

Dicovering spaces

For a local po/space X and a base point $x_0 \in X$, we define the universal dicovering space $\Pi : \tilde{X}_{x_0} \rightarrow X$. The image of Π is the future $\uparrow x_0$ of x_0 in X and X_{x_0} is a local po-space such that $|\vec{\pi}_1(\tilde{X}, [x_0], x_1)| = 1$ for the constant dipath $[x_0] \in \Pi^{-1}(x_0)$ and $x_1 \in \tilde{X}_{x_0}$. Moreover, dipaths and dihomotopies of dipaths (with a fixed starting point) in $\uparrow x_0$ lift uniquely to \tilde{X}_{x_0} . The fibers $\Pi^{-1}(x)$ are discrete, but the cardinality is not constant. We define dicoverings $P : \tilde{X} \rightarrow x_{x_0}$ and construct a map $\phi : \tilde{X}_{x_0} \rightarrow \tilde{X}$ covering the identity map. Dipaths and dihomotopies in \tilde{X} lift to \tilde{X}_{x_0} , but we give an example where ϕ is not continuous.