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Physical Layer Network Coding for FSK Systems

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Abstract—In this work we extend the existing concept of DeNoise and Forward (DNF) for bidirectional relaying to utilise non-coherent modulation schemes. This is done in order to avoid the requirement of phase tracking in coherent detection. As an example BFSK is considered, and through analysis the decision regions for the denoise operation in DNF are identified. The throughput performance of BFSK in DNF is compared to BPSK.

Index Terms—Physical layer network coding; frequency shift keying; non-coherent modulation.

I. INTRODUCTION

BIDIRECTIONAL relaying has been the focus of much research within wireless communication recently, [1]–[4]. Traditionally the three node scenario, where nodes A and B communicate with each other through a relaying node R, is considered. Examples of bidirectional relay protocols are Amplify-and-Forward, AF, and Decode-and-Forward, DF, [5], where DF is illustrated in Fig. 1(a). In [6] a concept called DeNoise-and-Forward, DNF, is presented. Here nodes A and B transmit their packets to the relay simultaneously. Assuming proper synchronisation, the signals are added in the air, which is referred to as analog network coding. The relay maps the resulting symbols to a binary message indicating that either equal or different symbols were received. The relay broadcasts this message, which makes an end node able to reconstruct its intended packet by knowing what it transmitted to the relay. Fig. 1(b) shows how packets can be exchanged in only two time slots, when using DNF. The mapping of received symbols to a binary message is effectively a remodulation performed in the physical layer, which removes the noise added during transmissions to the relay. This means that the packets are denoised, although decoding is not performed, hence the name.

In [6] BPSK modulation is applied in DNF, hence it is necessary to assume symbol synchronisation and coherent detection. The phase tracking required for coherent detection is impractical, hence non-coherent modulation schemes should be investigated. In this paper we investigate the use of BFSK modulation in DNF. Optimum decision regions are determined through analysis and the expected throughput is presented and compared to that of BPSK in DF and DNF respectively.

II. ANALYSIS OF DECISION REGIONS FOR BFSK

When analysing the decision regions we assume AWGN channels with no interference from other sources. We account for propagation loss and ergodic phase fading, where the phase, \( \phi \), is uniformly distributed between 0 and \( 2\pi \). Moreover, symbol synchronisation in joint transmissions is assumed.

FSK systems rely on envelope detection using quadrature receivers. Hence, the received signal is four dimensional and Gaussian noise components, \( \omega \), are added to each dimension respectively. The two possible received signals are represented as the following vectors:

\[
x_1 = \left( \sqrt{E_x \cos \phi_1 + \omega_1}, \sqrt{E_x \sin \phi_1 + \omega_2, \omega_3, \omega_4} \right) \quad (1)
\]

\[
x_2 = \left( \omega_1, \omega_2, (\sqrt{E_x \cos \phi_2 + \omega_3}), (\sqrt{E_x \sin \phi_2 + \omega_4}) \right) \quad (2)
\]

The envelope in both frequency bands can be calculated from dimensions 1 plus 2 and 3 plus 4, marked by \( \alpha \) and \( \beta \) respectively. Note that assuming AWGN, the envelope in a frequency band containing the signal is Rician distributed, while the envelope in a frequency band containing only noise is Rayleigh distributed.

In DNF there exist a significant difference between BPSK and BFSK. For BPSK the transmitted signals are either in phase or in reverse phase, which means that they can be added as scalars. In BFSK, however, they must be added as vectors due to the unknown phase difference. With two possible symbols we have four possible combinations in a joint transmission from nodes A and B. These are denoted \( x_{ij} \) where \( ij \) refers to the combination of \( x_1 \) and \( x_2 \) from Eqn. (1) and (2).

\[
x_{11} = \left( \sqrt{E_{xA} \cos \phi_{1A}} + \sqrt{E_{xB} \cos \phi_{1B} + \omega_1}, \sqrt{E_{xA} \sin \phi_{1A}} + \sqrt{E_{xB} \sin \phi_{1B} + \omega_2}, \omega_3, \omega_4 \right)
\]

\[
x_{12} = \left( \sqrt{E_{xA} \cos \phi_{1A} + \omega_1}, (\sqrt{E_{xA} \sin \phi_{1A} + \omega_2}), (\sqrt{E_{xB} \cos \phi_{2B} + \omega_3}), (\sqrt{E_{xB} \sin \phi_{2B} + \omega_4}) \right)
\]

\[
x_{21} = \left( \sqrt{E_{xA} \cos \phi_{1B} + \omega_1}, (\sqrt{E_{xA} \sin \phi_{1B} + \omega_2}), (\sqrt{E_{xB} \cos \phi_{2A} + \omega_3}), (\sqrt{E_{xB} \sin \phi_{2A} + \omega_4}) \right)
\]

\[
x_{22} = \left( \omega_1, \omega_2, (\sqrt{E_{xA} \cos \phi_{2A} + \sqrt{E_{xB} \cos \phi_{2B} + \omega_3}}), (\sqrt{E_{xA} \sin \phi_{2A} + \sqrt{E_{xB} \sin \phi_{2B} + \omega_4}}) \right)
\]
A. Conditional Distributions

The four possible analog coded symbols in BFSK do not follow the same type of distribution, hence the optimum decision regions can not be defined using Maximum Likelihood (ML) detection. Instead Maximum A posteriori Probability (MAP) detection is applied, where the conditional probability density functions of the possible symbols are compared. Note that in DNF we only discriminate between the symbols with equal frequencies and the symbols with different frequencies. Hence the two dimensional space in Fig. 2(b) should be divided into two regions based on the conditional PDFs.

In the case where the received symbol contains different frequencies the total signal is a two dimensional vector, whose elements both follow a Rician distribution. A signal vector is defined by the random variable $U = (U_i, U_j)^T$, where $U_i$ and $U_j$ are the envelopes in the two frequency bands respectively. Hence, the joint conditional PDF of $U$ is:

$$f_U(U|s_{ij}) = \frac{1}{\sigma} \exp\left(-\frac{U_i^2 + E_{sA}}{\sigma^2}\right)$$

$$I_0\left(\frac{U_i \sqrt{E_{sA}}}{\sigma}\right) I_0\left(\frac{U_j \sqrt{E_{sB}}}{\sigma}\right)$$

(3)

Where $s_{ij}$ is the transmitted symbol, and $ij$ is either 12 or 21. $I_0$ is the modified zero order Bessel function. Assuming that all symbols are equiprobable, the total joint PDF for symbols with different frequencies is:

$$f_U(U|s_{ij}, i \neq j) = \frac{1}{2} (f_U(U|s_{12}) + f_U(U|s_{21}))$$

(4)

When the two transmitters use the same frequency, the remaining frequency band contains only noise. These noise components, $\omega_i$, are orthogonal, hence the resulting envelope is Rayleigh distributed with parameter $\sigma$ since $\omega_i \sim \mathcal{N}(0, \sigma^2)$. This envelope is referred to as $U_k$, where $k = 2$ if $s_{11}$ is transmitted and vice versa.

$$f_U(U_k|s_{ij}, i = j) = \frac{U_k}{\sigma^2} \exp\left(-\frac{U_k^2}{2\sigma^2}\right)$$

(5)

The envelope in the used frequency band, $U_l$, where $l = 1$ if $s_{11}$ is transmitted, follows a composite distribution as stated earlier. This distribution is a Rician distribution where the mean value itself follows a distribution. This composite distribution can be expressed as follows.

$$f_{U_l}(U_l|s_{ij}, i = j) = \frac{1}{2} (f_{U_1}(U_l|s_{12}) + f_{U_1}(U_l|s_{21}))$$

(6)

The mean value is the noiseless envelope, $U$, whose distribution is a result of the uniform distribution of the phase difference, $\phi = \phi_{kB} - \phi_{kA}$, where $k$ refers to the transmitted frequency. The value of $\nu$ depends on $\phi_{kB}$ and not the individual values of $\phi_{kB}$ and $\phi_{kA}$, hence $\phi_{kB}$ is used as reference.

In order to derive the distribution of $\nu$, we first consider a probability mass function, PMF. This is a discrete expression of the distribution of $\nu$, i.e. it expresses the probability of experiencing a $\nu$ within a certain $\Delta \nu = [\nu_1; \nu_2]$. A certain $\Delta \nu$ corresponds to a certain $\Delta \phi$, whose relationship is expressed by the difference quotient $\frac{\Delta \phi}{\Delta \nu}$. Note that the probability of experiencing a $\nu$ within $\Delta \nu$ can be expressed as $\frac{d \phi}{d \nu}$, because $\phi$ is uniformly distributed between 0 and $2\pi$ and $\nu$ is symmetric around $\pi$ in this interval, as illustrated in Fig. 3. The PMF can thus be expressed as $\frac{d \phi}{d \nu}$ and for $\Delta \nu \rightarrow 0$ this becomes $\frac{d \phi}{d \nu}$, which expresses the PDF we are looking for. This is derived as follows:

$$\nu = \sqrt{(\sqrt{E_{sA}} + \sqrt{E_{sB}} \cos \phi)^2 + (\sqrt{E_{sB}} \sin \phi)^2}$$

$$\phi = \cos^{-1}\left(\frac{\nu^2 - E_{sA} - E_{sB}}{2 \sqrt{E_{sA} E_{sB}}}\right)$$

$$f_\nu(\nu) = \frac{d \phi}{\pi d \nu} = \frac{\nu}{\pi \sqrt{E_{sA} E_{sB}} \sqrt{1 - \left(\frac{\nu^2 - E_{sA} - E_{sB}}{2 \sqrt{E_{sA} E_{sB}}}\right)^2}}$$

(7)

By combining Eq. (5), (6) and (7) the joint conditional PDFs of symbols $s_{ij}$, when $i$ and $j$ are equal, can be expressed as $f_U(U|s_{ij}) = f_U(U_k|s_{ij}) \cdot f_{U_l}(U_l|s_{ij})$. Hence the total PDF of symbols with equal frequencies is then as follows:
with BFSK an increase of $\sim 0.33$ for using BFSK is more significant in DNF. However, the denoise operation saves a time slot compared to DF, hence the DNF scheme converges to a throughput of 0.5 compared to the 0.33 for DF. If fading was taken into account the relative performance of DNF and DF would be similar, however, a larger SNR would be required before converging to maximum throughput. This is the case for both modulation schemes.

### IV. CONCLUSION

The existing concept of De-Noise and Forward (DNF) is based on the coherent modulation scheme BPSK, where the required tracking of phase is impractical. Therefore, this work have extended the concept of DNF to utilise non-coherent modulation schemes, where we have considered BFSK. The decision regions have been identified through analysis. Results shows that BFSK in DNF yields a lower performance compared to BPSK in DF, as it requires a higher SNR before communication is possible. Hence being independent of the phase requires a larger SNR in order to obtain the same throughput as for BPSK.

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### REFERENCES


