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# A Centralized and Scalable Uplink Power Control Algorithm in Low SINR: A Case Study for UAV Communications

Xuesong Cai, István Z. Kovács, Jeroen Wigard, and Preben E. Mogensen

**Abstract**—Interference management through power control is essential to optimize the system capacity. With the introduction of aerial user equipments in cellular networks, resulting in an increase of line of sight links, power control is becoming more and more vital to enable the (uplink) high-throughput data streaming and protect the users on the ground. The investigation in [1] shows that in the high signal-to-interference-plus-noise (SINR) regime, geometrical programming (GP) can be used to efficiently and reliably solve the problem. In the low SINR regime, a series of GPs are solved by condensation. However, the condensation method proposed in [1] is non-scalable, which hinders its application to a large-scale network, e.g. a densified network, where many more cells could be jointly optimized. In this communication, by transforming the original problem into a standard form introducing auxiliary variables, a new condensation method is proposed. Its complexity linearly increases with the number of links increasing, which makes the power control practically solvable for both small- and large-scale networks. A case study for the up-link UAV communications in cellular networks is performed using the proposed algorithm.

**Index terms**— Interference management, power control, up-link, LTE and UAV.

## I. INTRODUCTION

Interference management has been investigated for decades. Many works, e.g. [1]–[3], have shown that through power control, significant gains can be achieved for the overall system capacity. As the network becomes densified and different types of users equipments are being involved in the fifth generation (5G) communication networks and beyond, interference has been considered the major limiting factor of the overall system capacity. For example, in the unmanned-aerial-vehicle (UAV) communications, interference in both up- and down-link becomes severe with the increasing height. The main reason is that the channel between the UAV and the terrestrial base

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stations (BSs) become more clearer, i.e., line-of-sight (LoS)-alike [4]–[8].

The power control problem usually has the form of maximizing the weighted-sum-rate of the system, with each receiver node (Rx) satisfying its power and quality of service (QoS) constraints. This is generally a non-convex problem and difficult to obtain the global optimality. Different algorithms have been proposed. Without the QoS constraints, the ADP (Asynchronous Distributed Pricing) algorithm was proposed in [9] where each Rx sends out a price and updates its transmitting power according to the prices sent by other links iteratively until convergence. In [3], the power allocation was obtained by tuning the current link's power while fixing other links' transmission power to maximize the system capacity in a Round-Robin (RR) manner (one by one) until convergence. In [10], the authors proposed to utilize binary power control (i.e. either transmitting with 0 power or maximum power), and results show that the performance loss to global optimality is insignificant. With QoS constraints the problem becomes more difficult.<sup>1</sup> In [2], by iteratively shrinking the polyblock, the proposed MAPEL algorithm can asymptotically approach the global optimality, although its complexity increases significantly with the number of link pairs increasing. In [11], the authors exploited the recent advances in deep learning and proposed an ensembling deep-neural-networks to tackle the problem. In [1], the authors approximated the problem as a geometrical programming (GP) in the high signal-to-interference-plus-noise (SINR) regime. This method has been considered as one of the best algorithms since GP can be solved efficiently and reliably [12].

However, in the low SINR regime, the convex-approximation in [1] is invalid. Therefore, a condensation method was also proposed in [1] to solve the original problem by solving a series of GP problems. Nevertheless, the condensation is performed for the power variables, which is non-straightforward and non-scalable. In other words, it is practically infeasible/impossible to use the condensation method proposed in [1] for a larger-scale network even with not so many link pairs.<sup>2</sup> However, as the network is becoming densified and the number of users significantly increasing, optimizing the power allocation for a certain (moderate to large) number of links are

<sup>1</sup>The case without QoS constraints can be considered a special case with QoS constraints.

<sup>2</sup>The detailed discussion can be found in Sect. III.

inevitable, e.g., for the uplink (UL) UAV communications. To solve the problem, the contributions of this communication are mainly three-folds. 1) A standard form of the original problem is proposed by introducing auxiliary variables. In the standard form, the condensation can be applied for auxiliary variables which is more intuitive. 2) A new condensation method is proposed, where the number of parameters to be calculated increases linearly with the number of links. Moreover, the proposed method is more straightforward as there is no coupling among those auxiliary variables compared to directly conducting the condensation for the power variables. The method can be easily scaled for large-scale networks. 3) In addition, by using the proposed method, a case study for up-link UAV communications in a (moderately) large-scale cellular network is also illustrated.

The rest of the communication is organized as follows. Sect. II elaborates the problem formulation and proposes the standard form. Sect. III discusses the condensation principle and proposes the novel condensation method. The case studies are presented in Sect. IV. Finally, conclusive remarks are included in Sect. V

## II. PROBLEM FORMULATION

Let us consider the power control problem in a wireless network with a set of  $\mathcal{N} = \{1, \dots, N\}$  of distinct link pairs (e.g., Fig. 1). Each link pair has a transmitter node (Tx) and a Rx. The channel gain matrix is denoted as  $\mathbf{G} = [G_{ij}]$  with  $G_{ij}$  indicating the channel gain between the  $i$ th Tx and the  $j$ th Rx. Note that  $G_{ij}$  is attributed to path loss, shadowing, fast fading, etc. The node pair  $(i, i)$  and node pairs  $(i, j), j \neq i$  are the serving link and interfering links, respectively. The transmit power  $p_i$  at the  $i$ th Tx is usually bounded between  $p_{i,\min}$  and  $p_{i,\max}$ . Moreover, the noise power measured at the  $i$ th Rx is denoted as  $n_i$ . Therefore, the received SINR  $\gamma_i$  at the  $i$ th Rx can be calculated as

$$\gamma_i(\mathbf{p}) = \frac{p_i G_{ii}}{\sum_{j \in \mathcal{N}, j \neq i} p_j G_{ji}}. \quad (1)$$

where  $\mathbf{p} = [p_1, \dots, p_N]$  is the compact vector notation of the transmitted power of all the Txs. We consider the data rate  $R_i$  (bit/sec/Hz) at the  $i$ th Rx node according to the modified Shannon capacity formula as

$$R_i(\mathbf{p}) = a \log_2(1 + b\gamma_i). \quad (2)$$

where  $a$  and  $b$  are constants no greater than 1. This is caused by different factors such as the coding gap to Shannon capacity, system efficiency, etc., and has been certified in [13] in LTE networks. Note that With  $a$  and  $b$  as 1, (2) becomes the Shannon capacity formula.

The objective of power control is to find the optimal transmitted power  $\mathbf{p}^*$  that leads to the maximum weighted sum rate for the whole network with possible QoS constraints for individual

link pairs. The optimization problem can be formulated as

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{N}} w_i R_i \\ & \text{subject to} && R_i \geq R_{i,\min}, \forall i \in \mathcal{N} \\ & && p_{i,\min} \leq p_i \leq p_{i,\max}, \forall i \in \mathcal{N} \end{aligned} \quad (3)$$

where  $w_i$  is the weight (importance) for the  $i$ th Rx node, and  $R_{i,\min}$  is the QoS constraints for the  $i$ th Rx (which can be formulated equivalently as  $\gamma_i \geq \gamma_{i,\min}$ ). As a special case with  $\mathbf{R}_{\min} = [R_{1,\min}, \dots, R_{N,\min}]$  as  $\mathbf{0}$ , the maximization problem (3) becomes an unconstrained problem in terms of QoS. By introducing auxiliary variables  $\mathbf{s} = [s_1, \dots, s_N]$  and  $r$ , we can further equivalently transform (3) to

$$\begin{aligned} & \text{maximize} && r \\ & \text{subject to} && \prod_{i=1}^N (1 + s_i)^{w_i} \geq r \\ & && \frac{p_i G_{ii}}{\sum_{j \in \mathcal{N}, j \neq i} p_j G_{ji}} \geq s_i, \quad \forall i \\ & && s_i \geq \gamma_{i,\min}, \quad \forall i \\ & && p_{i,\min} \leq p_i \leq p_{i,\max}, \quad \forall i \end{aligned} \quad (4)$$

One step further, we have

$$\begin{aligned} & \text{minimize} && r^{-1} \\ & \text{subject to} && \frac{r}{\prod_{i=1}^N (1 + s_i)^{w_i}} \leq 1 \\ & && s_i p_i^{-1} G_{ii}^{-1} \left( \sum_{j \in \mathcal{N}, j \neq i} p_j G_{ji} \right) \leq 1, \quad \forall i \\ & && s_i^{-1} \gamma_{i,\min} \leq 1, \quad \forall i \\ & && p_i^{-1} p_{i,\min} \leq 1, \quad \forall i \\ & && p_i p_{i,\max}^{-1} \leq 1, \quad \forall i. \end{aligned} \quad (5)$$

With the above transformation introducing nonnegative auxiliary variables  $\mathbf{s}$  and  $r$ , we can consider (5) be a standard form of the weighted sum rate maximization problem. The optimal power allocation  $\mathbf{p}^*$  is obtained when achieving the minimum  $r^{-1}$  (i.e.,  $r^*$ ), and the maximum weighted sum rate can be calculated as  $\log_2 r^*$ . It is worth noting that the proposed form (5) here is essential for the condensation in a large-scale network in the low SINR regime, as we can only consider the condensation for  $\mathbf{s}$  rather than  $\mathbf{p}$  which will be discussed in Sect. III.

## III. CONDENSATION METHOD IN THE LOW SINR REGIME

Before going to the low SINR regime, let us first consider the high SINR regime. In the high SINR regime, the maximum weighted sum rate is considered to be achieved with all the Rxs have high SINRs, which means that  $s_i^*$  or  $\gamma_i^*, \forall i$  is (much) larger than 1. Therefore, the term  $\frac{1}{\prod_{i=1}^N (1 + s_i)^{w_i}}$  in (5) can be well approximated as  $\prod_{i=1}^N s_i^{-w_i}$  so that (5) becomes a standard GP problem where a posynomial is to be minimized subject to upper bounded posynomial constraints and equality

monomial constraints [1], [14]. Briefly, a monomial has the form as

$$f(\mathbf{x}) = cx_1^{d_1}x_2^{d_2}\dots x_n^{d_n}. \quad (6)$$

where  $x_i$ 's and  $c$  are nonnegative variables and constant, respectively, and  $d$ 's are real constants. A posynomial has the form as the sum of several monomials. In the high SINR regime, the GP can be efficiently and numerically reliably solved using, e.g. the interior point method [12], to find the global optimal  $\mathbf{p}^*$ .

However, in the low SINR regime with severe interference, the approximation as done in the high SINR regime is not valid anymore, and obviously  $\frac{1}{\prod_{i=1}^N(1+s_i)^{w_i}}$  is not a posynomial. Therefore, a condensation method was proposed in [1] to solve a series of GP problems to find the power allocation satisfying the Karush-Kuhn-Tucker (KKT) conditions (which means that the final power allocation could be a local maxima) in the low SINR regime. The basic idea is to approximate the non-posynomial term in the denominator as a monomial at a given feasible power allocation point, get a new optimal power allocation for the currently approximated GP, then approximate the original problem at the new power allocation again to further get another power allocation. The process is proceeded iteratively until convergence. The monomial approximation proposed in [1] is based on the arithmetic-geometric mean inequality. Specifically, the approximated monomial  $\hat{g}(\mathbf{x})$  for a function  $g(\mathbf{x}) = \sum_i u_i(\mathbf{x})$  can be written as

$$\hat{g}(\mathbf{x}) = \prod_i \left( \frac{u_i(\mathbf{x})}{\alpha_i} \right)^{\alpha_i} \quad (7)$$

where  $u_i$  is a monomial component, and  $\alpha_i$  is calculated as  $\frac{u_i(\mathbf{x}_0)}{g(\mathbf{x}_0)}$  at the approximation point  $\mathbf{x}_0$ . Furthermore,  $\hat{g}(\mathbf{x})$  has to satisfy three conditions [1], [15] to guarantee the power allocation converge to a KKT point, which include: (a)  $\hat{g}(\mathbf{x}) \leq g(\mathbf{x})$  for all  $\mathbf{x}$ . This is to tighten the constraint so that the obtained new power allocation for the current approximated GP is always feasible for the original problem. (b)  $\hat{g}(\mathbf{x}_0) = g(\mathbf{x}_0)$ . This is to guarantee the monotonicity of the optimal values obtained in successive iterations. (c)  $\nabla \hat{g}(\mathbf{x}_0) = \nabla g(\mathbf{x}_0)$ . This is to guarantee the KKT conditions for the original problem are satisfied after convergence. The condensation (7) proposed in [1] satisfies the three conditions as discussed in [1], and simulations have shown its performance, e.g. in a small-scale network with 3 link pairs in [1] and up to 10 link pairs in [2]. Nevertheless, we would like to note that there is a major problem when (7) is applied in a large-scale network with a certain number of link pairs. The reason is that to conduct condensation (7), one has to firstly rewrite  $g(\mathbf{x})$  in the form of summing several monomials. For a small network, this could be done practically. However, the number of monomial terms increasing exponentially with the link number increasing, which means that it is practically difficult to conduct (7) in a larger scale network. As an example, considering the term  $g(\mathbf{s}) = \prod_{i=1}^N(1+s_i)^{w_i}$ , it has  $2^N$  monomial terms. With 20 link pairs, there will be more than 1 million monomial terms meaning more than 1 million  $\alpha$ 's have to be calculated. Moreover,  $2^N$  is for the variables  $\mathbf{s}$  in the form (5) as proposed

in this work. With the condensation applied directly for power variables ( $g(\mathbf{p}) = \prod_{i=1}^N(1 + \frac{p_i G_{ii}}{\sum_{j \in \mathcal{N}, j \neq i} p_j G_{ji}})^{w_i}$  as done in [1]), the number of monomial  $u_i$ 's increases much faster than  $2^N$ . Thus, a scalable condensation method that can be applied for a larger-scale network is in necessity for 5G and beyond communications.

As the proposed form (5) is general, we only need to focus on the condensation for auxiliary variables

$$g(\mathbf{s}) = \prod_{i=1}^N(1+s_i)^{w_i} \quad (8)$$

Before proposing the final condensation for  $g(\mathbf{s})$ , we firstly see the function  $h_i(s_i) = 1 + s_i$ . Consider a monomial  $\hat{h}_i(s_i) = c_i s_i^{d_i}$  that satisfies the conditions (b) and (c) with  $h_i$  at a given  $s_{i0}$ , we have

$$\hat{h}'_i(s_i) \Big|_{s_{i0}} = h'_i(s_i) \Big|_{s_{i0}}, \hat{h}_i(s_{i0}) = h_i(s_{i0}). \quad (9)$$

which is equivalently as

$$\frac{d}{ds_i} \ln(\hat{h}_i) \Big|_{s_{i0}} = \frac{d}{ds_i} \ln(h_i) \Big|_{s_{i0}}, \hat{h}_i(s_{i0}) = h_i(s_{i0}). \quad (10)$$

According to (10), it is straightforward to find  $d_i = \frac{s_{i0}}{1+s_{i0}}$  and  $c_i = (1+s_{i0})s_{i0}^{-d_i}$ . To show that  $h_i$  and  $\hat{h}_i$  satisfy the condition (a), i.e.  $\hat{h}_i(s_i) \leq h_i(s_i)$  for all  $s_i$ , we construct the difference function

$$l(s_i) = \hat{h}_i(s_i) - h_i(s_i) \quad (11)$$

It can be calculated that

$$l'(s_i) = \left( \frac{s_{i0}}{s_i} \right)^{\frac{1}{1+s_{i0}}} - 1, \quad \Downarrow \quad (12)$$

$$\ln(l'(s_i)) = \frac{1}{1+s_{i0}} \ln\left( \frac{s_{i0}}{s_i} \right)$$

where when  $s > s_{i0}$ ,  $l'$  is negative, and vice versa. Thus,  $l(s_i)$  is maximized at  $s_{i0}$  as 0, which means that condition (a) holds for  $\hat{h}_i(s_i)$  and  $h_i(s_i)$ . Finally we can write the condensation function  $\hat{g}(\mathbf{s})$  for  $g(\mathbf{s})$  at  $\mathbf{s}_0 = [s_{10}, \dots, s_{N0}]$  as

$$\hat{g}(\mathbf{s}) = \prod_{i=1}^N c_i^{w_i} s_i^{w_i d_i}. \quad (13)$$

It can be known that  $\hat{g}(\mathbf{s})$  and  $g(\mathbf{s})$  satisfy conditions (a) and (b), since each  $\hat{h}_i(s_i)$  and  $h_i(s_i)$  satisfy conditions (a) and (b). Condition (c) also holds for  $\hat{g}(\mathbf{s})$  and  $g(\mathbf{s})$ , which can be directly checked by comparing their gradients. It is worth noting that the calculation for a  $d_i$  is only related to  $s_i$  as  $d_i = \frac{s_{i0}}{1+s_{i0}}, \forall i$  (decoupled from all the other  $s_j, j \neq i$ ), and the multiplicative constant  $c = \prod_{i=1}^N c_i^{w_i}$  can be calculated directly as  $c = g(\mathbf{s}_0) (\prod_{i=1}^N s_{i0}^{w_i d_i})^{-1}$  after all  $d_i$ 's are obtained. This means that the proposed condensation method is easy and straightforward to be done.

To conclude, by exploiting the standard form as proposed in (5), the condensation method in (13) is proposed for the general power control problem. Furthermore, the number

of calculated parameters in the condensation scales linearly with  $N$  (which is actually  $N + 1$ ). This method makes the power control problem in the low SINR regime be practically solvable using a series of GPs for both small-scale and (very) large-scale networks. The pseudocode in Algorithm 1 illustrates the process for the problem (5) using the novel condensation method.<sup>3</sup>

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**Algorithm 1** Solving the power control problem (5) using the proposed condensation method (13).

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**Input:** An initial feasible power allocation  $\mathbf{p}$ .

**Output:** A power allocation that satisfies KKT conditions for problem (5).

- 1: **Repeat:**
  - 2: Calculate  $\mathbf{s}_0$  as  $\mathbf{s}_0 = [\gamma_1(\mathbf{p}), \dots, \gamma_N(\mathbf{p})]$ .
  - 3: Conduct the condensation proposed in (13) at  $\mathbf{s}_0$  for  $g(\mathbf{s})$  in 8.
  - 4: Solve the resulted GP problem using the interior method, and update  $\mathbf{p}$  as  $\mathbf{p}^*$  obtained in this step.
  - 5: **Until** the power difference between two successive iterations satisfies  $\|\mathbf{p}_{\text{new}} - \mathbf{p}_{\text{old}}\| < \epsilon$  with  $\epsilon$  a pre-defined tolerance.
- 

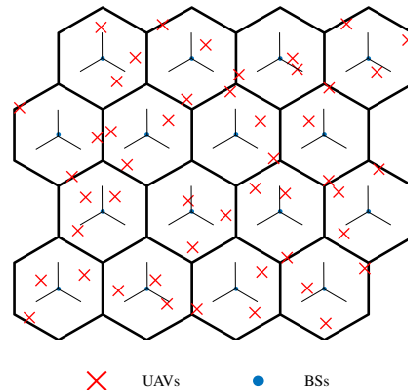


Fig. 1: An example topology of UAVs and sectorized cells in the case study for UL transmission of cellular-UAVs.

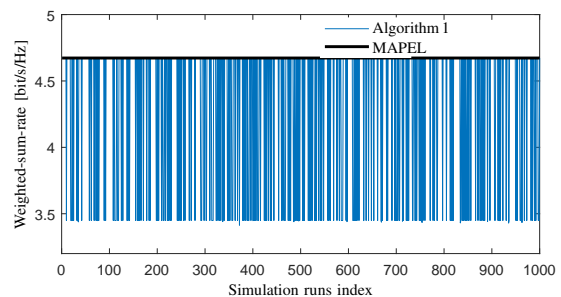


Fig. 2: The weight sum rate obtained by using Algorithm 1 and MAPEL, respectively, for **Example 1** [2].

#### IV. PERFORMANCE EVALUATION AND CASES STUDY

##### A. Case 1: Probability of achieving global optimality

In the low SINR regime, Algorithm 1 not necessarily converges to the global optimal power allocation. To study the probability, the ground truth of global optimum has to be obtained. Here, we resort to the MAPEL algorithm in [2]. Note that although MAPEL can obtain the global power allocation, its computation complexity increases drastically with the network size increasing [2], [11]. Thus we choose the same small-scale network as the **Example 1** presented in [2], which is a network with four link pairs. The channel gain matrix is

$$G = \begin{bmatrix} 0.4310 & 0.0002 & 0.2605 & 0.0039 \\ 0.0002 & 0.3018 & 0.0008 & 0.0054 \\ 0.0129 & 0.0005 & 0.4266 & 0.1007 \\ 0.0011 & 0.0031 & 0.0099 & 0.0634 \end{bmatrix}, \quad (14)$$

power upper bounds are  $[0.7, 0.8, 0.9, 1.0]$  mW, noise power is  $0.1 \mu\text{W}$  for all links, and the weights are  $[\frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}]$ . Fig. 2 illustrates the obtained weighted sum rate using Algorithm 1 with 1000 random power initializations, and the black horizontal line indicates the global maximum weighted sum rate obtained by using the MAPEL algorithm. The probability of Algorithm 1 achieving the global optimality is calculated as 73.4% in this case, which is slightly larger than 70.8% presented in [2] when using the condensation method (7).

##### B. Case 2: Close-to real-world up-link (UL) UAV communications in cellular networks

Recently, UAV is gaining its popularity in multiple applications due to its low cost and flexibility [5], [8]. The cellular networks, e.g. LTE, are considered promising to provide critical and non-critical communications to UAVs. Nevertheless, due to the clearance of the channel between UAVs and terrestrial BSs [5], [8], both the down-link and UL experience severe interference [4], [7], [16], which limits the system capacity significantly. To gain insights into the UL communication for cellular-UAVs, we study the power control for UAVs in a cellular network. As illustrated in Fig. 1, a network with 48 cells ( $4 \times 4$  sectorized hexagons) is considered in the simulation.<sup>4</sup> The distance between neighboring BSs is set as 2 km, and the heights of BSs are 35 m. In each sector the half power beam-widths (HPBW) of the sector antenna in azimuth and elevation domains are set as  $120^\circ$  and  $13^\circ$ , respectively, and the down-tilt angle is properly set (as  $8.5^\circ$  in this case) to optimize the ground coverage. An UAV with 60 m height is randomly put in each cell. The maximum transmission power of an UAV is set as 23 dBm, the noise power spectrum density is calculated at 290 K, and the weights are set identical for all UAVs as  $\omega_i = \frac{1}{N}, \forall i$  meaning that the weighted sum rate is the average value for all UAVs. In addition, we assume the

<sup>3</sup>An initial feasible  $\mathbf{p}$  with QoS constraints can be found using the method as discussed in Sect. III-B in [11]. Without QoS constraints, setting an initial feasible  $\mathbf{p}$  is trivial.

<sup>4</sup>The number of monomial terms in (7) is much higher than  $2^{48}$ .

Table I: The simulation configuration of the case study for UL UAV communications.

<i>Main parameters in the simulation</i>	
Network scale	48 cells
Cell type	Sectorized hexagon
BS spacing	2 km
BS height	35 m
HPBW of sector antenna	(120°, 13°)
Down-tilt angle	8.5°
UAV height	60 m
Max. transmit power per UAV-UE	23 dbm
Schedule assumption	One UAV-UE/cell/TTI

UAVs are using omnidirectional antennas. The channel model is from the results in [8].<sup>5</sup> Table I summarizes the important parameters configured in the case study.

In the simulation, one UAV per cell/sector is scheduled in the same TTI (transmission time interval) for all cells. A random distribution of 48 UAVs in the 48 cells is denoted as a topology, and totally 100 topologies are realized. For each topology Algorithm 1 is performed without QoS constraints with 100 random power initializations. As a comparison, we also exploit the standard 3GPP LTE UL open loop power control (OLPC) mechanism [16] with  $P_0 = -90.8$  dBm and  $\alpha = 0.8$ . Fig. 3(a) illustrates the average performance achieved utilizing the Algorithm 1 without constraints and 3GPP OLPC, respectively, and Fig. 3(b) illustrates the corresponding cumulative distribution functions (CDFs) of the performance of individual UAVs, respectively. It can be observed that Algorithm 1 can significantly increase the overall system performance compared to the OLPC scheme. However, the fairness among the UAVs is worse, as it can be observed from Fig. 3(b) that around 40% of UAVs are sacrificed with very low transmission rates. This is because some UAVs (e.g. at the cell edges) will cause severe interference to other UAVs if they want to achieve a better SINR, and they are muted to maximize system performance. Nevertheless, certain QoS constraints can be set in Algorithm 1 to increase the fairness. It is worth noting that assuming QoS constraints for the UAVs is non-trivial as there could be no feasible power solutions. To show the potential of Algorithm 1, we set the minimum QoS constraints for the UAVs in each topology as obtained from the OLPC scheme. In this way, a feasible power initialization can be easily chosen as the power allocation in OLPC. The overall system performance and performance CDF of individual UAVs are also illustrated in Fig. 3(a) and Fig. 3(b), respectively. It can be observed that based on the OLPC constraints, Algorithm 1 can further increase the performance of the system and individual UAVs. Moreover, the CDF of OLPC in the low rate region is kept (slightly to the right) in the CDF of Algorithm 1 assuming OLPC QoS constraints. In addition, it is easy to understand that the overall system performance with QoS

<sup>5</sup>Fast fading is not considered in this case, as the channel with UAV in the sky has a large K-factor [6]. On the other hand, we are not attempting to accurately reproduce the channel, and the obtained results can be considered as an upper-bound performance.

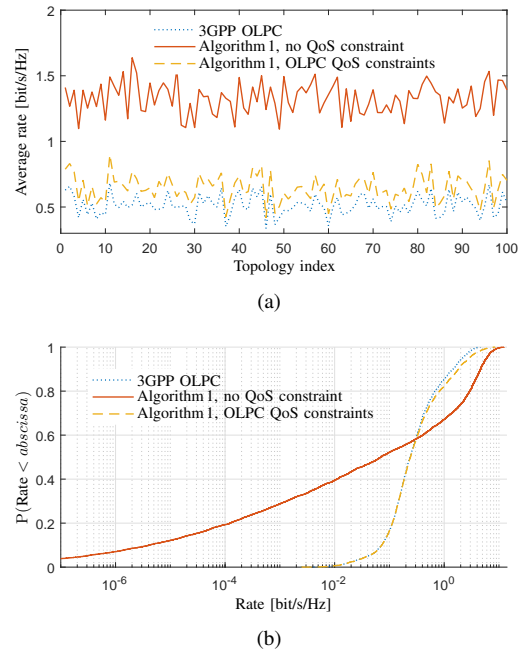


Fig. 3: Case study for UL transmission of cellular-UAVs. (a) Average performance for all UAVs. (b) Performance of individual UAVs.

constraints is lower than that without QoS constraints. The system performances averaged across the 100 topologies are calculated as 1.33 bit/s/Hz and 0.64 bit/s/Hz for Algorithm 1 without and with QoS constraints, respectively. Compared to 0.51 bit/s/Hz obtained using the OLPC scheme, the system gains are 312% and 25%, respectively.

The case study has shown the potential of Algorithm 1 in significantly increasing the overall performance. However, compared to the required UL speed (50 Mbps/18 MHz, i.e., 2.8 bit/s/Hz) to support the enhanced UAV communication in LTE [17, Table I], the obtained capacity is far from enough.<sup>6</sup> Thus, advanced techniques, e.g. directional antennas or beamforming [18], have to be further utilized. Moreover, some schedule algorithms, e.g. the proportional fair principle [19], can also be applied jointly to improving the fairness without too much loss in the overall performance.

## V. CONCLUSIONS

In this communication, a standard form for the power control problem aiming to maximize the weighted sum rate of the system with power and QoS constraints was presented by introducing auxiliary variables. Based on the standard form, a novel condensation method was proposed, which enables the solution through solving a series of GPs in the low SINR regime. Moreover, the proposed condensation method can be straightforwardly scaled with linearly increasing complexity. Its performance in achieving the global optimality has been verified in a small-scale network, i.e., case 1 in

<sup>6</sup>Considering the bandwidth efficiency, coding gap, fast fading, etc., the practically required speed should be much higher than 2.8 bit/s/Hz.

this communication. Furthermore, by applying it to the UL transmission of cellular-UAVs, results show its potential in increasing the system performance. However, there are still issues to be addressed. For example, much higher data rates are still required to enable the enhanced high-throughput uplink UAV communications. The fairness among UAVs needs to be improved. Techniques such as beamforming and schedule principles are possible solutions together with the proposed method. In addition, partially decentralizing the algorithm to decrease the system load is also practically important. Our future work will investigate these points thoroughly.

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