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A Grammar of Praxis

an Exposé of "A New Logical Foundation for Psychology", a Few Additions, and Replies to Alaric Kohler and Alexander Poddiakov

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A Grammar of Praxis: An Exposé of "*A New Logical Foundation for Psychology*", a few additions, and replies to Alaric Kohler and Alexander Poddiakov.

Jens Mammen[1]

Dualisms and dualities

My recent book (Mammen, 2017) tries to solve, or at least create preconditions for solutions of, some basic and long-standing problems in psychology, and in fact also in other fields of science and ideas.

The problems are variations of a common theme which perhaps can be referred to as problems of dualism. But although having much in common they are also rather diverse. In the history of psychology they have for many years been known as the crisis in psychology, although there may in fact be many crises.

You can just think of the so-called mind-body or mind-world problem, the split between the natural sciences on one side and the human or social sciences on the other side, the split between a causal and a hermeneutical or moral understanding of humans. Many of these dualities have been sharpened after the European Renaissance with its successful introduction of a mechanistic, mathematically supported, understanding of nature including humans, leaving no place for a traditional, and common sense, understanding of the soul as a human domain with its own logic and with specific relations to objects of knowledge and affections, beyond mechanism.

There have been many attempts to handle these dualities or even eliminate them. The elimination attempts have not been successful, and there is no doubt some rational core in the dualisms which reflect some deep structure in reality and human life.

A listing of the attempts would be a history of philosophy and ideas and is of course far beyond the ambitions in this short exposé. A little more is presented in Mammen (2017).

In a psychological context many attempts have referred to some sort of hierarchical organization of matter with the lower ones belonging to the realm of causality and natural science while the higher ones belong to the realm of human and social sciences, but with no success in understanding their logical and functional relations, cf. my critique of Vygotsky in Mammen (2016; 2017, pp. 14-16).

There have also been proposals of applying the concept of complementarity, borrowed from physics, accepting two mutually excluding but internally consistent and comprehensive frames of understanding human life (Wagoner, Chaudhary & Hviid, 2014), a stance also known from philosophy as aspect-dualism. This is perhaps productive as far as the phenomena are respected without reductionism and exclusions, but also far from a synthetic understanding of what after all is

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one world, and also far from our common sense understanding where e.g. understanding of causality and responsibility in human actions is much more intertwined.

The dualisms are, however, not only pervading psychology but all fields of human life. One generalization of mechanistic understanding of humans is the idea of people being functional units or modules with capacities and competences, preferences and measurable goals which seem to be mainstream in economics, sociology, political science and instrumentalist administrative practices as e.g. New Public Management, and is penetrating most of the educational, social and health sector in Western societies. The "perspective of the soul", i.e. humans' relations to what can't just be evaluated and measured on scales, but is relations to irreplaceable persons and objects, rooted in coexistence, in love and solidarity, relations of affective bonds, of belonging and owning, of loss and grief, etc. can't be contained in this "formalist" frame of reference. This ruling reductionism is causing overt dissatisfaction among lots of people feeling alienated in a cynical world, losing their "roots" and reacting with what may seem irrational chauvinism, nationalism, etc.

The remedy is not to supply the mechanistic understanding of man with a humanistic superstructure, be it language, semiotics, hermeneutics, ideologies or religion, as long as the basic embedment of man in nature through the body with its senses is understood mechanistic. There will be a missing dimension in the basis which can never be reconstructed in the superstructure. It will forever just be a mechanistic dish in humanistic dressing.

What is needed is an understanding of a duality already at the most basic practical interface between man as an agent and the world of objects, including persons. There have been attempts at such an understanding through history of ideas. I think Spinoza, Marx, and Ilyenkov (Surmava, 2018), Heidegger (Engelsted, 2017b), and some existentialist thinkers are examples among many, and perhaps also some ancient and mediaeval ones. Recently Niels Engelsted (2017a; 2018) has treated this subject pointing back to Aristotle. But I also think there is an obvious reason why this necessary way of thinking, integrating the soul as an agent in the basic practical interface, never became the ruling one.

One main reason could very well be that this way of thinking and communicating ideas could never compete in clarity, consistency, simplicity, compactness and applicability with the wonderful and even beautiful language of *mathematics* applied by mechanicism, and natural science in general. Since antiquity mathematics has been considered the queen of science (God or the gods being the kings), and the narrative tells that over the entrance to Plato's Academy was written that only knowledge to geometry allowed access.

Many European philosophers since the renaissance were deeply impressed by the elegant mathematics of Galilean and later Newtonian physics, and e.g. Descartes and Kant did not question the universal validity of mechanicism based on this very solid ground.

Assistance from modern mathematics

However, it is exactly in *mathematics* that we can search the counterstroke to the dominating mechanicism. This may seem paradoxical as the picture of man we are to defend is certainly not

fitting in the kind of mathematics we are used to for counting and measuring, or in general quantifying, human life. This is in fact true. But today, and accelerating through the 20th century, mathematics is no longer a formal "machine", but has been forced to give up its program of pure formalism and the definition of its objects in universal terms independent of existence and agency. Today mathematics, and especially its foundation in mathematical logic, is rather a philosophical discipline, but as a price also suffering of importing problems leading to contradictions in this very foundation, not quite unlike the apparent contradictions in physics between theory of relativity and quantum mechanics. In an increasing degree mathematics is sharing fate with the "real sciences" as it loses its status as pure formalism.

At the end of 19th century mathematics had problems with handling infinite sets and e.g. the concept of infinite convergence, calling for some more strict definitions of existence. This resulted in a set of fundamental sentences, so-called axioms, stating what was meant with e.g. a set of mathematical objects and gave some rules for how to handle them. Referring to the German mathematicians Ernst Zermelo and Abraham Fraenkel it was named the *Zermelo-Fraenkel* axioms, or abbreviated *ZF*. Finite sets could be defined by naming all members, but infinite sets had to be defined by the members having some common properties or by being generated by some repetitive rule. The members of a set had all to be mutually different by having *different properties*. Mathematical *existence* was so to say the same as definition by distinct properties. In a way this was just a precision of the classical concept of a set defined by its members and their properties, and some mathematicians believed or hoped that this was the final completion of the logical foundation of mathematics.

But there were still problems. There were obvious examples of infinite sets were the method of explicit definition could not always define some member narrow enough to "zoom in" on just that one as different from all others. Metaphorically, the fishing net of definitions was not selective enough to catch just one fish. This could only be done by harpoon not knowing in advance precisely what you caught. More technically speaking, you could have a set of sets, and *without* explicit selective definitions in advance *still* be able to select a new set with just one member from each. It was felt absurd to claim that such a set did *not exist*, just because you could not in advance tell *how to construct it*.

It was a serious step to claim the existence of such sets anyway, thus going beyond existence rooted *alone* in defining properties or features. You could of course do that in the *real* material world. To take an apple on the tree did not demand that you in advance had defined it by its properties in contrast to the properties of all other apples on the tree. But mathematics should be about *ideal* objects and not material ones, and there should be no human or divine hand *acting as a selector beyond properties*.

About 1904 Ernst Zermelo took this great step, already prepared by Felix Hausdorff, another German mathematician, because not taking it was evaluated even worse, and *the claim of existence* of these sets not selected by explicit rules or defining properties was called the *Axiom of Choice* or just *AC* (originally *das Auswahlaxiom*). But of course it caused much discussion until it finally was

accepted by nearly everyone. Today the founding axioms of mathematics are not only *ZF*, but *ZFC*, with *C* for "choice".

But AC is not without problems. It defines a "freedom" to go beyond rules and definitions, like when we take an apple without any advance rule, but still restricted by the finite number of apples on the tree. Perhaps we just take it quite at random or, in most cases, by improvising a local rule when we come close to a few apples. But AC is perhaps more *radical* in defining a sort of a more "infinite freedom" not known from the real material world.

Something points to AC as being too free. Just as when "freedom" is taken out of "freedom, equality and brotherhood", it turns against itself and reduces to the jungle law as we witness today.

There are several examples of AC being destructive to the *order* defining useful parts of mathematics. It contradicts the Axiom of Determinacy, useful in many contexts, e.g. theory of two-player games as chess and checkers. It contradicts the very useful Theory of Measures used e.g. in statistics.

As these other axioms and theories are not given up, and AC neither, we must say that mathematics today has a *contradictory* logical foundation (Loft, 2019), and that we are waiting for some possible a little more "disciplined" version of AC or a version which is not an appendix to ZF but more integrated. But still AC is so important that it would be premature to give it up now, just as we do not exclude any of the great theories from physics because they ultimately contradict each other for the time being. Too much explanatory or generative power would be lost in relation to the rich domains of study.

So, as a conclusion, using modern mathematics founded in ZFC in the study of human life is not automatically reducing it to formalism or machinery, but could perhaps in an exact way also map free human agency, and because of the generality of mathematics, not only map freedom in a psychological, but also in a broader, context.

Bridging psychology and mathematics

There is an astonishing, but in fact not accidental, parallel in psychology to the kinds of sets discussed in mathematics. At one hand there are the sets defined from difference in properties or features of objects and on the other hand the ones defined by choosing or selecting *beyond* such differences, although perhaps supported by them.

The two kinds of sets correspond to the selection of or focusing on objects in our environment by using our senses as defining criteria, on basis of differences between objects, and respectively, the way we select and keep objects or persons by choice beyond such criteria but rooted in coexistence. I am not bound to my close friends, my wife and my children only because of their properties, but because of our coexistence and connecting threads in space and time, defining them as irreplaceable.

The two kinds of sets, or in psychological context categories, have I called *sense categories* and *choice categories*, respectively. They are mutually *excluding* as categories in the sense that no category, which is not empty, can be both a sense category and a choice category. But a sense category and a choice category can contain some *common* objects because any object in a choice category is also contained in some sense category.

As the categories can be considered sets they can be combined using the usual set theoretical operations forming *intersections* and *unions*. On this basis an *axiomatic system* is proposed consisting of the below 11 axioms (Mammen, 2017, p. 88). "Ù" is a symbol for the world of objects. Two categories are "disjunct" when they contain no common objects. For details and background is referred to Mammen (2017, pp. 57-88).

Ax. 1: There is more than one object in Ù
Ax. 2: The intersection of two sense categories is a sense category
Ax. 3: The union of any set of sense categories is a sense category
Ax. 4 (Hausdorff): For any two objects in Ù there are two disjunct sense categories so that one object is in the one and the other object in the other one
Ax. 5 (perfectness): No sense category contains just one object
Ax. 6: No non-empty choice category is a sense category
Ax. 7: There exists a non-empty choice category
Ax. 8: Any non-empty choice category contains a choice category containing only one object
Ax. 9: The intersection of two choice categories is a choice category
Ax. 10: The union of two choice categories is a choice category
Ax. 11: The intersection of a choice category and a sense category is a choice category

Axioms 1-5 state that sense categories are organized in a structure called a *perfect topologi*.

This is well-known as the way the real line is organized by unions of open intervals and is the basis for describing lawful physical interactions as *continuous*. So this is mainstream natural science and just tells that the senses are functioning in the same way as physical devises and as described in classical *psychophysics* (Dzhafarov, 2017, pp. 109-111).

Axiom 6 states the mutual exclusion of the two kinds of categories. Axioms 7-10 about choice categories is also a well-known structure. Finally is axiom 11 an expression of the interaction or mutual framing of the two kinds of categories.

As we of course can *combine* the categories when defining or selecting categories from the world we define a *decidable category* this way (Mammen, 2017, p. 85):

Def. 1: A decidable category is a union of a sense category and a choice category.

This definition includes all sense categories and choice categories themselves according to the axioms.

Until now this is not looking as very advanced mathematics. The basic set theoretical operations are today already introduced in Danish elementary school. But the innocent looking axioms are e.g. implying that sense categories are always *infinite* if not empty, and therefore also that $\grave{U}$ is infinite. Further, the axioms are hiding a conundrum: Are the two kinds of categories *exhaustive* in the sense that there is not necessarily a *third* kind of category? Could we within the frames of ZFC claim that *any* possible selection of objects in the world forming a set could, in principle, be a decidable category as defined in Def. 1? In this case we really have a conceptual frame for a *complete basis* for any kind of superstructure of categories in the world, defined by signs or otherwise. There will always be *restrictions* in how the individual in practice applies this complete system of categories, but the restrictions are not found in the axiomatic system, but are *empirical* questions (Mammen, 2017, pp. 86-88).

As already suggested above this *completeness* is in fact the case and formulated in this theorem:

Th. 11 (completeness): There exists a space in $\grave{U}$ where any subset in $\grave{U}$ is a decidable category.

Here a "space" is just referring to a structure in $\grave{U}$ formed by sense categories and choice categories. The number "11" is due to presentation of other theorems 1-10 in Mammen (2017).

I formulated Th. 11 (not under this name) as a hypothesis, alongside its negation, in Mammen (1983, p. 406-407)[2], but it was not before 1994 I found a mathematician who could prove it (Hoffmann-Jørgensen, 2000). The proof is much too technical to present here, but the important in this context is that Hoffmann-Jørgensen could not prove it from ZF alone but had to use AC, the axiom of choice, as well.

Hoffmann-Jørgensen guessed in 1994 that AC was necessary for the proof, or in other words that Th. 11 implied AC (*Hoffmann's Conjecture*, Mammen, 2017, p. 86) and that Th.11 therefore was a new version of AC. But that could not be proved despite attempts the following years in both Aarhus and Moscow (Mammen et al., 2000, p. 168).

Recently the question has, however, been partially settled by a mathematician at Copenhagen University, Asger Törnquist. The very interesting and promising result is, that although it seems obvious, without being proved yet, that ZF is not sufficient for proving Th. 11, AC is not necessary.[3] This means that Th. 11, although pointing to *existence* of sets *beyond definition by properties* as also AC, is "weaker" or not so "wild" as AC. As Th. 11 further is easy to interpret in relation to reality outside mathematics it is hard to believe it is as destructive as AC. Th. 11 is also unprecedented in the *simplicity* of the axiomatic system Ax. 1-11 behind it, which is explainable at elementary school level, although the proofs are certainly not.

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[2] An error p. 407, pt. a) is corrected in later editions, 1989 and 1996. The link included in the references is to the 1996-edition.

[3] Törnquist (2019) has shown that The Ultrafilter Lemma implies the existence of maximal perfect topologies which according to Hoffmann-Jørgensen (2000) implies Th.11. If Th. 11 further implied AC, then The Ultrafilter Lemma also implied AC, which Halpern & Levi in 1971 have shown is not the case (Moore, 1982, pp. 242, 353). Therefore Th. 11 does not imply AC.

This again means that we not only see that mathematics can be useful for solving questions in psychology, but that the reverse can also be the case, as psychology can provide a "*model*" from *reality* securing some *consistency* in a mathematical system and thus prevent contradictions. Anyway this is a hope.

The synthesis

So, what have we accomplished by making this *bridge* between psychology and mathematics? One result is, that we have *enriched* the traditional mechanistic or "*psychophysical*" basis for our bodily and active contact with the world of objects, following the logic of axioms 1-5, with the logic of *free agency* beyond defining properties, following the logic of axioms 6-11, and being basis for our bonds and deep affections to the world, and also basis for our *practical being in the world* in general (Mammen, 2017, pp. 45-54, Engelsted, 2017b).

It was not necessary to turn our back to *mechanicism* and all its indisputable accomplishments. Rather we demonstrated that its logic is *included* in our interface with the world, but that mechanicism is *not complete* in itself. It can e.g. never alone reach the single object from the infinite universe of objects. A final move beyond definitions is needed, a *leap* to the singular, an infinite leap from the perspective of definitions, but a finite one from the perspective of coexistence.

Only by also including the logic of choice categories can we reach a completeness of our basic interface with the world, cf. Th. 11. It is as if free agency and relations to the singular and irreplaceable are *invited in* to fill the gaps in the logic of mechanicism.

Choice categories is the key to the *humanities* which is now shown to be an indispensable completion of *natural science*, and of course *vice versa*! The two fields are most beautifully framing and completing each other, and in fact also closely intertwined as expressed in Ax.11.

At last also *psychology* in this way is satisfying Kant's demand that any serious science should have a mathematical basis (Valsiner, 2017).

It is important for psychology in this way to find its place among the sciences, but at the same time important to overcome the internal schism between natural and human science or *Natur- vs. Geisteswissenschaften*, so central to its long-lasting "crisis".

On a more concrete level this new conceptual frame of reference opens for a deeper understanding of human affections and lasting bonds and their interaction with cognition in general. Without this, phenomena as fundamental as *love* and *grief* could not be understood.

In Mammen (2017) is also discussed some societal and political consequences of this picture of man in contrast to a more functionalist and instrumental one (see also Mammen & Mironenko, 2015), and also an attempt to understand some popular reactions against these more mechanic reductionisms, and the turn to conservative values, nationalism, *Blut und Boden*, etc. and away from more inclusive and cosmopolitan values.

We can't just fight these protests by demanding that people should *cut* their roots and bonds and become cosmopolitical. What is rather needed, is that we all *expand* our roots and bonds, and become cosmopolitical.

Image courtesy of Michael Leunig https://www.leunig.com.au/contact/frequently-answered-questions

A grammar of human praxis

What was accomplished with the above synthesis was not only a *theoretical frame* for overcoming *dualism*, or rather transforming it to a picture or map of a *real duality* in man's basic and practical encounter with the world of objects. It is also an effective *tool* for psychology and its analytical, critical, and practical endeavors. You could compare it with the role of *grammar* in our understanding, appropriation, and command of language. Although we follow its rules from early childhood very few of us are able to explain the general rules or make them explicit. They just function as a silent or tacit background. Most of us are also able to learn a second language later in

life just by extensive listening and talking. But already in school we usually have to learn grammar as a tool for appropriating foreign languages because we don't have the time and opportunity for "natural learning". And for effective teaching the instructor has to know the often complicated rules. Often they are hated by the pupils, and in Danish schools is not only German but even Danish grammar often more hated than mathematics.

In the scientific study of language is grammar of course indispensable as an analytic tool even if the perfect practitioners of the language don't know explicitly any of the rules they are strictly following, and would have hard times if they were forced to it.

The *axiomatic system* proposed here can in the same way be considered a *grammar* of the practical encounter with the world of objects we all experience every waken hour in our life. And the analogy is somewhat supported by the fact that the set theoretical operations used in the axioms correspond to the semantic connectives "and" and "or" (or "and/or"). Also *the subject-predicate* structure, explicit in most European languages, reflects the duality of choice and sense categories, respectively. Niels Engelsted gives a very clear and inspiring review of this with important general perspectives in Engelsted (2017b).

But working silently in the background this grammar of praxis may not be immediately recognizable by its practitioners, and as with linguistic grammar the complicated and formalized rules may be felt "foreign" in relation to their living domain. I think they even may be felt foreign by many psychologists, especially because we have experienced so many examples of using mathematics to force unjustified quantification upon domains of human life resulting in destructive reductionism. But what is presented here is not that kind of mathematics!

We should rather see the axiomatic system as an analytical tool digging out the fundamental dimension of *choice categories* in our life so important for our free agency and our relations to the world of objects and persons *beyond* sensory based distinctions and evaluations. And although this is also going beyond mechanicism and functionalism (Mammen & Mironenko, 2015), the bridge to other sides of human life, traditionally understood in frames of natural science, is not broken but integrated in a consistent system.

Despite the parallel between practical and linguistic grammar, there is also an asymmetric relation between them. The point of view in Mammen (2017) is that the practical grammar is basic and a precondition for development of linguistic grammar, which then *in a second move*, together with culturally developed semantic systems, may support and enrich the practical grammar with a conceptual level of meaning, both in a social and cognitive context. This dependence of a specific and basic human practical grammar could explain why other animals than humans can't develop a genuine referential language and a human conceptual system. I hope to return to these questions in a later publication.

About the importance of introduction of *choice categories* you could, with the risk of metaphorical simplification, say that much psychology has been rooted in a spectator or consumer perspective, a "*logic of the eye*", where the channel to the world has been the receptive senses, and objects are

considered equivalent if they appear or function similar or comparable. The introduction of choice categories supplies this with a "*logic of the hand*" which (as the whole body) can reach out in the world, seeking and selecting single objects, keeping them, owning or belonging to them as irreplaceable and with affective value or reverence.

The two "logics" are, however, also framing each other in many less emotionally involving tasks in everyday practice.

Perhaps the most compact overview of the analytical force of the duality of sense and choice categories to a broad domain of human praxis is briefly presented in a recent publication (Mammen & Gozli, 2018) where it is related to the three levels of activity in A. N. Leontiev's theory: Activity proper, acts or actions, and operations.

For more detailed and concrete examples se Chapter 6 in Mammen (2017) and Engelsted (2017b) , especially on "double entry book-keeping".

Contradictions, time, and threads we live in. Reply to Alaric Kohler

One of the two articles in this issue commenting my book (Mammen, 2017) is Alaric Kohler's (2019) to which I will reply below. I shall comment on the other commentator Alexander Poddiakov (2019) in a later section.

Concerning Kohler I am in the unusual situation that I not only agree in the review but also in the few critical remarks which all are constructive and with important suggestions.

The review is not only covering the main points in the book but is adding important perspectives and gives a fine and inviting introduction to the whole text. The review is also placing themes from the book in a context of other authors' work of which I think Piaget's is especially interesting. By this the reviewer is partially repairing one of the admitted weaknesses of the book, criticized by Kohler (as his 3rd point): That it could have substantiated many of its ideas by further reference to authors' with comparable or supporting statements. I agree with Kohler in the hope expressed earlier in the review that this could be a collective endeavor.

Another critique, or "wonder" (Kohler's 1st point), is about my reference to classical concepts from *dialectical logic* such as "jump" or "leap", "*Aufhebung*", the "transformation of quantity into quality", and "emergence". I rather consistently characterize these concepts as being without *explanatory* value as also with the concept of "holism".

I will maintain this point of view but admit that I should have explained it better, because I still think these concepts are useful as *descriptive* and also *normative* concepts. The normative aspect is in general a request for conceptual *openness* to aspects of *discontinuity* and *non-additivity* in the phenomena to be explained, and not being restricted by mechanistic prejudices. But this openness is no explanation in itself, only a necessary *precondition*. Being open for the fact that fluid water by being quantitatively heated becomes a gas with qualitative other properties is a precondition for the explanation, which refers to molecular forces and not to any "dialectical law".

With *Aufhebung* the normative aspect goes a little further. This is rather a specific request for solving apparent *contradictions* by *expanding* the conceptual or material frame to *contain* the opposites. In the example used by Kohler (in his 2ⁿᵈ point) is it a contradiction between open and closed cell membranes which can't be solved within a local context, cf. "Maxwell's demon". The solution is to expand the *context* with an *external* energy source defining an active flow of energy.

In ancient geometry there was a *contradiction* between the demand that relations between length of lines could be represented by a number, and the demand that all numbers were rational, i.e. fractions between whole numbers. To solve it you had to *expand* or *generalize* the concept of number with the irrational numbers. Now the relation between the length of the sides and the diagonal in a square also became a number, and the contradiction was not eliminated but "*aufgehoben*". The contradiction had been vehicle for an important invention or creative generalization. Referring to *Aufhebung* solved no problem in itself, but was a demand for solving the problem by *invention* or *discovery*, going beyond the premises for the contradiction. In fact most generalizations in mathematics, e.g. from real numbers to complex numbers, or from Euclidean to Riemann-Lobachevskyan geometry (Valsiner, 2017) are results of such an *Aufhebung* of contradictions (Witt-Hansen, 1963).

Kohler's 2ⁿᵈ point of critique turns more generally to the problem of *contradiction*. I think you here should distinguish between different kinds of contradictions. Of course you have to respect some *plain* contradictions as definitive within a *closed and static* conceptual frame. Otherwise you undermine e.g. the concept of indirect proof or of counterexamples in mathematics, and of course some kinds of self-contradiction are disqualifying arguments as invalid. We can't remove classical logic as a common ground, or "court of appeal", from a discourse without ending in nonsense.

But as Kohler rightly states, the concept of contradiction changes meaning when *time* is included and we are referring to *processes* or development. To say that an egg changes from being raw to being hard-boiled, is to say both that it is raw and that it is not, which would be a contradiction if stated at the same time, and if "it" referred to the same egg. The contradiction is, however, "aufgehoben" by *expanding* the context of the two statements from one point of time to two points or to an interval. Note that if the egg was identified alone by its properties there would be no contradiction, because "it" then referred to two different eggs. It is only because the egg as a *choice category* is *one* object, and the properties despite that as *sense categories* are *different*, that we can define the concept of change or development, at all. To think that the *concept of change* could be rooted in sense categories alone would be a simple logical flaw.

If this is an example of dialectic logic, it is not very different from classical logic applied with some common sense.

However, by pointing to the necessity of including *time* and processes in the discussion Kohler points to the perhaps strongest argument for introducing the *duality* of sense and choice categories: Without this duality we could not establish a well-founded concept of change or process!

In Mammen (2017) there have been occasional references to this kind of reasoning. But when "building" the axiomatic system, change in time has not been an explicit premise. What was described was rather a static system of objects with static properties.

One reason for that choice was a wish to make the presentation as simple as possible, as it was already evaluated complicated for the common psychological reader because of the mathematics introduced. Another reason was the chosen "plot" in the presentation: To take the standpoint of my opponents, believing in the sufficiency of "*pure sense categories*", or Leibniz' "identity of the indiscernible", and then step by step show that it was incomplete and had to be supplemented with choice categories.

"Pure sense categories" can't be introduced in a world with changing objects without immediately contradicting itself, because the concept of change, as shown above, could not be established without already also introducing choice categories. In other words, I would have to presuppose the conclusion before the argument if departing from a changing world and not from the "abstraction" of a static world. A choice between two evils!

I chose to depart from the standpoint of my opponents because that in many ways was the traditional, well-known and common, conceptual frame, not only in psychology, and reflecting the dominance of mechanistic thinking since the European Renaissance.

But exactly the same axiomatic system could as well have been build departing from a changing world of objects, not only with changing properties but also with changing positions in space, defining choice categories as *trajectories* or *"threads" in time and space*, and with the possibility of changing properties .

The mathematics building the bridge between the "static" and the "dynamic" case is briefly presented in Mammen (2017, pp. 83-84).

The generalization of choice categories to threads in time and space brings the axiomatic system more in accordance with our phenomenology than the static interpretation. These threads are in many ways basic in our life and define our "being-in-the-world" as first of all *historic*. It is the coexistence and history of places, objects and persons which define their *meaning* for us, both their cultural or societal meaning and their more personal sense, as described by A. N. Leontiev, calling the trajectories a "*fifth quasidimension*" in the world in addition to the one temporal and the three spatial dimensions. [4]

Sense categories are helping us orienting and acting in this objective and invisible space of threads. But they have their own importance also when informing about properties in the world, as already investigated in depth in psychology.

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[4] It should be noted that the duality of sense and choice categories also has an interpretation on the level of non-human *animals* to catch the general concepts of *intentionality* and *search*, but without yet presupposing choice categories as trajectories defining a historical depths in the world (Mammen, 2017, pp. 33-35). In the dawn of mankind it is suggested that exchange of personal gifts may have played a role in the transition to real human life (Mammen, 2017, pp. 37-43).

First of all sense and choice categories are supporting, completing and framing each other in a changing world. See also Mammen (1993).

Irreplaceability, reductionism, and creativity. Reply to Alexander Poddiakov

As with Alaric Kohler I agree with Alexander Poddiakov in the central points in his commentaries. The first important issue is about the implications of the *irreplaceability* of persons claimed in the book (e.g. Mammen, 2017, p. 52). Poddiakov agrees in the importance of this "existential irretrievability" of the other, also pointed to by Heidegger. But at the same time he points to the fact, that this acknowledgement of irreplaceability may not in itself imply a relation of love and solidarity, but also the opposite, ultimately in hate and deliberate killing. Heidegger himself was an example among many when he did not stand aloof from Nazism's mass homicide.

 I agree, and I should perhaps have stressed that irreplaceability is not a sufficient, but only a *necessary condition* for the deep and lasting affection of love. It should perhaps also have been mentioned, that this affection or "sentiment" as a lasting bond does not exclude negative emotions as anger and jealousy, rather strengthen them. Also *coexistence* as an important condition for affections, but still presupposing irreplaceability, is stressed several times in the book (Mammen, 2017, pp. 39-40, 52-53, 93).

The second main theme treated by Poddiakov is about the possibility of true duplicates of persons and of the possibility of coping minds to another medium than the person to which it is referred. I agree with Poddiakov that none of these very hypothetical scenarios are possible in the real world. I even doubt the questions can be answered affirmatively, or even be defined, without postulating two persons occupying the same trajectory in space and time, as the same mind would not be possible without the same experiences. In this case it would be meaningless to claim they are two and not one person. Just the same could be said about the person and the postulated other medium carrying the same mind. None of these hypothetical questions are treated in Mammen (2017), but I think they are included in the commentary as a support to the anti-reductionism in the book.

The third theme is about "brain reductionism", that is the question of reducing the mind to the brain. Again I agree with Poddiakov that this is also not possible, and I think the question is closely related to the above second theme, and is a key to its answer. As we agree that the mind is a relation between the person and the world, the mind can't of course be the same as the brain, and even a hypothetical (but in fact impossibe) duplication of the brain would not be a duplication of the mind.

The last issue to be discussed is *creativity*. Poddiakov agrees that the axiomatic system presented in Mammen (2017) provides a conceptual frame for understanding human free agency or "*free will*". The question is, if this frame just *allows* creativity, by not excluding it, or if it also is a frame for understanding the *dynamics and evolution* of creativity.

Creativity is a broad concept used within many spheres of activity, e.g. within artistic domains. But Poddiakov's examples are pointing at creativity understood as capacity for invention and discovery, and close to what may be called scientific or technological creativity, resulting in new general insights or useful artifacts. Still this is a very broad field of investigation and discourse, involving

motivation, imagination, and knowledge of possibilities and problems in the concrete domain in question, e.g. knowledge of aerodynamics and available technology in the example of invention of the first aircrafts, referred to by Poddiakov.

In an attempt to answer the question, and making the concepts of sense and choice categories relevant in this context, I have to focus on *analytic power* being a necessary condition for this kind of "scientific" creativity, acknowledging that there are many other necessary conditions, as already mentioned, of which persistence and diligence should not be forgotten.

My point is that analytic power, serving as a vehicle for new *general* knowledge of dynamics and laws *behind* the superficial sensory appearance of phenomena, can't be implemented *alone* by analysis of these sensory appearances themselves, however long and comprehensive. By logical necessity it has to be combined with identification and securing of objects as *choice categories* over time and in different material contexts. This is also a transition from an observational way of getting knowledge to a more *experimental* one.

This question is discussed at some length in Mammen (2017, pp. 45-51) under the headings of "What is Empirical Knowledge Beyond Adaptation?", "What is Knowledge of Laws of Nature Beyond Patterns of Regularity?", etc. A key concept is here "*double-entry bookkeeping*", i.e. the simultaneous mapping of the phenomena studied on sense and choice categories as an integrated frame of reference.[5] See also Mammen (1983, pp. 274-279), where A. N. Leontiev on this background is criticized for not being able to understand human activity as a "creative transformation of nature". About "double-entry bookkeeping" see also Engelsted (2017b, pp. 102-104).

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[5] Cf. the discussion of understanding *processes* in time in my reply to Alaric Kohler.

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