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Retrieval of Effective Permittivity and Permeability of Periodic Structures on Dielectric and Magnetic Substrates

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***Abstract*— This paper presents the retrieval of effective permittivity and permeability of periodic structure on dielectric and magnetic substrates. The retrieval approach is based on investigating the equivalent circuits of periodic structures. For demonstration, a single square loop-based periodic structure is served as an example to elaborate the retrieval process. First, the equivalent circuit of the free-standing single square loop-based structure is modelled with inductor and capacitor, where the values of these components are determined by the simulated Z-matrix of the free-standing structure; the effects of supporting substrates (dielectric or magnetic substrates) are then considered, where the compensating principles are deduced from inductive grids and capacitive patches periodic structures to compensate the corresponding values in the former equivalent circuit on purpose. The compensating values are also determined from the simulated S-parameter of the structure with supporting substrates. The formulas of effective permittivity and permeability are readily deduced and obtained from the original and compensating values of components in the equivalent circuits. The proposed retrieval process provides a new understanding of effective permittivity and permeability of periodic structures.**

Index Terms*— *Effective permittivity, effective permeability, periodic strcutures, inductive grids, capacitive patches.

I. INTRODUCTION

Since most of microwave or millimeter-wave components, such as: antennas, filters, amplifiers, etc., are printed circuit board (PCB)-based structures that the metallic patterns are usually printed on one surface of dielectric substrate, the effective permittivity is a one of the key parameters to characterize the wavelength in free space and in PCB-based structure. For a periodic structure that metallic patterns are printed on one surface of supporting dielectric substrate, there is a generalized and simplified relation about the effective permittivity and relative permittivity of the substrate, $\varepsilon_{eff} = (\varepsilon_r+1)/2$, which is widely acknowledged and used in electromagnetic fields [1], [2]. The complete formulae of effective permittivity and permeability can be deduced from Maxwell's equations mathematically [3].

In this paper, we alternatively provide a retrieval approach for effective permittivity and permeability of periodic structure on dielectric and magnetic substrates. The retrieval approach avoids the complicated calculations of Maxwell's equations, but is based on equivalent circuits and full-wave simulations. To elaborate the retrieval approach, a square loop-based periodic structure is selected as an example. To validate the effectiveness and correctness of the proposed retrieval approach, the relation between effective permittivity and relative permittivity of a dielectric substrate is deduced according to the proposed retrieval approach, it is found the obtained relation is highly consistent with the identity of $\varepsilon_{eff} = (\varepsilon_r+1)/2$. Then, the relation of effective permeability and relative permeability of a magnetic substrate is deduced based on the proposed retrieval approach as well, where it is interestingly found that the obtained relation is absolutely different with the equation that the effective permittivity follows. It should be noted that the proposed retrieval approach is a generalized tool that can be applicable to any other periodic structure scenario.

II. RETRIEVAL PROCESS

In this section, the retrieval of effective permittivity and permeability of periodic structure on dielectric and magnetic substrates is elaborated in detail.

A. Equivalent circuit of the free-standing square loop-based periodic structure

Without loss of generality, a single square loop-based periodic structure as shown in Fig. 1(a) is selected as the example to elaborate the retrieval process since the inductor and capacitor are equally dominated in the equivalent circuit. The equivalent components of the single loop-based periodic structure are established as shown in Fig. 1(b), where a series of L and C is shunt in the equivalent circuit. The inductor L is caused by the induced current on the surface of the square loop, while the gaps of adjacent loop squares contribute to the capacitor C. According to the equivalent circuit of the square loop, the Z-parameter elements of this two-port network are all identical and can be expressed as follows [4]:

$$Z_{11} = Z_{12} = Z_{21} = Z_{22} = j\left(\omega L - \frac{1}{\omega C}\right) \tag{1}$$

The values of the circuit components L and C can be determined with the help of simulated Z-matrix of the free-standing structure carried out in High Frequency Structure Simulator (HFSS). It should be noted that the two ports should be de-embed into the two surfaces of the free-standing structure as shown in Fig. 1(c) to obtain the desired Z-parameter. When L=3.25nH, C=0.056pF, the imaginary part of the impedance matrix, obtained from equivalent circuit, has a great agreement with the ones obtained from full-wave simulation as presented in Fig. 2.

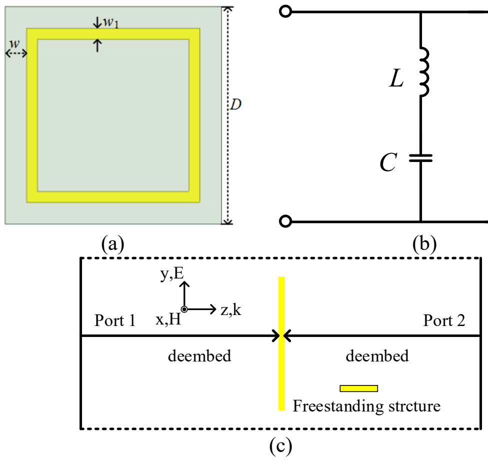


Fig. 1. The geometry of freestanding square loop-based periodic structure. (a) Front view. (b) Its equivalent circuit. (c) Deembed Ports. (Dimensions: D=10mm, w=1mm, w_1=0.5mm)

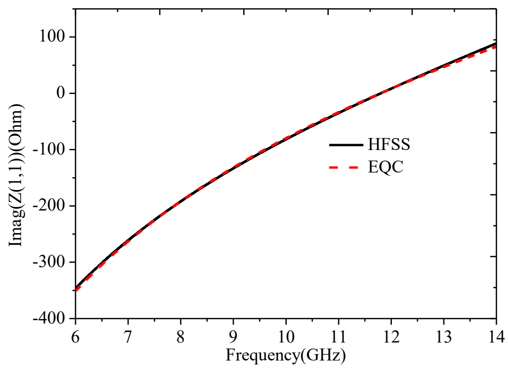


Fig. 2. The imaginary part of the impedance matrix obtained from full wave simulation and equivalent circuit (EQC). ($L = 3.25nH, C = 0.056pF$)

B. *Compensating principles*

Before considering the effects of supporting substrate on the equivalent circuits of freestanding structure, the impedances of inductive grids and capacitive patches are recalled to obtain some guidelines to compensate the values in the equivalent circuit of the free-standing structure.

The impedance of inductive grids as shown in Fig. 3(a) under normal incidence wave was recalled and given by [5]:

$$Z_{inductive} = j\frac{\eta_{eff}}{2}\alpha \tag{2}$$

where $\eta_{eff} = \sqrt{\varepsilon_{eff}/\mu_{eff}}$ is the effective wave impedance, ε_{eff} is the effective permittivity, and μ_{eff} is the effective permeability of the inductive grid unit cell. In Eq. (1), α is a parameter which is closely related to the dimensions of the unit cell [6]. For the inductive strips shown in Fig. 3(a), α is defined as:

$$\alpha = \frac{k_{eff}D}{\pi}\ln\left(1\Big/\sin\left(\frac{\pi w}{2D}\right)\right) \tag{3}$$

where k_{eff} is the wave number of the incident wave vector in the host medium. D and w are denoted in Fig. 3(a).

By substituting Eq. (3) into Eq. (2), after simplifying, yields to:

$$Z_{inductive} = j\omega\frac{D}{2\pi}\mu_0\mu_{eff}\ln\left(1\Big/\sin\left(\frac{\pi w}{2D}\right)\right) \tag{4}$$

It is concluded that only an inductor is enough to mimic the transmission characteristics of inductive grid-based periodic structure. The inductance L is expressed as:

$$L = \frac{D}{2\pi}\mu_0\mu_{eff}\ln\left(1\Big/\sin\left(\frac{\pi w}{2D}\right)\right) \tag{5}$$

Therefore, the equivalent circuit of inductive grid-based periodic structure can be modeled with a shunt inductor as demonstrated in Fig. 3(c), where β is phase constant, and Z_1, l are characteristic impedance and electric length of the supporting substrate, respectively.

Applying the approximate Babinet principle, we obtain the impedance for capacitive patches as shown in Fig. 3(b) under normal incidence from Eq. 2 as follows:

$$Z_{capacitive} = -j\frac{\eta_{eff}}{2\alpha} \tag{6}$$

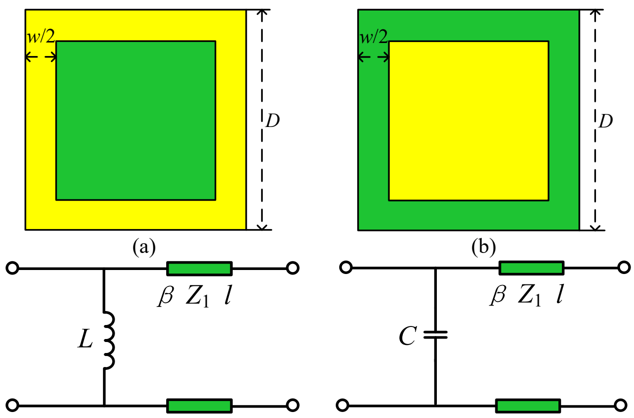
By substituting Eq. (3) into Eq. (6), after simplifying, then it's expressed as:

$$Z_{capacitive} = \frac{1}{j\omega\frac{2D\varepsilon_0\varepsilon_{eff}}{\pi}\ln(1/\sin(\frac{\pi w}{2D}))} \tag{7}$$

Likewise, we can use a capacitor to mimic the transmission characteristics of capacitive patches periodic structure, and the value of capacitance C is expressed as:

$$C = \frac{2D\varepsilon_0\varepsilon_{eff}}{\pi}\ln(1/\sin(\frac{\pi w}{2D})) \tag{8}$$

The equivalent circuit of capacitive patches therefore can be modeled with a shunt capacitor as shown in Fig. 3(d).



(c) (d)

Fig. 3. Geometries of two frequency selective surfaces. (a) Inductive strips, (b) Capacitive patches, (c) Equivalent circuit of inductive strips, (d) the equivalent circuit of capacitive patches. (The yellow and green parts stand for copper and supporting substrate, respectively.)

Although Eq.s (5) and (8) are deduced from the periodic inductive grids and capacitive patches under normal incidence, the physical significance that Eq.s (5) and (8) demonstrate are generalized. It is concluded that the values of L and C are only associated with the permeability and permittivity of supporting substrate, respectively, once the dimensions of the inductive grids or capacitive patches are determined.

C. *Retrieval of effective permittivity*

For demonstration, the rare square loop is printed on a dielectric substrate with a thickness of h = 2 mm, and a relative permittivity of ε_r = 5 as shown in Figs. 4 (a) and (b). Based on the conclusion that the capacitance is only affected by the supporting dielectric substrate, an extra parallel capacitor C_1 thus is added to compensate the capacitor C. The updated equivalent circuit that considers the supporting dielectric substrate is shown in Fig. 4(c). The parametric values of transmission line β, Z_1, l can be calculated from the relative permittivity and thickness of the supporting dielectric substrate directly using the formula $\beta = 2\pi f\sqrt{\varepsilon_r}/c$ and $Z_1 = Z_0/\sqrt{\varepsilon_r}$, where l is the electric length of the transmission line; c and Z_0 are the light velocity and characteristic impedance in free space, respectively.

It should be noted that the values of L and C in the updated equivalent circuit shown in Fig. 4 (a) should remain unchanged with the counterparts in the equivalent circuit shown in Fig. 1 (b). Only the value of C_1 needs to be determined. The S-parameter of the structure of Fig. 4 (a) is easily simulated by using full-wave simulation as shown in Fig. 5. Also, the S-parameter of equivalent circuit of Fig. 4 (c) is readily obtained once the capacitor C_1 is imposed a value. After optimizations, the value of C_1 is determined and equal to 0.12 pF, where the S-parameters from full-wave and equivalent circuit simulations are almost identical. The highly consistent results reveal the effectiveness of the compensating capacitance.

If the permittivity of supporting dielectric substrate is varied, only the value of compensating capacitor C_1 needs to be optimized and determined with the values of C and L fixed accordingly. On the other hand, the relation between the compensating capacitance C_1 and properties of the supporting dielectric substrate can be deduced from Eq. (7) and expressed as follows

$$C_1 = (\varepsilon_{eff} - 1)\cdot C \tag{9}$$

where C is the capacitor obtained from free-standing square loop [as shown in Fig.2]. Since the value of C_1 is only associated with the relative permittivity (ε_r) of supporting dielectric substrate, and the value of C is predetermined, it is interestingly found that the relation between effective permittivity and relative permittivity of supporting dielectric substrate can be deduced from Eq. (9).

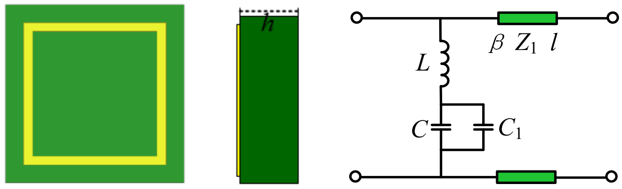


Fig. 4. Square loop structure with the supporting substrate. (a) Front view. (b) Side view. (c) Its modified equivalent circuit with the dielectric supporting substrate. (Dimension: h=2mm.)

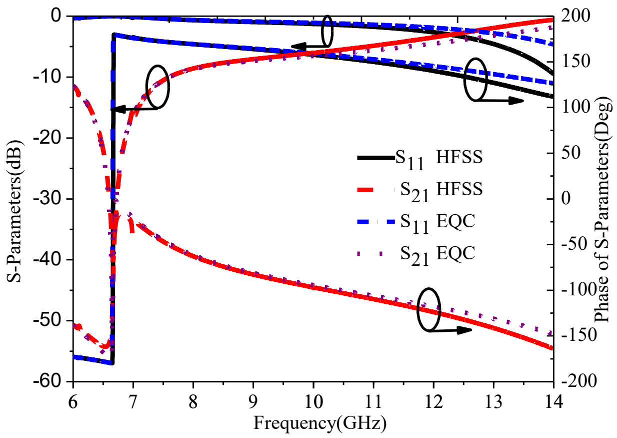


Fig. 5. S-Parameters obtained from full wave simulation and equivalent circuit (EQC). ($L = 3.25nH, C = 0.056pF, C_1 = 0.12pF$)

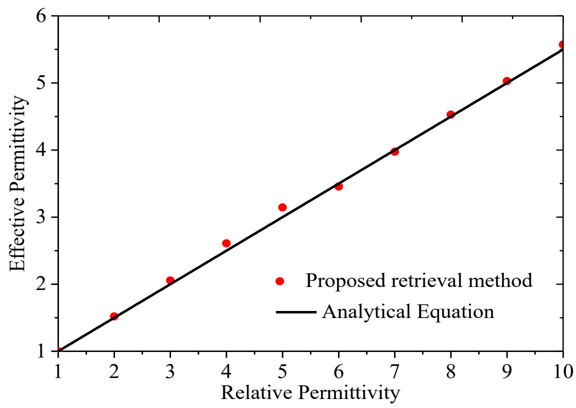


Fig. 6. The relation between effective permittivity and relative permittivity of supporting dielectric substrate. (h=2mm).

Fig. 6 plots the relation between effective permittivity and relative permittivity of supporting dielectric substrate, obtained according to Eq. (9). On the other hand, it is widely acknowledged that the effective permittivity of a periodic structure that the metallic pattern is only printed on one side of supporting dielectric substrate (one can take the structure in Fig. 4(a) as an example to understand it) is $\varepsilon_{eff} = (\varepsilon_r+1)/2$, which is plotted in Fig. 6 as well for comparison. As seen in Fig. 6, the relation between the effective permittivity is highly consistent with the widely used formula ($\varepsilon_{eff} = (\varepsilon_r+1)/2$), which verify the correctness of the retrieval effective permittivity from the equivalent circuit view point. It should be noted, however, that the widely used equation ($\varepsilon_{eff} = (\varepsilon_r+1)/2$) would be invalid when the thickness of supporting dielectric substrate exceeds or is less than a certain value. In contrast, we can always obtain the value of C_1 whatever the properties (thickness, permittivity) of supporting dielectric substrate are to obtain the relation between effective permittivity and relative permittivity of supporting dielectric substrate from Eq. (9).

D. *Retrieval of effective permeability*

There are very few literatures to investigate the effective permeability of periodic structures. Therefore, the investigations on effective permeability would provide a reference for the future research. For consistency, the supporting dielectric substrate used in Fig. 4 is replaced by a magnetic substrate with a relative permeability of $\mu_r = 1$. Based on the conclusion that the inductance is only affected by the supporting magnetic substrate, an extra cascaded inductor Lc is therefore added to compensate the inductor L. the updated equivalent circuit is presented in Fig. 7. The parametric values of the transmission line are calculated as follows: $\beta = 2\pi f\sqrt{\mu_r}/c$, $Z_2 = Z_0\sqrt{\mu_r}$.

It should be also noted that the values of L and C in the updated equivalent circuit shown in Fig. 7 should remain unchanged with the counterparts in the equivalent circuit shown in Fig. 1 (b). Only the value of L_c needs to be determined. Likewise, the value of L_c is determined by comparing the S-parameters of the periodic structure obtained from full-wave and equivalent circuit simulations. When L_c equals to 2.119nH, the S-parameters obtained from the equivalent circuit and full-wave simulations agree well as shown in Fig. 8.

Similarly, the relation between the compensating inductance L_c and properties of the supporting magnetic substrate can be deduced from Eq. (4) and expressed as follows:

$$L_c = \left(\mu_{eff} - 1\right)\cdot L \tag{10}$$

where L is the capacitor obtained from free standing square loop [as shown in Fig.2]. Since the value of L_c is only associated with the relative permeability (μ_r) of supporting magnetic substrate, and the value of L is predetermined, it is also found that the relation between effective permeability and relative permeability of supporting magnetic substrate can be deduced from Eq. (10).

Fig. 9 plots the relation between effective permeability and relative permeability of supporting magnetic substrate, obtained according to Eq. (10) [red in dot]. It is interestingly found that the relation does not fulfill the similar relation of $\mu_{eff} = (\mu_r+1)/2$. A logarithm function is employed here to fit them to obtain an analytical expression which is given by:

$$\mu_{eff} = 0.32\times\log 2(\mu_r)+1 \tag{11}$$

The curve of Eq. (11) is also plotted in Fig. 9 for comparison, where a fairly great agreement is observed.

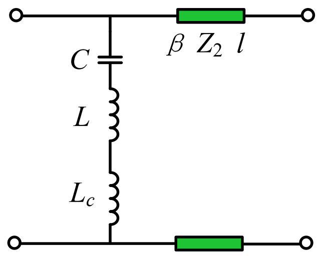


Fig. 7. Updated equivalent circuit with supporting magnetic substrate.

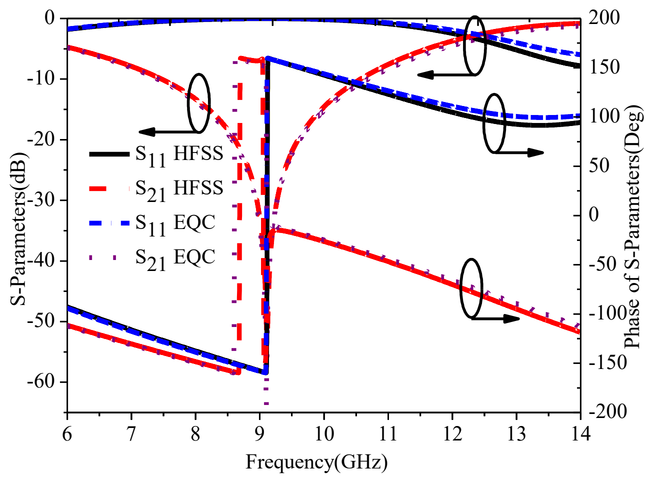


Fig. 8. S-Parameters obtained from full wave simulation in HFSS and equivalent circuit (EQC). ($L = 3.25nH, C = 0.056pF, L_c = 2.119nH$)

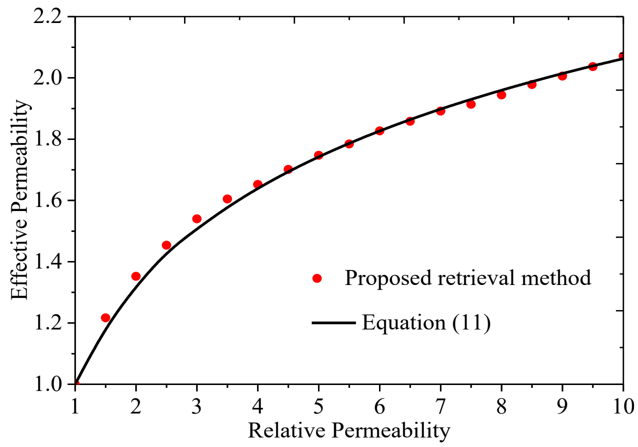


Fig. 9. The relation between effective permeability and relative permeability of supporting magnetic substrate. (h=2mm)

III. CONCLUSIONS

This paper describes the retrieval of effective permittivity and permeability of periodic structure on dielectric and magnetic substrates in detail. The retrieval approach is based on equivalent circuits and full-wave simulations. The retrieval approach is solidly validated by the highly consistent results with the widely acknowledged identity $\varepsilon_{eff} = (\varepsilon_r+1)/2$. And it is also applied to deduced the relation between effective permeability and relative permeability of a magnetic substrate. It is interestingly found that the relation about effective permeability is absolutely different with the identity that effective permittivity follows. It should also be noted that the proposed retrieval approach is generalized and can be applicable to any other periodic structures.

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REFERENCES

[1] R. C. Compton, L. B. Whitbourn, and R. C. McPherdan, "Strip gratings at a dielectric interface and application of Babinet's principle," *Appl. Opt.* vol. 23, no. 18, pp. 3236-3242, Sept. 1984.

[2] P. Callaghan, E. A. Parker, and R. J. Langley, "Influence of supporting dielectric layers on the transmission properties of frequency selective surfaces," *IEE Proceeding H, Microwaves, Antennas and Propagation,* 138, 5, 1991, pp. 448-454.

[3] Z. Li, and D. W. Prather, "Calculation of effective permittivity, permeability, and surface impedance of negative-refraction photonic crystals," vol. 15, no. 13, *Opt Express,* Jun. 2007.

[4] D. M. Pozar, "*Microwave Engineering*," *3rd*, John Wwiley & Sons, Inc., New York, 2005.

[5] O. Luukkonen, *et al*., "Simple and accurate analytical model of planar grids and high-impedance surface comprising metal strips or patches," *IEEE trans. Antennas and Propag.,* vol. 56, no. 6, pp. 1624-1635, June 2008.

[6] S. A. Tretyakov, *Analytical Modeling in Applied Electromagnetics*. Norwood, MA: Artech House, 2003.