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Integrating the methods of mathematical modelling and engineering design in projects

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In this paper, we show how an integration of the engineering design method and the mathematical modelling method can be applied in engineering. This is exemplified through two student projects in the first-year modules 'Dynamics and Vibrations' and 'Models, Mechanics and Materials', which are compulsory in the Sustainable Design engineering programme at Aalborg University, Denmark. We first describe and discuss the definitions of the two methods and argue that they have many similarities and that the differences appear to vanish once they are combined in an introductory engineering project. We argue that when students experience how the two methods are applied in a project, they may develop a better holistic understanding of the problems they may encounter as future engineers. They are thus better equipped to solve future real-life problems by having applied mathematics and engineering sciences as integrated activities.

Keywords: Engineering design, mathematical modelling, problem-based learning.

Introduction

One of the major functions of design engineers is to solve problems for the society in which they live. Design engineers work on products and systems that involve adapting and using engineering and mathematical techniques and they usually work with a team of engineers and other designers to develop conceptual and detailed designs that ensure a product works and is suitable for its purpose. In this paper, we will describe and discuss an example of how the engineering design process, mathematical modelling, and problem-solving activity are integrated through introductory first-year projects in the Sustainable Design engineering programme at Aalborg University (AAU) in Copenhagen, Denmark. Underlying the example is an assumption that by combining the engineering design process and the mathematical modelling process in an engineering context, students will be much more prepared to tackle the real-life problems that they might encounter in their future profession as engineers.

As we will show in this paper, the integration of the two methods conforms to the method of Problem-Based Learning (PBL) that AAU has adopted since it was founded in 1974. As argued by Kolmos, Holgaard, and Dahl (2013), there is not one single AAU PBL model, but nevertheless, the programmes at AAU are all organized around shared PBL principles described by Barge (2010) and Askehave et al. (2015). These principles are problem orientation, project organization, integration of theory and practice, participant direction, a team-based approach, and collaboration and feedback. The students therefore work in teams on open problems and the work includes all the steps from problem identification and problem analysis to problem solving. PBL is thus a student-centred learning method that uses real problems as a stimulus or starting point for the acquisition and integration of new knowledge. The teacher acts as an initiator and facilitator in the collaborative process of knowledge transfer and development. Parallel to the projects, the students also undertake modules which typically follow a more traditional style with lectures and exercises. The major characteristics of PBL projects include adaption to students' *prior* knowledge and experience, *integration* of knowledge, and teaching in relevant *contexts*.

The purpose of this paper is to show how to integrate the teaching of mathematics and engineering mechanics within the framework of PBL in order to enhance the students' understanding of both subjects as well as to introduce them to real-life situations, where the real problems they meet are mostly combinations of engineering, technology, and mathematics. When working on such problems in the real world, engineers, designers and applied mathematicians work together as a team to create new products. We aim to "transfer" these situations to the classrooms so that the students can develop an early acquaintance with the "real thing". This requires carefully designed teaching scenarios that help make both mathematics and engineering interesting to learn.

The main research questions of this paper are therefore: Can we design didactical situations that integrate mathematics and engineering mechanics through design projects that resemble practical problems? Is it possible to teach mathematics in connection with engineering courses so that the students can capture the essence of both subjects through design projects? To answer these questions, we investigate mathematics teaching through design projects, using mathematical modelling as a didactic tool. The problem that we address here is the lack of interest in mathematics courses among engineering students (Härterich et al., 2012). This issue may eventually lead to poor understanding and performance on engineering science courses that depend on the mathematical concepts taught in traditional mathematics modules. The significance of the paper is therefore that it suggests a method that might solve this problem by offering teaching experiments that integrate some real-life problems with some central concepts in engineering mathematics. The aim of the student projects is to make mathematics more interesting for engineering students and to improve their understanding of engineering and mathematics courses.

The processes of mathematical modelling and engineering design

Mathematical modelling as a design process

Mathematical modelling is used in a variety of disciplines. A mathematical modelling competence is considered central in both engineering and mathematics education. When doing mathematical modelling, a part of reality is encoded into a set of mathematical rules and equations. There are in fact many "models" or descriptions of the mathematical modelling process. Due to page restrictions, we show only one example of such a model (Blomhøj & Jensen, 2003). The one chosen was developed for use in education and is thus relevant. Every mathematical modelling process must begin with a problem or an observation, which is a process that fits well with a PBL project which also starts with open problems that require a solution. The problem can be either a well-defined physical question requiring a mathematical solution or a loosely described technical problem requiring a solution but with no obvious choice of mathematical model. However, since a mathematical model is an abstraction and mathematics itself is an abstract discipline, the starting point of the modelling process is to decide which aspects of the “real world” to observe and which to ignore.

The mathematical modelling process is as creative as the engineering design process, as engineers need to model devices and processes if they want to design these devices and processes. Just like the design of a certain product, a model of a specific physical situation may be good or bad, simplistic or sophisticated, aesthetic or ugly, useful or useless, but a model or design cannot be considered true or false. A mathematical model is therefore designed to correspond to a prototype, which may be a physical, biological, social, or psychological entity or yet another conceptual one. According to Tayal (2013), the engineering design process is a sequence of steps that a designer takes to go from identifying a problem or need to developing a solution that solves the problem or satisfies the need. If we just replace the words “design” and “designer” with “modelling” and “modeller”, we arrive at a definition of mathematical modelling that is close to the one described above by Blomhøj and Jensen (2003). The engineering modelling process is therefore the set of steps that a modeller takes to go from first identifying a problem or need to ultimately creating and developing a solution that solves the problem or meets the need. Thus, the two processes have *identical* objectives and, in fact, can be broken down into a series of *similar* steps, as seen in Table 1. The table constitutes the theoretical framework of this article. Both processes are different from the scientific method although they also have some things in common. As argued by Dowling, Carew, and Hadgraft (2013), the two processes begin with an open task, but while the engineering process begins with a problem or a need, the scientific method begins with a question. Furthermore, engineers look for suitable solutions while scientists look for suitable hypotheses to answer the question, that is, ultimately some generalized knowledge. Thus, engineers create new products using a more pragmatic modelling approach in which the models are convenient approximations to specific parts of reality.

Table 1: A comparison of the modelling and design processes

The mathematical modelling process	**The engineering design process**
1a) Begins with a problem	**1b)** Begins with a problem
2a) Select relevant objects, relations, and data and idealize these	**2b)** Do background research to define suitable criteria and constraints for problem solution
3a) Translate the objects into a mathematical representation	**3b)** Specify requirements for solutions
4a) Use mathematical methods to arrive at results	**4b)** Evaluate the solutions against the criteria. Are there alternative solutions?
5a) Interpret the results in relation to the initial question	**5b)** Choose a suitable solution and build a prototype
6a) Evaluate the model	**6b)** Make recommendations; test and redesign as necessary
7a) Communicate the results	**7b)** Communicate the results

The table shows that the mathematical modelling process and the engineering design process are quite analogous. Below, we design two teaching situations that can provide some justification for Table 1 as these situations integrate mathematics and engineering mechanics in realistic design projects.

Mathematics in an engineering context: An example

The study of differential equations has always been a major part of the mathematics curriculum in engineering education. This is expected as many engineering problems involve equations that relate the changes in some key variables to each other. Therefore, differential equations are used to investigate a wide variety of problems in sciences and engineering.

Below we present a classical problem from engineering dynamics that illustrates the use of Table 1 in differential equations, specifically second-order differential equations. The mechanical system is shown in Figure 1. Besides being a standard textbook problem, it is also seen in real-life problems such as modelling the suspension system of a car or the vibration of a wind turbine blade. Such real-life problems fit a PBL education model well as they can be quite open and can thus be approached by a problem-based strategy. At the same time, they also fit the framework shown in Table 1, as items 1a and 1b in Table 1 indicate~~s~~ that the starting point is a problem. The problems require analysis and simulation before a solution is reached. The analysis and simulation can be identified as items 2–5 for both processes in Table 1:

- Create the idealization and formulate constraints (items 2a and 2b)
- Encode the system into mathematical language, specify the requirements (items 3a and 3b)
- Solve the resulting equation(s) and check the results (items 4a and 4b)
- Interpret the results and build a prototype (items 5a and 5b)

To apply Table 1, we begin by assuming constant parameters for the spring and the damper (items 2a and 2b). Using Newton’s second law, the mechanical system can be described by the second-order linear differential equation with constant coefficients (items 3a and 3b):

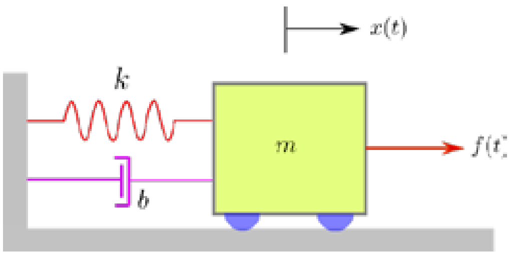

Figure 1: A mechanical system

$$m\ddot{x} + b\dot{x} + kx = f(t) \tag{1}$$

Here $x(t)$ is the position of the mass m, b is the damping constant, k is the spring constant, and $f(t)$ is the force applied on the mass. The students have already completed a mathematics module involving *linear* second-order differential equations. The purpose is to illustrate the different solutions of Equation (1) by changing the values of the constants b and k. By plotting the response

$x(t)$ for some chosen values of b and k (items 4a and 4b), students can see the different behaviours of the system and gain a better understanding of the underlying mathematics of the mechanical system, thus accomplish items 5a and 5b. To choose appropriate values of b and k requires that tests and a redesign be carried out, thus accomplishing items 6a and 6b. It is therefore pedagogically sound to base the teaching of *linear* second-order differential equations on systems whose behaviour students *already* intuitively understand. This illustrates that PBL, through working with open problems, can help the students achieve a better understanding of mathematics by using mathematical modelling as a didactic principle in teaching engineering mechanics and mathematics itself through the process illustrated in Table 1. Finally, the students accomplish items 7a and 7b through communicating the results to their peers and teachers as part of the module.

The students' ability to solve linear differential equations by using standard methods or formulas is therefore insufficient to understand fully the link between the mathematics learned and the other modules and projects. Students may eventually ask how we know that the parameters of the spring and the damper are constants. There are typically two different answers, we can give the students:

- Real springs and dampers have approximately linear behaviour.
- By assuming constant parameter values for the spring and damper, we can use the standard methods to solve linear differential equations, as we have a complete theory of linear differential equations, and not, e.g., a complete theory of *nonlinear* differential equations.

These answers can, however, be disputed by the fact "almost all systems are nonlinear to some extent" (Hilborn, 2000). Besides, many of the student projects in which the first author was involved as a supervisor involve nonlinear phenomena. However, as the students were only introduced to linear second-order differential equations in their mathematics module and are still in their first year of study, we will at this point assume linear behaviour of the systems involved.

Mathematical modelling in action

Model of a Lego van and a caravan: Example 1

In a project in the second-semester module "Dynamics and Vibrations" for the Sustainable Design engineering programme, the students should construct a mathematical model of the popular van and caravan playset by the Danish company Lego. In the context of Table 1, the problem of the project is to determine the response of the caravan if the van moves on a straight road with a constant acceleration a. This corresponds to items 1a and 1b in Table 1. Ideally, the caravan should follow the *kinematics* of the van as closely as possible. The project can illustrate the use of PBL as a framework in teaching an engineering mechanics module, where mathematics and mechanics are integrated in a realistic project through the process of mathematical modelling. The PBL project thus conforms to Table 1, as it starts with a problem that requires mathematical tools to arrive at acceptable results. A simplified model of the system is shown in Figure 2. Many students immediately see the similarity of this concrete system with the ideal one in Figure 1: The constant acceleration of the van gives rise to a constant force on the caravan. Some also realize that the *flexible linkage* between the van and the caravan can be *modelled* as a spring and a damper.

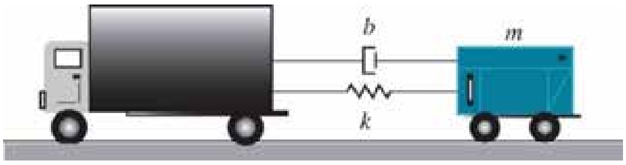

Figure 2: A model of the Lego project

If we let $x_1(t)$ and $x_2(t)$ denote the positions of the van and the caravan respectively, Newton's second law (Hibbeler, 2017) leads to the differential equation of the model:

$$m\ddot{x}_2 + b(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) = 0 \quad (2)$$

As the van's acceleration is constant, we have $\dot{x}_1 = at$ and $x_1 = \frac{1}{2}at^2$, where the initial conditions are assumed to be zero. Equation (2) can finally be written as

$$m\ddot{x}_2 + b\dot{x}_2 + kx_2 = abt + \frac{1}{2}akt^2 \quad (3)$$

Comparing Equations (3) and (1), the students see that $bat + \frac{1}{2}kat^2$ corresponds to the force $f(t)$. We are led to a second-order non-homogeneous differential equation where the unknown function is $x_2(t)$. This is exactly the type of differential equation, which the students have met in their mathematics module; they now meet it in an *engineering context*. Thus, Table 1's items 2a, 2b, 3a, and 3b are now satisfied. The students now see an old friend in action! Thus, we anticipate that this will offer some justifications to the students in answer to their eternal question "Why do we need to study differential equations?" in the context of their experience.

Student simulation of the model

The students were given the values $m = 0.50$ kg and $a = 1$ m/s^2 for which they ran a simulation of the model for two different combinations of the spring constant k and the damping constant b:

- $k = 10$ N/m and $b = 0$ (no damping)
- $k = 10$ N/m and $b = 0.50$ N · s/m (with damping)

The students used MATLAB to plot the position $x_2(t)$ of the caravan in the two cases (Figure 3).

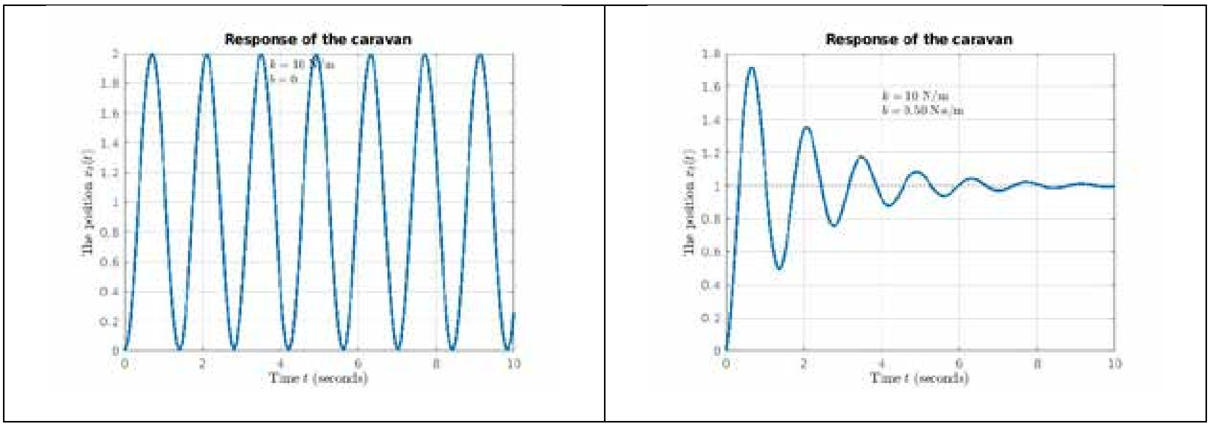

Figure 3: Response of the caravan. Left: no damping; Right: with damping

The case without damping obviously leads to an unacceptable model of the system. In contrast, the presence of damping ensures a smooth response of the caravan. Many students now realize why a damper should be included as a part of modelling the linkage, even though it is not out there! Thus, by changing the value of b (and of k for that matter) we get two different *designs* and *that* will result in two different mathematical *models*. By referring to items 4 to 6 for both processes in Table 1, we see that these are accomplished, as we are testing the models and comparing designs. The design process and the mathematical modelling process are therefore closely related to the extent that, in real-life engineering practice, separating the two processes would not be easy. It is therefore artificial to separate modules in mathematical modelling and engineering design in teaching situations, as engineering programmes ought to correspond to and be compatible with realistic situations that the future engineers might encounter.

The connection between mathematical modelling and engineering design: Example 2

To further illustrate the relation between the mathematical design process and the engineering design process, we mention here an example taken from a project by the first author introduced in the first-semester module “Models, Mechanics and Materials”. Briefly, the problem is to find the internal forces in the three cables of the hanging lamp (see Figure 4) and to redesign it if possible. The students themselves should provide realistic values for the weight and dimensions of the hanging lamp. They should also measure the lengths and diameters of the wires. The students should first make a model of the hanging lamp and then try to redesign it. This is an open problem with several solutions.

Figure 4: A hanging lamp

The modelling process consists of writing the equilibrium equations of the lamp using statics. The students discovered that they could not find the internal forces in the three wires: They got two equations with three unknowns. By arriving at an indeterminate system of equations, the students then had a justification for why the middle wire was *redundant*. Some other students added the extra equation using deformation theory in the topic of strength of materials. In that way, they could determine the internal forces in the three wires by writing them down in matrix form. Changing the geometry and materials of the wires (i.e., design) leads to another system of equations (i.e., a new model). Other students chose to remove the middle wire, thus arriving at a consistent system of equations with a unique solution. Thus, the design the students choose will affect the mathematical model of the lamp. Conversely, the mathematical model of the lamp will influence its design and redesign. Here, we can see that the items of each pair in Table 1 interact together in a fusion process to produce a solution to the problem. The main purpose of this course project is to relate topics from mathematics, specifically matrices and linear equations, to engineering mechanics and materials in the framework of Table 1. We believe that through these kinds of course projects, mathematics teaching will be more motivating for the students, so they can see the relevance of the mathematical topics they encounter in their study programme.

Discussion and conclusion

In the paper, we have used Table 1, which incorporates the engineering design and mathematical modelling processes as a theoretical framework for two student projects. The items in the table can be identified in the processes the students go through in two design projects. Here we also saw that the items in both the mathematical modelling process and the engineering design process became visible. We believe that this identification strengthens the argument that *engineering* mathematics should be taught in *an engineering* context, through well-designed teaching situations that allow engineering students to work with real-life problems. As stated in the introduction, design engineers work on projects that involve adapting and using engineering and mathematical techniques, and *in reality,* these are intertwined. We also believe that this strategy can be generalized to other situations where STEM integration is involved.

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