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# The Lothbrok approach for SPARQL Query Optimization over Decentralized Knowledge Graphs

Christian Aebeloe<sup>a,\*</sup>, Gabriela Montoya<sup>a</sup> and Katja Hose<sup>a</sup>

<sup>a</sup> *Department of Computer Science, Aalborg University, Selma Lagerlöfs Vej 300, DK-9220 Aalborg Ø, Denmark*  
E-mails: caebel@cs.aau.dk, gmontoya@cs.aau.dk, khose@cs.aau.dk

**Abstract.** While the Web of Data in principle offers access to a wide range of interlinked data, the architecture of the Semantic Web today relies mostly on the data providers to maintain access to their data through SPARQL endpoints. Several studies, however, have shown that such endpoints often experience downtime, meaning that the data they maintain becomes inaccessible. While decentralized systems based on Peer-to-Peer (P2P) technology have previously shown to increase the availability of knowledge graphs, even when a large proportion of the nodes fail, processing queries in such a setup can be an expensive task since data necessary to answer a single query might be distributed over multiple nodes. In this paper, we therefore propose an approach to optimizing SPARQL queries over decentralized knowledge graphs, called LOTHBROK. While there are potentially many aspects to consider when optimizing such queries, we focus on three aspects: cardinality estimation, locality awareness, and data fragmentation. We empirically show that LOTHBROK is able to achieve significantly faster query processing performance compared to the state of the art when processing challenging queries as well as when the network is under high load.

Keywords: LOTHBROK, Peer-to-Peer, characteristic sets, query optimization, cardinality estimation, data locality, SPARQL, RDF, knowledge graphs

## 1. Introduction

Due to the popularity of decentralized knowledge graphs on the Web, more and increasingly large knowledge graphs encoded in RDF are becoming available [1]. Furthermore, RDF knowledge graphs made available today are becoming exceedingly large. For instance, Wikidata [2] and Bio2RDF [3] contain more than 14 billion triples each. As a result, data providers experience an increasing burden of maintaining access to the datasets; and without any monetary incentives to do so, datasets often end up becoming unavailable [4–6] and outdated [7].

In recent years, several decentralized systems [6–11] have been proposed to alleviate the aforementioned burden from the data providers by reducing the computational load required to keep the data available, albeit using different methods to do so. For instance, Linked Data Fragments (LDF)-based approaches [9–13] reduce the computational load on the server by distributing some of the query processing effort to the client, ensuring that the server only processes requests with low time complexity. On the other hand, Peer-to-Peer (P2P) systems [6–8] remove the centralized point of failure that a server represents and replicate the data across several nodes in a decentralized fashion, ensuring that even if the uploading node fails, the data is still accessible. For instance, RDFPeers [14] uses a structured overlay over a P2P network that relies on Dynamic Hash Tables (DHTs) to determine where to replicate

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\*Corresponding author. E-mail: caebel@cs.aau.dk.

certain data. However, in situations where nodes frequently leave or join the network (i.e., churn), and data is often uploaded to the network, nodes have to go through a costly adjustment process to update the overlay and redistribute the data. Instead, systems like PIQNIC [6] and COLCHAIN [7] use unstructured P2P systems as foundation, where there is no global control over where data is replicated, making the network more stable under churn.

COLCHAIN builds upon PIQNIC and divides the entire network into communities of nodes that not only replicate the same data, but also collaborate on keeping certain data (fragments) up-to-date. This is done by using blockchain technology [15–18] where *chains* of updates maintain the history of changes to the data fragments. By linking such update chains to the data fragments in a community, COLCHAIN allows community participants to collaborate on keeping the data up-to-date while using consensus to make malicious updates less likely and allowing users to roll-back updates to an earlier version on request. Furthermore, the decentralized nature of COLCHAIN also increases the availability of the uploaded data by replicating the data on nodes within the community.

Nevertheless, while PIQNIC and COLCHAIN already use decentralized indexes [19] to determine where data is located during query time, subgraphs needed to answer a query are usually scattered across multiple nodes. Furthermore, the indexes provide limited information that prevents the nodes from considering locality and accurately estimating join cardinalities when optimizing queries. As a result, such systems often experience an unnecessarily large amount of intermediate results when processing a query. This problem is exacerbated by the decentralized nature of the systems, since the intermediate results have to be transferred between nodes, causing a significant communication overhead.

While there are potentially many aspects to consider when optimizing queries in a decentralized setup, we will focus on three such aspects: cardinality estimation, locality awareness, and data fragmentation. Suboptimal solutions to any of these three aspects can lead to an increased communication overhead and lower performance. For instance, while fragmenting large knowledge graphs into smaller fragments ensures that nodes do not have to replicate entire knowledge graphs, using a fragmentation technique that spreads out the data relevant to a single (sub)query across several fragments can increase the communication overhead since nodes might have to send an excessive number of requests to obtain all relevant data to answer a particular query [20–23]. On the other hand, inaccurate cardinality estimations can lead to a suboptimal join strategy that increases the amount of intermediate results and therefore runtime [24, 25]. And while several approaches have proposed reasonably accurate cardinality estimation techniques [24–26] over knowledge graphs, and for federated engines in particular [25, 27–29], such approaches cannot easily be transferred to a decentralized setup since nodes in a decentralized setup lack a global overview of the network and the data is scattered across multiple nodes. Finally, considering locality of the data when processing queries can help ensure that larger subqueries are delegated to nodes that can process them without communicating with other nodes, lowering the data transfer overall.

Nevertheless, while an optimization approach that maximizes the degree to which entire queries can be processed by a single node could decrease the communication overhead, a study [20] found that processing entire queries on one node can actually decrease the overall performance when the network is under heavy load, and that it is equally important to balance out the query load between nodes. As such, there is a need for a more holistic approach to query optimization that is able to delegate the processing of subqueries to other nodes in the network, thus reducing the communication overhead to the extent possible. For instance, query optimization techniques that are based on star-shaped subqueries have previously been shown to increase performance by at least an order of magnitude [10–12, 30]. This, and the fact that conjunctive subqueries are relatively efficient to process [31], means that decomposing and processing queries based on star-shaped subqueries can significantly reduce the communication overhead in decentralized systems.

In this paper, we therefore extend our work on PIQNIC [6] and COLCHAIN [7] in three aspects that work together to reduce the communication overhead when processing SPARQL queries, and in doing so, improve query processing performance in an approach that we call LOTHBROK. LOTHBROK adapts Characteristic Sets [10–12, 24] to fragment data in decentralized P2P systems. Furthermore, LOTHBROK builds upon Prefix-Partitioned Bloom Filters (PPBFs) [19] and proposes a new indexing scheme called Semantically Partitioned Bloom Filters (SPBFs) to obtain more accurate cardinality estimations. Lastly, LOTHBROK also introduces a locality-aware query optimization strategy that takes advantage of the SPBF indexes and is able to delegate the processing of (sub)queries to neighboring nodes in the network holding relevant data. We evaluate LOTHBROK thoroughly using LargeRDFBench [32], a benchmark suite for federated RDF systems that comprises 13 datasets with over a billion triples and includes 40

queries of varying complexity and sizes of intermediate results. Furthermore, we evaluate LOTHBROK using synthetic data and queries from WatDiv [33] to test the scalability of LOTHBROK under load. In summary, we make the following contributions:

- A data fragmentation technique that builds on Characteristic Sets [24]
- SPBF indexes adapted to the characteristic set fragmentation technique
- A cardinality estimation approach over decentralized RDF fragments using the SPBF indexes to provide more accurate cardinality estimations
- A locality-aware query optimization algorithm that uses SPBF indexes to delegate subqueries to neighboring nodes and reduce the communication overhead
- A thorough experimental evaluation of the impact of the presented techniques on query processing performance using real-world data from a well-known benchmark suite, and large-scale synthetic datasets

The paper is structured as follows: Section 2 discusses related work while Section 3 describes background information. Then, Section 4 presents LOTHBROK, Section 5 details how LOTHBROK optimizes queries, and Section 6 describes the query execution approach, while Section 7 presents our experimental evaluation. Lastly, Section 8 concludes the paper with an outlook to future work.

## 2. Related Work

The availability problem has prompted significant amount of research in the areas of decentralized query processing and decentralized architectures for knowledge graphs. In this section, we therefore discuss existing approaches related to LOTHBROK; client-server architectures, federated systems, and P2P systems.

### 2.1. Client-Server Architectures

SPARQL endpoints are Web services providing an HTTP interface that accepts SPARQL queries and remain some of the most popular interfaces for querying RDF data on the Web. However, several studies [4, 5] have found that such endpoints are often unavailable and experience downtime.

Linked Data Fragment (LDF) interfaces, such as Triple Pattern Fragments (TPF) [9], attempt to increase the availability of the server by shifting some of the query processing load towards the client while the server only processes requests with low time complexity. For instance, TPF servers only process individual triple patterns while the TPF clients process joins and other expensive operations. Today, several TPF clients exist that rely on either a greedy algorithm [9], a metadata based strategy [34], or star-shaped query decomposition combined with adaptive query processing techniques [35] to determine the join order of the triple patterns in a query. However, while in all these approaches the server can handle more concurrent requests in comparison to SPARQL endpoints without becoming unresponsive, TPF naturally incurs a large network overhead when processing queries since intermediate bindings from previously evaluated triple patterns are transferred along with subsequently evaluated triple patterns to limit the amount of intermediate results, one by one. Furthermore, studies found that the performance of TPF is heavily affected by the type of triple pattern (i.e., the position of variables in the triple pattern) [13] and the shape of the query [36, 37].

Several different systems have since been proposed to lower the network overhead. For instance, Bindings-Restricted TPF (brTPF) [38] bulks bindings from previously evaluated triple patterns such that multiple bindings can be attached to a single request. While this reduces the number of requests made for a triple pattern, it still incurs a somewhat large data transfer overhead, since each request still evaluates a single triple pattern. hybridSE [39] combines a brTPF server with a SPARQL endpoint and takes advantage of the strengths of each approach; subqueries with large numbers of intermediate results are sent to the SPARQL endpoint to overcome the limitations posed by LDF systems. However, hybridSE often answers complex queries using the SPARQL endpoint and is thus vulnerable to server failure.

To further limit the network overhead, Star Pattern Fragments (SPF) [11] clients send conjunctive subqueries in the shape of stars (star patterns) to the server and process more complex patterns locally on the client. Such

conjunctive subqueries can be processed relatively efficiently by the server [31], which results in the transfer of significantly fewer intermediate results than in systems like TPF and brTPF. On the other hand, Smart-KG [12] ships predicate-family partitions (i.e., characteristic sets) to the client and processes the entire query locally; however, triple patterns with infrequent predicate values (according to a certain threshold) are sent to and evaluated by the server. While this takes advantage of the distributed resources that the clients possess, Smart-KG often ends up transferring excessive amounts of data unnecessarily since entire partitions of a dataset are transferred regardless of any bindings from previously evaluated star patterns. WiseKG [10] combines SPF and Smart-KG and uses a cost model to determine which strategy (SPF or Smart-KG) is the most cost-effective to process a given star-shaped subquery. Like SPF and Smart-KG, WiseKG processes more complex patterns on the client. Nevertheless, all the aforementioned LDF approaches rely on a centralized server or a fixed set of servers that are subject to failure.

Lastly, different from LDF approaches, SaGe [40] decreases the load on the server by suspending queries after a fixed time quantum to prevent long-running queries from exhausting server resources; the queries can then be restarted by making a new request to the server. However, SaGe processes entire, and possibly complex, queries on the server, and as stated above, such servers are subject to failure.

## 2.2. Federated Systems

Federated systems enable answering queries over data spread out across multiple independent SPARQL endpoints [41–45] or LDF servers [46] offering access to different datasets. While such approaches spread out query processing over several servers, lowering the load on each individual server, they sometimes generate suboptimal query execution plans that increase the number of intermediate results and the load on individual servers [47]. As such, several approaches [25, 27–29, 48, 49] have attempted to optimize federated queries in different ways. For instance, [44] builds an index over time by remembering which endpoints in the federation can provide answers to which triple patterns. Furthermore, [48] decomposes queries into subqueries that can be evaluated by a single endpoint. While [48] uses a similar query decomposition strategy as LOTHBROK, they target federations over SPARQL endpoints, and as previously mentioned, such endpoints suffer from availability issues. On the other hand, [25, 49] estimate the selectivity of joins to produce more efficient join plans. For instance, [25] uses characteristic sets [24] and pairs [50] to index the data in the federation and combines this with Dynamic Programming (DP) to optimize query execution plans. Furthermore, [46] proposes an interface for processing federated queries over heterogeneous LDF interfaces. To achieve this, the query optimizer is adapted to the characteristics of the different interfaces as well as the locality of the data, i.e., knowledge of which nodes hold which data. Inspired by these approaches, LOTHBROK fragments knowledge graphs based on characteristic sets and uses a similar cardinality estimation technique to optimize join plans in consideration of data locality in the network.

## 2.3. Peer-to-Peer Systems

Peer-to-Peer (P2P) systems [6–8, 14, 23, 51, 52] tackle the availability issue from a different perspective: by removing the central point of failure completely and replicating the data across multiple nodes in a P2P network, they can ensure the data remains available even if the original node that uploaded the data fails. As such, they consist of a set of nodes (often resource limited) that act both as servers and clients, maintaining a limited local datastore. The structure of the network, i.e., connections between the nodes, as well as data placement (data allocation), varies from system to system. For instance, some systems [8, 14, 51] enforce data placement by applying a structured overlay over the network, such as Dynamic Hash Tables (DHTs) [53]. On the other hand, PIQNIC [6] imposes no structure on top the network; nodes are connected randomly to a set of neighbors that are shuffled periodically with another node’s neighbors to increase the degree of joinability between the fragments of neighboring nodes. Lastly, COLCHAIN [7] extends PIQNIC and divides the entire network into smaller communities of nodes that collaborate on keeping certain data available and up-to-date. By applying community-based ledgers of updates and relying on a consensus protocol within a community, COLCHAIN lets users actively participate in keeping the data up-to-date.

Each P2P system has different ways of processing queries. For instance, due to the lack of global knowledge over the network, basic P2P systems have to flood the network with requests for a given horizon to increase the likelihood of receiving complete query results. To counteract this, distributed indexes [19, 29, 54] like Prefix-Partitioned Bloom

Filter (PPBF) indexes [19] determine which nodes may include relevant data for a given query and thus allow the system to prune nodes from consideration during query optimization. Yet, the aforementioned systems still experience a significant overhead partly caused by inaccurate cardinality estimations, query optimization that does not consider the locality of data, as well as data fragmentation that splits up closely related data. For instance, PIQNIC and COLCHAIN both use a predicate-based fragmentation strategy that creates a fragment for each predicate. This, together with the replication and allocation strategy used, means that data relevant to a single query is distributed over a significant number of fragments and nodes.

However, while an approach that maximizes the degree to which entire queries can be processed by one node can lower the communication overhead, distributing some of the query processing load across multiple nodes is equally important when optimizing queries in a decentralized context [20] to avoid overloading individual nodes. As such, LOTHBROK limits the communication overhead by fragmenting data based on characteristic sets and introducing a new indexing scheme that lets nodes take advantage of the fragmentation to more accurately estimate subquery cardinality and distribute the processing of subqueries to nodes in the network based on data locality. Furthermore, since fragments are created based on characteristic sets, entire star patterns can be processed efficiently by single nodes, further distributing the query processing load, lowering the communication overhead at the same time, and increasing the query throughput.

### 3. Background

A commonly used format for storing semantic data is the Resource Description Framework (RDF) [55]. RDF structures data as triples, defined as follows.

**Definition 1** (RDF Triple). *Let  $I$ ,  $B$ , and  $L$  be the disjoint sets of IRIs, blank nodes, and literals. An RDF triple is a triple  $t$  of the form  $t = (s, p, o) \in (I \cup B) \times I \times (I \cup B \cup L)$ , where  $s$ ,  $p$ , and  $o$  are called subject, predicate, and object.*

Given the definition of an RDF triple, a *knowledge graph*  $\mathcal{G}$  is a finite set of RDF triples. The most popular language to query knowledge graphs is SPARQL [56]. A SPARQL query consists of one or more *triple patterns*. A triple pattern  $t$  is a triple of the form  $t = (s, p, o) \in (I \cup B \cup V) \times (I \cup V) \times (I \cup B \cup L \cup V)$  where  $V$  is the set of all variables. A Basic Graph Pattern (BGP) is a set of conjunctive triple patterns. Without loss of generality, we focus our discussion in the main part of this paper on BGPs and describe in Section 5 how our approach can support other operators, such as UNION and OPTIONAL; our experimental evaluation in Section 7 includes queries with a variety of SPARQL operators including UNION and OPTIONAL.

A complex BGP  $P$  can be decomposed into a set of *star patterns*. A star pattern  $P'$  is a set of triple patterns that share the same subject, i.e.,  $\forall t_1 = (s_1, p_1, o_1), t_2 = (s_2, p_2, o_2)$  such that  $t_1, t_2 \in P'$ , it is the case that  $s_1 = s_2$ . Note that while star patterns can be defined as both subject-based and object-based star patterns, for ease of presentation, we focus on subject-based star patterns only since subject-subject joins are much more common in real query loads [57]; LOTHBROK can trivially be adapted to object-based star patterns by using the same principles presented in this paper for object-object joins rather than subject-subject joins.

**Definition 2** (Star Decomposition [11]). *Given a BGP  $P = \{t_1, \dots, t_n\}$  with subjects  $S_P = \{s_1, \dots, s_m\}$ , the star decomposition of  $P$ ,  $\mathcal{S}(P) = \{P_s(P) \mid s \in S_P\}$ , is a set of star patterns  $P_s(P)$  for each  $s \in S_P$ , such that  $P = \bigcup_{s \in S_P} P_s(P)$  where  $P_s(P) = \{(s', p', o') \mid (s', p', o') \in P \wedge s' = s\}$ .*

The answer to a BGP  $P$  over a knowledge graph  $\mathcal{G}$  is a set of *solution mappings*, defined as follows.

**Definition 3** (Solution mapping [7, 9]). *Given a BGP  $P$  and a knowledge graph  $\mathcal{G}$ , the sets  $I_{\mathcal{G}}$ ,  $B_{\mathcal{G}}$ , and  $L_{\mathcal{G}}$  are the sets of IRIs, blank nodes, and literals in  $\mathcal{G}$ , and  $V_P$  is the set of variables in  $P$ , a solution mapping  $\mu$  is a partial mapping  $\mu : V_P \mapsto (U_{\mathcal{G}} \cup B_{\mathcal{G}} \cup L_{\mathcal{G}})$ .*

Given a BGP  $P$  and a solution mapping  $\mu$ , the notation  $\mu[P]$  denotes the triple (patterns) obtained by replacing variables in  $P$  according to the bindings in  $\mu$ . Furthermore, given a knowledge graph  $\mathcal{G}$  and BGP  $P$ ,  $[[P]]_{\mathcal{G}}$  denotes the set of solution mappings that constitute the answer to  $P$  over  $\mathcal{G}$ , i.e.,  $\forall \mu \in [[P]]_{\mathcal{G}}, \mu[P] \in \mathcal{G}$ , and  $\forall T \in \mu[P]$ ,  $T$  is a set of *matching* triples to  $P$ , denoted  $T[P]$ . Furthermore,  $\text{dom}(\mu)$  returns the *domain* of  $\mu$ , i.e., the set of variables that are bound in  $\mu$  and  $\text{vars}(P)$  returns the variables in  $P$ .

### 3.1. Peer-to-Peer

In its simplest form, an unstructured P2P system consists of a set of interconnected nodes that all maintain a local datastore managing a set of (partial) knowledge graphs, where each node maintains a local view over the network, i.e., a set of *neighboring* nodes (nodes within the local view over the network).

Formally, we define a P2P network  $N$  as a set of interconnected nodes  $N = \{n_1, \dots, n_n\}$  where each node maintains a local datastore and a local view over the network. The data uploaded to a node in  $N$  is replicated throughout the network. Furthermore, in line with previous work [7, 19], each node maintains a distributed index describing the knowledge graphs reachable within a certain number of steps (also known as hops), called the *horizon* of a node. A node  $n$  is defined as follows:

**Definition 4** (Node [6, 19]). *A node  $n$  is a triple  $n = (G, I, N_n)$  where:*

- $G$  is the set of knowledge graphs in  $n$ 's local datastore
- $I$  is  $n$ 's distributed index
- $N_n$  is a set of neighboring nodes

While maintaining the structure of the network is important for P2P systems, it is not relevant for the data and query processing techniques that this paper is focusing on. As such, we do not go into detail on network topology, data replication and allocation, and periodic shuffles. Instead, we refer the interested reader to related work such as [6, 7] for more details. In the following, we define data fragmentation and introduce a running example.

In line with previous work [6, 7], and to avoid having to replicate large knowledge graphs throughout the network, LOTHBROK divides knowledge graphs into smaller disjoint *fragments*, i.e., partial knowledge graphs, which can be replicated more easily. Fragments can be obtained using a *fragmentation* function. A fragmentation function is a function that, given a knowledge graph, returns a set of disjoint fragments, and is formally defined as follows:

**Definition 5** (Fragmentation Function [6, 7]). *A fragmentation function  $\mathcal{F}$  is a function that maps a knowledge graph  $\mathcal{G}$  to a set of knowledge graph fragments, i.e.,  $\mathcal{F} : \mathcal{G} \mapsto 2^{\mathcal{G}}$ .*

Different fragmentation functions can have different granularities. For instance, the most coarse-granular fragmentation function is  $\mathcal{F}_C(\mathcal{G}) = \{\mathcal{G}\}$ , i.e., the fragmentation function does not split up the original knowledge graph. COLCHAIN [7] as well as PIQNIC [6] use a *predicate-based* fragmentation function for  $\mathcal{G}$ , i.e.,  $\mathcal{F}_P(\mathcal{G}) = \{\{(s', p', o') \mid (s', p', o') \in \mathcal{G} \wedge p' = p\} \mid \exists s, o : (s, p, o) \in \mathcal{G}\}$ , which creates a fragment for each unique predicate in  $\mathcal{G}$ . LOTHBROK uses a fragmentation function based on characteristic sets [24] (i.e., predicate families) that is detailed in Section 4.2.

The fragments created by the fragmentation function are replicated and allocated at multiple nodes in the network to ensure availability in case the original provider of the knowledge graph becomes unavailable and to enable load balancing. The replication and allocation factor are parameters of the underlying network; for instance, in PIQNIC [6], fragments are replicated and allocated across the node's neighbors, and nodes index all fragments available within a certain horizon. On the other hand, COLCHAIN [7] replicates and allocates fragments at nodes that participate within the same communities. Since this paper focuses on data fragmentation and query optimization, we omit details on data replication and allocation and refer the interested reader to related work [6, 7] for details.

Consider, as a running example, the unstructured P2P network in Figure 1a consisting of five nodes ( $N = \{n_1, \dots, n_5\}$ ) that replicate a total of five fragments ( $f_1, \dots, f_5$ ). In this example, each node maintains a set of two neighbors and each fragment is replicated across two nodes. For instance, node  $n_5$  has  $\{n_2, n_4\}$  as its set of neighbors, and replicates the fragments  $\{f_2, f_4, f_5\}$  in its local datastore. While the running example is based on an unstructured network, such as the one presented in [6], LOTHBROK could be adapted to more structured setups, such as the one presented in [7].

### 3.2. Distributed Indexes

To speed up query processing performance, systems like PIQNIC [6] and COLCHAIN [7] use distributed indexes [19, 54] to efficiently identify nodes holding relevant data for a given SPARQL query. The indexes capture

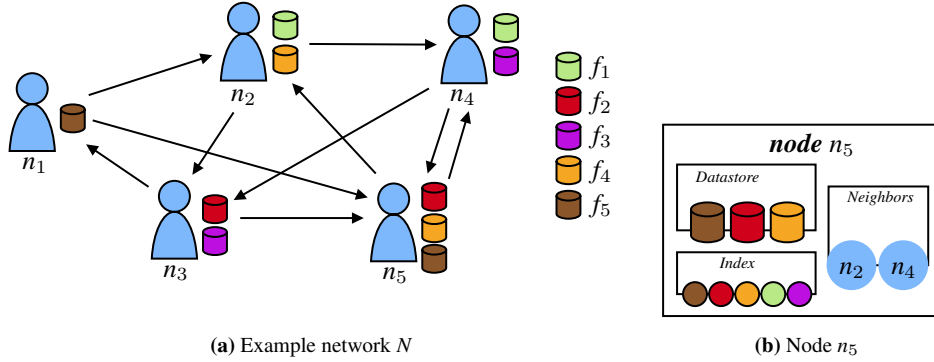


Fig. 1. (a) Example of an unstructured P2P network  $N = \{n_1, \dots, n_5\}$  and (b) architecture of a single node  $n_5$  that indexes data within a horizon of 2 nodes.

information about the fragments stored locally at the node itself as well as information about fragments that can be accessed via its neighbors.

In [7, 19], a distributed index is formally defined as consisting of two mappings; (1) from a triple pattern to the set of fragments containing relevant data to the triple pattern, and (2) from a fragment to the set of nodes that store the fragment. Furthermore, to build the indexes for a node's local view over the network, nodes share partial indexes, i.e., partial mappings, for the fragments that they have access to, called *index slices*. In line with [7, 19], we define distributed indexes and index slices in the following.

**Definition 6** (Distributed Index [7, 19]). *Let  $\mathcal{N}$  be the set of nodes within a network,  $n$  be a node such that  $n \in \mathcal{N}$ ,  $\mathcal{T}$  be the set of all possible triple patterns, and  $\mathcal{F}$  be the set of fragments that  $n$  has access to within its local view over the network. A distributed index on  $n$  is a tuple  $I_n = (\nu, \eta)$  with  $\nu : \mathcal{T} \mapsto 2^{\mathcal{F}}$  and  $\eta : \mathcal{F} \mapsto 2^{\mathcal{N}}$ . For a triple pattern  $t$ ,  $\nu(t)$  returns the set of fragments in  $\mathcal{F}$  that  $t$  matches. For a fragment  $f \in \mathcal{F}$ ,  $\eta(f)$  returns the nodes on which  $f$  is located.*

Given a node  $n$ ,  $n$ 's distributed index is denoted  $I_n$ . Given the definition of a distributed index, we define a *node mapping* as a mapping from a triple pattern  $t$  in a BGP  $P$  to a set of nodes that contain relevant fragments to  $t$ , as follows:

**Definition 7** (Node Mapping [7, 19]). *For any BGP  $P$  and distributed index  $I$ , there exists a function  $match(P, I)$  that returns a node mapping  $M : P \mapsto 2^{\mathcal{N}}$ , such that  $\forall t \in P$ ,  $M(t)$  returns the indexed nodes that have fragments holding data matching the triple  $t$ .*

An index slice for a fragment is a partial mapping from triple patterns to the fragments that contain relevant triples to the triple patterns, as well as a mapping from the fragment to the nodes that replicate it, and is defined as follows:

**Definition 8** (Index Slice [7, 19]). *Let  $f$  be a fragment. The index slice of  $f$ ,  $s_f$ , is a tuple  $s_f = (\nu', \eta')$ , where  $\nu'(t)$  returns  $\{f\}$  if there exists a triple in  $f$  that matches  $t$ , and  $\eta'(f)$  returns the set of all nodes that contain  $f$  in their local datastore. The function  $s(f)$  returns the index slice describing  $f$ .*

Index slices for the fragments that a node has access to are combined into a distributed index for that particular node using the  $\oplus$  operator<sup>1</sup>. The distributed index is then used to check the relevancy and overlap of fragments during query time to optimize the query. Given a set of slices  $S$ , the index obtained by combining the slices in  $S$ ,  $I(S)$ , can be computed using the formula in Equation 1 [7, 19].

$$I(S) = \left( \bigoplus_{s \in S} s.\nu', \bigoplus_{s \in S} s.\eta' \right) \quad (1)$$

<sup>1</sup> $\oplus$  is defined in [7, 19] as  $(f \oplus g)(x) = f(x) \cup g(x)$  if  $f$  and  $g$  are defined at  $x$ ;  $(f \oplus g)(x) = f(x)$  if  $f$  is defined at  $x$ ;  $(f \oplus g)(x) = g(x)$  if  $g$  is defined at  $x$ .



While the definition of distributed indexes allows for several different types of indexes, the index slices used in PIQNIC [6] and COLCHAIN [7] correspond to Prefix-Partitioned Bloom Filters (PPBFs) [19], which extend regular Bloom filters [58]. A Bloom filter  $\mathcal{B}$  for a set  $S$  of IRIs such that  $|S| = n$  is a tuple  $\mathcal{B} = (\hat{b}, H)$  where  $\hat{b}$  is a bitvector of size  $m$  and  $H$  is a set of  $k$  hash functions [19]. Each hash function in  $H$  maps the elements from  $S$  (i.e., IRIs) to a position in  $\hat{b}$ ; these positions are thus set to 1 whereas the positions not mapped to by a function in  $H$  are 0. In other words, [19] represents the combined set of subjects and objects in a fragment in a prefix-partitioned bitvector. Looking up whether an element  $e$  is in  $S$  using the Bloom filter for  $S$  is done by hashing  $e$  using the hash functions in  $H$  and checking the value of each position in  $\hat{b}$ . If at least one of those positions is set to 0, it is certain that  $e \notin S$ . However, if all corresponding bits are set to 1, it is not certain that  $e \in S$ , since it could be a false positive caused by hash collisions, i.e., different values are mapped to the same positions in the underlying bitvector. In this case, we say that  $e$  may be in  $S$ , denoted  $e \in S$ .

To check the compatibility of two fragments relevant for conjunctive triple patterns, we check whether or not they produce any join results. To do this, we could check whether or not the intersection of the bitvectors describing the subjects and objects of the fragments is empty (i.e., if they have some IRI in common). Given two Bloom filters  $\mathcal{B}_1 = (\hat{b}_1, H)$  and  $\mathcal{B}_2 = (\hat{b}_2, H)$ , the intersection of  $\mathcal{B}_1$  and  $\mathcal{B}_2$  is approximated by the logic AND operation between  $\hat{b}_1$  and  $\hat{b}_2$ ,  $\mathcal{B}_1 \cap \mathcal{B}_2 \approx \hat{b}_1 \& \hat{b}_2$ .

To avoid exceedingly large bitvectors, PPBFs partition the bitvector based on the prefix of the IRIs. A PPBF is formally defined in [19] as follows.

**Definition 9** (Prefix-Partitioned Bloom Filter [19]). A PPBF  $\mathcal{B}^P$  is a 4-tuple  $\mathcal{B}^P = (P, \hat{B}, \theta, H)$  where

- $P$  a set of prefixes
- $\hat{B}$  is a set of bitvectors such that  $\forall \hat{b}_1, \hat{b}_2 \in \hat{B} : |\hat{b}_1| = |\hat{b}_2|$
- $\theta : P \rightarrow \hat{B}$  is a prefix-mapping function
- $H$  is a set of hash functions

For each  $p_i \in P$ ,  $\mathcal{B}_i = (\theta(p_i), H)$  is the Bloom Filter that encodes the names of the IRIs with prefix  $p_i$  and is called a partition of  $\mathcal{B}^P$ .

Consider the example where the IRI `dbr:Copenhagen` is inserted into a PPBF, visualized in Figure 2a. In this case, the IRI is matched to the prefix `dbr`, and the IRI is hashed using each hash function in the PPBF; each corresponding bit in the bitvector for the `dbr` prefix is thus set to 1.

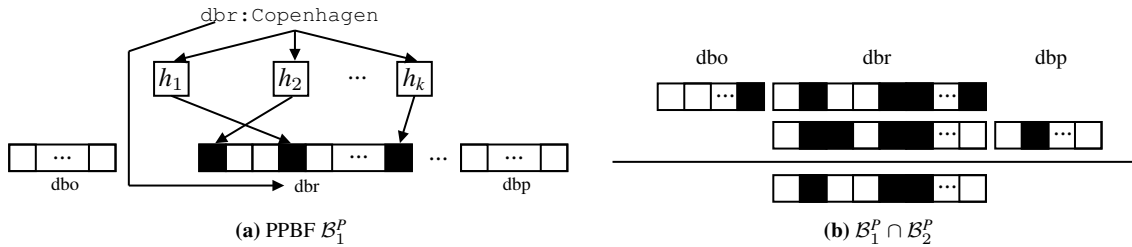


Fig. 2. Example of (a) inserting an IRI into a PPBF  $\mathcal{B}_1^P$  and (b) intersection between two PPBFs  $\mathcal{B}_1^P \cap \mathcal{B}_2^P$  [19].

Like for regular Bloom filters, we say that an IRI  $i$  with prefix  $p$  may be in a PPBF  $\mathcal{B}^P$ , denoted  $i \in \mathcal{B}^P$ , if and only if all positions given by  $h(i)$  such that  $h \in H$  are set to 1 in the bitvector  $\theta(p)$ . PPBFs are used by PIQNIC and COLCHAIN to prune non-overlapping fragments of joining triple patterns from the query execution plan (i.e., the  $match(P, I)$  function in Definition 7). This is done by finding the intersection of the two PPBFs to check whether or not they overlap; if the intersection of the two PPBFs is empty, the corresponding fragments do not produce any join results. The PPBF intersection is defined in [19] as follows.

**Definition 10** (Prefix-Partitioned Bloom Filter Intersection [19]). The intersection of two PPBFs with the same set of hash functions  $H$  and bitvectors of the same size, denoted  $\mathcal{B}_1^P \cap \mathcal{B}_2^P$ , is  $\mathcal{B}_1^P \cap \mathcal{B}_2^P = (P_\cap, \hat{B}_\cap, \theta_\cap, H)$ , where  $P_\cap = \mathcal{B}_1^P.P \cap \mathcal{B}_2^P.P$ ,  $\hat{B}_\cap = \{\mathcal{B}_1^P.\theta(p) \& \mathcal{B}_2^P.\theta(p) \mid p \in P_\cap\}$ , and  $\theta_\cap : P_\cap \rightarrow \hat{B}_\cap$ .

Consider the example intersection visualized in Figure 2b. As described above, the intersection of two PPBFs is the bitwise AND operation on the bitvectors for the prefixes that  $\mathcal{B}_1^P$  and  $\mathcal{B}_2^P$  have in common. In this example,  $\mathcal{B}_2^P$  does not have a bitvector with the prefix `dbp`, thus this partition is omitted from the intersection. Similarly, the bitvector partition with the `dbo` prefix is omitted. Since both PPBFs have bitvectors for the `dbp` prefix, the resulting PPBF has one partition for the `dbp` prefix that is a result of the bitwise AND operation between the two corresponding partitions in  $\mathcal{B}_1^P$  and  $\mathcal{B}_2^P$ .

#### 4. The LOTHBROK Approach

Differently from PIQNIC and COLCHAIN, LOTHBROK uses a fragmentation strategy based on characteristic sets. To accommodate efficient query processing over such fragments, as well as to enable locality-awareness and more accurate cardinality estimation, LOTHBROK introduces an indexing scheme that maps star patterns to fragments rather than triple patterns. In the remainder of this section, we provide a brief overview of the LOTHBROK architecture and how LOTHBROK optimizes SPARQL queries over decentralized knowledge graphs, followed by a formal definition of the fragmentation and indexing approach. Query optimization with details on how to exploit locality-awareness and join ordering are explained in Section 5.

##### 4.1. Design and Overview

LOTHBROK introduces three contributions, that altogether decrease the communication overhead and in doing so increases query processing performance. First, LOTHBROK creates fragments based on characteristic sets such that entire star patterns can be answered by a single fragment. This is beneficial since, as we discussed in Section 1, such star patterns are relatively efficiently processed by the nodes [31] and reduce the communication overhead. The characteristic set of a subject value (entity) is the set of predicates that occur in triples with that subject. As such, LOTHBROK creates one fragment per unique characteristic set and each fragment thus contains all the triples with the subjects that match the characteristic set of the fragment. Consider, for instance, the example network in Figure 1 and query  $Q$  shown in Figure 3a. Table 3b shows the characteristic sets of each fragment in the network. Using this fragmentation method, each fragment can provide answers to entire star patterns; for instance,  $P_3 \in \mathcal{S}(Q)$  can be processed over just  $f_5$ , since it is the only fragment containing triples with both predicates present in  $P_3$ . The formal definition of the fragmentation approach is presented in Section 4.2.

```
select * where {
  ?person dbo:nationality ?country . # tp1 (P1)
  ?person dbo:author ?publication . # tp2 (P1)
  ?country dbo:capital ?capital . # tp3 (P2)
  ?country dbo:currency ?currency . # tp4 (P2)
  ?publication dbo:publisher ?publisher . #tp5 (P3)
  ?publication dbp:language ?language . #tp6 (P3)
}
```

(a) Query  $Q$

| Fragment | CS                                                                                      |
|----------|-----------------------------------------------------------------------------------------|
| $f_1$    | { <code>dbo:nationality</code> , <code>dbo:author</code> , <code>dbo:deathDate</code> } |
| $f_2$    | { <code>dbo:nationality</code> , <code>dbo:author</code> }                              |
| $f_3$    | { <code>dbo:capital</code> , <code>dbo:currency</code> , <code>dbo:population</code> }  |
| $f_4$    | { <code>dbo:capital</code> , <code>dbo:currency</code> }                                |
| $f_5$    | { <code>dbo:publisher</code> , <code>dbo:language</code> }                              |

(b) CSs of each fragment in the running example

Fig. 3. (a) Example SPARQL query  $Q$  and (b) corresponding characteristic sets in the example network.

Second, to accommodate processing entire star patterns over individual fragments, and to encode structural information that can be used for cardinality estimation and locality awareness, LOTHBROK introduces a novel indexing scheme, called Semantically Partitioned Blooms Filter (SPBF) Indexes, that builds upon the Prefix-Partitioned Bloom Filter (PPBF) indexes presented in [19]. In particular, SPBFs partition the bitvectors based on the IRI's position in the fragment, i.e., whether it is a subject, predicate, or object. For instance, in the running example, the SPBF for  $f_5$  contains a partition encoding all the subjects with the characteristic set

$\{dbr:publisher, dbr:language\}$ , as well as partitions encoding all the objects in  $f_5$  that occur in a triple with each predicate. The formal definition of SPBF indexes is discussed in Section 4.3.

Third, LOTHBROK proposes a query optimization technique that takes advantage of the fragmentation based on characteristic sets and the SPBF indexes to estimate cardinalities and consider data locality while optimizing the query execution plan. First, LOTHBROK builds a *compatibility graph* using the SPBF indexes that describes, for a given query, which fragments are compatible with one another for each star join in the query (i.e., which fragments may produce results for the joins). Then, LOTHBROK builds a query execution plan using a Dynamic Programming (DP) algorithm that considers the compatibility of fragments in the compatibility graph and the locality of the fragments in the index.

In the remainder of this section, we detail data fragmentation (Section 4.2) and indexing (Section 4.3) in LOTHBROK. Section 5 details the query optimization approach used by LOTHBROK.

#### 4.2. Data Fragmentation

As discussed in Section 1, star-shaped subqueries can be processed relatively efficiently over a fragment [31], thus they can also help achieving a better balance between reducing the communication overhead and distributing the query processing load [10–12]. To facilitate processing such star patterns on single nodes, we propose to fragment the uploaded knowledge graphs based on *characteristic sets* [10, 12, 24]. Formally, a characteristic set is defined as follows:

**Definition 11** (Characteristic Set [10, 12, 24]). *The characteristic set for a subject  $s$  in a given knowledge graph  $\mathcal{G}$ ,  $C_{\mathcal{G}}(s)$ , is the set of predicates associated with  $s$ , i.e.,  $C_{\mathcal{G}}(s) = \{p \mid (s, p, o) \in \mathcal{G}\}$ . The set of characteristic sets of a knowledge graph  $\mathcal{G}$  is  $C(\mathcal{G}) = \{C_{\mathcal{G}}(s) \mid (s, p, o) \in \mathcal{G}\}$ .*

In other words, the characteristic set of a subject is the set of predicates (i.e., predicate combination) used to describe the subject, i.e., that occur in the same triples as the subject. For instance, if the triples  $(dbr:Denmark, dbr:capital, dbr:Copenhagen)$  and  $(dbr:Denmark, dbr:currency, dbr:Danish_Krone)$  are the only ones with subject  $dbr:Denmark$ , then this subject is described by the characteristic set  $\{dbr:capital, dbr:currency\}$ .

Characteristic sets were first introduced in [24], used for cardinality estimation and, in extension of that, join ordering. WiseKG [10] and Smart-KG [12] used the notion of characteristic sets for fragmentation of knowledge graphs in LDF systems to balance the query load between clients and servers. In this paper, we use characteristic set based fragments as an alternative to the purely predicate-based fragmentation used by for example PIQNIC. We define the characteristic set based fragmentation function as follows:

**Definition 12** (Characteristic Set Fragmentation Function). *Let  $\mathcal{G}$  be a knowledge graph, then the characteristic set fragmentation function of  $\mathcal{G}$ ,  $\mathcal{F}_C(\mathcal{G})$ , is defined using the notation introduced in Definition 11, as:*

$$\mathcal{F}_C(\mathcal{G}) = \{\{(s, p, o) \mid (s, p, o) \in \mathcal{G} \wedge C_{\mathcal{G}}(s) = C_i\} \mid C_i \in C(\mathcal{G})\} \quad (2)$$

That is, the characteristic set fragmentation function creates a fragment for each characteristic set in the knowledge graph. In the characteristic sets shown in Figure 3b,  $f_4$  thus contains all triples of all subjects that are described by the characteristic set  $\{dbr:capital, dbr:currency\}$ .

LOTHBROK nodes can then use these fragments to process entire star patterns. However, for relatively unstructured knowledge graphs, using fragmentation purely based on characteristic sets can lead to an unwieldy number of fragments. For instance, in our experimental evaluation in Section 7, fragmenting the data from LargeRDF-Bench [32] using Equation 2 led to 181,859 distinct fragments, most of which contain very few subjects. Usually, these fragments are created for subjects that are unique due to one or two predicates, while the remaining predicates could fit into larger fragments.

Consider, for instance, in the running example, the situation where the following five characteristic sets are found in the uploaded knowledge graph; for illustration purposes we have extended the notation with the number of subjects covered by each characteristic set:

$$\begin{aligned}
CS_1 &= (\{\text{dbo}:\text{nationality}, \text{dbo}:\text{author}, \text{dbo}:\text{deathDate}\}, 500) \\
CS_2 &= (\{\text{dbo}:\text{nationality}, \text{dbo}:\text{author}\}, 500) \\
CS_3 &= (\{\text{dbo}:\text{publisher}, \text{dbo}:\text{language}\}, 1000) \\
CS_4 &= (\{\text{dbo}:\text{nationality}, \text{dbo}:\text{author}, \text{dbo}:\text{language}\}, 2) \\
CS_5 &= (\{\text{dbo}:\text{nationality}\}, 1)
\end{aligned}$$

In this case, a separate fragment is created for  $CS_4$  even though it does not carry very much information because it describes only two subjects. While this is not such a big issue in terms of space, it affects the lookup time when optimizing the join order and estimating the cardinalities, since the query processor has to consider potentially thousands of such small fragments. As such, and similar to [24], we merge infrequent characteristic sets into fragments with a larger number of subjects. After fragmenting datasets using Equation 2, we apply a strategy with two sequential steps for fragments with infrequent characteristic sets.

First, we merge a fragment  $f_1$  with characteristic set  $CS_1$  into a fragment  $f_2$  with characteristic set  $CS_2$  if  $CS_1 \subseteq CS_2$  by adding the triples of  $f_1$  to  $f_2$ ; if there are multiple candidates for  $f_2$ , we select the one with the smallest set of predicates. In the example above, for instance, we merge  $CS_5$  into  $CS_2$  by adding the subject from the fragment with  $CS_5$  to the fragment with  $CS_2$ .

Second, we split fragments  $f$  with infrequent characteristic sets into two separate fragments,  $f_1$  and  $f_2$ , such that  $f_1$  and  $f_2$  can be merged into other fragments with more frequent characteristic sets. In the example above, we thus split the fragment with  $CS_4$  into two smaller fragments  $f'_4$  and  $f''_4$  such that  $f'_4$  has the characteristic set  $\{\text{dbo}:\text{nationality}, \text{dbo}:\text{author}\}$  and  $f''_4$  has the characteristic set  $\{\text{dbo}:\text{language}\}$ ;  $f'_4$  is then merged into the fragment with  $CS_2$  and  $f''_4$  is merged into the fragment with  $CS_3$ . For example, in the example above, we end up with the following fragments:

$$\begin{aligned}
CS_1 &= (\{\text{dbo}:\text{nationality}, \text{dbo}:\text{author}, \text{dbo}:\text{deathDate}\}, 500) \\
CS_2 &= (\{\text{dbo}:\text{nationality}, \text{dbo}:\text{author}\}, 503) \\
CS_3 &= (\{\text{dbo}:\text{publisher}, \text{dbo}:\text{language}\}, 1002)
\end{aligned}$$

#### 4.3. Semantically Partitioned Bloom Filter Indexes

The indexing schema presented in [19] (Definition 6) represents the set of subject and object values as prefix-partitioned bitvectors based on Bloom filters [58] called Prefix-Partitioned Bloom Filters (PPBFs). However, PPBFs encode the entire set of subjects and objects in a fragment as a single set and ignore the position (subject or object) of the IRIs in the triples; as such, in a situation where two fragments, for instance, use the same IRIs in the object position, the intersection of the two PPBFs is non-empty. Then, if the corresponding triple patterns in the query are joined with a subject-object join, the fragments are not pruned since the PPBFs overlap; however, since we are looking for a subject-object join rather than an object-object join, these fragments could have been pruned without affecting the query completeness. Furthermore, PPBF indexes do not include the predicate values in the index slices, rather they associate the predicate value with the index slice itself (Definition 9), thus maintaining information about the links between the subjects, predicates, and objects. This is possible since the implementations of PIQNIC and COLCHAIN use the predicate-based fragmentation function; however, LOTHBROK allows for fragments with several distinct predicates.

Hence, to efficiently estimate whether or not fragments join for a particular query and to maintain the connection between the subjects, predicates, and objects for fragments with multiple predicates, we propose an indexing schema called *Semantically Partitioned Bloom Filters* (SPBFs), which builds upon PPBF as baseline. As the triples contained in fragments defined based on characteristic sets (Section 4.2) share the same subjects, SPBFs encode the

subject values in a single prefix-partitioned bitvector, while there is one prefix-partitioned bitvector for each predicate in the fragment that encodes the objects occurring in triples with that predicate. For instance, in the running example, each subject within  $f_2$  occurs in triples with both `dbo:nationality` and `dbo:author` as predicates. The SPBF for  $f_2$  contains one partition describing the subject values, one partition describing the object values connected with the `dbo:nationality` predicate, and one partition describing the object values connected with the `dbo:author` predicate. Formally, an SPBF is defined as follows:

**Definition 13** (Semantically Partitioned Bloom Filter). *An SPBF  $\mathcal{B}^S$  is a 5-tuple  $\mathcal{B}^S = (P, \mathcal{B}_s, B_o, \Phi, H)$  where:*

- $P$  is a set of distinct predicate values
- $\mathcal{B}_s$  is the prefix-partitioned bitvector that summarizes the subjects
- $B_o$  is the set of prefix-partitioned bitvectors that summarize the objects
- $\forall \mathcal{B}_i \in \{\mathcal{B}_s\} \cup B_o, \mathcal{B}_i = (P_i, \hat{B}_i, \theta_i)$  where:
  - \*  $P_i$  is a set of prefixes
  - \*  $\hat{B}_i$  is a set of bitvectors such that  $\forall \hat{b}_1, \hat{b}_2 \in \hat{B}_i : |\hat{b}_1| = |\hat{b}_2|$
  - \*  $\theta_i : P_i \rightarrow \hat{B}_i$  is a prefix-mapping function
- $\Phi : P \rightarrow B_o$  is a predicate-mapping function such that  $\forall p \in P : \Phi(p) \in B_o$
- $H$  is a set of hash functions

Similarly to prefix-partitioned bitvectors, we say that an IRI  $i$  at position  $\rho \in \{s, p, o\}$  may be in an SPBF  $\mathcal{B}^S$ , denoted  $i \in^p \mathcal{B}^S$ , if and only if  $i \in \mathcal{B}^S.\mathcal{B}_s$  if  $\rho = s$ ,  $\exists p \in \mathcal{B}^S.P : i \in \mathcal{B}^S.\Phi(p)$  if  $\rho = o$ , or  $i \in \mathcal{B}^S.P$  if  $\rho = p$ . Furthermore,  $\mathcal{B}_p(\mathcal{B}^S)$  denotes a function that computes and returns the prefix-partitioned bitvector that contains all predicates in  $\mathcal{B}^S.P$ . Given a fragment  $f$ ,  $\mathcal{B}^S(f)$  describes the SPBF for  $f$ .

Consider again the running example from Figure 1. Figure 4 shows the SPBFs of fragments  $f_1$  (Figure 4a) and  $f_4$  (Figure 4b). The SPBF for  $f_1$  contains a prefix-partitioned bitvector that encodes all the subject values in  $f_1$ ,  $\mathcal{B}^S(f_1).\mathcal{B}_s$ , as well as a prefix-partitioned bitvector for each predicate that encodes the object values that are connected with the predicates, i.e., the partition  $\mathcal{B}^S(f_1).\Phi(\text{dbo:author})$  that describes the objects that are connected with the `dbo:author` predicate, and so on. Similar for the SPBF for  $f_4$ ,  $\mathcal{B}^S(f_4)$ .

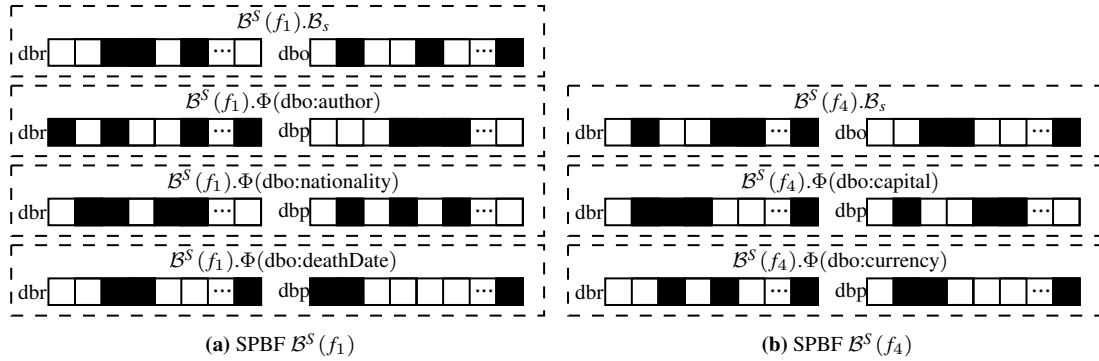


Fig. 4. SPBFs of  $f_1$ ,  $\mathcal{B}^S(f_1)$  (a) and  $f_4$ ,  $\mathcal{B}^S(f_4)$  (b) in the running example.

A distributed index as defined in Definition 6 and [7, 19] associates triple patterns in the query with fragments that contain relevant data to the triple patterns. However, since LOTHBROK partitions data based on characteristic sets, we adapt the definition of a distributed index to the fragmentation based on characteristic sets and SPBF indexes. Let  $\text{relevantFragment}(P, f)$  be a function that returns `true` if  $\forall t = (s, p, o) \in P, s \in V$  or  $s \in^s \mathcal{B}^S(f), p \in V$  or  $p \in^p \mathcal{B}^S(f)$ , and  $o \in V$  or  $o \in \mathcal{B}^S(f).\Phi(p)$ , or `false` otherwise. We define an SPBF index as follows:

**Definition 14** (Semantically Partitioned Bloom Filter Index [7, 19]). *Let  $n$  be a node and  $\mathcal{N}$  be the set of nodes within  $n$ 's local view of the network,  $\mathcal{P}$  be the set of all possible star patterns, and  $\mathcal{F}$  be the set of fragments stored by at least one node in  $\mathcal{N}$ . The SPBF index on  $n$  is a tuple  $I_n^S = (v, \eta)$  with  $v : \mathcal{P} \mapsto 2^{\mathcal{F}}$  and  $\eta : \mathcal{F} \mapsto 2^{\mathcal{N}}$ .  $v(P)$  returns the set of fragments  $F$  such that  $\forall f \in F, \text{relevantFragment}(P, f) = \text{true}$ .  $\eta(f)$  returns the set of nodes  $N$  such that  $f \in n_i.G, \forall n_i \in N$  and  $n_i \in \mathcal{N}$ .*

In other words, an SPBF index maps a star pattern to the fragments that may contain all the constants within the star pattern, and the fragments to the nodes that store them. Furthermore, since LOTHBROK, like PIQNIC and COLCHAIN, builds partial indexes, i.e., slices, for each fragment that are combined to form the node's distributed index, we define an SPBF index slice as follows:

**Definition 15** (SPBF Slice). *Let  $f$  be a fragment. The SPBF slice describing  $f$  is a tuple  $s_f^S = (v', \eta')$  where  $v'(P)$  returns  $\{f\}$  if and only if  $\text{relevantFragment}(P, f) = \text{true}$ , and  $\eta'(f)$  returns the set of all nodes that contain  $f$  in its local datastore.*

The function  $s^S(f)$  finds the SPBF slice describing  $f$ . The SPBF slice describing a fragment is the SPBF obtained from the respective fragment. For instance, in the running example, the SPBF slice of  $f_1$  corresponds to the SPBF obtained from  $f_1$ , i.e., the one in Figure 4a. In Section 5, we detail how SPBF indexes are used to optimize queries using cardinality estimations and the locality of the data.

## 5. Query Optimization

To optimize queries over the network, LOTHBROK first determines which fragments are compatible i.e., produce join results for the given query. This is to prune fragments that would not contribute to the overall query result. To do this, LOTHBROK builds a graph that includes the fragments that are compatible for star patterns in the given query, called a *compatibility graph*. In other words, the nodes in a compatibility graph are fragments, and the edges connect the compatible ones.

Compatibility graphs encapsulate two things. First, the fragments within a compatibility graph are the fragments that contribute to the overall query result, i.e., fragments that do not contribute to the result are pruned. Second, different branches of a compatibility graph for the same subqueries can be processed in parallel. Take, for instance, again query  $Q$  in Figure 3a. In this case, assuming the join order  $P_2 \bowtie P_1 \bowtie P_3$  (details on join order optimization in Section 5.3) and the compatibility of the fragments given in Figure 5g (details on compatibility graphs in Section 5.1), then subquery  $P_2 \bowtie P_1$  could be processed concurrently over  $\{f_1, f_4\}$  and  $\{f_2, f_3\}$  since  $f_1$  only depends on the intermediate results from  $f_4$  and  $f_2$  only depends on the intermediate results from  $f_3$ . Hence, it could be beneficial to process  $P_1 \cup P_2$  by delegating the subquery to nodes  $n_2$  and  $n_3$  concurrently such that  $n_2$  processes  $[[P_2]]_{f_4} \bowtie [[P_1]]_{f_1}$  locally and  $n_3$  processes  $[[P_2]]_{f_3} \bowtie [[P_1]]_{f_2}$  locally, and using the combined (by union) results as intermediate bindings when processing  $[[P_3]]_{f_5}$  on node  $n_1$ .

To this end, LOTHBROK applies Dynamic Programming (DP) similar to [24, 25] to build a query execution plan specifying join delegations and parallel processing of subqueries. To further decrease the network overhead, we adapt the cost function in the DP algorithm to consider data *locality* and cardinality estimations available using the SPBF indexes. In other words, the cost function estimates, given a query execution plan, how many intermediate results processing the join on a particular node incurs, and selects the execution plan that incurs the least data transfer overhead.

In summary, given a BGP  $P$ , LOTHBROK optimizes  $P$  by applying the following steps:

1. Select the relevant fragments for each star pattern in  $P$  using the SPBF index.
2. Build the compatibility graph  $G^C$  for  $P$  (Section 5.1) by checking the overlap of the corresponding bitvector partitions in the SPBF index.
3. Build a query execution plan using Dynamic Programming (DP) on  $P$  and  $G^C$  in consideration of cardinality estimations (Section 5.2) and data locality (Section 5.3).

The output of the above steps is a query execution plan. In the remainder of this section, we go into details with source selection using compatibility graphs, cardinality estimation, and the query optimization strategy using Dynamic Programming. In Section 6, we describe how a query execution plan is processed.

### 5.1. Fragment and Source Selection

As mentioned above, query optimization in LOTHBROK exploits fragment *compatibility*. To achieve this, nodes build a *compatibility graph* describing which fragments are compatible for a given query. Two fragments are said to be compatible for a given query if the intersection of the corresponding SPBF partitions is non-empty. A compatibility graph is thus an undirected graph where nodes are the relevant fragments for the star patterns in the query (determined using the SPBF index) and edges describe the compatible ones.

Recall the function  $\mathcal{B}^S(f)$  that returns the SPBF for a fragment  $f$ , and let  $\text{vars}(P)$  be a function that returns all the variables in a star pattern  $P$ . Furthermore, given an SPBF  $\mathcal{B}^S$ , a star pattern  $P$ , and a variable  $v$ , let  $\mathcal{B}(\mathcal{B}^S, P, v)$  denote a function that returns  $\mathcal{B}^S$  if  $v$  can only occur once in  $P$ ,  $\mathcal{B}^S \cdot \mathcal{B}_s$  if  $v$  is the subject in  $P$ ,  $\mathcal{B}^S \cdot \Phi(p)$  if  $v$  is the object with predicate  $p$ , i.e.,  $(s, p, v) \in P$ , or  $\mathcal{B}_p(\mathcal{B}^S)$  if  $v$  is a predicate in  $P$ . Then, a compatibility graph of a BGP  $P$  and SPBF index  $I^S$  is formally defined as follows.

**Definition 16** (Compatibility Graph). *Given an SPBF index  $I^S$  and a BGP  $P$ , the compatibility graph  $G^C$  of  $P$  over  $I^S$  is a tuple  $G^C(P, I^S) = (F, C)$  such that  $\forall P_1, P_2 \in \mathcal{S}(P)$  where  $\text{vars}(P_1) \cap \text{vars}(P_2) \neq \emptyset$  and  $\forall v \in \text{vars}(P_1) \cap \text{vars}(P_2)$ , it is the case that  $\forall f_1 \in I^S.v(P_1), f_2 \in I^S.v(P_2)$  where  $\mathcal{B}(\mathcal{B}^S(f_1), P_1, v) \cap \mathcal{B}(\mathcal{B}^S(f_2), P_2, v) \neq \emptyset$ ,  $(f_1, f_2) \in C$  and  $f_1, f_2 \in F$ . Furthermore,  $\forall P' \subseteq P$  where  $\text{vars}(P') \cap \text{vars}(P - P') = \emptyset$  (i.e., for Cartesian products), it is the case that  $\forall f_1 \in F$  such that  $f_1 \in I^S.v(P_1)$  for some  $P_1 \in \mathcal{S}(P')$  and  $\forall f_2 \in F$  such that  $f_2 \in I^S.v(P_2)$  for some  $P_2 \in \mathcal{S}(P - P')$ ,  $(f_1, f_2) \in C$ .*

For instance, in the running example, let  $\mathcal{B}^S(f_1) \cdot \Phi(\text{dbo} : \text{nationality}) \cap \mathcal{B}^S(f_4) \cdot \mathcal{B}_s \neq \emptyset$ , i.e.,  $f_1$  and  $f_4$  produce join results, and  $\mathcal{B}^S(f_1) \cdot \Phi(\text{dbo} : \text{nationality}) \cap \mathcal{B}^S(f_3) \cdot \mathcal{B}_s = \emptyset$ , i.e.,  $f_1$  and  $f_3$  do not overlap. Then, the compatibility graph for query  $Q$  in Figure 3a contains an edge between  $f_1$  and  $f_4$ , but no edge between  $f_1$  and  $f_3$ . We denote the empty compatibility graph (i.e., where  $F$  and  $C$  are empty sets) as  $G_\emptyset^C$ . Algorithm 1 defines the  $G^C(P, I^S)$  function in lines 1-16 that computes a compatibility graph given a BGP  $P$  and SPBF index  $I^S$ .

Figure 5 shows how Algorithm 1 builds the compatibility graph for query  $Q$  in Figure 3a. In the following, we go through each intermediate step of the algorithm, describing the intermediate compatibility graphs built in the process. First, the  $G^C$  function selects the star pattern in  $\mathcal{S}(P)$  with the lowest estimated cardinality in line 2 (cardinality estimation is detailed in Section 5.2). Assume in the running example, that  $P_2$  is the star pattern with the lowest estimated cardinality (Section 5.2), and that it is therefore selected in line 2 as the first star pattern. Furthermore, assume that  $f_1$  is only compatible with  $f_4$  and  $f_2$  is compatible with  $f_3$ .

Then, the relevant fragments for the selected star pattern are found using the  $I^S.v$  function from the SPBF index (Definition 14) and iterated over in the for loop in lines 5-8; for each of these fragments, the function calls the  $\text{buildBranch}(P, I^S, f, P', P_e)$  function in lines 17-30 that builds the (sub)graph starting from the current fragment. In the example, the loop in lines 5-8 iterates over  $\{f_3, f_4\}$ , since these are the fragments relevant for  $P_2$ .

The  $\text{buildBranch}(P, I^S, f, P', P_e)$  function defines a recursive function that builds a sub-graph starting from a specific fragment and star pattern. In the first iteration in the running example (i.e., for  $f_3$ ),  $\text{buildBranch}$  is called with  $P = P_1 \cup P_3$ ,  $f = f_3$ , and  $P' = P_2$  as parameters. First, if  $P$  does not contain any star patterns that join with  $P'$ , i.e., if  $P'$  is the outer-most star pattern in the join tree or for a Cartesian product, the function returns the compatibility graph just containing  $f$  without any edges (lines 18-19). In the example, since  $P_1$  joins with  $P_2$ , the algorithm does not enter the if statement in line 19.

Instead, the for loop in lines 21-29 iterates through the star patterns  $P'' \in P$  that join with  $P'$ , i.e., star patterns that have at least one variable in common. For each fragment  $f'$  relevant for  $P''$  (again found using the SPBF index), the function checks the compatibility of  $f$  and  $f'$  for each join variable  $v$  in line 24, i.e., whether or not  $f$  and  $f'$  may produce join results for each join variable, by intersecting the corresponding partitioned bitvectors in  $\mathcal{B}^S(f)$  and  $\mathcal{B}^S(f')$ . If the fragments may produce join results, a recursive call is made in line 25 with the  $P = P - P''$ ,  $f = f'$ , and  $P' = P''$  as parameters. In the example, the for loop in line 21 has only one iteration for  $P'' = P_1$ , i.e., the only star pattern in  $\mathcal{S}(P)$  that joins with  $P_2$ . Hence, the for loop in line 24 checks the compatibility of each fragment relevant for  $P_1$  ( $f_1$  and  $f_2$ ) with  $f_3$  (since  $f = f_3$  in this call to the function). Since  $f_2$  is compatible with  $f_3$  (cf. the join cardinalities in Table 1), a recursive call is made in line 25 with  $P = P_3$ ,  $f = f_2$ , and  $P' = P_1$ .

Since  $P_3$  joins with  $P_1$ , the for loop in line 24 checks the compatibility of  $f_5$  and  $f_2$  and makes another recursive call to the function in line 25 with  $P = \emptyset$ ,  $f = f_5$ , and  $P' = P_3$ . In this iteration of the function,  $P$  is empty, thus

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**Algorithm 1** Compute the Compatibility Graph of a BGP over an SPBF index
 

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**Input:** A BGP  $P = P_1 \cup \dots \cup P_n$ ; an SPBF index  $I^S = (v, \eta)$   
**Output:** A compatibility graph  $G^C$

```

1: function  $G^C(P, I^S)$ 
2:    $P' \leftarrow P_k$  where  $P_k \in \mathcal{S}(P)$  and  $\text{card}_B(P_k) \leq \text{card}_B(P_j) \forall P_j \in \mathcal{S}(P)$ ;
3:    $P_\epsilon \leftarrow P'$ ;
4:    $F, C \leftarrow \emptyset$ ;
5:   for all  $f \in I^S.v(P')$  do
6:      $G_\epsilon^C \leftarrow \text{buildBranch}(P - P', I^S, f, P', P_\epsilon)$ ;
7:      $F \leftarrow F \cup G_\epsilon^C.F$ ;
8:      $C \leftarrow C \cup G_\epsilon^C.C$ ;
9:   if  $P - P_\epsilon \neq \emptyset$  then
10:     $G_\epsilon^C \leftarrow G^C(P - P_\epsilon, I^S)$ ;
11:    if  $G_\epsilon^C = G_\emptyset^C$  then return  $G_\emptyset^C$ 
12:    for all  $f_1 \in F, f_2 \in G_\epsilon^C.F$  do
13:       $C \leftarrow C \cup \{(f_1, f_2)\}$ ;
14:       $F \leftarrow F \cup G_\epsilon^C.F$ ;
15:       $C \leftarrow C \cup G_\epsilon^C.C$ ;
16:   return  $(F, C)$ ;
17: function  $\text{BUILDBranch}(P, I^S, f, P', P_\epsilon)$ 
18:   if  $P = \emptyset$  or  $\forall P'' \in \mathcal{S}(P) : \text{vars}(P') \cap \text{vars}(P'') = \emptyset$  then
19:     return  $(\{f\}, \emptyset)$ ;
20:    $F, C \leftarrow \emptyset$ ;
21:   for all  $P'' \in \mathcal{S}(P)$  s.t.  $\text{vars}(P') \cap \text{vars}(P'') \neq \emptyset$  do
22:      $P'_\epsilon \leftarrow P_\epsilon \cup P''$ ;
23:      $V \leftarrow \text{vars}(P') \cap \text{vars}(P'')$ ;
24:     for all  $f' \in I^S.v(P'')$  s.t.  $\forall v \in V : \mathcal{B}(\mathcal{B}^S(f), P', v) \cap \mathcal{B}(\mathcal{B}^S(f'), P'', v) \neq \emptyset$  do
25:        $G_\epsilon^C \leftarrow \text{buildBranch}(P - P'', I^S, f', P'', P'_\epsilon)$ ;
26:       if  $G_\epsilon^C \neq G_\emptyset^C$  then
27:          $F \leftarrow F \cup G_\epsilon^C.F \cup \{f\}$ ;
28:          $C \leftarrow C \cup G_\epsilon^C.C \cup \{(f, f')\}$ ;
29:        $P_\epsilon \leftarrow P_\epsilon \cup P'_\epsilon$ ;
30:   return  $(F, C)$ ;

```

---

the graph  $(\{f_5\}, \emptyset)$  is returned in line 19. This graph is visualized in Figure 5a and contains only  $f_5$  with no edges. Since this compatibility graph is non-empty, it is added to the output graph in lines 26-28 together with  $f_2$  (since  $f = f_2$  in this iteration of `buildBranch`) and the edge between  $f_5$  and  $f_2$ . This graph is visualized in Figure 5b and returned by the current iteration of the `buildBranch` function. Upon receiving the graph in Figure 5b, the function adds  $f_3$  (since  $f = f_3$  in the current iteration) and an edge between  $f_2$  and  $f_3$  in lines 26-28, resulting in the compatibility graph shown on Figure 5c that is returned in line 30.

In the next iteration of the for loop in line 5, the `buildBranch` is called with  $P = P_1 \cup P_3$ ,  $f = f_4$ , and  $P' = P_2$ . Following the same procedure as described above for  $f_3$ , we first build the subgraph containing only  $f_5$  shown in Figure 5d. Then,  $f_1$  is added to the graph along with an edge between  $f_1$  and  $f_5$  (since they produce join results), resulting in the subgraph shown in Figure 5e. Next,  $f_4$  is added along with an edge between  $f_4$  and  $f_1$ , resulting in the compatibility graph for  $f_4$  shown in Figure 5f. After merging this in lines 7-8 with the compatibility graph in Figure 5c, the resulting compatibility graph can be seen in Figure 5g.

The if statement in lines 2-5 ensures that subqueries with star patterns that do not join (i.e., in the case of Cartesian products) are included in the compatibility graph. This is done by keeping track of the considered star patterns in  $P$  using the accumulator  $P_\epsilon$  defined in line 3 and updated in line 29. The example query contains no Cartesian products and so the compatibility graph on Figure 5g is returned by the algorithm.



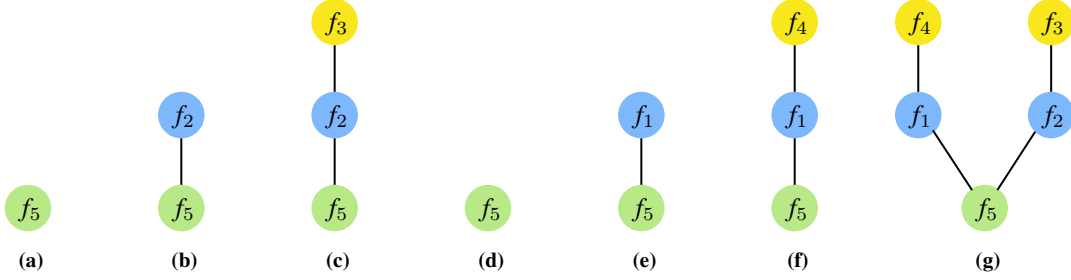


Fig. 5. Recursively building the compatibility graph for the query in Figure 3a by applying Algorithm 1 resulting in  $G^C(Q, I_{n_1}^S)$ . Yellow nodes denote the fragments relevant for  $P_2$ , blue nodes the fragments relevant for  $P_1$ , and the green nodes the fragments relevant for  $P_3$ .

The output of Algorithm 1 in the example is the compatibility graph shown in Figure 5g, specifying that  $f_1$  is compatible with  $\{f_4, f_5\}$  and  $f_2$  is compatible with  $\{f_3, f_5\}$ .

## 5.2. Cardinality Estimation

In Section 4.2 we have described how LOTHBROK fragments knowledge graphs based on characteristic sets. Furthermore, in Section 4.3 we described how SPBF indexes connect the objects in a fragment to the predicates they occur in triples with. Since the SPBF of a fragment includes partitioned bitvectors describing the subjects and objects (Definition 14), we can estimate the number of values within these partitioned bitvectors and use those estimations to obtain cardinality estimations in a similar way as [24, 25]. To achieve this, we first define the estimated number of values in a partitioned bitvector.

Given a partitioned bitvector  $\mathcal{B}$  and  $\hat{b} \in \mathcal{B}.\hat{B}$ , let  $t(\hat{b})$  be a function that returns the number of bits in  $\hat{b}$  that are set. Then, the estimated cardinality of a partitioned bitvector  $\mathcal{B}$ , denoted  $card^P(\mathcal{B})$ , is the sum of the estimated cardinality for all bitvector partitions in  $\mathcal{B}.\hat{B}$  [19, 59] and is formally defined as follows:

$$card^P(\mathcal{B}) = \sum_{\hat{b} \in \mathcal{B}.\hat{B}} \frac{\ln(1 - t(\hat{b})/|\hat{b}|)}{|\mathcal{B}.H| \cdot \ln(1 - 1/|\hat{b}|)} \quad (3)$$

Consider, for instance, again the running example introduced in Section 3.1 and the SPBF for  $f_4$ ,  $\mathcal{B}^S(f_4)$ , in Figure 4b. Assume that  $|\mathcal{B}^S(f_4).H| = 5$  and that  $|\hat{b}| = 20000$  for all  $\hat{b} \in \mathcal{B}^S(f_4).\hat{B}$ . Since the partitioned bitvector for the predicate `dbo:capital` in  $f_4$  (Figure 4b) has two partitions, `dbp` and `dbp`, obtaining the estimated cardinality for  $\mathcal{B}^S(f_4).\Phi(\text{dbo:capital})$  is the sum of estimating the cardinality of both prefix partitions. Let the number of set bits in the bitvector for the `dbp` prefix be 736 and the number of set bits in the bitvector for the `dbp` prefix be 249. Then, the estimated cardinality using Equation 3 is:

$$card^P(\mathcal{B}^S(f_4).\Phi(\text{dbo:capital})) = \frac{\ln(1 - 736/20000)}{5 \cdot \ln(1 - 1/20000)} + \frac{\ln(1 - 249/20000)}{5 \cdot \ln(1 - 1/20000)} \approx \frac{-0.0375}{-0.00025} + \frac{-0.0125}{-0.00025} \approx 150 + 50 \approx 200$$

Table 1 shows the estimated cardinalities of each partitioned bitvector in the running example.

To estimate the cardinality of star-shaped subqueries, we utilize the fact that the subjects are described by a single partitioned bitvector. For a star-shaped subquery asking for the set of unique subject values described by a given set of predicates (i.e., queries with the `DISTINCT` keyword), the cardinality can be estimated as the sum of the number of subjects in each fragment that includes all the predicates in the query. For instance, the cardinality of  $P_1$  in the query in Figure 3a is the number of distinct subject values in  $f_1$  and  $f_2$ .

Table 1  
Estimated cardinalities for the SPBFs  $\mathcal{B}^S(f_1)$ ,  $\mathcal{B}^S(f_2)$ ,  $\mathcal{B}^S(f_3)$ , and  $\mathcal{B}^S(f_4)$  for the running example in Figure 1

| Partitioned Bitvector                                                                   | card <sup>P</sup> | Partitioned Bitvector                                                                   | card <sup>P</sup> |
|-----------------------------------------------------------------------------------------|-------------------|-----------------------------------------------------------------------------------------|-------------------|
| $\mathcal{B}^S(f_1).\mathcal{B}_s$                                                      | 1000              | $\mathcal{B}^S(f_3).\mathcal{B}_s$                                                      | 100               |
| $\mathcal{B}^S(f_1).\Phi(\text{dbo:author})$                                            | 5000              | $\mathcal{B}^S(f_3).\Phi(\text{dbo:capital})$                                           | 100               |
| $\mathcal{B}^S(f_1).\Phi(\text{dbo:nationality})$                                       | 1000              | $\mathcal{B}^S(f_3).\Phi(\text{dbo:currency})$                                          | 150               |
| $\mathcal{B}^S(f_1).\Phi(\text{dbo:deathDate})$                                         | 1000              | $\mathcal{B}^S(f_3).\Phi(\text{dbo:population})$                                        | 100               |
| $\mathcal{B}^S(f_2).\mathcal{B}_s$                                                      | 2000              | $\mathcal{B}^S(f_4).\mathcal{B}_s$                                                      | 200               |
| $\mathcal{B}^S(f_2).\Phi(\text{dbo:author})$                                            | 3000              | $\mathcal{B}^S(f_4).\Phi(\text{dbo:capital})$                                           | 200               |
| $\mathcal{B}^S(f_2).\Phi(\text{dbo:nationality})$                                       | 2000              | $\mathcal{B}^S(f_4).\Phi(\text{dbo:currency})$                                          | 500               |
| $\mathcal{B}^S(f_1).\Phi(\text{dbo:nationality}) \cap \mathcal{B}^S(f_3).\mathcal{B}_s$ | 0                 | $\mathcal{B}^S(f_2).\Phi(\text{dbo:nationality}) \cap \mathcal{B}^S(f_3).\mathcal{B}_s$ | 100               |
| $\mathcal{B}^S(f_1).\Phi(\text{dbo:nationality}) \cap \mathcal{B}^S(f_4).\mathcal{B}_s$ | 50                | $\mathcal{B}^S(f_2).\Phi(\text{dbo:nationality}) \cap \mathcal{B}^S(f_4).\mathcal{B}_s$ | 0                 |
| $\mathcal{B}^S(f_5).\mathcal{B}_s$                                                      | 8000              | $\mathcal{B}^S(f_1).\Phi(\text{dbo:author}) \cap \mathcal{B}^S(f_5).\mathcal{B}_s$      | 500               |
| $\mathcal{B}^S(f_5).\Phi(\text{dbo:publisher})$                                         | 8000              | $\mathcal{B}^S(f_2).\Phi(\text{dbo:author}) \cap \mathcal{B}^S(f_5).\mathcal{B}_s$      | 1000              |
| $\mathcal{B}^S(f_5).\Phi(\text{dbo:language})$                                          | 9000              |                                                                                         |                   |

Given a star pattern  $P$  and a fragment  $f$ , the cardinality of  $P$  over  $f$ , assuming that  $f$  is a relevant fragment for  $P$ , is the number of values in the partitioned bitvector on the subject position in  $\mathcal{B}^S(f)$ , and is formally defined as:

$$\text{card}_D(P, f) = \text{card}^P(\mathcal{B}^S(f).\mathcal{B}_s) \quad (4)$$

For queries not including the `DISTINCT` keyword, we need to account for duplicates by considering, on average, the number of triples for each non-variable predicate value in  $P$  that each subject value is associated with. Given a star pattern  $P$  and fragment  $f$ , let  $\text{preds}(P)$  denote the non-variable predicate values in  $P$  (in the case of a variable on the predicate position in  $P$ , we consider the average number of predicate occurrences in the characteristic set). The cardinality of  $P$  is thus estimated as follows [24, 25]:

$$\text{card}_S(P, f) = \text{card}_D(P, f) \cdot \prod_{p_i \in \text{preds}(P)} \frac{\text{card}^P(\mathcal{B}^S(f).\Phi(p_i))}{\text{card}^P(\mathcal{B}^S(f).\mathcal{B}_s)} \quad (5)$$

Henceforth, we will refer to the more generalized function  $\text{card}$  rather than  $\text{card}_D$  and  $\text{card}_S$  to be equivalent to  $\text{card}_D$  for queries with the `DISTINCT` modifier and  $\text{card}_S$  for queries without. Using Equations 4 or 5, the cardinality of a star pattern  $P$  over a node  $n$ 's SPBF index is, for all queries (both with and without the `DISTINCT` keyword), the aggregated cardinality over each relevant fragment to  $P$ , and is formally defined as follows:

$$\text{card}_n(P) = \sum_{f \in I_n^S, \eta(P)} \text{card}(P, f) \quad (6)$$

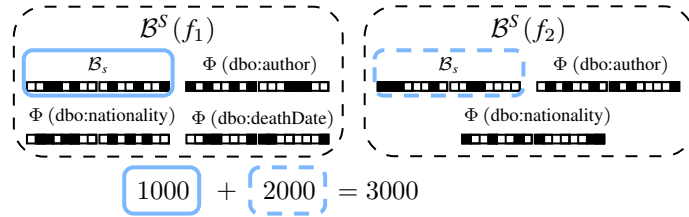


Fig. 6. Estimating the cardinality of  $P_1$  with the `DISTINCT` modifier as the number of subjects in  $f_1$  and  $f_2$  found using Equation 4.

Consider, for instance, in the running example, the star-shaped BGP  $P_1$  in Figure 3a and the estimated cardinalities of the partitioned bitvectors for each fragment in Table 1. Assume in this case that the `DISTINCT` keyword is given in the query. Then,  $card_{n_1}(P_1)$  is computed as the aggregated estimation of subject values in  $f_1$  and  $f_2$ ,  $card_{n_1}(P_1) = 1000 + 2000 = 3000$ . This is visualized in Figure 6.

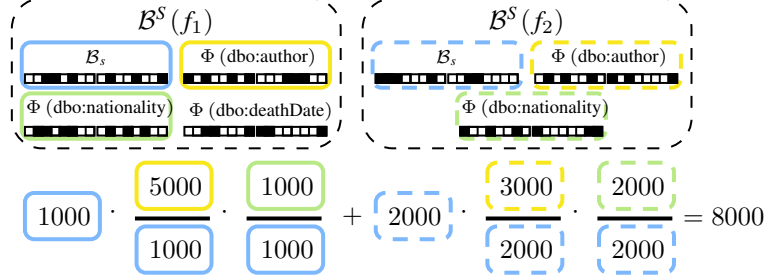


Fig. 7. Estimating the cardinality of  $P_1$  without the `DISTINCT` modifier. Outlines show which bitvector each value is computed from.

If, instead, the `DISTINCT` keyword was not included in the query, the cardinality  $card_{n_1}(P_1)$  is, for each relevant fragment ( $f_1$  and  $f_2$ ), the number of subject values within the fragment multiplied with the average number of triples with each predicate  $p_i \in preds(P_1)$  that each subject value is associated with,  $card_{n_1}(P_1) = 1000 \cdot (5000/1000) \cdot (1000/1000) + 2000 \cdot (3000/2000) \cdot (2000/2000) = 5000 + 3000 = 8000$ . Figure 7 visualizes the above computations and shows which bitvector each value is computed from.

Until now, the cardinality estimations presented in this section are useful for estimating the cardinality of individual star patterns in a query [24, 25]. However, to estimate the cardinality of arbitrary BGPs, [50] introduced characteristic pairs that describe the connections between IRIs described by different characteristic sets. In our case, however, we rely on the SPBFs of the relevant fragments to compute characteristic pairs without storing additional information; by intersecting the partitioned bitvectors on the positions corresponding to the join variable, we can estimate the selectivity of a given join and use that to estimate the cardinality of the join.

Formally, for queries including the `DISTINCT` keyword, given two star patterns  $P_k$  and  $P_l$  that join on a variable  $v$  such that  $(s, p, v) \in P_k$  and  $v$  is the subject of all triple patterns in  $P_l$ , and two fragments  $f_k$  and  $f_l$  that are relevant for  $P_k$  and  $P_l$  respectively, the cardinality of the join is estimated as the number of IRIs on the subject position in  $f_k$  multiplied by the selectivity of the join, i.e., the chance that each subject in the right side corresponds to a value in the join. This is defined as follows:

$$card_D(P_k, P_l, p, f_k, f_l) = card^P(\mathcal{B}^S(f_k).B_s) \cdot \left( \frac{card^P(\mathcal{B}^S(f_k).B_s \cap \mathcal{B}^S(f_l).B_s)}{card^P(\mathcal{B}^S(f_k).B_s)} \right) \quad (7)$$

For queries that do not include the `DISTINCT` keyword, we again consider the average predicate occurrences for each triple pattern in both  $P_k$  and  $P_l$ , similar to Equation 5 [25, 50]:

$$card_S(P_k, P_l, p, f_k, f_l) = card_D(P_k, P_l, p, f_k, f_l) \cdot \prod_{p_k \in P_k - \{p\}} \left( \frac{card^P(\mathcal{B}^S(f_k).B_{p_k})}{card^P(\mathcal{B}^S(f_k).B_s)} \right) \cdot \prod_{p_l \in P_l} \left( \frac{card^P(\mathcal{B}^S(f_l).B_{p_l})}{card^P(\mathcal{B}^S(f_l).B_s)} \right) \quad (8)$$

Processing a join between two star patterns over a node  $n$ 's SPBF is the aggregated cardinality over each pair of relevant fragments to the two star patterns, and is for all queries formally defined as follows:

$$card_n(P_k, P_l, p) = \sum_{f_k \in I_n^S \cdot \eta(P_k) \wedge f_l \in I_n^S \cdot \eta(P_l)} card(P_k, P_l, p, f_k, f_l) \quad (9)$$

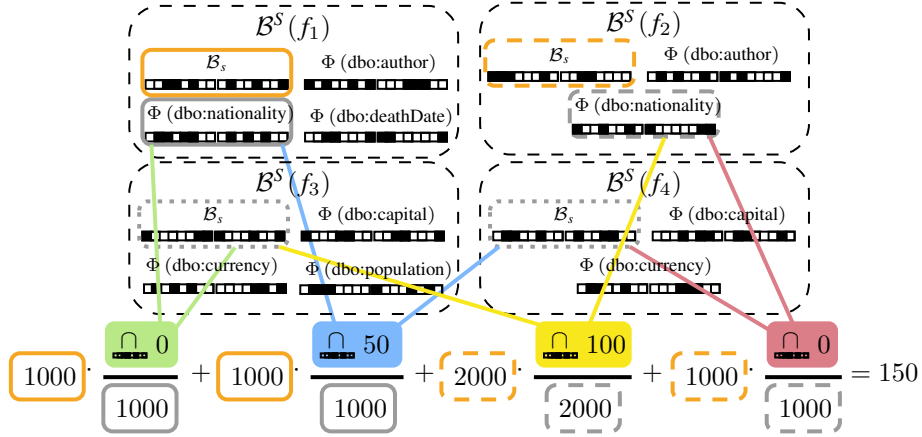


Fig. 8. Estimating the cardinality of  $P_1 \bowtie P_2$  with the DISTINCT modifier.

For instance, consider the join between  $P_1$  and  $P_2$  in Figure 3a where the DISTINCT keyword is given in the query. Here, the cardinality  $card_{n_1}(P_1, P_2, \text{dbo:nationality})$  is the aggregated cardinality of the partitioned bitvectors obtained by intersecting the partitioned bitvector on the object position for the `dbo:nationality` for each  $f_k \in \{f_1, f_2\}$  with the partitioned bitvector on the subject position for each  $f_i \in \{f_3, f_4\}$ . That is, given the cardinalities of the intersections shown in Table 1,  $card_{n_1}(P_1, P_2, \text{dbo:nationality}) = 1000 \cdot (0/1000) + 1000 \cdot (50/1000) + 2000 \cdot (100/2000) + 2000 \cdot (0/1000) = 150$ . We have visualized this computation in Figure 8.

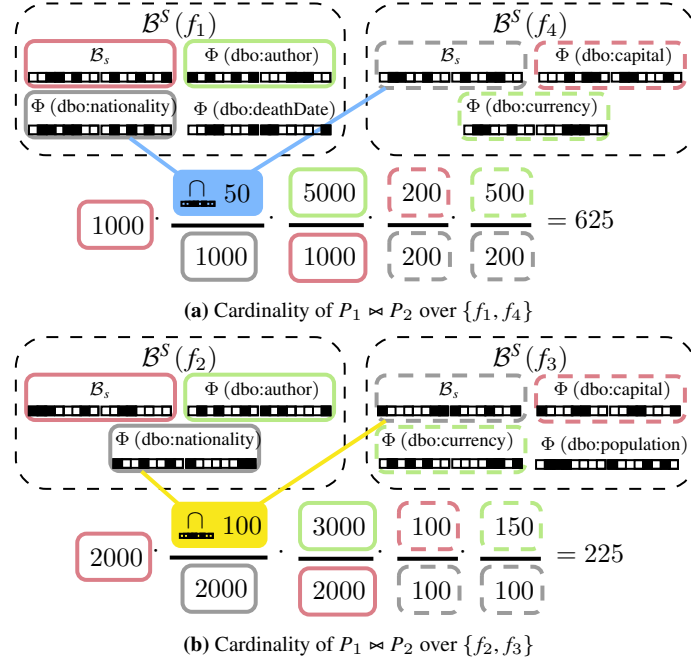


Fig. 9. Estimating the cardinality of  $P_1 \bowtie P_2$  without the DISTINCT modifier over (a)  $\{f_1, f_4\}$  and (b)  $\{f_2, f_3\}$ . The output of Equation 9 is thus the sum of the two formulas ( $625 + 225 = 850$ ).

In the case where the DISTINCT keyword is not included in the query, the join between  $P_1$  and  $P_2$  in Figure 3a given the partitioned bitvector cardinalities in Table 1 yields the following equation:  $card_{n_1}(P_1, P_2, \text{dbo:nationality}) = 1000 \cdot (50/1000) \cdot (5000/1000) \cdot (200/200) \cdot (500/200) + 2000 \cdot (100/2000) \cdot (3000/2000) \cdot (100/100) \cdot (150/100) = 625 + 225 = 850$ . Figure 9 visualizes the above computation; Figure 9a

shows the computation over  $\{f_1, f_4\}$  and Figure 9b over  $\{f_2, f_3\}$ . The outlines show which partitioned bitvector is used to compute each number, e.g., the value 50 is found by computing the cardinality of the bitvector intersection  $\mathcal{B}^S(f_1). \Phi(\text{dbo : nationality}) \cap \mathcal{B}^S(f_4). \mathcal{B}_s$ . The result is the sum of the formulas.

### 5.3. Optimizing Query Execution Plans

Based on the compatibility graph (Section 5.1), the locality of fragments, and cardinality estimations (Section 5.2), LOTHBROK builds a *query execution plan* that specifies which subqueries can be processed in parallel and which joins are delegated to which nodes as well as the join order. A query execution plan is defined as follows:

**Definition 17** (Query Execution Plan). A query execution plan  $\Pi$  consists of the execution plan and the node that processes the plan, called a delegation. A query execution plan can be one of four types:

- Join  $\Pi = \Pi_1 \bowtie^n \Pi_2$  where  $\Pi_1$  and  $\Pi_2$  are two (sub)plans and  $n$  is the node the join is delegated to.
- Cartesian product  $\Pi = \Pi_1 \times^n \Pi_2$  where  $\Pi_1$  and  $\Pi_2$  are two (sub)plans and  $n$  is the node the Cartesian product is delegated to.
- Union  $\Pi = \Pi_1 \cup^n \Pi_2$  where  $\Pi_1$  and  $\Pi_2$  are two (sub)plans and  $n$  is the node the union is delegated to.
- Selection  $\Pi = [[P]]_f^n$  where  $P$  is a star pattern,  $f$  is the fragment that  $P$  is processed over, and  $n$  is the node the selection is delegated to.

Since unions are not explicitly executed by any node, instead the partial results of each subplan in the union are transferred to the nodes that use those intermediate results, we simply omit the specification of delegations for unions from the description below. Furthermore, we assume that query execution plans are always left-deep, i.e., the right side of a join can only consist of a selection or a union of selections. For instance, the execution plan for query  $Q$ ,  $\Pi = ((([[P_2]]_{f_4}^{n_2} \bowtie^{n_2} [[P_1]]_{f_1}^{n_2}) \cup ([[[P_2]]_{f_3}^{n_3} \bowtie^{n_3} [[P_1]]_{f_2}^{n_3})]) \bowtie^{n_1} [[P_3]]_{f_5}^{n_1})$  (Figure 12g) specifies that the join  $[[P_2]]_{f_4} \bowtie [[P_1]]_{f_1}$  is delegated to  $n_2$  and processed in parallel with  $[[P_2]]_{f_3} \bowtie [[P_1]]_{f_2}$  on  $n_3$  (specified by the union), the result of which is transferred to  $n_1$  and joined with  $[[P_3]]_{f_5}$ .

Since our cost function includes the estimated cardinality of a particular subplan, we first extend the framework for cardinality estimation described in Section 5.2 to enable cardinality estimation of an entire query execution plan. This is straightforward for Cartesian products, unions, and selections; for Cartesian products it is the multiplication of the cardinality of the operands, for unions it is the sum of the cardinality of the operands, and for selections it is the cardinality of the star pattern over a specific fragment defined in Equations 4 and 5. Given the reasoning above, we define the cardinality of a query execution plan  $\Pi$ ,  $\text{card}(\Pi)$ , covering all types of  $\Pi$ , as follows:

$$\text{card}(\Pi) = \begin{cases} \text{card}(\Pi_1) \cdot \text{card}(\Pi_2), & \text{if } \Pi = \Pi_1 \times^n \Pi_2 \\ \text{card}(\Pi_1) + \text{card}(\Pi_2), & \text{if } \Pi = \Pi_1 \cup^n \Pi_2 \\ \text{card}(P, f), & \text{if } \Pi = [[P]]_f^n \\ \text{card}(\Pi_1 \bowtie^n \Pi_2), & \text{if } \Pi = \Pi_1 \bowtie^n \Pi_2 \end{cases} \quad (10)$$

To generalize Equation 9 such that we can compute the cardinality of any join  $\Pi = \Pi_1 \bowtie^n \Pi_2$  (e.g., including joins between a BGP with multiple star patterns and a star pattern), we consider two cases: (1) where  $\Pi_2$  is a union  $\Pi_2 = \Pi_2' \cup \Pi_2''$ , and (2) where  $\Pi_2$  is a selection  $\Pi_2 = [[P]]_f^{n_1}$ . The cardinality of the join can thus be estimated using the following formula:

$$\text{card}(\Pi_1 \bowtie^n \Pi_2) = \begin{cases} \text{card}(\Pi_1 \bowtie^n \Pi_2') + \text{card}(\Pi_1 \bowtie^n \Pi_2''), & \text{if } \Pi_2 = \Pi_2' \cup \Pi_2'' \\ \text{card}^{\bowtie}(\Pi_1, P, f), & \text{if } \Pi_2 = [[P]]_f^{n_1} \end{cases} \quad (11)$$

The function  $\text{card}^{\bowtie}(\Pi, P, f)$  in the second case of Equation 11 computes the cardinality of the join for a particular selection on the right side of the join,  $[[P]]_f$ . To achieve this estimation, we consider the estimated cardinality of  $\Pi$  and the selectivity of the join similar to Equation 7. To avoid a significant overestimation due to the possible

correlation between multiple join variables in the same join, we only consider the most selective join variable for any specific join. Recall the  $\mathcal{B}(\mathcal{B}^S, P, v)$  function that returns the partitioned bitvector in  $\mathcal{B}^S$  that corresponds to  $v$ 's position in  $P$ , and let  $S(\Pi, P)$  denote the set of star patterns in  $\Pi$  that join with  $P$  and  $F(\Pi, f)$  denote the set of fragments in  $\Pi$  that join with  $f$ . For instance, for the execution plan in Figure 12d and the compatibility graph in Figure 5g,  $S(\Pi, P_3) = \{P_1\}$  and  $F(\Pi, f_5) = \{f_1, f_2\}$ . Furthermore, given two star patterns  $P_1$  and  $P_2$ , let  $v(P_1, P_2) = \{v \mid v \in \text{vars}(P_1) \cap \text{vars}(P_2)\}$ , i.e., the set of join variables. The cardinality of the join between a plan  $\Pi$  and a selection  $[[P]]_f$  is, given the `DISTINCT` keyword, generalized from Equation 7 as follows:

$$\text{card}_D^s(\Pi, P, f) = \text{card}(\Pi) \cdot \min_{P' \in S(\Pi, P) \wedge v \in v(P, P')} \left( \frac{\sum_{f' \in F(\Pi, f)} \text{card}^P(\mathcal{B}(\mathcal{B}^S(f), P, v) \cap \mathcal{B}(\mathcal{B}^S(f'), P', v))}{\sum_{f' \in F(\Pi, f)} \text{card}^P(\mathcal{B}(\mathcal{B}^S(f'), P', v))} \right) \quad (12)$$

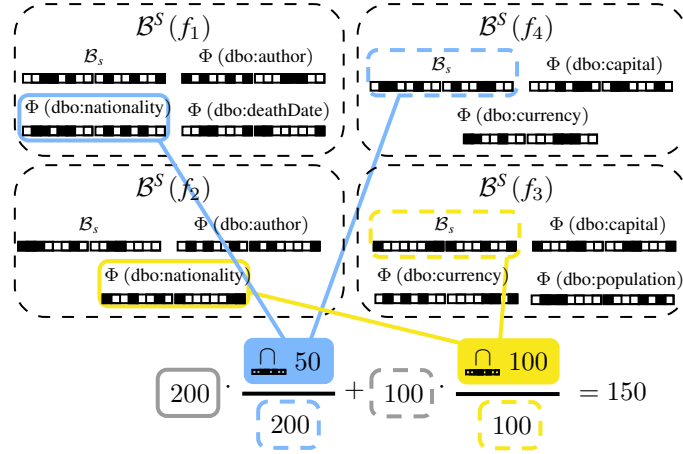


Fig. 10. Estimating the cardinality of  $\Pi = ([P_2]_{f_4}^{n_2} \bowtie^{n_2} [P_1]_{f_1}^{n_2}) \cup ([P_2]_{f_3}^{n_3} \bowtie^{n_3} [P_1]_{f_2}^{n_3})$  with the `DISTINCT` keyword using the cardinalities from Table 1 and Equation 12.

As an example, consider computing the cardinality  $\text{card}(\Pi)$  of the plan  $\Pi$  visualized in Figure 12d using the `DISTINCT` keyword. Since  $\Pi$  is a union, we compute the cardinality of  $\Pi_1 = [P_2]_{f_4}^{n_2} \bowtie^{n_2} [P_1]_{f_1}^{n_2}$  and  $\Pi_2 = [P_2]_{f_3}^{n_3} \bowtie^{n_3} [P_1]_{f_2}^{n_3}$  and let  $\text{card}(\Pi) = \text{card}(\Pi_1) + \text{card}(\Pi_2)$ . Using Equation 12 on  $\Pi_1$  and  $\Pi_2$ , we get the formula  $\text{card}(\Pi) = 200 \cdot (50/200) + 100 \cdot (100/100) = 150$  as visualized in Figure 10 (the gray values are the cardinalities of the left selections in each join obtained using Equation 4).

For queries without the `DISTINCT` keyword, we once again consider the average predicate occurrences. However, since the predicate occurrences in  $\Pi$  are already considered in  $\text{card}(\Pi)$  in Equation 12, we only consider the average number of occurrences in  $f$  for each triple pattern in  $P$  that does not join with  $\Pi$  on the object. The cardinality of the join between a plan  $\Pi$  and selection  $[[P]]_f$ , without the `DISTINCT` keyword, is computed as:

$$\text{card}_S^s(\Pi, P, f) = \text{card}_D^s(\Pi, P, f) \cdot \prod_{p \in \text{preds}(P): (s,p,o) \in P \wedge o \notin v(P, P') \forall P' \in S(\Pi, P)} \left( \frac{\text{card}^P(\mathcal{B}^S(f), \Phi(p))}{\text{card}^P(\mathcal{B}^S(f), \mathcal{B}_s)} \right) \quad (13)$$

Once again, computing the cardinality of  $\Pi$  in Figure 12d not including the `DISTINCT` keyword is  $\text{card}(\Pi) = \text{card}(\Pi_1) + \text{card}(\Pi_2)$ . Using Equation 13 on each of these yields the equation  $\text{card}(\Pi) = 500 \cdot (50/200) \cdot (5000/1000) + 150 \cdot (100/100) \cdot (3000/2000) = 625 + 225 = 850$ . Figure 11 visualizes this computation.

Using the cardinality estimation shown in Equation 10, Algorithm 2 shows how the transfer cost of a query execution plan  $\Pi$  on a node  $n$  is computed taking into account the locality of the fragments. First, if  $\Pi = [[P]]_f^{n_i}$ , i.e.,  $\Pi$  is a selection, the algorithm checks whether  $n = n_i$  (line 4); if they are equal it is 0 (since it incurs no transfer

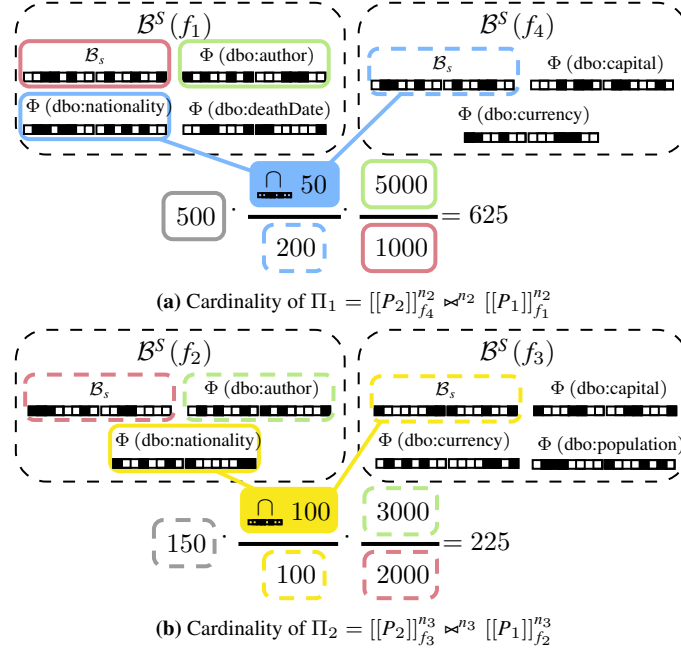


Fig. 11. Estimating the cardinality of  $\Pi$  in Figure 12d without the `DISTINCT` modifier for (a)  $\Pi_1 = [[P_2]]_{f_4}^{n_2} \bowtie^{n_2} [[P_1]]_{f_1}^{n_2}$  and (b)  $\Pi_2 = [[P_2]]_{f_3}^{n_3} \bowtie^{n_3} [[P_1]]_{f_2}^{n_3}$ . The output of Equation 10 is thus the sum of the two formulas ( $625 + 225 = 850$ ).

cost), otherwise the transfer cost of  $\Pi$  is equal to the cardinality of the selection (Equation 5). For instance, the transfer cost of the execution plan shown in Figure 12c ( $[[P_3]]_{f_5}^{n_1}$ ) on  $n_1$  is 0 since  $f_5$  is available on  $n_1$ .

If, instead,  $\Pi = \Pi_1 \cup \Pi_2$ , i.e.,  $\Pi$  is a union, the transfer cost is the sum of the transfer costs for  $\Pi_1$  and  $\Pi_2$  (line 6). For instance, the transfer cost of the execution plan shown in Figure 12a ( $[[P_1]]_{f_1}^{n_2} \cup [[P_1]]_{f_2}^{n_3}$ ) on  $n_1$  is  $5000 + 3000 = 8000$ , since neither  $f_1$  or  $f_2$  is available on  $n_1$ .

---

**Algorithm 2** Compute the transfer cost of a query execution plan

---

**Input:** A query execution plan  $\Pi$ ; a node  $n$   
**Output:** The estimated transfer cost  $cost$

```

1: function TRANSFERCOST( $\Pi, n$ )
2:    $cost \leftarrow 0$ ;
3:   if  $\Pi = [[P]]_f^{n_i}$  then
4:     if  $n \neq n_i$  then  $cost \leftarrow card(P, f)$ ;
5:   else if  $\Pi = \Pi_1 \cup \Pi_2$  then
6:      $cost \leftarrow transferCost(\Pi_1, n) + transferCost(\Pi_2, n)$ ;
7:   else if  $\Pi = \Pi_1 \times^{n_i} \Pi_2$  then
8:      $cost \leftarrow transferCost(\Pi_1, n_i) + transferCost(\Pi_2, n_i)$ ;
9:     if  $n_i \neq n$  then  $cost \leftarrow cost + card(\Pi)$ ;
10:  else if  $\Pi = \Pi_1 \bowtie^{n_i} \Pi_2$  then
11:    if  $\Pi_2 = \Pi'_2 \cup \Pi''_2$  then
12:       $cost \leftarrow transferCost(\Pi_1 \bowtie^{n_i} \Pi'_2, n) + transferCost(\Pi_1 \bowtie^{n_i} \Pi''_2, n)$ ;
13:    else if  $\Pi_2 = [[P]]_f^{n_j}$  then
14:       $cost \leftarrow transferCost(\Pi_1, n_i)$ ;
15:      if  $n_i \neq n_j$  then  $cost \leftarrow cost + card_S^{\infty}(\Pi_1, P, f)$ ;
16:      if  $n \neq n_i$  then  $cost \leftarrow cost + card(\Pi)$ ;
17:  return  $cost$ ;

```

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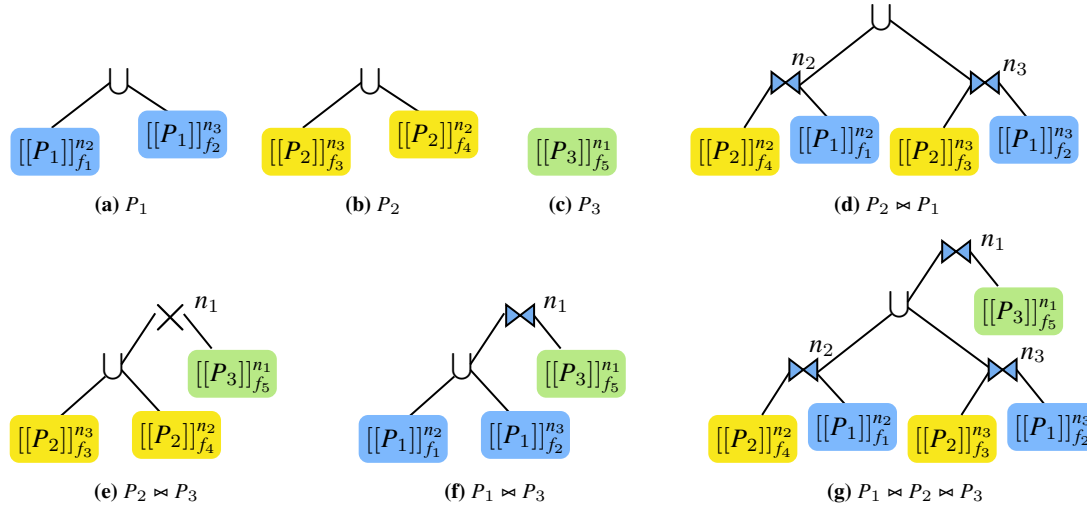


Fig. 12. Best query execution plan for each subquery in the DP table (Table 2).

Otherwise, if  $\Pi = \Pi_1 \times^{n_i} \Pi_2$ , i.e.,  $\Pi$  is a Cartesian product, the transfer cost is the sum of the transfer costs for  $\Pi_1$  and  $\Pi_2$  (line 6), plus the cardinality of the Cartesian product if it is delegated to a different node than the one processing the (sub)plan, i.e., if  $n \neq n_i$  (since they have to be transferred from  $n_i$  to  $n$ ).

Finally, if  $\Pi = \Pi_1 \bowtie^{n_i} \Pi_2$ , i.e.,  $\Pi$  is a join, we once again take advantage of the fact that the right side of a join is always either a selection or a union of selections; in the latter case, we aggregate the transfer cost over each subplan in the union (line 12). However, if the right side of the join is a selection  $\Pi_2 = [[P]]_f^{n_j}$ , we start by estimating the transfer cost of the left side of the join (line 14); if  $n_j \neq n_i$ , we further add in line 15 the cardinality of the join (since these results should have to be sent back to  $n_i$ ). Furthermore, if  $n_i \neq n$ , we add in line 16 the cardinality of the execution plan to the cost, since the results have to be transferred from  $n_i$  to  $n$ .

Given the transfer cost in Algorithm 2, the cost of processing a query execution plan  $\Pi$  on a node  $n$  is the transfer cost plus the cardinality of  $\Pi$ . This is formally defined as follows:

$$cost_n(\Pi) = transferCost(\Pi, n) + card(\Pi) \quad (14)$$

Given the cost function in Equation 14, we compute the cost of each possible join delegation and apply Dynamic Programming (DP) to achieve the execution plan with the lowest cost. Table 2 shows the best execution plan in the DP table for each (sub)plan for processing query  $Q$  (Figure 3a) on node  $n_1$  in the running example; Figure 12 visualizes the execution plans in Table 2.

Table 2  
Entries in the DP table for query  $Q$  (Figure 3a)

| Subquery                      | Execution Plan                                                                                                                                           | Cardinality | Cost      |
|-------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-----------|
| $P_1$                         | $[[P_1]]_{f_1}^{n_2} \cup [[P_1]]_{f_2}^{n_3}$                                                                                                           | 8,000       | 8,000     |
| $P_2$                         | $[[P_2]]_{f_3}^{n_3} \cup [[P_2]]_{f_4}^{n_2}$                                                                                                           | 650         | 650       |
| $P_3$                         | $[[P_3]]_{f_5}^{n_1}$                                                                                                                                    | 9,000       | 9,000     |
| $P_2 \bowtie P_1$             | $([[P_2]]_{f_4}^{n_2} \bowtie^{n_2} [[P_1]]_{f_1}^{n_2}) \cup ([P_2]_{f_3}^{n_3} \bowtie^{n_3} [[P_1]]_{f_2}^{n_3})$                                     | <b>850</b>  | 1,700     |
| $P_2 \bowtie P_3$             | $([[P_2]]_{f_3}^{n_3} \cup [[P_2]]_{f_4}^{n_2}) \times^{n_1} [[P_3]]_{f_5}^{n_1}$                                                                        | 5,850,000   | 5,850,650 |
| $P_1 \bowtie P_3$             | $([[P_1]]_{f_1}^{n_2} \cup [[P_1]]_{f_2}^{n_3}) \bowtie^{n_1} [[P_3]]_{f_5}^{n_1}$                                                                       | 1,688       | 9,688     |
| $P_2 \bowtie P_1 \bowtie P_3$ | $(([[P_2]]_{f_4}^{n_2} \bowtie^{n_2} [[P_1]]_{f_1}^{n_2}) \cup ([P_2]_{f_3}^{n_3} \bowtie^{n_3} [[P_1]]_{f_2}^{n_3})) \bowtie^{n_1} [[P_3]]_{f_5}^{n_1}$ | 154         | 1,004     |



## 6. Query Execution

Until now, we have described in Section 5 how LOTHBROK obtains a query execution plan using compatibility graphs and locality information provided by the SPBF indexes. In this section, we detail how LOTHBROK evaluates a query given a query execution plan.

Given a BGP  $P$ , a compatibility graph  $G^C = G^C(P, I^S)$ , and a query execution plan  $\Pi$  over  $P$  and  $G^C$ , LOTHBROK processes  $P$  by processing the operations specified in  $\Pi$  and, in doing so, delegating joins and Cartesian products to the nodes specified in  $\Pi$ . The intermediate results from previous steps are used as input to subqueries at a later stage in the query execution plan. In case of a distributed join, the intermediate results are transferred along with the partial query to use local bind joins similar to [11, 38]. To formalize how star patterns in the query execution plan are processed over the fragments, we define a so-called *selector* function in line with related work [7, 11, 38]. The selector function returns the results of processing the star pattern over a fragment given a set of solution mappings, i.e., the set of stars in the fragment that constitute the answer to the star pattern, as follows:

**Definition 18** (Selector Function [7, 11, 38]). *Given a node  $n$ , a star pattern  $P$ , and a finite set of distinct solution mappings  $\Omega$ , the star pattern-based selector function for  $P$  and  $\Omega$ , denoted  $s_{(P,\Omega)}$  is for every fragment  $f$  in  $n$ 's local datastore defined as follows.*

$$s_{(P,\Omega)}(f) = \begin{cases} \{t \in T \mid T \subseteq f \wedge T[P]\} & \text{if } \Omega = \emptyset \\ \{t \in T \mid T \subseteq f \wedge \exists \mu \in [[P]]_f, \mu' \in \Omega : \mu[P] = T \wedge \mu' \subseteq \mu\} & \text{otherwise.} \end{cases}$$

In line with [7, 11, 38], and to avoid long-running requests on each node, we apply pagination to the results of star pattern requests, i.e., we group the results into reasonably sized pages to avoid excessive data transfer. The page size used in our experimental evaluation (Section 7) is the page size recommended by related work [7, 11, 38], i.e., 100. However, for ease of presentation, we assume that all results can fit into one page when presenting the approach to query processing. Furthermore, to avoid underestimating costs caused by the selector function returning some duplicate values (e.g., when the same subject has multiple object values for a specific predicate), our implementation always uses  $card_S$  (Equation 5) and  $card_S^*$  (Equation 13) for cardinality estimations, regardless of whether or not the DISTINCT keyword is given. Last, given a star pattern  $P$ , a node  $n$ , a fragment  $f_i$ , and a finite set of solution mappings  $\Omega$ ,  $sel_n(f_i, P, \Omega)$  denotes the result of invoking  $s_{(P,\Omega)}(f_i)$  on  $n$ .

Let  $I_n^S$  denote a node  $n$ 's SPBF index. The `evaluatePlan` function in Algorithm 3 defines a recursive function that processes a query execution plan on a node  $n$  by using the selector function defined in Definition 18 for selections in the plan and making recursive calls to the nodes specified in the plan.

Consider, for instance, the query execution plan  $\Pi$  shown in Figure 12g for query  $Q$  in Figure 3a processed by node  $n_1$  in the running example. Figure 13 shows an overview of which parts of the query are sent to which node during query processing. Since  $\Pi$  is of type join, the function enters the if statement in line 6. Here, the function first makes a recursive call (since the join was delegated to node  $n_1$ ) with the left-most subplan, i.e.,  $\Pi_1 = ([P_2]]_{f_4} \bowtie^{n_2} [[P_1]]_{f_1}) \cup ([P_2]]_{f_3} \bowtie^{n_3} [[P_1]]_{f_2})$  (visualized in Figure 12d), in line 7.

Since  $\Pi_1$  is of type union, Algorithm 3 in lines 10-11 makes two recursive calls for the two subplans  $\Pi_1 = [[P_2]]_{f_4} \bowtie^{n_2} [[P_1]]_{f_1}$  and  $\Pi_2 = [[P_2]]_{f_3} \bowtie^{n_3} [[P_1]]_{f_2}$ . Note that these two recursive calls can be processed concurrently and indeed is done so in the implementation of LOTHBROK. This step is shown in Figure 13a where  $\Pi_1$  is sent to node  $n_2$  and  $\Pi_2$  is sent to node  $n_3$ . Since both subplans follow the same structure, and thus the same evaluation process, we will only explain what happens when processing  $\Pi_1$ .

When processing the plan  $\Pi_1$  from above, Algorithm 3 first calls the `evaluatePlan` on node  $n_2$  for the subplan  $[[P_2]]_{f_4}$ , i.e., the selection for  $P_2$  over  $f_4$ , in line 7. The `takeOne` function in line 16 selects a random node with the fragment in its local datastore if the node that processes the subquery does not store the fragment locally. In this case, since  $n_2$  stores  $f_4$ , it calls the selector function for  $P_2$  over  $f_4$  locally in line 17. The 500 results of processing  $P_2$  over  $f_4$  (cf. Table 1) are then joined with the singleton set of bindings  $\Omega$  that includes the empty mapping (i.e., a mapping compatible with any mapping) in line 18 and returned in line 19.

---

**Algorithm 3** Evaluate a join plan
 

---

**Input:** A join plan  $\Pi$ ; a node  $n$ ; a set of solution mappings  $\Omega$   
**Output:** A set of solution mappings  $\Omega$

```

1: function EVALUATEPLAN( $\Pi, n, \Omega = \{\emptyset\}$ )
2:   if  $\Pi = \Pi_1 \times^{n_i} \Pi_2$  then
3:      $\Omega_1 \leftarrow \text{evaluatePlan}(\Pi_1, n_i, \Omega)$ ;
4:      $\Omega_2 \leftarrow \text{evaluatePlan}(\Pi_2, n_i, \Omega)$ ;
5:      $\Omega \leftarrow \Omega_1 \times \Omega_2$ ;
6:   else if  $\Pi = \Pi_1 \bowtie^{n_i} \Pi_2$  then
7:      $\Omega \leftarrow \text{evaluatePlan}(\Pi_1, n_i, \Omega)$ ;
8:      $\Omega \leftarrow \text{evaluatePlan}(\Pi_2, n_i, \Omega)$ ;
9:   else if  $\Pi = \Pi_1 \cup \Pi_2$  then
10:     $\Omega_1 \leftarrow \text{evaluatePlan}(\Pi_1, n, \Omega)$ ;
11:     $\Omega_2 \leftarrow \text{evaluatePlan}(\Pi_2, n, \Omega)$ ;
12:     $\Omega \leftarrow \Omega_1 \cup \Omega_2$ ;
13:   else if  $\Pi = [[P]]_f$  then
14:     $N \leftarrow I_n^S \cdot \eta(f)$ ;
15:    if  $n \in N$  then  $n_i \leftarrow n$ ;
16:    else  $n_i \leftarrow \text{takeOne}(N)$ ;
17:     $\phi \leftarrow \text{sel}_{n_i}(f, P, \Omega)$ ;
18:     $\Omega \leftarrow \Omega \bowtie \{\mu \mid \text{dom}(\mu) = \text{vars}(P) \text{ and } \mu[P] \in \phi\}$ ;
19:   return  $\Omega$ ;
  
```

---

Upon receiving the 500 results in line 7, Algorithm 3 makes another recursive call in line 8 to `evaluatePlan` on node  $n_2$  for the subplan  $[[P_1]]_{f_1}$ , i.e., the selection for  $P_1$  over  $f_1$  with the 500 intermediate results in  $\Omega$ . Again,  $n_2$  calls the local selector for  $P_1$  over  $f_1$  using the intermediate results in  $\Omega$  as bindings. This results in 625 intermediate results in  $\Omega$  that are the result of processing  $P_1 \bowtie P_2$  over  $f_1$  and  $f_4$ , which are returned by the function in line 19.

While  $n_2$  found the 625 results from processing  $[[P_2]]_{f_4} \bowtie^{n_2} [[P_1]]_{f_1}$  in the recursive call in line 10,  $n_3$  found the additional 225 results of processing  $[[P_2]]_{f_3} \bowtie^{n_3} [[P_1]]_{f_2}$  in the recursive call in line 11 following the same steps as described above for  $n_2$ . In line 12, these results are combined and 850 bindings are returned in line 19, which is visualized on Figure 13a as  $n_2$  returning 625 results to  $n_1$  and  $n_3$  returning 225 results to  $n_1$ .

The 850 intermediate results in  $\Omega$  found by processing  $([[P_2]]_{f_4} \bowtie^{n_2} [[P_1]]_{f_1}) \cup ([[P_2]]_{f_3} \bowtie^{n_3} [[P_1]]_{f_2})$  in line 7 are used as bindings for the recursive call made in line 8 for the subplan  $[[P_3]]_{f_5}$ . This is visualized in Figure 13b.

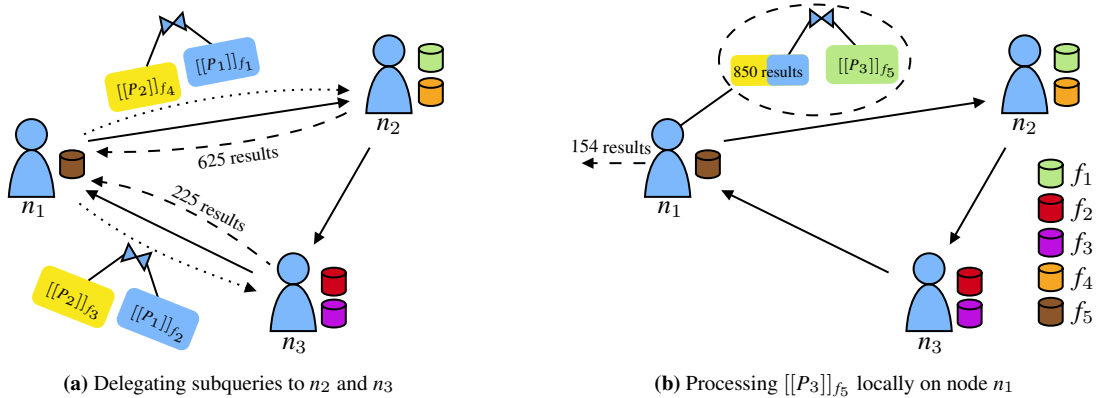


Fig. 13. Processing  $\Pi$  in Figure 12g on  $n_1$  by (a) delegating  $[[P_2]]_{f_4} \bowtie^{n_2} [[P_1]]_{f_1}$  to  $n_2$  and  $[[P_2]]_{f_3} \bowtie^{n_3} [[P_1]]_{f_2}$  to  $n_3$  concurrently and (b) processing the join between these 850 results and  $[[P_3]]_{f_5}$  locally on  $n_1$  to achieve the 154 results (solid arrows denote neighbors, dotted arrows subquery delegation, and dashed arrows transferring of intermediate results).  $n_1$  can send intermediate results to  $n_3$  since it is within its horizon.

Since  $n_1$  stores  $f_5$  locally, it calls the local selector for  $P_3$  over  $f_5$  and  $\Omega$  in line 17. The 154 results of processing  $P_3$  over  $f_5$  are joined with  $\Omega$  in line 18 and returned as the final results in line 19.

As mentioned above, our implementation uses pagination of the results meaning, for instance, when processing the subplan  $[[P_2]]_{f_4}$  in line 7, the 500 results would be split into multiple pages. In the implementation of LOTHBROK, nodes at subsequent steps in the pipeline start processing joins as soon as they receive some intermediate bindings. For instance, in the running example,  $n_1$  starts processing the join between  $P_2 \bowtie P_1 \bowtie P_3$  locally as soon as it receives results for  $P_2 \bowtie P_1$  from either  $n_2$  or  $n_3$ .

## 7. Experimental Evaluation

The experimental evaluation compares LOTHBROK with two state-of-the-art approaches building on P2P systems: PIQNIC [6] and COLCHAIN [7] with the query optimization approach outlined in [19]. To do this, we implemented the fragmentation, indexing, and cardinality estimation approach as a separate package in Java 8 and modified PIQNIC’s and COLCHAIN’s query processors to use it. Like COLCHAIN and PIQNIC, LOTHBROK’s query processor is implemented as an extension to Apache Jena<sup>2</sup>. Fragments in our implementation are stored as HDT files [60], allowing for efficient processing of the star patterns. We provide all source code, experimental setup (queries, datasets, etc.), and the full experimental results on our website<sup>3</sup>.

### 7.1. Experimental Setup

In this section, we detail the experimental setup, including a characterization of the used datasets and queries, the hardware and software setup, experimental configuration, as well as the evaluation metrics.

*Datasets and Queries.* To test the scalability of the approaches when the network is under heavy load, and to assess the impact of the query pattern on performance and network usage, we ran experiments with the synthetic WatDiv [33] benchmark using different dataset sizes: 10 million triples to 1 billion triples. Furthermore, to test LOTHBROK in a realistic setting where users would upload several interlinked datasets to a network, and ask queries with varying complexity, we ran experiments using a well-known benchmark suite for federated RDF engines called LargeRDFBench [32]. LargeRDFBench comprises 13 different, interlinked datasets with over a billion triples in total. To provide a fair comparison between the systems with and without LOTHBROK, we created an equal number of fragments for both fragmentations: characteristic sets (Section 4.2) and predicate-based. To do this, we iteratively merged the characteristic set fragments with the fewest number of subjects into larger fragments following the approach outlined in Section 4.2 until the number of fragments equalled the number of predicate-based fragments. The characteristics of the datasets are shown in Table 3. Furthermore, to assess the impact of reducing the number of characteristic sets on query completeness, we ran similar experiments where we did not create an equal number of fragments for LOTHBROK, i.e., where we created one fragment for each characteristic set that describes at least 50 subjects and provide the results on our website<sup>3</sup>; since these results are quite similar to the ones presented in this section, we will not report on them further.

LargeRDFBench includes 40 different queries [32] that are divided into five different categories of varying complexity and result set sizes: Simple (S), Complex (C), Large Data (L), and Complex and High Data Sources (CH).

For WatDiv, we used WatDiv *star query loads* from [11] consisting of 1-3 star patterns, called the `watdiv-1_star`, `watdiv-2_star`, and `watdiv-3_star` query loads, as well as a query load consisting of path queries, i.e., queries where each star pattern only has one triple pattern, called the `watdiv_path` query load. Each of these query loads consists of 6,400 different queries. Furthermore, we combine the aforementioned query loads into a single query load called `watdiv-union`. Last, we created a query load with 19,968 queries from the WatDiv stress testing query templates (156 per node) called `watdiv-sts`. The complete set of queries is available on our website<sup>3</sup>. Figure 14 shows an overview of the following characteristics of each load [11, 61]: Triple pattern

<sup>2</sup><https://jena.apache.org>

<sup>3</sup><https://relweb.cs.aau.dk/lothbrok>

Table 3  
Characteristics of the used datasets

| Dataset        | #triples      | #subjects   | #predicates | #objects    |
|----------------|---------------|-------------|-------------|-------------|
| LargeRDFBench  | 1,003,960,176 | 165,785,212 | 2,160       | 326,209,517 |
| LinkedTCGA-M   | 415,030,327   | 83,006,609  | 6           | 166,106,744 |
| LinkedTCGA-E   | 344,576,146   | 57,429,904  | 7           | 84,403,402  |
| LinkedTCGA-A   | 35,329,868    | 5,782,962   | 383         | 8,329,393   |
| ChEBI          | 4,772,706     | 50,477      | 28          | 772,138     |
| DBPedia-Subset | 42,849,609    | 9,495,865   | 1,063       | 13,620,028  |
| DrugBank       | 517,023       | 19,693      | 119         | 276,142     |
| GeoNames       | 107,950,085   | 7,479,714   | 26          | 35,799,392  |
| Jamendo        | 1,049,647     | 335,925     | 26          | 440,686     |
| KEGG           | 1,090,830     | 34,260      | 21          | 939,258     |
| LinkedMDB      | 6,147,996     | 694,400     | 222         | 2,052,959   |
| NYT            | 335,198       | 21,666      | 36          | 191,538     |
| SWDF           | 103,595       | 11,974      | 118         | 37,547      |
| Affymetrix     | 44,207,146    | 1,421,763   | 105         | 13,240,270  |
| watdiv10M      | 10,916,457    | 521,585     | 86          | 1,005,832   |
| watdiv100M     | 108,997,714   | 5,212,385   | 86          | 9,753,266   |
| watdiv1000M    | 1,092,155,948 | 52,120,385  | 86          | 92,220,397  |

count #TP (Figure 14a), join vertex count #JV (Figure 14b), join vertex degree DEG (Figure 14c), result cardinality #Results (Figure 14d), mean triple pattern selectivity  $SEL_G(tp)$  (Figure 14e), and join vertex type (Figure 14f).

*Experimental Configuration.* We compare the following systems: (1) PIQNIC [6] using PPBF indexes [19] (PIQNIC), (2) LOTHBROK on top of PIQNIC (LOTHBROK<sub>PIQNIC</sub>), (3) COLCHAIN [7] using PPBF indexes (COLCHAIN), and (4) LOTHBROK on top of COLCHAIN (LOTHBROK<sub>COLCHAIN</sub>). All configurations were run on networks with 128 nodes. To assess the scalability of LOTHBROK under load, we ran 156 `watdiv-sts` queries concurrently on each node over 8 different configurations where  $2^i$  nodes issue queries concurrently such that  $0 \leq i \leq 7$  (i.e., up to all 128 nodes). Furthermore, to analyze the impact of the query pattern on performance, we ran the WatDiv star query loads over each WatDiv dataset size such that for each star query load, each node issued 50 queries. Lastly, we tested the performance of LOTHBROK over each individual query in LargeRDFBench by running the queries sequentially in random order on three randomly selected nodes and report the average result.

*Hardware Configuration.* For all configurations and P2P systems, we ran 128 nodes concurrently on a virtual machine (VM) with 128 vCPU cores with a clock speed of 2.5GHz, 64KB L1 cache, 512KB L2 cache, 8192KB L3, and a total of 2TB main memory. To spread out resources evenly across nodes, all nodes were restricted to use 1 vCPU core and 15GB memory, enforced using the `-Xmx` and `-XX:ActiveProcessorCount` options for the JVM. Furthermore, to simulate a more realistic scenario, where nodes are not run on the same machine, we simulated a connection speed of 20 MB/s.

*Evaluation Metrics.* We used measured the following metrics:

- *Workload Time (WT):* The amount of time (in milliseconds) it takes to complete an entire workload including queries that time out.
- *Throughput (TP):* The number of completed queries in the workload divided by the total workload time (i.e., number of queries per minute).
- *Number of Timeouts (NTO):* The number of queries that timed out (timeout being 1200 seconds).
- *Query Execution Time (QET):* The amount of time (in milliseconds) elapsed between when a query is issued and when its processing has finished.
- *Query Response Time (QRT):* The amount of time (in milliseconds) elapsed between when a query is issued and when the first result is computed.
- *Query Optimization Time (QOT):* The amount of time (in milliseconds) elapsed between when a query is issued and when the optimizer has finished (i.e., when query execution starts).

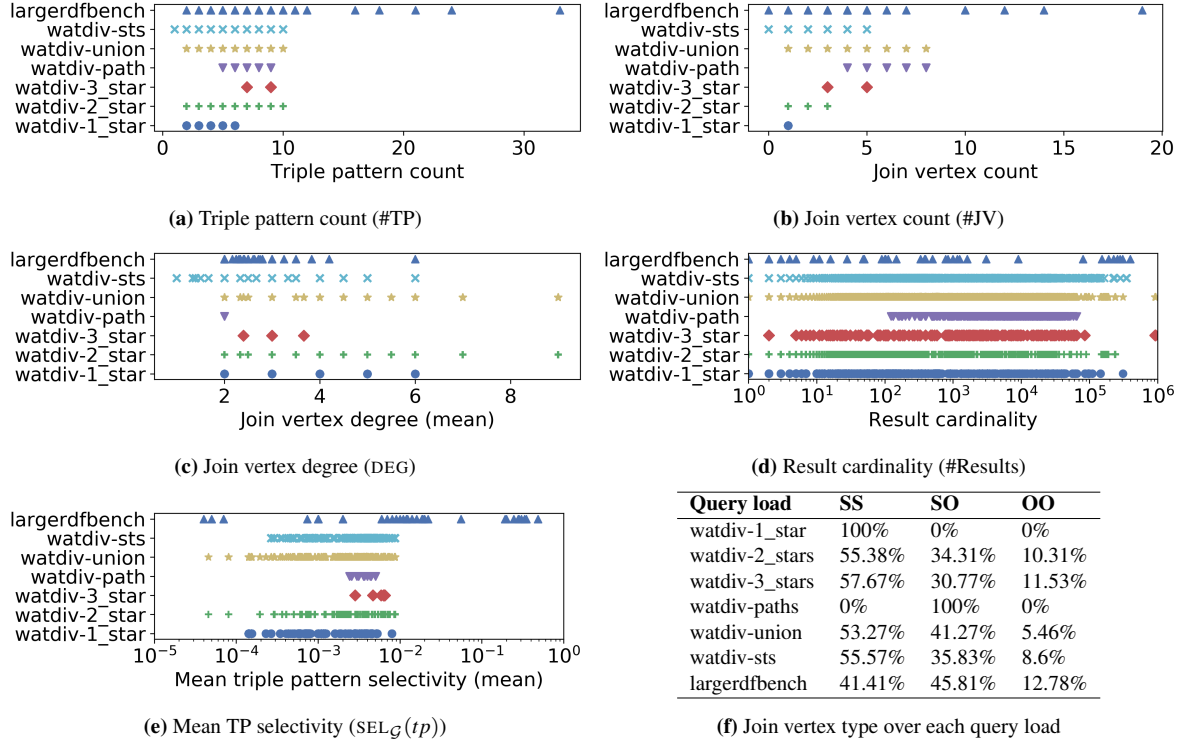


Fig. 14. Characteristics of all query loads (WatDiv query loads over watdiv100M; statistics over the watdiv10M and watdiv100M datasets can be found on our website<sup>3</sup>).

- *Number of Requests (REQ)*: The number of requests made between nodes when processing a query (including requests made from nodes that have been delegated subqueries).
- *Number of Transferred Bytes (NTB)*: The amount of data (in bytes) transferred between nodes when processing a query (including data transferred to and from nodes that have been delegated subqueries).
- *Number of Relevant Nodes (NRN)*: The number of distinct nodes that replicate fragments containing relevant data to a query.
- *Number of Relevant Fragments (NRF)*: The number of distinct fragments containing relevant data to a query.

**Software Configuration.** Unless otherwise specified, we used the following parameters when running the systems. For COLCHAIN, we used the following parameters recommended in [7]: Community Size: 20, Number of Communities: 200. For PIQNIC, we use the following parameters recommended in [6]: Time-to-Live (number of hops): 5, Number of Neighbors: 5. The replication factor for PIQNIC (i.e., the percentage of nodes replicating each fragment) was matched with the size of the communities in COLCHAIN to provide a better comparison. Nodes were randomly assigned neighbors throughout the network. The page size (i.e., how many results can be returned with each request, was set to 100. Furthermore, to limit the size of HTTP requests, the number of results that each system was allowed to attach to each request (i.e.,  $|\Omega|$  in Section 6) was set to  $|\Omega| = 30$ . The timeout for all systems and queries was set to 20 minutes (1,200 seconds).

## 7.2. Scalability under Load

In these experiments, we ran the watdiv-sts queries over each WatDiv dataset in configurations where  $2^i$  nodes issued 156 queries from the watdiv-sts query load concurrently such that  $0 \leq i \leq 7$ . Figures 15a-15c show the throughput (TP) of the watdiv-sts query load over each configuration in the scalability tests for the watdiv10M (Figure 15a), watdiv100M (Figure 15b), and watdiv1000M (Figure 15c) datasets in logarithmic

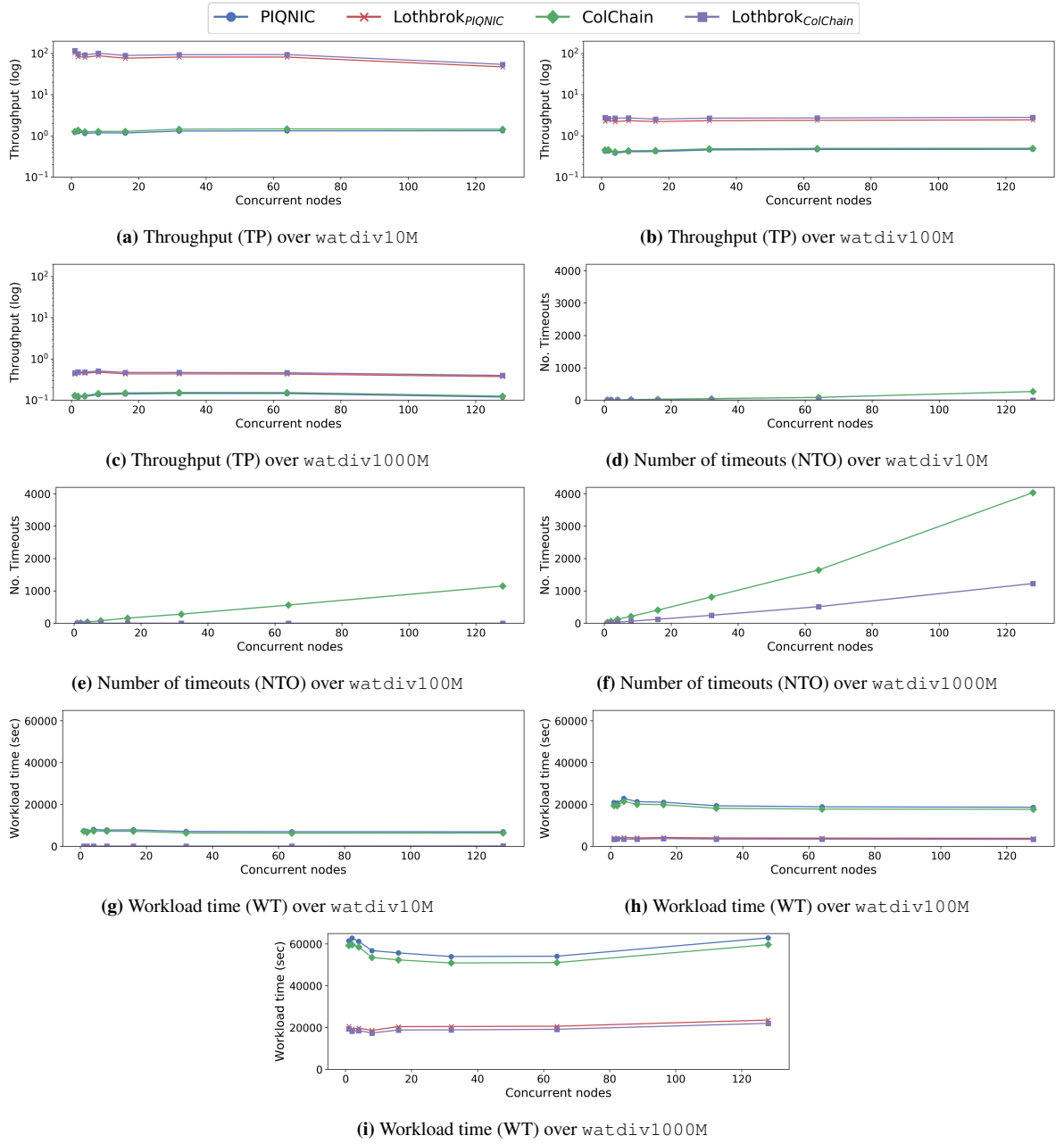


Fig. 15. Throughput (TP), number of timeouts (NTO), and workload time (WT) for *watdiv-sts* over the *watdiv10M*, *watdiv100M*, and *watdiv1000M* datasets.

scale. Clearly, LOTHBROK has a significantly higher throughput across all datasets and configurations compared to the approaches that do not include LOTHBROK (i.e., PIQNIC and COLCHAIN). In fact, for *watdiv10M*, this increase in throughput is close to two orders of magnitude. While the increase in throughput that LOTHBROK provides is smaller for both *watdiv100M* and *watdiv1000M*, LOTHBROK still increases the throughput by close to an order of magnitude for these datasets. Furthermore, while some results show that COLCHAIN has a slightly higher throughput than PIQNIC, both with and without LOTHBROK on top, this difference is relatively negligible. Last, the results show that the throughput of LOTHBROK is relatively stable when increasing numbers of nodes issue

queries concurrently. In fact, even when every node in the network issue queries concurrently, the throughput is relatively close to the highest throughput throughout the configurations.

Figures 15d-15f show the number of queries that timed out (TO) of the `watdiv-sts` query load over each configuration for each WatDiv dataset. As expected, the number of timeouts increases relatively linearly with the number of nodes issuing queries concurrently. This is due to the fact that when more nodes issue queries, more queries in total are executed, meaning the total number of the queries that time out increases. Generally, the queries that time out correspond to query templates that result in a large number of intermediate results, e.g., by using the `owl:sameAs` predicate. Furthermore, PIQNIC and COLCHAIN incur significantly more timeouts without LOTHBROK compared to with LOTHBROK. In fact, for both `watdiv10M` and `watdiv100M`, LOTHBROK experiences no timeouts while PIQNIC and COLCHAIN experience 267 timeouts for `watdiv10M` and 1,148 timeouts for `watdiv1000M`. Even for `watdiv1000M`, the number of timeouts experienced by LOTHBROK is just 1,151 while PIQNIC and COLCHAIN both experience 4,036 timeouts. Furthermore, PIQNIC and COLCHAIN incur the exact same number of timeouts.

Figures 15g-15i show the workload time (WT) for each configuration. In line with the throughput and number of timeouts, LOTHBROK incurs a significantly lower average workload time than PIQNIC and COLCHAIN across all experiments and datasets. The slight decrease in the workload time for fewer nodes can be attributed to the network being able to process more queries concurrently when the overall load is relatively low. Nevertheless, the average workload time only increases slightly even when all nodes issue queries concurrently.

Overall, our experimental results show that, even when the network is under heavy query processing load, LOTHBROK increases the query throughput and decreases the average workload time significantly compared to state-of-the-art decentralized systems. In fact, the increase in performance is up to two orders of magnitude. As a result, LOTHBROK is also able to finish more queries without timing out.

### 7.3. Impact of Query Pattern

To test the impact of the query pattern on the performance of LOTHBROK, we ran the `watdiv-1_star`, `watdiv-2_star`, `watdiv-3_star`, `watdiv-path`, `watdiv-union`, and `watdiv-sts` query loads on each system; the `watdiv-sts` queries consist of, on average, more selective star patterns compared to the other WatDiv query loads (Figure 14).

Figures 16a-16c show the execution time (QET) for each WatDiv query load over each WatDiv dataset, and Figures 16d-16f show the response time (QRT) for each WatDiv query load in logarithmic scale. Our results show that LOTHBROK has significantly better performance across all datasets for almost every query load. As expected, the improvement in performance is more significant for the query loads with a lower number of star patterns. This is due to the fact that since the star patterns within these queries represent a large part of the query, LOTHBROK has to issue fewer requests overall, lowering the network overhead. For instance, the queries in the `watdiv-1_star` query load can by LOTHBROK be answered by issuing 0.89 requests per 90 results<sup>4</sup>, whereas PIQNIC and COLCHAIN have to issue 9.27 requests per 90 results on average, for `watdiv1000M` in our experiments. In the `watdiv-3_star` query load, the improvement in performance is more modest across the datasets since each star pattern is a relatively small part of the query resulting in a higher number of requests; however, on average, we still see a performance increase of up to an order of magnitude.

We notice that for the `watdiv-path` query load, LOTHBROK actually has a slightly worse performance both in terms of QET and QRT compared to PIQNIC and COLCHAIN due to higher network usage. Figure 17 shows the number of relevant fragments (NRF) and the number of relevant nodes (NRN) for each query load over each dataset after optimization (similar figures are provided for NRF and NRN before optimization on our website<sup>3</sup>). Analyzing these results, we see that the decreased performance for `watdiv-path` is caused by LOTHBROK having a significantly larger number of relevant fragments and by extension a larger number of relevant nodes compared to PIQNIC and COLCHAIN. In fact, this is the case for all the WatDiv query loads (9 times larger for `watdiv-path` while up to 5 times larger for the other query loads); however, for the other query loads, this is compensated by the increased performance that the query optimization approach provides. This analysis is corroborated by the number

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<sup>4</sup>Even though one request can fetch up to 90 results, the average number of requests is lower than 1 since the nodes store some data locally.

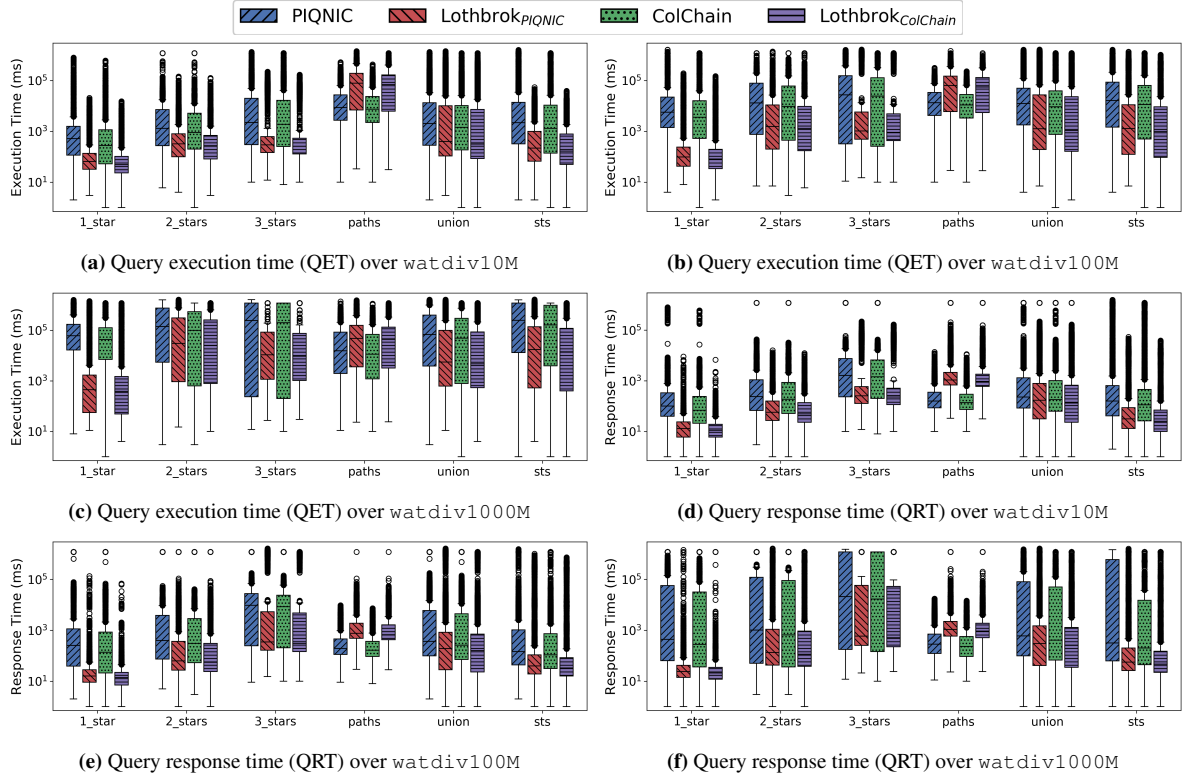


Fig. 16. Query execution time (QET) and query response time (QRT) for the WatDiv datasets and star queries.

of fragments pruned during optimization for each query load (figures provided on our website<sup>3</sup>); the *watdiv-path* query load has significantly less pruned fragments compared to the other query loads except *watdiv-1\_star*. For PIQNIC and COLCHAIN, the number of relevant fragments will always equal the number of unique predicates in the query since one fragment is created per predicate; however, due to fragmenting the data based on characteristic sets, LOTHBROK can encounter multiple fragments for each unique predicate in the query. Furthermore, the number of relevant fragments is, on average, more than twice as high for LOTHBROK over the *watdiv-path* query load than over the other query loads. This is because most of the path queries use common predicates like *owl:sameAs*.

Nevertheless, the slightly worse performance for LOTHBROK over *watdiv-path* is compensated by the significantly improved performance over the other query loads, so we still see a performance increase for the *watdiv-union* query load. As such, our experimental results show that LOTHBROK is generally able to increase performance over queries with star-shaped subqueries (i.e., all other queries than path queries) significantly and that the increase in performance depends on the shape of the query; queries with fewer but larger star patterns (cf. Figure 14c) show a bigger performance increase than queries with many but small star patterns.

#### 7.4. Network Usage

Figure 18 shows the network usage when processing WatDiv queries over each WatDiv dataset in terms of the number of requests (Figures 18a-18c) and the number of transferred bytes (Figures 18d-18f) in logarithmic scale. LOTHBROK incurs a significant lower network overhead for all query loads except *watdiv-path* despite the larger number of relevant fragments as discussed in Section 7.3. This is caused by LOTHBROK having to send significantly fewer requests for each star pattern since a star pattern can be processed entirely over the relevant fragments, even if there are more fragments (and thus nodes) to send the requests to. Again, the query loads with a smaller number of star patterns see a larger decrease in network usage since larger parts of the queries can be



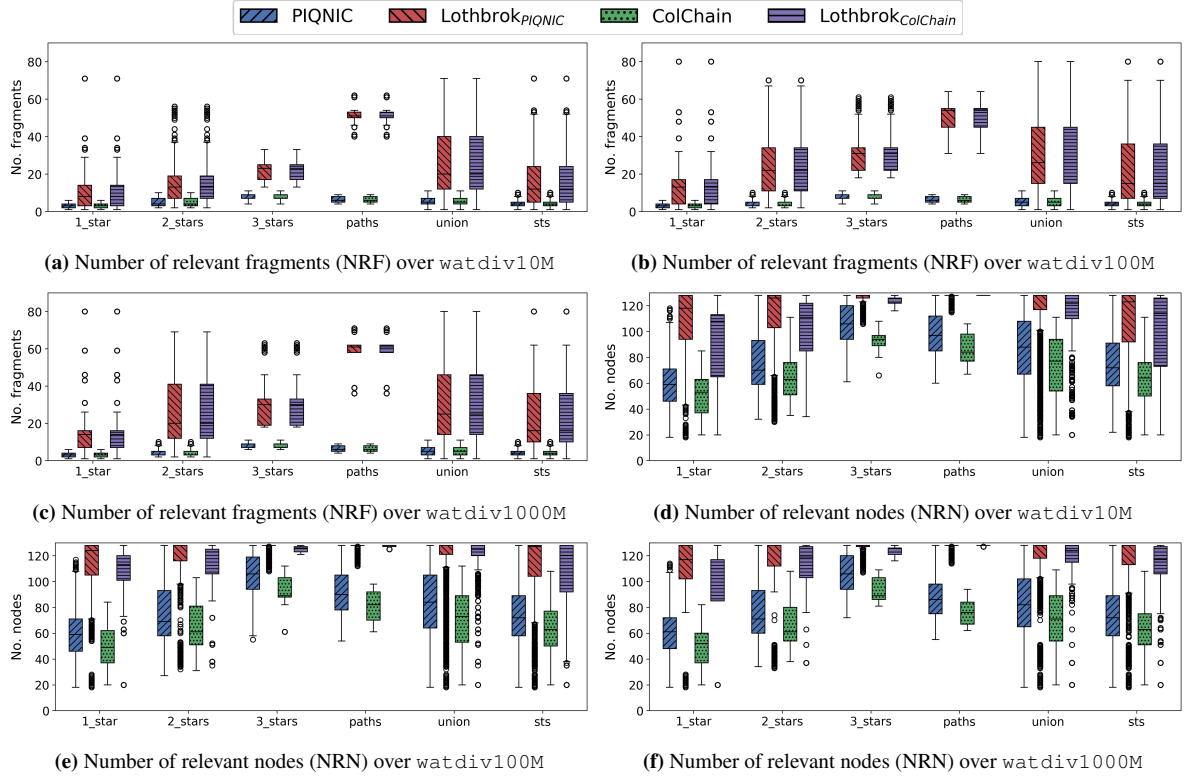


Fig. 17. Number of relevant fragments (NRF) and number of relevant nodes (NRN) for the WatDiv datasets and star queries.

processed by individual nodes. Since the queries in the *watdiv-path* query load do not benefit from the star pattern-based query processing, the network usage is slightly higher; however, even still, the *watdiv-union* shows an improvement in the network usage for LOTHBROK. These results are in line with the experiments shown in Sections 7.2 and 7.3 and support the hypothesis that LOTHBROK increases performance by lowering the network overhead when processing queries, compared to state-of-the-art systems such as PIQNIC and COLCHAIN.

### 7.5. Performance of Individual Queries

In these experiments, we ran the LargeRDFBench queries three times on each system sequentially to test the performance of those individual queries and report the average results. Figure 19 shows the execution time (Figure 19a), response time (Figure 19b), and optimization time (Figure 19c) for the C query load over LargeRDFBench in logarithmic scale. Similar figures for the other LargeRDFBench query loads are provided on our website<sup>3</sup>. The results in Figure 19 are similar to the remaining query loads; we show the C query load since this query load had the most diversity in the performance across the queries.

While, in our experiments, LOTHBROK provides an improvement for the execution time (Figure 19a) across all the queries in LargeRDFBench, the improvement varies based on the query shape in line with the findings of [10, 11] and the query shape experiments shown in Section 7.3. For instance, query C4 consists of one highly selective star pattern with 6 unique predicates. LOTHBROK is thus able to answer C4 with one request to the only fragment with that predicate combination, while PIQNIC and COLCHAIN have to send at least one request per triple pattern. Hence, LOTHBROK has around two orders of magnitude better performance for this particular query. On the other hand, query C5 consists of four star patterns, two of which contain only one triple pattern with one of them being the very common `rdfs:label` predicate. As a result, LOTHBROK has more than twice the number of relevant fragments for C5 compared to both PIQNIC and COLCHAIN. Nevertheless, LOTHBROK still has slightly improved

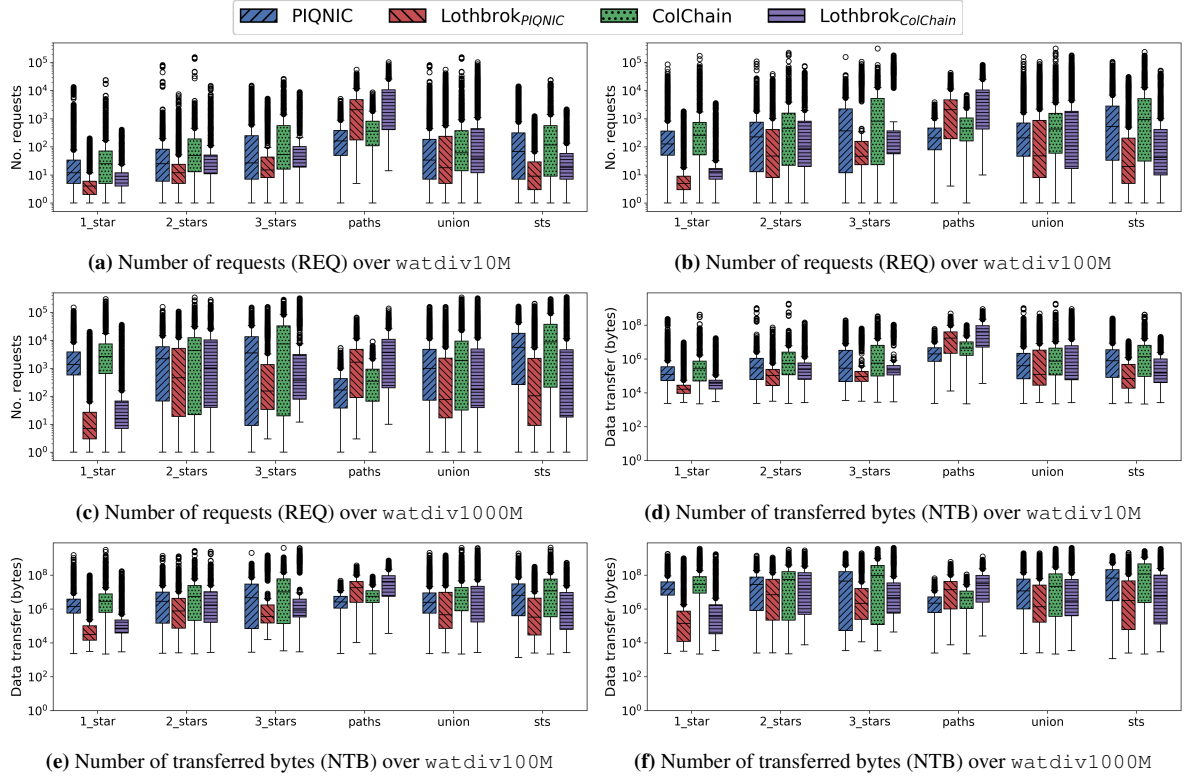


Fig. 18. Number of requests (REQ) and number of transferred bytes (NTB) for the WatDiv datasets and star queries.

performance for C5 compared to PIQNIC and COLCHAIN since the query still contains two star patterns with three triple patterns each, meaning the increased optimization and communication overhead that the additional relevant fragments entail is offset by the benefits of processing the star patterns over the individual fragments. The response times (Figure 19b) show a similar comparison between the systems as the execution times (Figure 19a) with the exception of query C4. Again, the reason being that LOTHBROK can process this query with a single request, and therefore the first result is obtained immediately after receiving the response to the request.

However, the optimization times (Figure 19c) differ quite significantly depending on the number of relevant fragments to the query. For instance, queries like C5 and C6 (that contain a star pattern consisting of a single triple pattern with a very common predicate) incur a significant number of relevant fragments for LOTHBROK (286 for C5 and 144 for C6) and thus a higher optimization time. This is the case, since a higher number of relevant fragments means a higher number of SPBFs have to be intersected which represents an overhead. In all of these cases, however, the benefits of processing entire star patterns over the fragments, in terms of decreased network overhead mean that the overall execution time is still lower for LOTHBROK. This is especially the case for C6, which contains a star pattern with 6 triple patterns that in PIQNIC and COLCHAIN have to be processed individually. On the other hand, queries like C4 that contain few very selective star patterns have a low optimization time for LOTHBROK, since each star pattern have very few relevant fragments. In the case of C4, PIQNIC and COLCHAIN have a relatively high number of relevant fragments due to one of the predicates being the common `owl:sameAs` predicate that occurs in multiple datasets. As a result, PIQNIC and COLCHAIN have a significantly higher optimization time for this query compared to LOTHBROK.

Figure 20 shows the number of transferred bytes (Figure 20a), the number of requests (Figure 20b), the number of relevant fragments (Figure 20c), and the number of relevant nodes (Figure 20d) for each LargeRDFBench query load in logarithmic scale. We provide figures displaying each measure in Figure 20 for each individual LargeRDFBench query on our website<sup>3</sup>. As with the experiments shown in Section 7.4, LOTHBROK clearly incurs a lower network

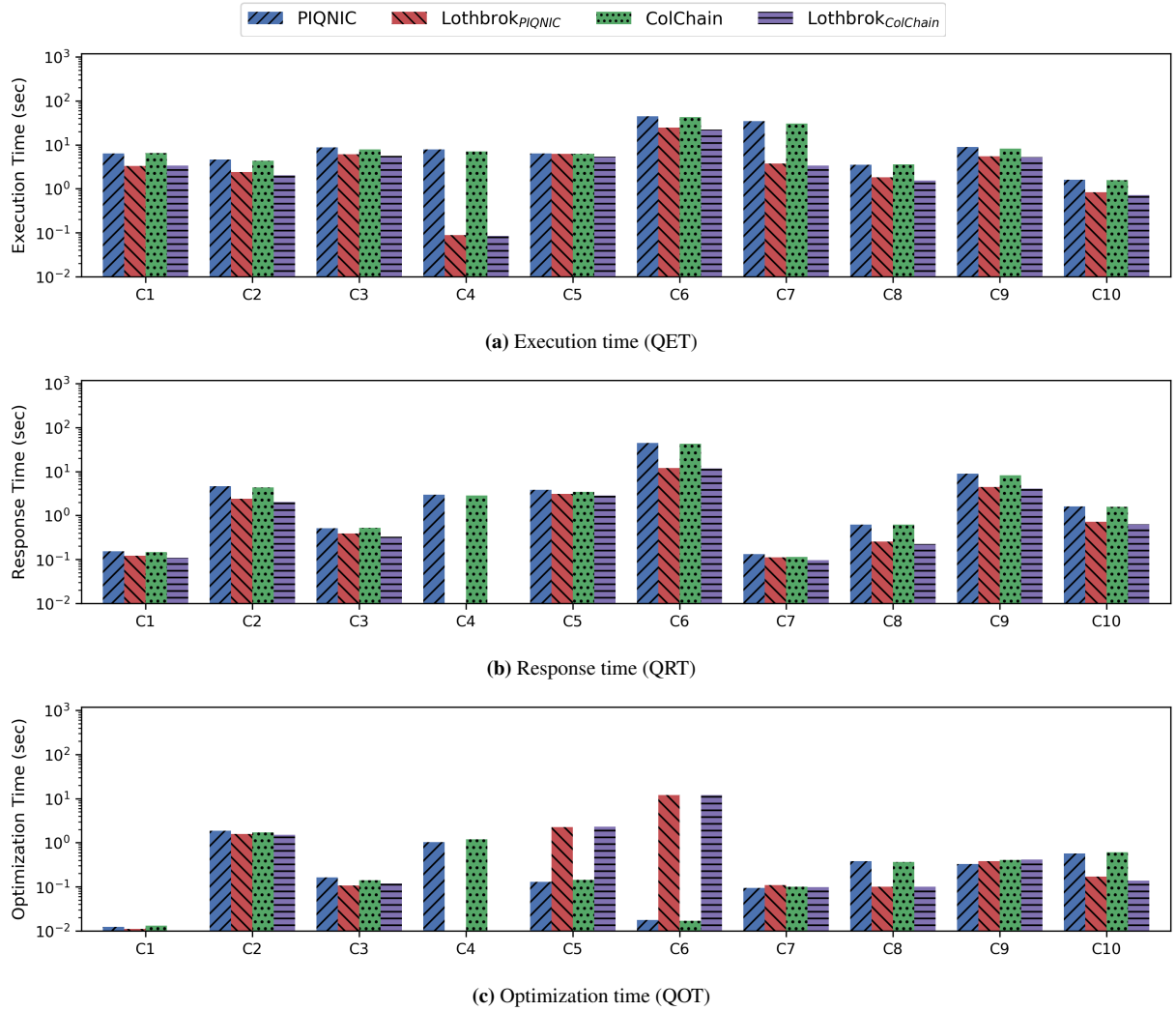


Fig. 19. Query execution time (a), response time (b), and optimization time (c) for the C query load over LargeRDFBench.

usage than both PIQNIC and COLCHAIN, both in terms of data transfer (Figure 20a) and the number of requests made (Figure 20b). This, together with the performance experiments, shows that LOTHBROK is able to reduce the network overhead significantly across all query loads and, in doing so, increase the performance overall.

Interestingly, while for most query loads, LOTHBROK has a higher number of relevant fragments (Figure 20c) in line with the experiments presented in Section 7.3, for the L query load, LOTHBROK has a lower number of relevant fragments in most queries. The reason is that the queries in this query load mostly use data from the quite structured `linkedTCGA` datasets which contain few similar characteristic sets, thus incurring a low number of relevant fragments per star pattern. On the other hand, for PIQNIC and COLCHAIN, the fact that some star patterns with a low number of triple patterns include common predicates like `rdf:type` increases the number of relevant fragments. The number of relevant nodes (Figure 20d) shows a similar trend to the number of relevant fragments since each fragment is replicated across 20 nodes; in some cases, however, where two relevant fragments are simultaneously replicated by some of the same nodes, the actual number of relevant nodes will be a bit lower than when the relevant nodes replicate exactly one relevant fragment.

Our results are similar for all query loads (figures provided on our website<sup>3</sup>) and show that even for the complex queries in query loads C and CH and the queries with a large number of intermediate results in query load L,

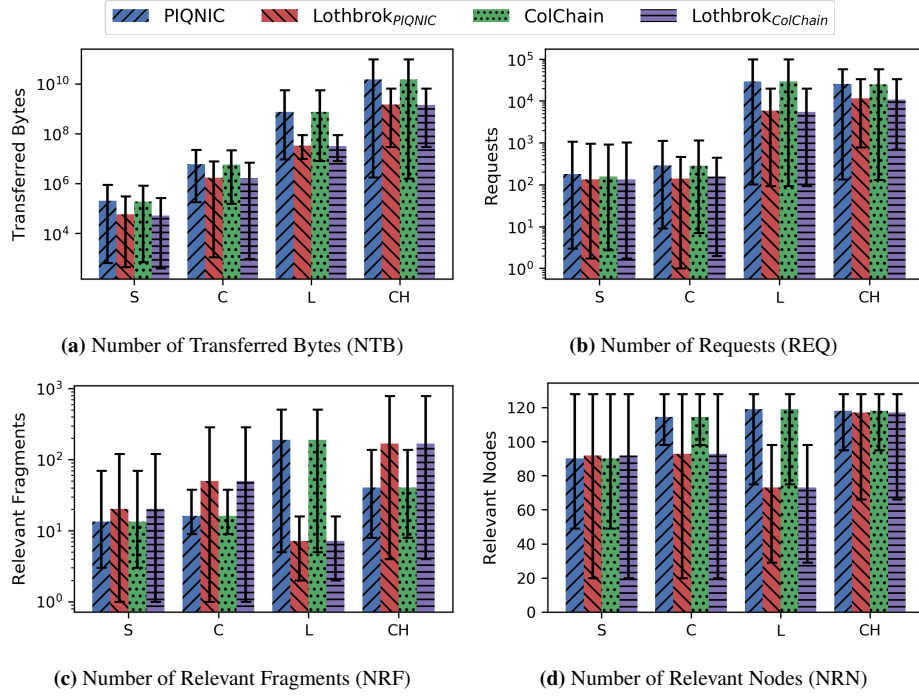


Fig. 20. Number of Transferred Bytes (NTB) (a), Number of Requests (REQ) (b), Number of Relevant Fragments (NRF) (c), and Number of Relevant Nodes (NRN) (d) for each LARDFBench query load.

LOTHBROK presents a significant performance increase because it lowers the communication overhead. For some queries, this is quite significant; for instance the queries C4 and S3 where LOTHBROK increases execution time by up to two orders of magnitude. Furthermore, some queries in the L and CH groups that timed out for PIQNIC and COLCHAIN, such as L3 and CH2, finished within the timeout of 1200 seconds for LOTHBROK. This is in line with the results presented in Section 7.2 and suggests that LOTHBROK is able to complete more queries within the timeout than the state-of-the-art systems.

### 7.6. Summary

Our experimental evaluations show that LOTHBROK significantly improves query performance while lowering the communication overhead compared to PIQNIC and COLCHAIN. LOTHBROK does so by distributing subqueries to other nodes such that the estimated network cost is limited as much as possible, and by processing entire star patterns over the individual fragments. In doing so, LOTHBROK decreases the network usage both in terms of the data transfer and number of requests, and increases performance by up to two orders of magnitude compared to the state of the art. Moreover, LOTHBROK does so while providing scalable performance under load; in fact, even when all nodes in the network issue queries concurrently, LOTHBROK maintains efficient query processing.

## 8. Conclusions

In this paper, we proposed LOTHBROK a novel query optimization approach for SPARQL queries over decentralized knowledge graphs. LOTHBROK builds upon recent work on decentralized Peer-to-Peer (P2P) systems [6, 7] and introduces a novel fragmentation technique based on characteristic sets [24], i.e., predicate families, as well as a novel indexing scheme that summarizes the sets of subjects and objects in a fragment using partitioned bitvectors. Furthermore, LOTHBROK proposes a query optimization strategy based on cardinality estimation, fragment

compatibility, and data locality that is able to delegate the processing of (sub)queries to other, neighboring nodes in the network that hold relevant data. We implemented our approach on top of two recent systems and evaluated LOTHBROK’s capabilities over well-known benchmarking suites containing real-world data and queries, as well as the performance of LOTHBROK under load using large-scale synthetic datasets and stress-testing query templates. The experimental results show that LOTHBROK significantly reduces the network overhead when processing queries in a P2P network and, in doing so, increases performance by up to two orders of magnitude.

While we presented a novel distribution of the workload across nodes in a P2P network, LOTHBROK also presents an opportunity to explore the effects of alternative strategies, e.g., for cost estimation, considering fragments optimized for object-object joins (Figure 14f), or alternative fragmentation and allocation strategies, e.g., based on SHACL/ShEx shapes [62, 63]. Furthermore, we plan to expand the range of supported queries to include aggregation and analytical queries [64, 65] and to expand the framework with support of provenance both for data [66–68], so that the system has information about the origin of the data it uses, as well as for queries [69] so that the system can explain how query answers were computed.

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