Risk-based Stochastic Scheduling of Resilient Microgrids Considering Demand Response Programs

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Abstract—In this paper, a risk-constrained stochastic framework is presented for joint energy and reserve scheduling of a resilient microgrid considering demand side management. The optimization problem is formulated to schedule the system operation in both normal and islanding modes by addressing the prevailing uncertainties of islanding duration as well as prediction errors of loads, renewable power generation and electricity price. In normal operation mode, where the grid-connection is available, the energy and reserve of local resources and energy trading with the main grid is scheduled to maximize the operator’s profit considering feasible islanding. In resilient operating mode, which is triggered by a disturbance in the main grid, the local resources should be scheduled to supply loads with the lowest emergency load shedding. To balance the economy and security requirements under uncertainties, the optimal scheduling is done properly through a security-constrained power flow method by considering system’s objectives and constraints. Moreover, to properly handle the uncertainties of the problem, conditional value-at-risk (CVaR) metric is incorporated with the optimization model to control the risk of profit variability. The proposed scheme is implemented on a test microgrid and various case studies are presented to verify its effectiveness in normal and resiliency operating conditions.

Index Terms—Resilient microgrid, demand response, optimal scheduling, stochastic framework, conditional value-at-risk (CVaR).

NOMENCLATURE

Indices and sets

\( (.)_{s,t} \) - At time \( t \) in scenario \( s \).
\( (.)_{\text{max}}, (.)_{\text{min}} \) - Minimum and maximum amount of a variable.
\( t, N_T \) - Index and number of timeslots in the scheduling horizon.
\( h, N_H \) - Index and number of scenario for islanding duration.
\( \tau \) - Timeslot index in island mode scheduling problems.
\( s, N_S \) - Index and number of normal operation scenarios.
\( i, N_G \) - Index and number of DG units.
\( w, N_W \) - Index and number of wind turbines.
\( h, n, r \) - Indices of system buses.

Parameters and constants

\( P_{j,t} \) - Demand of \( j \)-th group of customers (kW).
\( P_{r,j,s} \) - Price of selling electricity to \( j \)-th group of customer (\$/kWh).
\( P^\text{buy,sell}_{m,t} \) - Electricity market price for buying (selling) energy from (to) the main grid (\$/kWh).
\( \beta \) - Risk-aversion parameter.
\( \alpha \) - Per unit confidence level.
\( \lambda_{i,t}^{R,ap} \) - Bid of (up)-spinning reserve submitted by DG unit \( i \) at time \( t \) (\$/kWh).
\( \lambda_{i,t}^{R,dn} \) - Bid of (down)-spinning reserve submitted by DG unit \( i \) at time \( t \) (\$/kWh).
\( \lambda_{m,j}^{R,ap} \) - Up (down)-regulation market prices at time \( t \) (\$/kWh).
\( \pi_{s,i} \) - Occurrence probability of scenario \( s \) (islanding duration scenario \( h \)).
\( \pi_{s,i} \) - Start-up (shut-down) cost constants of DG unit \( i \) ($).
\( RU_{i}^{p}(RD_{i}) \) - Ramp-up/down rates of DG unit \( i \).
\( UT_{i}^{p}(DT_{i}) \) - Minimum up (down) time of DG unit \( i \).
\( C \) - Conductance (Susceptance) of line \( l \).
\( V \) - Value of lost load.
\( E_{l,t} \) - Expected energy not served

Active (reactive) power (kW).

Power exchange between the microgrid and the main grid (kW).

Active power bought (sold) from (to) the main grid at time \( t \) (kW).

Active (reactive) power flowing between bus \( n \) and \( r \).

Start-up (Shut-down) cost variables of DG unit \( i \) ($).

Up-spinning reserve deployed by DG unit \( i \) (customers in group \( j \)).

Down-spinning reserve deployed by DG unit \( i \) (customers in group \( j \)).

Up (down)-spinning reserve deployed by main grid (kW).

Non-spinning reserve deployed by DG unit \( i \).

Cross elasticity of period \( t \) to period \( h \).

Active and reactive power of emergency load shedding (kW).

Commitment status of DG unit \( i \), \{0, 1\}.

Start-up (shut-down) indicator of DG \( i \), \{0, 1\}.

Auxiliary variables, \{0, 1\}. 

I. INTRODUCTION

Microgrids, as main building blocks of smart grids, can be viewed as small-scale power systems with controllable loads, distributed energy resources (DERs) and ability of self-supply and islanding. Utilizing of microgrids, in which DERs are located near the end-use customers, can improve the resiliency of power systems by lowering the possibility of load shedding [1]. Resiliency represents the ability of a power system to withstand severe disturbances without experiencing any major disruption, and further enabling a quick recovery and restoration to the normal operation state [2]-[3]. Moreover, deploying microgrids with self-supply and islanding capabilities is considered as one of the most effective solutions for supplying local loads when a severe weather-related event occurs in the main grid and a power interruption is inevitable [4]. On the other hand, to make microgrids more flexible, they should be evolved into smart active networks by implementing innovative concepts such as demand response (DR) actions [5]-[6].

Multiple research works are conducted to solve the optimal energy management problem of microgrids under uncertainties considering DR programs [7]-[9]. Authors in [7] have proposed a two-stage real-time demand side management (DSM) method for a microgrid including different time scales under different uncertainties. The operation cost is minimized by applying a model predictive control-based dynamic optimization considering the uncertainties imposed by both supply and demand sides in the microgrid. In [8] a two-stage stochastic programming model has been proposed for optimal scheduling of commercial microgrids equipped with 100% renewable energy sources (RESs) to handle the existing uncertainties. In that model, the microgrid operator maximizes its profit by optimizing bidding strategy in the day-ahead market, and minimizes the imbalance cost through adjusting the DERs in the real-time balancing market. In [9] a potential game approach has been presented to distribute operational optimization for energy management of microgrid with high penetration of RESs and DR resources.

In none of the above references, microgrids resilience issues have been addressed in energy management models and resiliency benefits of microgrids have not been discussed. In [10], a stochastic scheduling model has been presented for enhancing the resiliency of microgrids considering feasible islanding and survivability of critical loads. The optimization problem has been formulated for both normal and emergency conditions where the normal operation is coordinated with the emergency operation to enable a feasible islanding. Moreover, in [11], a two-stage adaptive robust formulation has been presented for day-ahead scheduling of resilient-microgrid to minimize the damaging consequences of islanding events. In both of the two mentioned works, the prevailing uncertainties associated with unscheduled islanding events after a disturbance, which can significantly affect the operation of microgrid, have not been considered.

The impact of prevailing uncertainties of islanding duration on the scheduling of microgrids is addressed in a number of research works [12]-[14]. In [12], an optimal scheduling model has been proposed for minimizing the load curtailment of microgrids during extended islanded periods considering uncertainties in islanding duration, loads and generations. In [13], a two-stage stochastic framework has been presented for optimal scheduling of resilient microgrids. The framework minimizes the operation cost of microgrid while taking into account the prevailing uncertainties associated with wind power, electric vehicles and electricity prices. Moreover, a two-stage adaptive robust optimization model has been presented in [14] for scheduling of microgrids in both grid-connected and islanded modes. The objective is to minimize operating cost of microgrid under the worst-case scenarios associated with RESs and islanding events. The uncertainties associated with islanding duration periods, electricity demand and prices as well as output power of RESs introduce risk into microgrid operator scheduling problem. Therefore, risk measurement plays a significant role in optimization under uncertainties and provides valuable information to decision makers. In [15] an optimal energy management strategy has been proposed for a microgrid equipped with battery storage in a way to enhance the resiliency of the microgrid while maintaining its operational cost at a minimum level. Conditional value-at-risk (CVaR) as a risk measurement index has been used in the formulation to account for the uncertainty of RESs power and the electricity price. Also, in [16], a risk-constrained stochastic framework has been proposed for optimal scheduling of microgrids over unscheduled islanding periods. The objective of that work was to minimize the expected value of operation cost, while the risk caused by uncertainties in islanding duration, loads and renewable generation was addressed via CVaR approach. However, the impact of risk aversion on decision-making problem and also the effects of implementing DR programs on resilience improvement of microgrids have not been analyzed properly.

The authors in [17] have proposed a risk-constrained two-stage stochastic framework for joint energy and reserve scheduling of islanded microgrids where risk on profit variability is considered using CVaR. Likewise, in [18] a stochastic risk-constrained framework has been presented for optimal scheduling of microgrids in islanded mode to evaluate the influence of DR programs on security and economic issues, considering risk management strategy. In addition, the authors in [19] and [20] have proposed stochastic optimization frameworks to maximize the expected profit of a microgrid operator under uncertainties, where the trade-off between maximising the operator's profit and the risk of getting low profits in undesired scenarios has been modelled by CVaR method. The main focus of two mentioned studies was on investigation of the influence of consumers’ participation in DR programs and their emergency load shedding for different values of lost load on the expected profit, CVaR, expected energy not served and scheduled reserves of the microgrid. However, the operation of microgrid in grid-connected mode has not been considered in the mentioned works. Moreover, there is lack of systematically addressing the effect of uncertainties of microgrid islanding events on the economy and security constraints.

In this paper, a risk-constrained stochastic model is presented for optimal scheduling of a resilient-microgrid considering DR participants. The problem is formulated as a linear programming model incorporated with CVaR to manage the energy and reserve capacity in order to maximize expected profit of the operator. The presented model addresses the prevailing uncertainties of islanding duration after a disturbance as well as prediction errors of wind energy, demand and electricity price. Also, by incorporating security-constrained power flow in the proposed solution method, reliable operating conditions are guaranteed in an uncertain environment, especially during an islanded mode. In addition, by incorporating CVaR into the model, the impacts of risk-aversion on decision-making of the operator are evaluated for normal and resilient operations of microgrid. The scope of models in technical literature and the contribution of this work is
summarized in Table I. Compared to the existing studies, the main contributions of this paper can be summarized as follows:

- A risk-constrained stochastic optimization model is presented for joint energy and reserve scheduling of resilient microgrids considering DR programs. In the proposed model, both normal operation uncertainties (including uncertainties associated with output power of RESs, loads and electricity prices) and contingency-based uncertainties (including uncertainties of islanding duration events) are addressed, properly.

- The sensitivity of the microgrid profit, reliability indices and the operator decision making in cases with and without the results are determined by \( \pi \).

- Comprehensive case studies are presented to analyze the impact of islanding durations on decision making of the operator and resilient operation of microgrids. Also, the effect of standard deviation (SD) of islanding duration events on the on decisions is investigated.

The rest of this paper is organized as follows. The proposed optimal scheduling concept is described in Section II. Mathematical formulation of the studied problem is presented in Section III. Case studies together with simulation results are discussed in section IV. Finally, the major findings of the paper are concluded in Section V.

### Table I

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### II. DESCRIPTION OF THE PROPOSED SCHEDULING STRATEGY

Fig. 1 shows general structure of the under-study microgrid that consists of local units such as wind generation and dispatchable units, responsive and non-responsive loads. The dispatchable units in the microgrid could be micro-turbines, fuel cells, gas engines, etc. The microgrid is equipped with an energy management system (EMS) to schedule its local resources and to trade energy with the main grid. In this scheme, the customers are equipped with house energy management controllers and are able to respond to the electricity prices by adjusting their demand to reduce their consumption costs. To model the elastic behavior of the customers, economic DR model presented in [17] is used in this paper.

The operation of the microgrid is decomposed into normal and resilient operations. At the normal operation, microgrid is connected to the main grid, thus the EMS schedules the local DERs and energy exchange with the main grid to maximize the operator’s profit while considering a possible islanding event. However, when a severe disturbance event occurs in the main grid, microgrid can switch into resilient operation (i.e., islanded mode). In this mode, EMS schedules available local resources to supply local loads with the lowest mandatory load shedding. In this model, two categories of uncertainties are modeled: normal operation uncertainties and contingency-based uncertainties.

The uncertainties associated with wind energy, loads and electricity prices are considered as normal operation uncertainties while the uncertainties of islanding duration events are deemed as contingency-based uncertainties. In this study, normal probability distribution functions (PDFs) are employed for representing both normal and contingency-based uncertainties [13]. Monte-Carlo simulation (MCS) is also used for scenario generation based on random sampling from related PDFs and then K-means algorithm [19], [21] is applied to reduce the number of scenarios into a limited set representing well enough the uncertainties. By considering number of \( N_s \) scenarios for representing normal operation uncertainties and \( N_{ih} \) scenarios for modelling the contingency-based uncertainties, a total number of \( N_s \times N_{ih} \) scenarios will be considered for stochastic scheduling. Since the two groups of uncertainties are independent [22], the occurrence probability of a normal scenario \( s \) (\( \pi_s \)) and an islanding period that lasts for \( h \) time intervals (\( \phi_h \)) would be equal to \( \pi_s \times \phi_h \).

Fig. 2 depicts an overview of scheduling horizons of EMS in the proposed strategy. As shown, the time horizon is assumed to be comprised of 24 time periods in which a probabilistic islanding event occurs together with related PDF. It should be noted that islanding events is considered as a stochastic parameter that its probability is presented with related PDF that is calculated based on previous records of the islanding durations of the microgrid. The forecasted errors of islanding duration events are modeled using its associated PDF in which the mean values are equivalent to the forecasted values of stochastic variable. Here, the PDFs are divided into seven discrete intervals with different probability levels in which the mean values of PDFs are equivalent to the forecasted values of the islanding durations in each time period. The proposed stochastic optimization is solved to determine the optimal schedule of the microgrid resources over the estimated islanding durations. In the normal operation, unit commitment and power trading with the main grid are determined by considering responsive loads and their share in allocating reserve capacity. In addition, to ensure a feasible islanding following a disturbance event, the energy and reserve resources should be rescheduled by considering probability of disturbance occurrence and other uncertain parameters. In resilient operation mode, by...
deploying scheduled reserves during islanding mode, the amount of load curtailment should be minimized.

In the proposed model, an economic model is considered for participation of end-use customers in DR programs by using load curtailment and load shifting options [23]-[24]. In order to enhance the model practicality, the mandatory load shedding is applied to non-critical loads when sufficient generation is not available. Moreover, it is assumed that the responsive loads can provide up and down spinning reserve capacity when it is required.

\[ \text{OF_{NORMAL}} = \sum_{i=1}^{N_r} \sum_{s=1}^{N_s} (1-\phi_s) \sum_{t=1}^{T} \pi_t (F_1 - F_2 - F_3 - F_4) + \beta \xi + \frac{1}{(1-\alpha)} \sum_{i=1}^{N_r} \sum_{s=1}^{N_s} \pi_t \eta_i. \]  

The optimal scheduling is done properly through a unit commitment model, and AC power flow procedure by considering system’s objectives and constraints. Moreover, CVaR at the confidence level \( \alpha, (\alpha\text{-CVaR}) \), is incorporated with the optimization model to evaluate the profit risk associated with the operator’s decisions in different conditions. The proposed optimal scheduling is formulated as an efficient mixed-integer linear programming (MILP) model and solved by using commercially available software packages. The Benders decomposition method is also employed for promoting the computational tractability of the problem. The outcomes of the proposed model provide optimal scheduling of DERs and DR, reserves capacity allocated by dispatchable units, responsive loads and the main grid, expected energy not served (EENS) and also energy trading with the main grid while guaranteeing the resiliency of the microgrid.

III. PROBLEM FORMULATION

A. Objective Function

The objective of the proposed stochastic scheduling algorithm is to maximize the expected profit of the microgrid in both normal operating and islanded modes during the scheduling time horizon. As mentioned earlier, the scheduling process is updated several times in the study horizon. If the occurrence probability of islanding in hour \( t \) is considered as \( \phi_t \), then the microgrid will be operated in normal condition with a probability of \( (1-\phi_t) \). The objective function of normal mode (\( \text{OF}_{\text{NORMAL}} \)) and islanded mode (\( \text{OF}_{\text{ISLAND}} \)) over the entire possible realization scenarios of different uncertainties can be formulated as below:

\[ \text{Max } \{ \text{OF}_{\text{NORMAL}} + \text{OF}_{\text{ISLAND}} \} \]  

\[ \text{OF}_{\text{NORMAL}} = \sum_{i=1}^{N_r} \sum_{s=1}^{N_s} (1-\phi_s) \sum_{t=1}^{T} \pi_t (F_1 - F_2 - F_3 - F_4) + \beta \xi + \frac{1}{(1-\alpha)} \sum_{i=1}^{N_r} \sum_{s=1}^{N_s} \pi_t \eta_i. \]  

\[ \text{OF}_{\text{ISLAND}} = \sum_{i=1}^{N_r} \sum_{s=1}^{N_s} \phi_s \sum_{t=1}^{T} \pi_t [\sum_{j=1}^{N_r} (Pr_{j,s,t} - P_{j,s,t}) - F_2 - F_4] + \beta \xi + \frac{1}{(1-\alpha)} \sum_{i=1}^{N_r} \sum_{s=1}^{N_s} \pi_t \eta_i. \]  

where, functions \( F_1 \) to \( F_4 \) are defined as follows:

\[ F_1 = P_{\text{sell}}^{\text{on}} - P_{\text{sell}}^{\text{off}} - P_{\text{buy}} - \sum_{j=1}^{N_r} Pr_{j,s,t} P_{j,s,t} \]  

\[ F_2 = \sum_{i=1}^{N_r} [C_i (P_{i,t,s}) + \text{SUC}_{i,t,s} + \text{SDC}_{i,t,s}] \]  

\[ F_3 = \lambda R_{\text{on}}^R_{\text{on}} + \lambda R_{\text{on}}^R_{\text{off}} + \lambda R_{\text{on}}^R_{\text{on}} + \lambda R_{\text{on}}^R_{\text{off}} \]  

\[ F_4 = \sum_{i=1}^{N_r} [\lambda R_{\text{on}}^R_{\text{on}} + \lambda R_{\text{on}}^R_{\text{off}} + P_{\text{shed}} + P_{\text{VOLL}}] \]  

Function \( F_1 \) represents total income from trading energy with the main grid and revenue of selling energy to customers. \( F_2 \) denotes the costs of energy and reserves provided by dispatchable units together with their startup and shutdown costs. Function \( F_3 \) represents the cost of reserve capacity provided for the main grid and \( F_4 \) represents the cost of reserve allocated by responsive loads and cost of emergency load shedding. Moreover, the second terms in (2) and (3) denote CVaR of a candidate solution. Parameter \( \beta \) is used to model the tradeoff between the expected profit and the risk of profit variability. Also, auxiliary variable \( \xi \) is used to compute the value at risk, and \( \eta_i \) is the difference between microgrid operation cost in scenario \( s \) and \( \xi \) [13]. It should be noted that the operating cost of RESs is neglected in this study.

B. Problem Constraints

Linearized Power Flow Equations: These equations model the real-time operation of microgrid through AC power flow for each scenario and at each time interval. Equations (8) and (9) respectively represent the active and reactive power balance between production and consumption at node \( n \) as follows:

\[ P_{t,s}^{n} + P_{w,t,s}^{n} - P_{j,t,s}^{n} + P_{\text{shed}}^{n} = \sum_{r=1}^{N_r} \beta_{(n,r)}^{P} \]  

\[ Q_{t,s}^{n} + Q_{w,t,s}^{n} - Q_{j,t,s}^{n} + Q_{\text{shed}}^{n} = \sum_{r=1}^{N_r} \beta_{(n,r)}^{Q} \]  

By considering bus 1 as the slack bus, which is connected to the mains, \( P_{t,s}^{n} \) and \( Q_{t,s}^{n} \) must be added to the left side of (9) and (10), respectively. Also, \( \beta_{(n,r)}^{P} \) and \( \beta_{(n,r)}^{Q} \) are the active and reactive power flows between bus \( n \) and \( r \) at time \( t \) and scenario \( s \), respectively. In this study, linearized form of (10) and (11) is used with the following assumptions [26]: (i) over a typical range of voltage amplitude \((0.95 \text{ pu} \leq |V| \leq 1.05 \text{ pu})\), it can be assumed \(|V_{n,t,s} - V_{r,t,s}| \approx 0 \), and (ii) over a typical range of difference in voltage phase angle across branch \( n \) and \( r \), i.e., \(|\delta_{n,t,s} - \delta_{r,t,s}| \leq 10^\circ\), it is assumed that \( \sin(\delta_{n,t,s} - \delta_{r,t,s}) \approx \delta_{n,t,s} - \delta_{r,t,s} \) and \( \cos(\delta_{n,t,s} - \delta_{r,t,s}) \approx 1 \).
To satisfy network congestion, the active and reactive line flows should be limited as:
\[-I_A^{P_{\text{max}}}(n,r) \leq I_A^P(n,r) \leq I_A^{P_{\text{max}}}(n,r)\]  \hspace{1cm} (12)
\[-I_R^{Q_{\text{max}}}(n,r) \leq I_R^Q(n,r) \leq I_R^{Q_{\text{max}}}(n,r)\]  \hspace{1cm} (13)

Moreover, to ensure a safe operation in terms of allowed voltage magnitude and phase angle, the following constraints should be satisfied:
\[|V_{\text{n,t,s}}| \leq |V_{\text{max}}| \text{ and } -\pi \leq \delta_{\text{n,t,s}} \leq \pi\]  \hspace{1cm} (14)

Additionally, the following exchange power capacity constraints must be considered for buying/selling power from/to the main grid in each time interval.
\[P_{\text{buy}} = P_{\text{m,t,s}} - P_{\text{sell}}\]  \hspace{1cm} (15)
\[0 \leq P_{\text{buy}} \leq P_{\text{m,t,s}} \text{, } P_{\text{sell}} \leq P_{\text{m,t,s}}\]  \hspace{1cm} (16)
\[0 \leq P_{\text{sell}} \leq P_{\text{m,t,s}} \text{, } 1-u_{\text{t,s}}\]  \hspace{1cm} (17)

Demand Response Model and Constraints: Customers participate in DR programs that sheddable and shiftable loads by using load curtailment and load shifting options [27]. In this study, the economic model of responsive loads is extracted from [27] where the demand of customers is modeled based on elasticity concept, which is defined as demand sensitivity with respect to the price. Using this concept, the amount of demand after participation in DR is obtained as follow:
\[P_{\text{j,t}} = P_{\text{int}} \sum_{h=1}^{N_h} \left( \frac{P_{\text{j,h}}}{P_{\text{int},h}} + \frac{1}{1+(E_{\text{j,h}})^3} \right) E_{\text{j,h}}\]  \hspace{1cm} (18)

where, \(P_{\text{int},j}\) and \(P_{\text{int},j}\) denote the initial value of active power of load \(j\) and electricity price before applying DR program, respectively. When the microgrid faces a capacity shortage in a working scenario, the emergency load curtailment can be employed to maintain system security. Definitely, the amount of curtailed emergency load is less than maximum active power of the load [19].
\[0 \leq P_{\text{shed}} \leq P_{\text{max}}\]  \hspace{1cm} (19)

Also, by assuming a certain power factor for load \(j\) \((\cos\theta_j)\), the amount of curtailed reactive power is calculated as follow:
\[Q_{\text{shed}} = P_{\text{shed}} \tan(\cos^{-1}\theta_j)\]  \hspace{1cm} (20)

Dispatchable Distributed Generators Constraints: These constraints include power capacity limits of distributed generators (DGs) [17], ramping up/down limits (22)-(23), startup/shutdown costs limits (24), as well as minimum up/down time limits (25)-(26).
\[P_{\text{min}} \leq P_{\text{u,t,s}} \leq P_{\text{max}}\]  \hspace{1cm} (21)
\[P_{\text{i,t,s}} - P_{\text{u,t,s}} \leq RU_i \text{, } (1-Y_{\text{i,t,s}}) + P_{\text{min}} \cdot Y_{\text{i,t,s}}\]  \hspace{1cm} (22)
\[P_{\text{i,t,s}} - P_{\text{u,t,s}} \leq RD_i \text{, } (1-z_{\text{i,t,s}}) + P_{\text{min}} \cdot z_{\text{i,t,s}}\]  \hspace{1cm} (23)
\[SUC_{\text{i,t,s}} \leq CU_i \cdot Y_{\text{i,t,s}}; \quad SDC_{\text{i,t,s}} \leq CD_i \cdot z_{\text{i,t,s}}\]  \hspace{1cm} (24)
\[\sum_{h=t+UT_{-1}}^{t+UT_{-1}} u_{i,t,s} \geq UT_i \cdot Y_{i,t,s}\]  \hspace{1cm} (25)

Wind Power Constraints: the amount of utilized wind power in each scenario and at each examined interval is limited to the maximum available power.
\[0 \leq P_{\text{w,t,s}} \leq P_{\text{w}}\]  \hspace{1cm} (27)

Reserve Constraints: The limits of reserve services offered by dispatchable units and responsive loads determined by constraints (28)-(32)
\[0 \leq R_{\text{up},i,t,s} \leq P_{\text{up},i,t,s}\]  \hspace{1cm} (28)
\[R_{\text{down},i,t,s} \leq P_{\text{up},i,t,s} - P_{\text{min},i,t,s}\]  \hspace{1cm} (29)
\[0 \leq R_{\text{mon},i,t,s} \leq P_{\text{min}} \cdot (1-u_{i,t,s})\]  \hspace{1cm} (30)
\[0 \leq R_{\text{down},i,t,s} \leq P_{\text{max}} - P_{\text{min}} \cdot u_{i,t,s}\]  \hspace{1cm} (31)
\[0 \leq R_{\text{down},i,t,s} \leq P_{\text{max}} - P_{\text{min}} \cdot u_{i,t,s}\]  \hspace{1cm} (32)

Moreover, it is assumed that the microgrid can provide reserve services to the main grid. The amounts of these reserves are limited by:
\[0 \leq R_{\text{up},i,t,s} \leq P_{\text{max}} \cdot u_{i,t,s}\]  \hspace{1cm} (33)
\[0 \leq R_{\text{down},i,t,s} \leq P_{\text{max}} \cdot u_{i,t,s}\]  \hspace{1cm} (34)

C. The Problem Solution Methodology

To solve the proposed problem, both normal operation uncertainties and contingency-based uncertainties are modelled using MCS method according to their probability distributions and a set of 100 scenarios is generated for each stochastic parameter. Here, uncertainties associated with wind generation, market prices, loads and the uncertainties of islanding duration events are considered. The sets of generated scenarios of stochastic parameters are combined to build a scenario tree with 10^8 scenarios. To reduce the computation complexity of the optimization problem, K-means algorithm [21] as a proper scenario-reduction technique is applied to reduce scenario tree to 1000 scenarios. In the next step, these reduced scenarios are implemented to the stochastic optimization model to maximize the expected profit of the microgrid as well as to minimize the total customers’ payments with the optimal scheduling of supply and demand-side energy and reserve resources and optimal trading with the main grid. In this regard, in the first stage, decisions are submitted to the day-ahead market for the next day. In this stage, the status of unit commitment and outputs of committed DERs units as well as the hourly dispatched quantities and the hourly energy and reserve prices of the day-ahead market are determined. Then, the electricity prices, the demand loads and the RESs output power are updated based on their new information. In the second stage, new decisions are submitted and the real-time market is cleared for an hour. The decision variables of this stage are power generations in scenarios, reserves of dispatchable units, load demand after implementing DR programs, deployed reserves of DR, energy traded between the microgrid and the main grid, auxiliary variable used to compute the CVaR.
IV. SIMULATION AND NUMERICAL RESULTS

A. Test System and Main Assumptions

To demonstrate the effectiveness of the proposed method, it is implemented for scheduling of a typical microgrid which includes five dispatchable DG units, three wind turbines (WTs) as well as eight groups of responsive loads. More details about the test system can be found in [19]. The data associated with the installed dispatchable DGs are summarized in Table II (MT, FC and DE stand for micro-turbine, fuel cell and diesel engine, respectively) [18]. The hourly forecasted values of microgrid load, WTs output power and the electricity price are depicted in Fig. 3. Here, load and wind power data and are extracted from [19] and electricity prices are from Nordpool market [28]. Also, it is assumed that prediction errors of load, WTs output power and electricity price follow normal distributions with SD equal to 8%, 5% and 10% of the forecasted values, respectively [29], [30]. Moreover, the price elasticity of the responsive loads is extracted from [17]. Furthermore, it is assumed that islanding duration of the microgrid follows a normal distribution with a mean of 12 hours and different values of SDs. The probabilities associated with different islanding durations are depicted in Table III [30].

It should be mentioned that all required data has been gathered from appropriate resources to keep the consistency throughout the study and draw reliable conclusion while clearly justifying the contributions of this work compared to previous studies considering similar set of input information namely load and generation profiles. Also real energy market information (i.e., electricity prices are from Nordpool market) has been used to make fair cost/benefit analysis.

![Image](image.png)

Fig. 3. The hourly forecasted values of microgrid load, WTs output power and electricity price.

<table>
<thead>
<tr>
<th>Table II</th>
<th>TECHNICAL DATA OF DISPATCHABLE DG UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGs Type</td>
<td>$P_{max}$ (kW)</td>
</tr>
<tr>
<td>----------</td>
<td>----------------</td>
</tr>
<tr>
<td>MT1</td>
<td>25</td>
</tr>
<tr>
<td>MT2</td>
<td>25</td>
</tr>
<tr>
<td>FC1</td>
<td>20</td>
</tr>
<tr>
<td>FC2</td>
<td>20</td>
</tr>
<tr>
<td>GE</td>
<td>35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table III</th>
<th>DIFFERENT ISLANDING DURATION SCENARIOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>9</td>
</tr>
<tr>
<td>$\varphi_h$</td>
<td>0.006</td>
</tr>
</tbody>
</table>

The simulation process is presented as follows. At first, Monte-Carlo simulation method is used to generate 2000 scenarios for stochastic parameters which are then reduced to 27 final scenarios using K-means algorithm [21]. Accordingly, the reduced scenario set is applied to the proposed optimization problem to maximize the expected profit of the microgrid operator. The required coding and optimization algorithm is carried out on a PC with 4 GB of RAM and Intel Core i7 @ 2.60 GHz processor with GAMS software and CPLEX solver considering an optimality gap of 0.031. The computation time in different cases is less than two minutes which further illustrates the practical merits of the proposed strategy.

B. Numerical Results

To investigate the performance of the proposed method, the following four cases are defined. In all cases, the scheduling horizon is considered one day which is divided into 24 time intervals. Moreover, the values of lost load (VOLL) and confidence level, $\alpha$, are set to 1 $/kWh and 0.95, respectively.

Case 1: Optimal scheduling of microgrid in normal condition without considering DR actions. In this case, the microgrid operator maximizes its expected profit while only normal operation uncertainties are taken into account.

Case 2: Similar to Case 1 while DR programs are also included in the scheduling process.

Case 3: The microgrid operator determines the optimal resilient scheduling considering the islanding duration scenarios specified in Table III. In this case, DR actions are not considered.

Case 4: Similar to Case 3 but with participation of customers in DR programs.

Fig. 4 depicts the efficient frontiers for different cases. Here, the optimal solution is obtained only for 10 values of risk-aversion parameter $\beta$ by modifying this parameter from 0 (risk-neutral case) to 50 (risk-averse case). This figure shows that how the expected profit decreases as risk aversion increases, i.e., as the microgrid operator adopts increasingly risk-averse positions. Moreover, it shows that how CVaR, which represents the average expected profit of the worst-case scenarios, increases at the same time, i.e., the microgrid operator reduces its expected profit but also its profit volatility. Also, CVaR is negative in all cases that is due to profit in some scenarios is negative and there is a probability of experiencing financial losses. Therefore, based on the obtained efficient frontiers and negative values for CVaR, it can be deduced that achieving profit with an expected value acceptable for the operator could also show a non-negligible probability of experiencing negative profits or losses.

By comparing the results in different cases it is understood that when customers participate in DR program, the expected profit and CVaR increase. Since, in case of with DR, the operating cost of DG units decreases and hence the operator imports less energy from the main grid and therefore the rate of decrement in the expected profit is lower than that in the case without DR. Also, in cases with DR, the uncertainty in the system environment increases but the undesired scenarios are reduced and consequently the values of CVaR increases. Moreover, comparison of results in different cases in the same figure shows when a resilient scheduling according to the credible islanding contingencies is considered (i.e. cases 3 and 4), the expected profit decreases and CVaR increases in comparison with the normal operations (i.e. cases 1 and 2). In fact, the operator loses a part of its profit during islanded mode, due to increasing microgrid operation costs and/or increasing cost of mandatory load shedding in islanding durations. Therefore, the consideration of islanding event scenarios causes a relatively strong profit reduction in some unfavorable scenarios.
cases 1, 2, 3 and 4 are reduced by 39%, 18%, 19% and 16% but their associated CVaR are increased by 32%, 88%, 18% and 71%, respectively. These results show that by implementing DR, the dependency between profit and risk aversion of the operator reduces in both normal and resilient microgrid. Moreover, the impact of risk averse on the CVaR in the cases with DR is significantly higher than that of in other cases. The values of CVaR in cases 3 and 4, where microgrid resiliency is considered, have increased compared to cases 1 and 2 where microgrid is operated in normal condition. This happens due to the fact that ensuring a feasible islanding after a disturbance event, necessitates rescheduling of the energy and reserve resources according to the worst possible scenarios of islanding mode. Fig. 5 depicts the total operation cost of dispatchable units in different cases versus risk aversion during scheduling horizon. As can be seen, in lower risk aversion (i.e. $\beta < 0.25$), the operation cost of DGs in cases with DR actions is lower than those in cases without DR, while in higher values of risk aversion the opposite trend happens. As mentioned before, in lower values of $\beta$, the operator tries to supply more loads through the main grid to maximize its expected profit. Moreover, in cases with DR in which the customers shift their demands to off-peak hours, the provided power from DGs reduces at peak periods and as the result, the operation cost of DGs decreases. However, in higher degrees of risk aversion, the cost of DGs in cases with DR is more than those in other cases. The total cost of EENS and scheduled reserve versus risk aversion are illustrated in Fig. 6. As can be seen from Fig. 6 (a), when customers participate in DR programs, the total cost of scheduled reserve decreases. In fact, by implementing DR actions, three resources including DR, DGs and the main grid provide required reserve for the microgrid, competitively; which results to reserve cost decrement. Moreover, reserve cost in cases 3 and 4 are higher than those of in cases 1 and 2, respectively due the presence of uncertainties. Additionally, it is observed that the supplement of scheduled reserves depends on the microgrid operator’s risk attitude. A higher risk aversion yields a lower probability of mismatch between supply and demand and thus entails less required reserve. That is because when considering a higher risk aversion, DG units are scheduled in order to mitigate the probability of mismatch between supply and demand. Therefore, the number of worst scenarios reduces and as a result, a lower reserve is required to be scheduled to accommodate the uncertainties of the microgrid. In addition, Fig. 6 (b) shows that by increasing risk aversion, the cost of EENS increases in all cases non-monotonically. Moreover, in cases 2 and 4, due to allocating more reserve capacity through incorporating responsive loads, the amount of load shedding decreases which ultimately ends in lower EENS cost compared to the other two cases. Moreover, cost of EENS decreases during unscheduled islanding periods, due to higher reserve capacities allocated in these cases in comparison with normal operation cases. Also, Fig. 6 shows in

![Fig. 4. Operator’s expected profit versus CVaR for different values of $\beta$, (a) case 1, (b) case 2, (c) case 3, and (d) case 4.](image)

Since, the SD of energy price forecasts are considered higher than that of for responsive loads, trading energy with upstream network might cause the occurrence of more undesirable scenarios. Therefore, when customers participate in DR, the number of scenarios with negative profits decreases and consequently the values of CVaR in cases 2 and 4 are higher than those in cases 1 and 3, respectively. Additionally, by increasing the values of parameter $\beta$ from 0 to 50, the expected profit of

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Case1 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
<th>Case2 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
<th>Case3 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
<th>Case4 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>176 49 0</td>
<td>127 38 24</td>
<td>191 40 0</td>
<td>176 40 0</td>
</tr>
<tr>
<td>0.5</td>
<td>176 34 0</td>
<td>107 3 24</td>
<td>190 32 0</td>
<td>176 32 0</td>
</tr>
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<td>1</td>
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<td>95 0 23</td>
<td>189 30 0</td>
<td>176 30 0</td>
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<td>82 0 22</td>
<td>188 23 0</td>
<td>171 23 0</td>
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<tr>
<td>10</td>
<td>168 20 0</td>
<td>76 0 21</td>
<td>184 12 0</td>
<td>168 12 0</td>
</tr>
<tr>
<td>20</td>
<td>164 16 0</td>
<td>75 0 21</td>
<td>182 11 0</td>
<td>164 11 0</td>
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<tr>
<td>50</td>
<td>161 12 0</td>
<td>75 0 20</td>
<td>172 8 0</td>
<td>161 8 0</td>
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</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Case1 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
<th>Case2 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
<th>Case3 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
<th>Case4 $C^{DG}<em>{R}$ $C^{M}</em>{R}$ $C^{DR}_{R}$</th>
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<td>75 0 20</td>
<td>172 8 0</td>
<td>161 8 0</td>
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</table>
lower values of parameter $\beta$ that the microgrid reserve providers allocate more scheduled reserve, the amount of mandatory load shedding reduces, and so, the cost of EENS decreases.

Table IV illustrates more details about the impact of risk aversion on the costs of scheduled reserves provided by dispatchable DGs ($C^{DG}_{R}$), DR ($C^{DR}_{R}$) and the main grid ($C^{m}_{R}$).

As observed, in higher values of risk aversion, the available resources are scheduled such a way that the probability of mismatch between supply and demand mitigates and as a result the required reserve decreases. In fact, when the operator becomes more risk-averse, it is willing to sacrifice high profits in the best scenarios in the hope of avoiding low profits or even losses in the worst scenarios. Therefore, by decreasing the number of worst scenarios, the cost of scheduled reserves of all resources decreases.

![Fig. 5. Total cost of DG units in different cases versus risk aversion.](image)

![Fig. 6. Costs of different cases versus risk aversion, (a) cost of EENS, and (b) cost of scheduled reserves](image)

In fact, when islanding period lasts longer, the operator tends to schedule local resources instead of the main grid, and as the result, the purchasing energy from the main grid decreases. In order to investigate the impact of SD associated with islanding duration on the expected profit and CVaR, the proposed problem is solved for two values of SD, i.e., SD=1 hour and SD=2 hours, and the results are illustrated in Fig. 7.

The total amount of energy traded between the microgrid and the main grid over the 24-hour period is compared in Table IV. In this table, $E_1$ ($E_2$) represents the amount of energy bought (sold) from (to) the main grid. Also, $E_{net}$ represents the net energy provided from the main grid (i.e., $E_1$-$E_2$). As observed, by increasing risk aversion parameter ($\beta$), the operator tends to buy more energy blocks from the main grid while exporting energy most of the times to make more profit (see rows 2, 3 and 4). In addition, to investigate the impact of islanding duration on the microgrid resiliency operation, two values of SD of islanding durations is considered to be 1 to 2 hours.

C. Discussion

As clearly observed from the numerical results, it is deemed that consideration of uncertainties of islanding duration events has a significant effect on the decision-making problem of the microgrid operator. The obtained results confirmed that the expected profit of the operator decreases but load curtailment and

![Table IV. EXCHANGED ENERGY (KWH) BETWEEN THE MICROGRID AND THE MAIN GRID VERSUS RISK AVERSION IN DIFFERENT CASES.](image)
EENS indices increase when islanding events is considered. Also, when islanding events of the microgrid is considered, relatively more reserve should be allocated by DG and DR resources in order to hedge against the volatility of this uncertain parameter, especially in lower risk aversion. Moreover, the supplement of scheduled reserves depends on the operator's risk perspective meaning that a higher risk-aversion operator yields a lower required reserve capacity. Also, by increasing SD of islanding duration events, the operator tries to schedule based on local DG and DR resources and so the amount of energy provision from the main grid decreases.

\[ \text{Expected profit (a)} \]

\[ \text{CVaR (b)} \]

Fig. 7. The effect of standard deviation (SD) on the expected profit and CVaR in different risk aversion, (a) expected profit, (b) CVaR.

V. CONCLUSIONS

This paper presented a stochastic framework for optimal scheduling of a resilient microgrid with considering DR participation. Expected profit of the microgrid operator was maximized through a risk-constrained stochastic optimization model where the risk imposed by uncertainties related to islanding duration, WTs output power, electricity prices and loads was addressed via CVaR method. The proposed strategy was applied to a test microgrid and several case studies were presented. The results confirmed that the proposed strategy could enable the microgrid operator to determine the risk aversion and balance its profit according to risk factor in both normal and emergency-triggered operation modes. Moreover, the impact of implementing DR actions in normal and resiliency conditions were investigated. The main conclusions drawn out of this study can be highlighted as below:

- When islanding contingencies are considered in the microgrid scheduling, the expected profit decreases significantly compared to a normal operating condition. Moreover, by implementing DR actions, the dependency between profit and risk aversion of the operator reduces in both normal and resilient conditions.
- In a resilience microgrid, the value of CVaR in a certain risk aversion is higher than the one in normal condition.
- In a risk-neutral case the operator tends to buy more energy from the main grid. However, by increasing risk aversion, the operator tends to supply microgrid loads from more reliable dispatchable units rather than the main grid.

Future works mainly include extending the proposed model to a multi-microgrid systems and co-optimizing the customer’s revenue stream from their flexibility options and the energy procurement cost via a two- stage bi-level programming problem.

VI. REFERENCES


