivity and dimensions of the elements, and the
dynamics of the boundary conditions. For more information about the calculation of the effective thermal
capacity and thermal inertia of construction elements (such as walls), please read ISO standard 13786:2007
[6].

The thermal capacity and thermal inertia of a thermodynamic system are also sometimes referred to as
“thermal mass”. The denomination “thermal mass” is an analogy (although not directly related) to the
concept of “inertia” for mechanical systems. In mechanics, a system with a large mass has a large inertia and
it requires a lot of energy to accelerate it and change its velocity (speed or direction). In addition, a large
mass with large inertia in motion stores a lot of kinetic energy. This is very similar to thermodynamic systems
where massive systems (composed of large volumes of material with a high density and thus a large
volumetric and total thermal capacity) have a large “thermal mass” or “thermal inertia” and require a lot of
energy to change their temperature, and can store a lot of thermal energy.

6.2. Thermal Capacity in Simple (Thermodynamic) Terms

The concepts of thermal capacity in thermodynamics can be explained in simple terms with a “water bucket”
analogy. This analogy consists in assuming that a thermodynamic system defined in space (defined finite
volume with boundaries) is like a bucket of water. The amount of water in the bucket represents the amount
of internal (thermal) energy stored in the system. The level (height) of water in the bucket represents the
homogeneous average temperature of the system (see Figure 10).

\[
\text{Amount of water} = \text{amount of internal thermal energy}
\]

\[
\text{Level of water} = \text{system temperature}
\]

*Figure 10: The “water bucket” analogy of thermodynamics.*
If thermal energy is added to the system (heat gain), the system’s temperature will increase, similarly to the increase of water level in the bucket when water is added into it (see Figure 11).

![Diagram of thermal energy increase](image)

**Figure 11:** Increase of internal thermal energy inducing an increase of temperature in the system.

Reciprocally, if thermal energy is removed from the system (heat loss), its temperature will decrease (see Figure 12).

![Diagram of thermal energy decrease](image)

**Figure 12:** Decrease of internal thermal energy inducing a decrease of temperature in the system.
For a given change of the amount of water in the bucket, the change of the water level will depend on the diameter of the bucket. Similarly, for a given change of internal thermal energy inside the system (internal energy increase for heat gain, or internal energy decrease for heat loss), the change of the system’s temperature is proportional to its thermal capacity (see Figure 13).

\[ +\Delta E = \Delta \theta \times C_m \]

\[ C_m = \rho \times C_p \times V \]

**Figure 13:** For a given increase of internal thermal energy, the change of the system’s temperature depends on its thermal capacity.
For the same amount of water added into a bucket, the water level in the narrow bucket will increase more than in the large bucket. Similarly, for the same amount of injected thermal energy, a system with a small thermal capacity will have its temperature increasing more than a system with a large thermal capacity. (see Figure 14).

\[ \Delta E = C_m \times \Delta \theta \]

\[ C_{m1} < C_{m2} \]

\[ \Delta E_1 = \Delta E_2 \]

\[ \Rightarrow \Delta \theta_1 > \Delta \theta_2 \]

*Figure 14:* For a given increase of internal thermal energy, a low-thermal capacity system will have its temperature increasing more than a high-thermal capacity system.
6.3. Effects of Thermal Capacity and Thermal Inertia in Buildings

As mentioned before, the thermal capacity of a system will greatly determine its thermal inertia. In the case of buildings, constructions made of low-density materials such as wood will have a low thermal capacity and a low thermal inertia. One the contrary, constructions made of high-density materials such as concrete, stone or bricks, will have a thermal capacity and a high thermal inertia (see Figure 15).

![Figure 15: A low-thermal inertia wooden house (left); a high-thermal inertia stone house (right).](image)

On the one hand, low-thermal inertia buildings will be subjected to faster and higher temperature variations. Less time and energy is needed to change their indoor temperature. In the case of a heating system start-up phase in a cold building, the latter will rapidly reach an appreciable indoor temperature after a few dozens of minutes (see Figure 16). On the other hand, because the thermal storage capacity of the building is limited, if the heating system is turned off in winter (heating system’s failure for example), the indoor temperature will drop quickly*.

The opposite is observed for high-thermal inertia buildings. They need more time and energy to warm up when they start cold (see Figure 16). In the case of a high-thermal inertia building which would have a night-time set back (turning off heating system at night when there are no occupants), it is important to take the heat-up time into consideration and start the heating system before the first occupants arrive since it can take several hours to reach a comfortable indoor temperature. However, the heating system can be turned off several hours before the departure of the last occupants because the indoor temperature will only drop slowly. In general, high-thermal inertia buildings are less subjected to overheating during summer and present more stable indoor temperature*.

*All assertions in this section are based on the assumption that the considered buildings have the same level of insulation.
Figure 16: Indoor temperature increase as a function of time for a constant heating of a low-thermal inertia wooden house and a high-thermal inertia concrete house.
7. Thermal Conductivity

7.1. Definitions

There are 3 main modes of heat transfer (see Figure 17):

- **Conduction** heat transfer: Transfer of heat by means of molecular excitement and transmission of vibration from neighbor to neighbor molecules (heat diffusion) within a material without bulk motion of the matter (no mass transfer). Conduction heat transfer occurs in solids, but also in liquids and gases.

- **Convective** heat transfer (convection): Transfer of heat by the movement of fluid (mass transfer). Convection is usually the dominant form of heat transfer in liquids and gases.

- **Radiation** heat transfer: Transfer of heat in between 2 surfaces by means of electromagnetic radiation exchange (photons) without any contact or mass transfer.

![Figure 17: Examples of the 3 main modes of heat transfer.](image)

The thermal conductivity of a material is related to the conduction heat transfer mode. The thermal conductivity of a material is the ability of the latter to propagate heat energy by conduction. The SI unit of W/m.K is commonly used for the thermal conductivity of a material. The higher is the thermal conductivity and the easier it is to propagate heat through a material, therefore the more thermal energy is transferred through the latter when there is a temperature gradient. Consequently, the higher is the thermal conductivity of a material and the worse it is at being a good thermal insulator. The thermal conductivity of a material is commonly denoted “λ” or “k”.
One can see in Figure 18 (log scale) the thermal conductivity of various typical construction and building materials. One can notice that there is a clear correlation between material density and thermal conductivity. That can be explained by the fact that, apart from metals and ceramics, most of the building materials have a certain degree of porosity. Consequently, their thermal conductivity is mainly determined by the air and water content trapped inside these pores, which is directly correlated to their porosity and, therefore, to their density. For dry porous materials, low density is usually associated with high porosity and large air-filled cavities (trapped air). The latter having a very low thermal conductivity, they drive the entire thermal conductivity of the porous material down [8]. Conversely, compact materials (higher density) tend to not have low-thermal conductivity air-filled pores and thus present a higher thermal conductivity. In humid and hydrated porous materials such as wood, concrete, stone, and soil, it is water that is trapped in the pores. Because water is quite dense and has a very high thermal conductivity, higher content of liquid water in porous materials is usually associated with a higher density and a higher thermal conductivity. In the case of metals (which are both very dense and highly conductive), heat conductivity is primarily due to free electrons as freely moving valence electrons transfer both electric current and heat energy.

However, one can see in Figure 19 that for very low-density porous materials (light foams and insulation materials) the minimum thermal conductivity is not at the lowest density. Indeed, below a certain density threshold (equivalent to a certain compaction level) the insulating air-filled pores can become relatively large. This results in higher free convection occurring inside those pores (free convection is proportional to the pore size), which leads to a general increase of the thermal conductivity at very low densities. However, this is not necessarily the case for aerogel materials.
Conversely, from the density with the lowest thermal conductivity, higher compaction of porous materials (increase of density) leads to a collapsing of the air-filled pores, creating more and more contact bridges in between the conductive solid matrix (often fibers). This also results in a general increase in thermal conductivity.

**Figure 19:** Thermal conductivity of porous construction and building materials [7].
7.2. Effects of Thermal Conductivity in Buildings

Typically, insulating materials with low thermal conductivity (such as foams containing lots of gas-filled cavities) are used to build the envelope of buildings in order to thermally isolate their indoor environment from the outside. With a well-insulated envelope, the building minimizes the unwanted heat exchange (heat losses or gains) with the outdoor, which reduces the energy needs for heating and cooling, and improves the indoor thermal comfort.

On the other hand, materials with high thermal conductivity (such as metals) are typically used in building systems where the heat exchange has to be maximized. For instance, this is the case for radiator heating units or heat exchange.

7.3. Calculation of Heat Transfer and Thermal Resistance

In steady-state conditions (or assuming a steady-state situation: no change of temperature over time at any given point), the heat transfer through a planar construction element (assuming a 1D heat transfer, which is typically the case for building elements) can be easily calculated by the Fourier’s law of heat conduction. The Fourier’s law of heat conduction is a simplification of the heat equation for one-dimensional and steady-state conditions. The law states that the rate of heat transfer through a material is proportional to the negative temperature gradient in the body through which the heat flows and proportional to the section area at right angles to that temperature gradient. In the case of a one-dimensional heat transfer through a planar element, the Fourier’s law of heat conduction can be written with the following equation:

\[ \vec{q} = -\lambda \nabla \theta \]  

Where,
\[ \vec{q} \]: Vector of local heat flux density [W/m²]
\[ \lambda \]: Thermal conductivity [W/m.K]
\[ \nabla \theta \]: Temperature gradient [K/m]

This equation can be further simplified by considering the heat flux between 2 points (heat flux flowing from a hot point towards a cold point) separated by a given distance in a material:

\[ q = \lambda \frac{\Delta \theta}{\Delta x} \]  

Where,
\[ q \]: Heat flux density through the element [W/m²]
\[ \lambda \]: Thermal conductivity [W/m.K]
\[ \Delta \theta \]: Temperature difference between the 2 points in the material [K]
\[ \Delta x \]: Distance between the 2 points in the material [m]
This simple equation is commonly used to estimate the (steady-state) heat transfer and heat losses through building elements (estimating the total heat flux “Q” through the element by multiplying the equation (9) by the surface area of the element):

\[ Q = q \times A = \lambda \times A \frac{\Delta \theta}{\Delta x} \]

\[ Q = \lambda \times A \frac{\theta_{\text{hot}} - \theta_{\text{cold}}}{\Delta x} \]  

(10)

Where,

\(Q\): Heat flux (from hot point to cold point) [W]

\(q\): Heat flux density through the element [W/m²]

\(A\): Surface area of the element (or contact surface area) [m²]

\(\lambda\): Thermal conductivity [W/m.K]

\(\Delta \theta\): Temperature difference between the 2 points in the element [K]

\(\theta_{\text{hot}}\): Temperature at the hot point in the element [°C]

\(\theta_{\text{cold}}\): Temperature at the cold point in the element [°C]

\(\Delta x\): Distance between 2 points in the element (or thickness of the element) [m]

**Figure 20:** Steady-state one-dimensional heat transfer by conduction through a planar element.
The Fourier’s law of heat conduction can be rearranged as follows:

\[ q = \lambda \frac{\Delta \theta}{\Delta x} \]  
\[ \Leftrightarrow \Delta \theta = \frac{\Delta x}{\lambda} q \]  

Where,
- \( q \): Heat flux density through the element [W/m²]
- \( \lambda \): Thermal conductivity of the element’s material [W/m.K]
- \( \Delta \theta \): Temperature difference between the 2 points in the element [K]
- \( \Delta x \): Distance between 2 points in the element (or thickness of the element) [m]

One can then define a “thermal resistance” term “\( R_{th} \)” as follows:

\[ R_{th} = \frac{\Delta x}{\lambda} \]  

The Fourier’s law of heat conduction can thus be written as follows:

\[ \Delta \theta = R_{th} \times q \]  

Where,
- \( q \): Heat flux density through the element [W/m²]
- \( \Delta \theta \): Temperature difference between the 2 points in the element [K]
- \( R_{th} \): Thermal resistance of the element [m².K/W]

This equation (13) resembles another very famous equation from electrical engineering: the Ohm’s law:

\[ U = R \times i \]  

Where,
- \( i \): Current: rate of flow of electric charge or flux density of electric charge [A]
- \( U \): Potential difference [V]
- \( R \): Electrical resistance of an electric element [Ω]

The temperature difference “\( \Delta \theta \)” is analogous to the potential difference “\( U \)”, the heat flux density “\( q \)” is analogous to the current “\( i \)”, and the thermal resistance “\( R_{th} \)” is analogous to the electrical resistance “\( R \)” (see Figure 21).
In building energy engineering, the thermal resistance \( R_{th} \) of a construction element (a wall for instance) is also denominated as “R-value”. It is also very common to use the thermal transmittance [W/m\(^2\).K] of the construction element (U-value) which is the inverse of the thermal resistance:

\[
U\text{-value} = \frac{1}{R_{th}} = \frac{1}{R\text{-value}} = \frac{\lambda}{\Delta x} \tag{15}
\]

Where,

\( \lambda \): Thermal conductivity of the element’s material [W/m.K]

\( \Delta x \): Thickness of the element [m]

The Fourier’s law of heat conduction for the calculation of one-dimensional steady-state heat transfer (often heat losses through building element) can thus be written with “\( R_{th} \)“ and “U-value” as follows:

\[
q = \frac{\Delta \theta}{R_{th}} \tag{16}
\]

\[
q = U\text{-value} \times \Delta \theta \tag{17}
\]

Where,

\( q \): Heat flux density through the element [W/m\(^2\)]

\( \Delta \theta \): Temperature difference between the 2 points in the element [K]

\( R_{th} \): Thermal resistance of the element [m\(^2\).K/W]

\( U\text{-value} \): Thermal transmittance of the element [W/m\(^2\).K]

Figure 21: Analogy between Ohm’s law of electricity and Fourier’s law of heat conduction.
The Ohm’s law analogy is very useful since the same rules apply for electrical or thermal systems to calculate the total resistance of elements placed in parallel and/or in series (see Figure 22).

\[ R_{\text{total}} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \]

Figure 22: Calculation rules for equivalent resistance of resistances in parallel and/or in series are the same for electrical and thermal systems.

7.4. Measuring Thermal Conductivity

There are two main types of measurement method to determine the thermal conductivity of construction materials:

- **Steady-state** measurement methods: Those methods maintain constant (do not vary over time) and controlled boundary conditions around the test sample (constant temperature, constant temperature difference, constant thickness) in order to obtain a constant and homogenous heat flux through the test sample. Steady-state measurement methods are used in equipment such as the Guarded Hot Plate Apparatus [9], the Heat Flux Meter, or the Guarded Hot Box. The steady-state methods are relatively simple and easy to employ, and can usually be used for large test samples. However, those methods are usually time-consuming in the sense that the experimental setup requires several hours or days to reach steady-state conditions in order to conduct a proper measurement at a given temperature.

- **Dynamic (transient)** measurement methods: Those methods do not maintain constant and steady-state boundary conditions or heat flux to the test sample, but measure the change of temperature of the sample as a function of time when the latter is heated up. The thermal conductivity of the tested sample material is determined with dedicated thermodynamic models taking into account the particularities of the experimental test. Transient measurement methods are used in equipment such as the Laser Flash Analysis (LFA) Apparatus [10], the Hot Disk, and the Hot Wire. These methods are more complex than the steady-state ones and are usually restricted to relatively small samples. However, they are usually much faster than steady-state methods and enable the measurement of other material properties such as specific heat capacity and thermal diffusivity.
At Aalborg University – Department of Civil Engineering, the following equipment setups are commonly used to measure the thermal conductivity of construction materials and building elements:

- The Laser Flash Analysis Apparatus [10] (see Figure 23), uses a transient measurement method to test small size samples: from 6 mm to 25.4 mm in diameter, with a thickness ranging from 0.5 mm to 3 mm. This instrument can be used for the measurement of solid samples, but also slurries, powders or liquids, at temperatures ranging from 10 °C to 300 °C. It is primarily intended to measure materials with high thermal conductivity (see Figure 27).

- The Guarded Hot Plate Apparatus [9] (see Figure 24), uses a steady-state measurement method for medium size samples: 150 x 150 mm for insulating material or 500 x 500 mm for non-insulating material, with a thickness of the sample ranging from 10 mm to 120 mm. It can measure thermal conductivity within the temperature range 10 – 40 °C. It is primarily intended to measure materials with low and medium thermal conductivity (see Figure 27).

- Based on very similar measurement principles as the Guarded Hot Plate, the Guarded Hot Box (see Figure 25) and the Big Guarded Hot Box (see Figure 26) are experimental setups using a steady-state method for large test samples (full-size real construction elements). The Guarded Hot Box can be used to test 2 x 2 m samples, while the Big Guarded Hot Box can be used to test 4 x 4 m construction elements, which is the size of a full-scale building façade.

*Figure 23: Laser Flash Analysis (LFA) Apparatus LFA 447 (Netzsch Gerätebau GmbH) [5].*

Figure 25: The Guarded Hot Box setup at Aalborg University, Department of Civil Engineering.
Figure 26: The Big Guarded Hot Box setup at Aalborg University, Department of Civil Engineering.

Figure 27: Range of application for the different thermal conductivity measurement methods.
8. Thermal Diffusivity

8.1. Definitions

The thermal diffusivity of a material determines how fast heat is propagated through it. The thermal diffusivity can be considered as the effective thermal inertia or the inverse of the time constant of a material. In a material with a high thermal diffusivity, the thermal conductivity is very large in comparison to the volumetric heat capacity. Consequently, heat is propagated rapidly through it by conduction. Therefore, the material has a small time constant and it quickly reaches thermal equilibrium (homogenous temperature throughout the bulk of the material). The SI units of m²/sec or mm²/sec are commonly used for the thermal diffusivity of a material. The thermal diffusivity is the ratio of the thermal conductivity on the volumetric heat capacity (which is the density multiplied by the specific heat capacity):

\[ \alpha = \frac{\lambda}{C_V} \]  \hspace{1cm} (18)

\[ \alpha = \frac{\lambda}{\rho \cdot C_p} \]  \hspace{1cm} (19)

Where,

\( \alpha \): Thermal diffusivity \([\text{m}^2/\text{s}]\)
\( \lambda \): Thermal conductivity \([\text{W/m.K}]\)
\( \rho \): Density of the material \([\text{kg/m}^3]\)
\( C_p \): Specific heat capacity of the material \([\text{J/kg.K}]\)
\( C_V \): Volumetric heat capacity of the material \([\text{J/m}^3.\text{K}]\)

Consequently, the heat equation can be written with the thermal diffusivity as the only material parameter:

\[ \rho \times C_p \times \frac{\partial \theta}{\partial t} = \lambda \frac{\partial^2 \theta}{\partial x^2} \]  \hspace{1cm} (2)

\[ \Leftrightarrow \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} \]  \hspace{1cm} (20)
One can see in *Figure 28* (log scale) the thermal diffusivity of various typical construction and building materials. One can notice that metals have a very large thermal diffusivity compared to the other common building materials. This is due to their very high thermal conductivity.

*Figure 28: Thermal diffusivity of common construction and building materials [7].*
8.2. Measuring Thermal Diffusivity

The thermal diffusivity of materials is commonly measured with the Laser Flash Analysis Apparatus [10] (see Figure 29). Once the thermal diffusivity has been measured, it can be used to determine the other thermophysical properties of a material.

*Figure 29: Laser Flash Analysis (LFA) Apparatus LFA 447 (Netzsch Gerätebau GmbH) [5].*

Apart from the aforementioned main thermophysical properties of building materials, there are numerous other properties that can be of interest for civil engineers working on building energy efficiency, material durability, and indoor environment quality. Among them, the following can be measured at Aalborg University (see Figure 30):

- **Thermal stability** of material when subjected to large temperature variations: Can be measured with Thermogravimetric Analysis (TGA): measure the change in mass of a test sample under controlled heating conditions [4].
- **Fire resistance**: Measure the temperature inside the construction element as a function of time when exposed to direct fire or very high temperatures.
- **Hygrothermal** properties: Water vapor sorption and desorption capacity; how much humidity (water vapor) can be absorbed, transported, stored and released by a material. It can be measured on small test samples with a VSA apparatus [12].
- **Gas diffusion coefficient** (and porosity): how permeable to gas is a porous material. It can be measured with the ODA 20 (Oxygen Diffusion Apparatus), an instrument developed at Aalborg University.
- **In-situ measurement** of building element **thermal conductivity** and **effective thermal mass**: Can be measured or estimated with specific portable test setup comprising multiple temperature sensors and heat flux meters.

Figure 30: Fire resistance test (a); VSA apparatus for hygrothermal properties measurements (b); In-situ measurement setup (c); ODA 20 for measurement of gas diffusion in porous materials (d); Thermogravimetric Analysis apparatus (e).
References

[1] Building Material Characterization Laboratory of Aalborg University, Department of Civil Engineering, Aalborg, Denmark.
   https://buildingmaterials.civil.aau.dk

   https://www.mt.com/ch/en/home/applications/Laboratory_weighing/density-measurement.html

   https://www.tainstruments.com/products/dilatometers/


   https://www.iso.org/standard/40892.html


   https://doi.org/10.1016/j.rser.2016.11.145


Recent publications in the DCE Lecture Note Series

Hicham Johra. Simple data pre-processing of the laser flash analysis results from the LFA 447 apparatus. DCE Lecture Notes No. 72. Department of Civil Engineering, Aalborg University, 2019.

Hicham Johra. Description of the laser flash analysis method for thermal diffusivity measurement with the LFA 447. DCE Lecture Notes No. 73. Department of Civil Engineering, Aalborg University, 2019.

