Multivariate Stochastic Volatility: A Likelihood Approach

Turatti, Douglas Eduardo

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Likelihood-Based approximation for multivariate stochastic volatility models

Douglas Eduardo Turatti

Institut for Økonomi og Ledelse, Aalborg Universitet, Aalborg 9220, Denmark

Abstract

A likelihood-based estimation procedure for state-space models with multivariate stochastic volatility is developed. The methodology tackles the dimensionality problem by approximating the system as a sequence of conditionally independent univariate stochastic volatility models. A maximum likelihood approach for this sequence is proposed based on the Efficient Importance Sampling technique together with the Kalman Filter, yielding a computationally fast and accurate estimator. A Monte Carlo study for a time-varying vector autoregression with multivariate stochastic volatility suggests that the procedure has good finite-sample and asymptotic properties and results are comparable to the one-step estimation of the model parameters.

Keywords: Multivariate stochastic volatility, State-space models, Efficient importance sampling, Rao-Blackwellization, Time-varying vector autoregressions
JEL *Classification*: C15, C32

Email address: det@business.aau.dk (Douglas Eduardo Turatti)

1. Introduction

Time-varying volatility matrices are fundamental to asset allocation, risk management, and derivative pricing. Recently, macroeconomists have also been interested in time-varying covariance matrices to analyze possible changes to the transmission mechanism of monetary policy. In particular, the model of Cogley and Sargent (2005) has been popular in analyzing changes in idiosyncratic shocks and propagation mechanisms. Their specification allows for time-varying variances and covariances, and is especially interesting when combined in a state-space model.

The need to model time-varying volatilities of financial and macroeconomic data has created significant interest in estimating multivariate stochastic volatility (MSV) models. However, statistical inference for this class of models is challenging and prone to numerical shortcomings. The problem is due to the presence of multiple non-linear volatility states precluding the evaluation of the likelihood function. Thus, the usual estimation procedure requires highly intensive computational methods. A further hindrance is related to the dimensionality of the model, usually suffering from a flat likelihood function, multimodality and other matters related to its numerical optimization (or integration, in a Bayesian context). In general, statistical inference for MSV models is heavily affected by the so-called "curse of dimensionality".

This paper introduces an estimation procedure for models with MSV which tackles most of the dimensionality issue. We base our analysis on a Cholesky decomposition which allows an univariate approximated model using state-space and importance sampling methods. The model parameters are then obtained by a sequence of maximum likelihood estimations. The aim of this paper is twofold. The first one is to introduce our estimation procedure for state-space models with multivariate stochastic volatility. The second is to present a Monte Carlo (MC) study using the method.

2. State-Space models with multivariate stochastic volatility

The class of MSV models considered is given by:

$$y_t = X_t(\gamma)B_t + \Upsilon(H_t)\varepsilon_t, \qquad \varepsilon_t \sim N(0_N, I_N), \tag{1}$$

$$B_{t+1} = F_t(\gamma)B_t + \Upsilon(Q)\eta_t, \qquad \eta_t \sim N(0, I), \tag{2}$$

where $t = 1, \ldots, T$ and γ contain parameters. $X_t(.)$ and $F_t(.)$ are matrices of exogenous regressors or static parameters. y_t is the $N \times 1$ vector of observable variables at time t. B_t represents potential linear latent states following a Gaussian vector autoregression. $\Upsilon(.)$ represents the lower Cholesky factor, and Q is assumed to be a diagonal matrix. The stochastic volatility is introduced in the matrix H_t. Consider the LDL decomposition of the matrix H_t,

$$H_t = A^{-1}\Sigma_t A^{-1'}, \tag{3}$$

where A is a lower triangular unit matrix, and Σ_t is a diagonal matrix consisting of time-varying variance states,

$$\Sigma_t = \begin{bmatrix} \sigma^2_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2_{N,t} \end{bmatrix}. \tag{4}$$

It follows that the Cholesky factor of H_t is given by,

$$\Upsilon(H_t) = A^{-1}\Sigma_t^{0.5}. \tag{5}$$

The time-varying variance states are determined by log-normal stochastic volatility models,

$$\sigma^2_{i,t} = \exp(h_{i,t}), \quad h_{i,t} = \mu_i + \lambda_i h_{i,t-1} + \omega_i \upsilon_{i,t}, \quad \upsilon_{i,t} \sim N(0,1). \tag{6}$$

Given the structure of (5) the model implies time-varying variances and covariances, although the latter varies according to changes in the volatility's innovations. Nevertheless, this model is popular in the macroeconomics and finance literature, and it has been used by Cogley and Sargent (2005) for the analysis of monetary policy. Finally, time-varying parameter VAR models with stochastic volatilities, e.g. Cogley and Sargent (2005); heteroskedastic factor models, e.g. Han (2006); ARMA models with multivariate stochastic volatility, e.g. Stock and Watson (2007); and zero-mean MSV models can all be cast in the format (1)-(2).

3. An approximated procedure for multivariate stochastic volatility models

Statistical inference for MSV models is challenging. The presence of multiple nonlinear latent states prevents the instant computation of the likelihood function. Additionally, multiple static parameters implies a complicated model whose likelihood function may have multiple peaks, some of which are in uninteresting or implausible regions of the parameter space. Finally, from a numerical perspective it is easier to optimize several small-dimensional objective functions than a very large one. Our algorithm tackles these problems and delivers an equation-by-equation univariate approximation.

In the first step the state-space model (1)-(2) is written in a triangular form, i.e. according to the contemporaneous dependence in the system's equations. In a further step the equations are shown to be estimated as univariate stochastic volatility models. Finally, to estimate this N one-dimensional stochastic volatility models, we propose an estimator based on a simulated maximum likelihood via importance sampling. Recall that the measurement equation (1) is given by,

$$y_t = X_t(\gamma)B_t + A^{-1}\Sigma_t^{0.5}\varepsilon_t, \qquad \varepsilon_t \sim N(0_N, I_N). \tag{7}$$

Let $a_{i,j}$ be a generic term of the matrix A^{-1}. Given the triangular form of the matrix A

the system can be written as,

$$\begin{aligned} y_{1,t} &= X_t(\gamma)B_t + \sigma_{1,t}\varepsilon_{1,t}, & (8)\\ y_{2,t} &= X_t(\gamma)B_t + a_{2,1}\sigma_{1,t}\varepsilon_{1,t} + \sigma_{2,t}\varepsilon_{2,t}, & (9)\\ &\vdots \\ y_{N,t} &= X_t(\gamma)B_t + a_{N,1}\sigma_{1,t}\varepsilon_{1,t} + \cdots + a_{N,N-1}\sigma_{N-1,t}\varepsilon_{N-1,t} + \sigma_{N,t}\varepsilon_{N,t}. & (10)\end{aligned}$$

For the sake of simplicity we have omitted partitions in $X_t(\gamma)$ and B_t. A generic equation for $y_{i,t}$ is thus given by,

$$y_{i,t} = X_t(\gamma)B_t + a_{i,1}\epsilon_{1,t} + \cdots + a_{i,i-1}\epsilon_{i-1,t} + \sigma_{i,t}\varepsilon_{i,t}, \quad \forall i = 1,\ldots N, \tag{11}$$

where $\epsilon_{i-1,t} = \sigma_{i-1,t}\varepsilon_{i-1,t}$ for all $i = 1,\ldots N$. Our estimation strategy is based on developing an approximation for equation (11).

However, equation (11) cannot be easily estimated because the error terms in $a_{i,1}\epsilon_{1,t} + \cdots + a_{i,i-1}\epsilon_{i-1,t}$ are unknown and cannot be estimated.[1] If the values of the disturbances $\epsilon_{1,t},\ldots,\epsilon_{i-1,t}$ were directly observable, the model would collapse in a conditionally linear and Gaussian state-space model with stochastic volatility, which can be efficiently estimated by the Rao-Blackwellized Efficient Importance Sampling (RB-EIS) of Moura and Turatti (2014). We propose to replace these unknown disturbances by their conditional expectations. This approach has the advantage that conditional expectations can be computed, and it has been called a quasi-optimal procedure.[2] We propose to approximate (11) as,

$$y_{i,t} = X_t(\gamma)B_t + a_{i,1}\mathbb{E}[\epsilon_{1,t}|\psi_{t-1}] + \cdots + a_{i,i-1}\mathbb{E}[\epsilon_{i-1,t}|\psi_{t-1}] + \sigma_{i,t}\varepsilon_{i,t}, \tag{12}$$

where ψ_{t-1} represents the information set up to time $t-1$. Assuming the conditional expectations are available, equation (12) becomes dependent on a single stochastic volatility process. However, to estimate equation (12) it is still necessary to evaluate the likelihood function of a conditionally linear and Gaussian state-space model with univariate stochastic volatility. A likelihood estimator is provided based on the RB-EIS method.

3.1. Rao-Blackwellized efficient importance sampling

The likelihood function for (12) is given by,

$$L(\gamma|y_i) = \int \cdots \int \prod_{t=1}^{T} g(y_{i,t}|B_t, h_{i,t};\gamma)p(B_t|B_{t-1};\gamma)p(h_{i,t}|h_{i,t-1};\gamma)dB_1 \ldots dB_T\, dh_{i,1}\ldots dh_{i,T}, \tag{13}$$

where $g(.)$ and $p(.)$ are the measurement and transition densities respectively. Note that for a given log-volatility path, equation (12) becomes a linear and Gaussian state-space model. Thus, it is possible to analytically integrate out the conditionally linear states B_t

[1] States and disturbances cannot be estimated. It is only possible to consistently estimate aspects of their conditional (unconditional) distribution.

[2] This form of approximation replacing disturbances by conditional expectations has been employed in the econometrics literature before. See for example Harvey et al. (1992), where conditional expectations are used to estimate unobserved component models with ARCH disturbances.

using the Kalman Filter,

$$L(\gamma|y_i) = \int \cdots \int \prod_{t=1}^{T} g^*(y_{i,t}|h_{i,t};\gamma)\, p(h_{i,t}|h_{i,t-1};\gamma)\, dh_{i,1} \ldots dh_{i,T}, \tag{14}$$

where $g^*(y_{i,t}|h_{i,t};\gamma)$ is the likelihood contribution delivered by the Kalman filter. However, it is still necessary to evaluate the integral (14), and this will be done using the Efficient Importance Sampling (EIS) criteria proposed by Richard and Zhang (2007).

It is possible to rewrite (14) using the importance sampler $m_t(h_{i,t}|h_{i,t-1};\alpha_t)$,

$$L(\gamma|y_i) = \int \cdots \int \prod_{t=1}^{T} \frac{\varphi_t(y_{i,t}, h_{i,t};\gamma)}{m_t(h_{i,t}|h_{i,t-1};\alpha_t)} m_t(h_{i,t}|h_{i,t-1};\alpha_t)\, dh_{i,1} \cdots dh_{i,T}, \tag{15}$$

where $\varphi_t(\cdot) = g^*(y_{i,t}|h_{i,t};\gamma) p(h_{i,t}|h_{i,t-1};\gamma)$ is the integrand in (14), and α_t are auxiliary parameters determining the moments of the importance sampler m_t. The importance sampler can be decomposed as $m_t(h_{i,t}|h_{i,t-1};\alpha_t) = k_t(h_{i,t}|h_{i,t-1};\alpha_t)/\chi_t(h_{i,t-1};\alpha_t)$, with $\chi_t(h_{i,t-1};\alpha_t) = \int k_t(h_{i,t}|h_{i,t-1};\alpha_t)\, dh_{i,t}$ as its integrating constant. It is possible to transfer $\chi_{t+1}(h_{i,t};\alpha_{t+1})$ back to period-t to exploit the fact that it carries information on $h_{i,t}$:

$$L(\gamma|y_i) = \int \cdots \int \chi_1(\alpha_1) \prod_{t=1}^{T} \left[\frac{\varphi_t(y_{i,t}, h_{i,t};\gamma)\chi_{t+1}(h_{i,t};\alpha_{t+1})}{k_t(h_{i,t}|h_{i,t-1};\alpha_t)} \right] m_t(h_{i,t}|h_{i,t-1};\alpha_t) dh_{i,1} \cdots dh_{i,T}, \tag{16}$$

where $\chi_{T+1} \equiv 1$. Equation (16) can be estimated as:

$$\widehat{L}(\gamma|y_i) = \chi_1(\alpha_1) \frac{1}{S} \sum_{j=1}^{S} \prod_{t=1}^{T} w_j(\gamma, y_{i,t}) \varphi_t(y_{i,t}, h_{i,t}^{(j)};\gamma), \tag{17}$$

where $\{\{h_{i,t}^{(j)}\}_{t=1}^{T}\}_{j=1}^{S}$ denotes S i.i.d. paths drawn from the importance samplers $\{m_t(h_{i,t}|h_{i,t-1};\alpha_t)\}_{t=1}^{T}$, and $w_j(.)$ are importance weights defined as $w_j(\gamma, y_{i,t}) = \chi_{t+1}(h_{i,t}^{(j)};\alpha_{t+1})/k_t(h_{i,t}^{(j)}|h_{i,t-1}^{(j)};\alpha_t)$.

Richard and Zhang (2007) show that the MC variance of $\widehat{L}(\gamma|y_i)$ is minimized when k_t mimics $\varphi_t\chi_{t+1}$. Thus, the EIS' strategy is to construct a sequence of density kernels k_t by selecting auxiliary parameters α_t that closely approximates $\varphi_t\chi_{t+1}$. EIS tackles this problem by iterating on the following backward sequence of low dimensional least-squares going from T to 1:

$$\widehat{\alpha}_t = \min_{\alpha_t} \sum_{j=1}^{S} \left[\ln\left(\varphi_t(y_{i,t}, h_{i,t}^{(j)};\gamma) \cdot \chi_{t+1}(h_{i,t}^{(j)};\widehat{\alpha}_{t+1}) \right) - c_t - \ln k(h_{i,t}^{(j)}|h_{i,t-1}^{(j)};\widehat{\alpha}_t) \right]^2, \tag{18}$$

where draws $\{h_{i,t}^{(j)}\}_{j=1}^{S}$ come from a chosen initial sampler $m(h_{i,t}|h_{i,t-1};\alpha_t^0)$, and the constant c_t accounts for possible missing terms. As the log-volatilities are driven by Gaussian processes, the kernel $\ln k(h_{i,t}^{(j)}|h_{i,t-1}^{(j)};\widehat{\alpha}_t)$ is linear on natural parameters and (18) can be easily computed by OLS estimates.

Lastly, an EIS estimator for $\mathbb{E}[\epsilon_{1,t}|\psi_{t-1}],\ldots,\mathbb{E}[\epsilon_{i-1,t}|\psi_{t-1}]$ can be given by,

$$\mathbb{E}[\widehat{\epsilon_{i,t}|\psi_{t-1}}] = \frac{\sum_{j=1}^{S} w_j(\gamma, y_{i,t})\,\mathbb{E}[\widehat{\epsilon_{i,t}^{(j)}|\psi_{t-1}}]}{\sum_{j=1}^{S} w_j(\gamma, y_{i,t})}, \tag{19}$$

where each conditional expectation $\mathbb{E}[\epsilon_{i,t}^{(j)}|\psi_{t-1}]$ can be computed by the Kalman Filter with disturbances included in the state-vector once the volatility processes are sampled.[3]

3.2. Equationwise RB-EIS

Finally, with the estimator (19) the equationwise procedure for MSV models is completed:

1. Write the model in a triangular form.
2. Estimate the parameters of the first equation by the RB-EIS method. Obtain an estimator for the conditional expectation $\mathbb{E}[\epsilon_{1,t}|\psi_{t-1}]$ using formula (19).
3. Plug this estimator in the second equation. Estimate the parameters by the RB-EIS method and obtain an estimate for $\mathbb{E}[\epsilon_{2,t}|\psi_{t-1}]$.
4. Proceed to the next equation. Repeat this process until all parameters are estimated.

This algorithm allows for an equation-by-equation univariate approximation while incorporating information from the whole system sequentially. In addition, given the structure of the dependence in the error terms, the procedure is asymptotically subject to only an approximation error. In the next section, we perform a Monte Carlo experiment to investigate its finite-sample properties.

4. Monte Carlo study

A Monte Carlo study is performed on the basis of a TVP-VAR(1)-MSV with 2 and 3 variables. The model is given by,

$$\begin{aligned}
y_{1,t} &= \beta_{1,t} + \beta_{11,t}y_{1,t-1} + \beta_{12,t}y_{2,t-1} + \beta_{13,t}y_{3,t-1} + \epsilon_{1,t}, && (20)\\
y_{2,t} &= \beta_{2,t} + \beta_{21,t}y_{1,t-1} + \beta_{22,t}y_{2,t-1} + \beta_{23,t}y_{3,t-1} + \epsilon_{2,t}, && (21)\\
y_{3,t} &= \beta_{3,t} + \beta_{31,t}y_{1,t-1} + \beta_{32,t}y_{2,t-1} + \beta_{33,t}y_{3,t-1} + \epsilon_{3,t}, && (22)
\end{aligned}$$

where the vector of random innovations is $\epsilon_t \sim N(0_3, H_t)$, and $H_t = A^{-1}\Sigma_t A^{-1'}$. The state-variables $\boldsymbol{B}_t$ and Σ_t follow random walk processes,

$$\begin{aligned}
\beta_{ii,t} &= \beta_{ii,t-1} + q_{ii}\eta_{ii,t}, \quad \eta_{ii,t} \sim N(0,1), \quad \forall i = 1,2,3, && (23)\\
\beta_{ij,t} &= \beta_{ij,t-1} + q_{ij}\eta_{ij,t}, \quad \eta_{ij,t} \sim N(0,1), \quad \forall i,j = 1,2,3 \ \& \ i \neq j, && (24)\\
\beta_{i,t} &= \beta_{i,t-1} + q_i\eta_{i,t}, \quad \eta_{i,t} \sim N(0,1), \quad \forall i = 1,2,3, && (25)\\
h_{i,t} &= h_{i,t-1} + \omega_i\upsilon_{i,t}, \quad \upsilon_{i,t} \sim N(0,1), \quad \forall i = 1,2,3. && (26)
\end{aligned}$$

[3]There are other possible estimators for these conditional expectations. For example, by estimating the model without MSV using the Kalman Filter.

We examine the properties of our procedure in 2 cases: (I) high parameter variation intensity of time-varying parameters and stochastic volatility; (II) moderate parameter variation intensity of time-varying parameters and stochastic volatility.[4] The data generating processes (DGP) are summarized in table (1).

Table 1: Data Generating Processes

DGP	q_{ii}	q_{ij}	q_i	ω_i	$a_{2,1}$	$a_{3,1}$	$a_{3,2}$
I	0.04	0.03	0.04	0.04	0.3	0.3	0.3
II	0.03	0.02	0.03	0.03	0.3	0.3	0.3
III	0.03	0.02	0.03	0.03	0.6	-	-
IV	0.03	0.02	0.03	0.03	1	-	-

The samples sizes are $T = 250, 500, 1000$. The number of replications is set to 250, and 100 draws are used to evaluate the likelihood function.

Table (2) presents averages and standard deviations of the parameter estimates for DGP I and II. We note that for small sample sizes ($n = 250$), all the parameters have large standard deviations, but for $n = 500$ and $n = 1000$ they become much smaller. However, the sample average is close to the DGP even for a small size as 250. Most importantly all parameters seem to exhibit convergence to the DGP as the sample size increases, and at $n = 1000$ the parameter estimates are very close to the real ones with small standard deviations.

A second MC study aims to compare the estimates of our equationwise procedure and the one-step estimation of the full model for DGP III and IV from a finite-sample perspective. For this experiment we work with a small TVP-VAR(1) with 2 variables to minimize the curse of dimensionality when estimating the full model. The samples sizes are $T = 250, 500$. For a fair comparison, in the joint estimation the number of draws was increased to 200. The one-step estimation will be carried out by the Rao-Blackwellized Efficient Importance Sampling.

Table (3) shows the results of the second MC study. Results indicate that the average of the estimates are very similar, especially for $T = 500$. This provides strong support for the equationwise procedure as a sound estimator and comparable to the one-step procedure in small and large sample sizes.

5. Conclusions

This paper proposes an estimation procedure for MSV models, which takles most of its dimensionality problem. The method is based on an equation-by-equation univariate approximation combined with the RB-EIS technique. The resulting algorithm can be applied to a range of models commonly used in finance and macroeconomics.

A Monte Carlo study shows that the equation-by-equation procedure has good finite-sample and asymptotic properties: most parameter estimates are very close to the DGP in large sample sizes, and their sample standard deviations become smaller. Another MC study compares the equation-by-equation procedure with the one-step estimation of

[4] Stability checks were performed at every point in time, and explosive paths for y_t were discarded.

Table 2: Properties of the equationwise procedure

Parameters	DGP	$T=250$	$T=500$	$T=1000$
			DGP I	
q_1	0.04	0.042 (0.044)	0.039 (0.031)	0.039 (0.018)
q_{11}	0.04	0.034 (0.015)	0.035 (0.010)	0.036 (0.005)
q_{12}	0.03	0.028 (0.015)	0.028 (0.009)	0.028 (0.005)
q_{13}	0.03	0.026 (0.013)	0.028 (0.008)	0.028 (0.005)
ω_1	0.04	0.076 (0.050)	0.061 (0.033)	0.049 (0.020)
q_2	0.04	0.043 (0.043)	0.041 (0.029)	0.038 (0.019)
q_{21}	0.03	0.027 (0.017)	0.028 (0.009)	0.029 (0.006)
q_{22}	0.04	0.034 (0.015)	0.036 (0.010)	0.036 (0.006)
q_{23}	0.03	0.028 (0.016)	0.028 (0.009)	0.028 (0.006)
ω_2	0.04	0.074 (0.050)	0.061 (0.034)	0.050 (0.020)
$a_{2,1}$	0.30	0.306 (0.102)	0.312 (0.070)	0.295 (0.053)
q_3	0.04	0.041 (0.042)	0.041 (0.027)	0.040 (0.018)
q_{31}	0.03	0.027 (0.015)	0.028 (0.009)	0.028 (0.006)
q_{32}	0.03	0.028 (0.016)	0.028 (0.009)	0.028 (0.006)
q_{33}	0.04	0.034 (0.015)	0.036 (0.009)	0.039 (0.006)
ω_3	0.04	0.073 (0.051)	0.059 (0.032)	0.050 (0.021)
$a_{3,1}$	0.30	0.300 (0.104)	0.296 (0.067)	0.304 (0.060)
$a_{3,2}$	0.30	0.307 (0.102)	0.302 (0.071)	0.302 (0.048)
			DGP II	
q_1	0.03	0.033 (0.034)	0.028 (0.022)	0.030 (0.015)
q_{11}	0.03	0.025 (0.013)	0.025 (0.009)	0.027 (0.005)
q_{12}	0.02	0.018 (0.012)	0.018 (0.009)	0.019 (0.005)
q_{13}	0.02	0.014 (0.010)	0.018 (0.008)	0.018 (0.005)
ω_1	0.03	0.059 (0.043)	0.047 (0.029)	0.038 (0.017)
q_2	0.03	0.034 (0.042)	0.029 (0.021)	0.029 (0.014)
q_{21}	0.02	0.017 (0.013)	0.019 (0.008)	0.019 (0.005)
q_{22}	0.03	0.024 (0.013)	0.026 (0.008)	0.027 (0.005)
q_{23}	0.02	0.017 (0.012)	0.018 (0.007)	0.019 (0.005)
ω_2	0.03	0.058 (0.041)	0.047 (0.026)	0.038 (0.016)
$a_{2,1}$	0.30	0.299 (0.090)	0.302 (0.060)	0.294 (0.043)
q_3	0.03	0.033 (0.034)	0.030 (0.020)	0.030 (0.014)
q_{31}	0.02	0.016 (0.014)	0.019 (0.009)	0.019 (0.005)
q_{32}	0.02	0.018 (0.014)	0.018 (0.008)	0.019 (0.005)
q_{33}	0.03	0.025 (0.014)	0.027 (0.009)	0.028 (0.006)
ω_3	0.03	0.057 (0.039)	0.044 (0.024)	0.037 (0.014)
$a_{3,1}$	0.30	0.300 (0.089)	0.295 (0.062)	0.299 (0.046)
$a_{3,2}$	0.30	0.307 (0.085)	0.304 (0.064)	0.302 (0.043)

The table presents the DGP parameters followed by sample averages of estimated parameters from 250 series. Values in parentheses are sample standard deviations of the estimates.

the model parameters, and it shows that the difference between them is small in finite samples, and in large samples they yield similar results. For future research it would be interesting to incorporate the procedure in a Gibbs Sampling algorithm.

Table 3: Finite-Sample properties of the equationwise procedure

Parameters	DGP	$T = 250$	$T = 500$	$T = 250$	$T = 500$
		DGP III			
		Equationwise procedure		**One-Step procedure**	
q_1	0.03	0.035 (0.037)	0.036 (0.026)	0.033 (0.037)	0.031 (0.026)
q_{11}	0.03	0.025 (0.015)	0.025 (0.009)	0.025 (0.014)	0.025 (0.009)
q_{12}	0.02	0.017 (0.014)	0.018 (0.008)	0.017 (0.013)	0.018 (0.008)
ω_1	0.03	0.054 (0.038)	0.046 (0.026)	0.054 (0.039)	0.045 (0.026)
q_2	0.03	0.038 (0.046)	0.032 (0.030)	0.033 (0.046)	0.029 (0.033)
q_{21}	0.02	0.019 (0.016)	0.019 (0.009)	0.018 (0.015)	0.018 (0.009)
q_{22}	0.03	0.024 (0.014)	0.026 (0.010)	0.022 (0.014)	0.025 (0.010)
ω_2	0.03	0.059 (0.042)	0.045 (0.028)	0.059 (0.043)	0.045 (0.028)
$a_{2,1}$	0.60	0.587 (0.082)	0.589 (0.058)	0.590 (0.080)	0.591 (0.059)
		DGP IV			
		Equationwise procedure		**One-Step procedure**	
q_1	0.03	0.039 (0.047)	0.031 (0.021)	0.032 (0.039)	0.030 (0.023)
q_{11}	0.03	0.025 (0.015)	0.026 (0.009)	0.025 (0.013)	0.026 (0.008)
q_{12}	0.02	0.017 (0.012)	0.017 (0.007)	0.017 (0.011)	0.017 (0.010)
ω_1	0.03	0.055 (0.040)	0.044 (0.023)	0.056 (0.042)	0.043 (0.023)
q_2	0.03	0.044 (0.050)	0.036 (0.032)	0.031 (0.043)	0.029 (0.034)
q_{21}	0.02	0.022 (0.019)	0.022 (0.011)	0.017 (0.018)	0.017 (0.007)
q_{22}	0.03	0.026 (0.014)	0.028 (0.010)	0.022 (0.014)	0.025 (0.010)
ω_2	0.03	0.059 (0.043)	0.044 (0.028)	0.059 (0.044)	0.044 (0.027)
$a_{2,1}$	1.00	0.990 (0.084)	0.989 (0.060)	0.993 (0.082)	0.992 (0.061)

The table presents the DGP parameters followed by sample averages of estimated parameters from 250 series. Values in parentheses are sample standard deviations of the estimates.

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