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Published in:
IEEE Antennas and Wireless Propagation Letters

DOI (link to publication from Publisher):
10.1109/LAWP.2022.3215582

Publication date:
2022

Link to publication from Aalborg University

Citation for published version (APA):

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Effective Sparse Recovery Framework for Ultra-Wideband Robust Plane Wave Generator

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Abstract—This letter proposes a novel ultra-wideband plane wave generator (PWG) design methodology suitable for a low-cost small anechoic chamber. Several novel strategies are proposed to handle various crucial concerns in practice to make the proposed methodology reliable and robust. Specifically, the sub-array-based amplitude-only excitation method is capable of reducing the required number of control channels without the need for phase shifters, which can significantly simplify the feeding network complexity. One major concern of the PWG application is the non-negligible multipath reflections in small anechoic chambers. Thus, a novel regression approach, imposing the constraint of maximum energy outside the quiet zone (QZ), is implemented to suppress the reflection level. For the design of the PWG layout, a compressed sensing-based adaptive backtracking orthogonal matching pursuit (ABOMP) algorithm is proposed. In the simulation, a spherical test zone supporting the ultra-wideband frequency over 2.4-5 GHz is synthesized with the proposed methodology, and the effectiveness and robustness are comprehensively investigated.

Index Terms—Antenna measurement, constrained regression, compressed sensing, plane wave generator.

I. INTRODUCTION

HIGHLY integrated active antenna system (AAS) will no longer provide any connected radio frequency (RF) port for testing, entailing the cost-effective over-the-air (OTA) interface testing solution in urgent demand [1-3]. The antennas under test (AUT) are expected to be measured under the ideal plane wave illuminations, and the test zone shall be wideband and large enough to support various sizes of communication devices, especially the sub-6 GHz massive Multiple-Input Multiple-Output (MIMO) base stations (BSs) [4]. Plane wave generator (PWG) has been developed and discussed for many years and was permitted for the AAS OTA conformance testing by the 3GPP organization [5].

The basic principle of PWG is to coherently synthesize the plane wave field in the quiet zone (QZ) via properly allocating excitations for the contained antennas. Various methodologies, e.g., in [6-12], have been developed to determine the optimal locations and excitations of PWG antennas for a high-quality and cost-effective plane wave synthesis. For example, a systematic guideline on the PWG geometrical design and the stability of the solution are studied in [9]. Applying the sparse representation theory, [10] proposed a novel PWG design method with fewer radiating sources. Most of them, however, are limited to narrowband applications and require rigorous amplitude and phase feeding control, except for the tapering methods [11]. However, it is typically applied for the PWG with a large aperture and is not ideal for the small anechoic chambers [8]. In addition, although several commercial PWG systems have been reported in recent years [12], the design details were not given. This letter aims to propose a novel methodology to derive the geometrical layout and the excitation coefficients for the wideband PWG suitable in a small anechoic chamber for the sub-6 GHz scenarios, where the chamber dimension is the dominant cost factor.

Compressed sensing has been widely utilized in various engineering fields of research [13-15] and is employed to determine a sparse PWG array in this letter. The sparse recovery generally results in non-uniform arrays, and such arrangements are demonstrated to support ultra-wideband PWG performance [8] and reduce array truncation effects [17]. Furthermore, this strategy effectively reduces the number of required antenna elements, reducing the cost of the PWG system. Different from the conventional amplitude and phase control for each PWG element in the feeding network, the amplitude-only excitation method (i.e., with the same phase excitation for all PWG elements) and the sub-array excitation principle (i.e., without the need for individually exciting each PWG elements) are adopted in this letter, significantly reducing the number of required excitation channels and the feeding network complexity. It is crucial since the individual excitation method is rather complicated and cost-prohibitive for wideband applications [12]. Furthermore, a novel constrained regression approach is proposed for excitation computation. This strategy could suppress the energy outside the QZ and enable the stable generated field inside the QZ in small anechoic chambers.

Notations: \(|x|\) is to calculate the integer part of \(x\), \((\cdot)^T\) is for transpose, \((\cdot)^H\) is for Hermitian transpose, \(\|\cdot\|_p\) and \(\|\cdot\|_\infty\) are the transpose, Hermitian transpose, \(p\)-norm, and infinity norm operations, respectively.

II. PROBLEM FORMULATION

The PWG configuration, generating a spherical test zone with the diameter of \(D_Q\), is shown in Fig. 1, and all the antenna elements with the same digital label, i.e., located on the same concentric circle, are considered as a sub-array. A two-dimensional circular QZ with a diameter of \(D_Q\) is defined for algorithm implementation. The center of PWG is
A. General System Model

Let us consider a PWG composed of U sub-arrays, and the uth sub-array contains $L_u$ radiating elements. With the coherent superposition of the electromagnetic waves radiated from the sub-arrays of PWG, the generated field in the QZ can be expressed as: $T = HG$, where $T \in \mathbb{C}^{M \times 1}$ is the vector containing the generated complex field values of the $M$ sampling points in the QZ. $H = [h_1, ..., h_U] \in \mathbb{C}^{M \times U}$ is the polarized transfer matrix. $h_u = [h_{1,u}, ..., h_{M,u}]^T \in \mathbb{C}^{M \times 1}$ is the $u$th column vector of $H$. $h_{m,u}$ represents the coherent sum of the transfer coefficients from the $L_u$ elements of the $u$th sub-array to the $m$th sampling point in the QZ. $G \in \mathbb{C}^{U \times 1}$ is the vector of real number excitations for the U sub-arrays.

The objective function to obtain the excitation vector is:

$$
\min_{G} \|HG - e \cdot \exp(j\phi)\|_2.
$$

(1)

de = [1, ..., 1]^T \in \mathbb{C}^{M \times 1}$ and $\phi$ is the phase of the target field. Note that we are concerned with the uniformity of the generated field, while the field phase value $\phi$ is not important. Obviously, the calculated amplitude-only excitation vector $G$ varies when different $\phi$ values are set for $e$ in regression, resulting in different synthesized fields. We therefore find the optimal phase value $\phi^{opt}$ and optimal excitation $G^{opt}$ minimizing the field deviations, and the phase term $\exp(j\phi)$ can be moved to the PWG side:

$$
\inf_{\phi^{opt}, G^{opt}} \left\{ f(H \cdot \exp(-j\phi)G) \min_{G} \|H \cdot \exp(-j\phi)G - e\|_2 \right\},
$$

(2)

where $\inf_{S} f(S)$ denotes the infimum of the subset $S$. $f(\cdot)$ is a metric measuring the synthesized field deviation defined as:

$$
\alpha_{max} \text{ and } \alpha_{min} \text{ are the maximum and minimum amplitude in the QZ, respectively, while } \psi_{max} \text{ and } \psi_{min} \text{ are the maximum and minimum phases in the QZ, respectively. } \varepsilon \text{ is a scaling factor set to 0.1 in this work and can be flexibly adjusted according to the performance requirements [8].}
$$

B. Constrained Regression

A penalty function in [10] is proposed to mitigate the unwanted energy outside the QZ. It is a competent choice in narrowband, but the parameters should be adjusted for wideband scenarios. In this letter, we proposed a novel constrained regression strategy. The basic idea is that if the maximum energy outside the QZ is lower than a threshold $\delta_{thr}$, electromagnetic reflections caused by the imperfect absorber in the small anechoic chamber can be mitigated. (1) can be rewritten with a non-linear constraint:

$$
\min_{G} \|H \cdot \exp(j\phi)G - e\|_2
$$

s.t. \( \|H' \cdot \exp(j\phi)G\|_\infty < \delta_{thr} \) (4)

Where $H'$ is the transfer matrix between the PWG sub-arrays and the zone (with the aperture of $D\Omega$) outside the QZ. To solve (4), a logarithmic barrier function can be introduced. (4) can be written as:

$$
\min_{G} \left\{ \left\| H' \cdot \exp(j\phi)G - e \right\|_2, t > 0, p > 0 \right\}
$$

(5)

(5) can be solved via the quasi-Newton method [18], where $t$ and $p$ are hyper-parameter. Specifically, the convex optimization can be performed for (1) with the preset $\phi$. The obtained excitation will be used as the initial values in the quasi-Newton method to calculate the final optimal amplitude excitations. Then $\phi$ is reset, and we repeat this procedure until all possible phase values are traversed. $G^{opt}$ can then be obtained. Note that $\phi^{opt}$ is only used to calculate $G^{opt}$, and we do not have to set it in practical PWG. It should be stressed that the transfer matrices $H$ and $H'$, phase term $\phi$ for computation needs, and excitation coefficients $G$ are frequency dependent, and the optimal excitation coefficients $G^{opt}$ are determined with different values at different frequencies.

C. Sparse Recovery for PWG Layout

The sparse recovery for the sub-array-based PWG is a 0-norm minimization problem and is, therefore, NP-hard [13]. In [11], a brute force method is adopted. However, the optimal number of antennas in each sub-array is typically unknown. For this reason, combined with the above theories, a legible adaptive backtracking orthogonal matching pursuit (ABOMP) approach for the sparse PWG design is derived.

1) Over-complete dictionary construction: Regarding the candidate PWG sub-array, we first define $\hat{r}_s$ and $\Delta \hat{r}$ as starting radius and radius interval, respectively. The number of candidate sub-arrays can be characterized as: $\hat{U} = \lfloor (D\Omega/2 - \hat{r}_s)/\Delta \hat{r}\rfloor + 1$. For the $u$th candidate sub-array, its radius is: $\hat{r}_u = \hat{r}_s + (\hat{u} - 1) \cdot \Delta \hat{r}$, $\hat{u} = 1, ..., \hat{U}$. Since the number of radiating elements contained in a candidate sub-array is quite problematic to be determined, we applied the non-redundant sampling strategy [6] to determine the number of antennas contained in each candidate sub-array. Specifically, there should be $L_{\hat{u}} = 2M_{\hat{u}} + 1$ antennas uniformly deployed on the $\hat{u}$th concentric circle candidate sub-array with the radius of $\hat{r}_u$ [6]. $M_{\hat{u}} = \lfloor \pi D\Omega \chi_1 \sin(\theta_{\hat{u}})/\lambda_{\hat{u}} \rfloor$, where $\chi_1 = 1 + (\chi_1 - 1)\sin(\theta_{\hat{u}})^2/3$, and $\theta_{\hat{u}} = \tan^{-1}(\hat{r}_u/z_0)$. $\lambda_{\hat{u}}$ is the wavelength of the lowest frequency of the designed wideband. $\chi_1$ is an adjustable parameter. The possible numbers of antennas on the $\hat{u}$th candidate sub-array can therefore be derived with different $\chi_1$ values. To construct an over-complete dictionary, we preset a small $\chi_1$ value as $\chi_1^2$ and a large $\chi_1$ value as $\chi_1^2$ to derive the minimum $L_{\hat{u}}^{min}$ and
maximum $L_{\text{max}}$ numbers of antennas in the $\hat{u}$th candidate sub-array. Thus $L_{\hat{u}}$ can be the integer ranging from $L_{\text{min}}^{\hat{u}}$ to $L_{\text{max}}^{\hat{u}}$. In other words, there are $L_{\text{max}}^{\hat{u}} - L_{\text{min}}^{\hat{u}} + 1$ possible antenna layouts for the $\hat{u}$th candidate sub-array. The transfer coefficient vector between the $\hat{u}$th candidate sub-array with $\hat{L}_{\hat{u}}$ antennas and the quiet zone can be denoted as $h(\hat{L}_{\hat{u}})$. A matrix composed of the transfer coefficient vectors of all possible layouts of the $\hat{u}$th candidate sub-array can be denoted by $\hat{H}(\hat{u}) = [h(\hat{L}_{\text{min}}^{\hat{u}}), ..., h(\hat{L}_{\text{max}}^{\hat{u}})]$. Therefore, by treating the transfer coefficient vector $h(\hat{L}_{\hat{u}})$ of the $\hat{u}$th candidate sub-array with $\hat{L}_{\hat{u}}$ uniformly distributed antennas as an atom in the dictionary, an over-complete dictionary, including $\hat{U}$ candidate sub-arrays, can be expressed in the cell array form: $\xi = \{\hat{H}(1), ..., \hat{H}(\hat{U})\}$. When we calculate the contribution of one candidate sub-array, for example the $\hat{u}$th candidate sub-array, in synthesizing the plane wave field in the following two parts, all possibilities in $\hat{H}(\hat{u}) = [h(\hat{L}_{\text{min}}^{\hat{u}}), ..., h(\hat{L}_{\text{max}}^{\hat{u}})]$ should be considered, and the layout with the most significant contribution will be selected.

2) Initial support sub-arrays generation: After constructing the over-complete dictionary, the orthogonal matching pursuit (OMP) algorithm is implemented. By calculating the correlation values between the residual vector and the transfer coefficient vectors of the $\hat{U}$ candidate sub-arrays with all possible numbers of radiators, one can select the sub-array with a certain number of radiating elements which is capable of minimizing the objective function (1) in each iteration. The obtained sub-arrays are referred to as support sub-arrays in the following. After a sub-array with a certain structure is selected, other candidate sub-arrays whose radius differences from the selected sub-array are smaller than the isolation distance $r^*$ are no longer available to avoid the antenna coupling. This procedure is quite similar to that in [10] [19].

3) Backtracking examination: In this part, the support sub-arrays are adjusted individually through the backtracking examination. For each initial support sub-array under examination, a unique optional sub-array region is constructed and is composed of the candidate sub-arrays that will not be coupled with the other support sub-arrays. Combine one possible support sub-array in the optional set with the support sub-arrays that are not being examined as a temporary sub-array set. At multiple frequencies, perform the constrained regression calculation for the temporary sub-array set and record the field errors with equation (3). If there is a certain optional sub-array that is more competent in minimizing the average field deviation over the wideband, it will replace the sub-array being examined as the support sub-array. Repeat this procedure for each support sub-array until the wideband field deviation no longer changes.

D. Summary

In the proposed strategies, the over-fitting problem [8] could be significantly relieved with the constrained regression strategy (also with the amplitude-only excitation constraint). Note that $L_{\text{max}}^{\hat{u}}$ should be restricted to avoid coupling. Actually, in backtracking examination, the possible sub-array can be rotated at multiple angles about the center of PWG for a comprehensive investigation. Though computationally heavy in the design period, the proposed framework effectively generates the ultra-wideband sparse PWG. In application, the PWG can be directly employed with the already obtained excitation codebook at different frequencies.

III. Numerical Simulation

In this section, the ultra-wideband performance, ranging from 2.4 GHz to 5 GHz, of synthesizing the desired field in the QZ with our proposed framework is comprehensively investigated. For the dimensions of sampling zones and PWG, the configuration of $D_O = 2m$, $D_Q = 0.6m$, $D_P = 1.2m$, $U = 8$, and $z_0 = 2.1m$, are set to be simulated. $\lambda_1 = 1.0$ and $1.7$ are used to derive $L_{\text{min}}^{\hat{u}}$ and $L_{\text{max}}^{\hat{u}}$. For algorithm implementation, the starting radius, radius interval, and isolation distance are determined as: $\hat{r}_s = 0.5\lambda_0$, $\Delta \hat{r} = 0.1\lambda_0$, and $r^* = 0.5\lambda_0$, respectively. The fields presented are about the main polarization and $\delta_{\text{thr}} = -3$ dB. The above settings are adopted for all simulations unless otherwise stated. For comparison, the popular simple least squares method (LSM) is applied to the ABOMP as well, and the complex excitations are therefore employed. Also, the configuration in [11] is used to examine the proposed method.

The generated PWG layout, including 8 sub-arrays and 83 antennas, is shown in Fig. 3, as well as the excitations at different frequencies. With the generated sparse PWG, the fields synthesized at 2.6 GHz, 3.5 GHz, and 5 GHz are shown in Fig. 2. It can be observed that the obtained PWG design is capable of supporting the ultra-wideband plane wave field synthesis, with the maximum peak-to-peak amplitude and phase deviations of 0.89 dB and 7.7$^\circ$, respectively. Adopting the constrained regression, the maximum undesired field outside
The QZ is limited to no more than $-3$ dB, which is an adequate result in the small anechoic chamber. Over the designed ultra-wideband, the maximum cross-polarized field below $-25$ dB is achieved, indicating negligible cross-polarization interference.

To evaluate the system robustness, random amplitude and phase noises are considered to be the uniformly distributed random variables, which range from $-0.15$ dB to $0.15$ dB and from $-1.5^\circ$ to $1.5^\circ$ respectively, and are added to the excitations. Fig. 5 shows the maximum field deviations within the QZ at ultra-wideband frequency points in terms of the theoretical value without excitation noises and the practical value. The practical value is calculated as the average of multiple results of the maximum field deviations excited by the feeding coefficients with noises. It can be observed that the errors most affected by noises are only as low as $0.2$ dB in amplitude and $0.5^\circ$ in phase. In particular, for the PWG layout and excitation suitable for ultra-wideband, the maximum cross-polarized field below $-25$ dB, case II) practical amplitude $3$ dB, case II) practical phase $3^\circ$, are further simulated with $\delta_{thr} = -1$ dB. In addition to the maximum field deviations over ultra-wideband, the maximum root mean square (RMS) field deviation is presented as well in Table I. It can be demonstrated that the proposed strategies can be well generalized and extended to various scenarios. In particular, configuration III synthesizes the satisfying plane wave field in the QZ with the same size as in [11], while fewer antennas and excitation channels are required.

**IV. Conclusion**

This letter presents a novel framework for determining the PWG layout and excitation suitable for ultra-wideband applications. Adopting the amplitude-only excitation method with the sub-array design, the feeding network can be effectively simplified, and the required number of feeding channels is significantly reduced. The simulations of several PWG configurations show that the designed PWG offers adequate and robust plane wave fields within the QZ over $2.4$ GHz-5 GHz. Furthermore, the proposed constrained regression strategy could suppress the undesired field outside the QZ, enabling the PWG to be applied to the small anechoic chamber.
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