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# Topology optimization - Improved checker-board filtering with sharp contours

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**Summary** In topology optimization it is mandatory to use a filtering technique in order to prevent checker-boarder solutions. The paper examines a new filtering principle and demonstrates an improved sharpness in the contours. This was not realized in the original proposal of the filter. Furthermore the paper offers an explanation of the advantage of the filtering technique.

## Introduction

Topology optimization is an important field in mechanics, and the book of Bendsøe and Sigmund [1] gives an excellent overview of the field. In this paper we concentrate on a small subject namely the problem of distributing a given volume of material in order to reach the maximum stiffness. Only a single loadcase is considered, and the problem is plane and linear. The problem is discretized by the finite element method.

The basic idea is to introduce a so-called relative material density  $\rho \in [0; 1]$  for each element. In order to penalize intermediate densities which from a construction point of view is not suitable the finite element solution is made by a reduced stiffness proportional to  $\rho^p$ , where  $p$  typically is 3. In this way intermediate densities are not very effective. The method is denoted SIMP (Solid Isotropic Material with Penalization), and detailed explanations can be found in [1]. The analysis continues through a number of iterations leading finally to an optimal design. In this paper we use the Optimal Criteria Method which is very easy to implement and suitable see [1]. An alternative optimization method can be based on the Method of Moving Asymptotes (MMA), and in this way optimization problems with several loadcases and nonlinear mechanical behaviour can be treated, see [1].

In the optimal solution all elements have the same strain density, and the solution is also a 0-1 density distribution i.e. elements are either with no (small) density or full density. If no special precautions are made the solutions may have so-called checker-boarder patterns i.e. areas where the density jumps from 0 to 1 between neighboring elements. In this way an intermediate material density can be created without penalization. In order to avoid this non-physical phenomenon a filtering technique can be implemented, see the pioneering work of Sigmund [2].

## Checker-boarder filter

In the optimization algorithm the derivative of the strain density  $w$  with respect to density variations are needed. The basic idea of Sigmund [2] was to filter these sensitivities as shown in (1)

$$\widehat{\frac{\partial w_i}{\partial \rho_i}} = \sum_{j=1}^N \frac{\partial w_j}{\partial \rho_j} \rho_j (R - r_j) / \rho_i \sum_{j=1}^N (R - r_j) \quad (1)$$

where the summation from 1 to  $N$  involves the neighboring elements as shown in Figure 1. The sensitivities are weighted according to the difference between a chosen averaging radius  $R$  and the

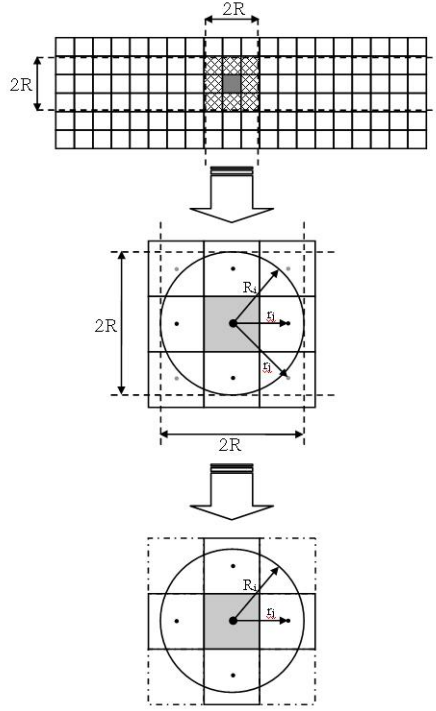


Figure 1: Elements participating in filtering for element  $i$

distance  $r_j$  from the center of element  $i$  to the center of element  $j$  times the density of the element  $\rho_j$ . In this filtering method variations in densities will have a large influence leading to less sharp contours between voids and solid material. The filter technique has proven effective, but in the literature different improvements are discussed, see e.g. [4] and [5].

### Improved filter

The filter technique used in this paper was original proposed by Back-Pedersen see [3], but the paper did not fully recognized all the positive consequences. This might be due to the fact that a more complicated problem was investigated involving several loadcases.

The basic idea is to remove the influence of density on the results. The sensitivities in the optimal solutions will be equal both in elements with large and small densities. However, the density weighting will change a smooth pattern along boundaries between dense parts and voids. The proposed filtering process only secures continuous sensitivity solutions, and the checker-boarder pattern is solely due to the finite element discretization. In (2) the filtering formula is given:

$$\frac{\widehat{\partial w_i}}{\partial \rho_i} = \sum_{j=1}^N \frac{\partial w_j}{\partial \rho_j} v_j (R - r_j) / \sum_{j=1}^N v_j (R - r_j) \quad (2)$$

where the notation is similar to the notation used in (1). In the filter we have included the influence of different element volumes to have consistency for different element sizes.

## Example

The example is solved with an algorithm similar to the 99-line program in [1] where more details can be found. In Figure 2 an example with a cantilever beam is given. The design depends on

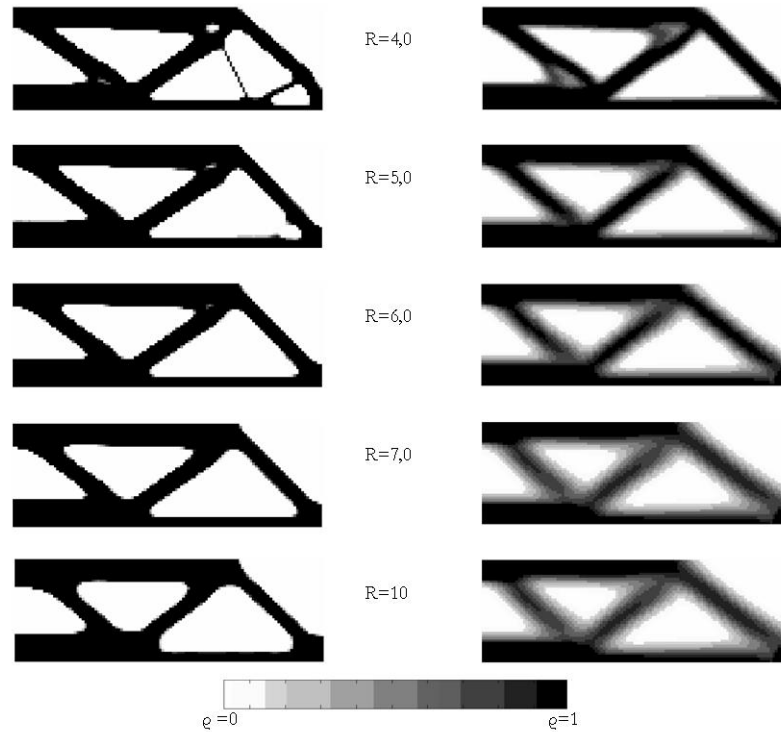


Figure 2: Different designs depending on  $R$ . To the left the improved filter and to the right Sigmund's filter.

the chosen material volume and the filtering radius  $R$ . A large value of  $R$  will give a more simple design on the cost of more material. The advantage of the improved filter is that the material distribution becomes closer to the 0-1 design. The number of elements with intermediate densities is very small with the new filter. This can improve the transfer of design data from the optimization program to the CAD environment.

A big difference between the original filter and the improved is the ability of the filter to operate with sharp transitions from 0 to 1 density. This is illustrated in Figure 3 and Figure 4. In Sigmund's filter the transition will normally involve an element with intermediate density, and this can be seen in the gray parts of the design. Further it may be noticed that the  $R$ -value has a very reduced influence on the cost of the design in the new filter compared to Sigmund's filter.

## Conclusion

The paper discusses a new filtering technique in topology optimization. The idea is to filter the derivative of the strain densities with respect to density solely as a geometric filtering not including the material density. In this way the checker-boarder pattern is avoided, and it is regarded solely as a finite element discretization problem. The new filter results in a design with very limited intermediate densities, and the contours are sharper. This will ease the transfer of data from the

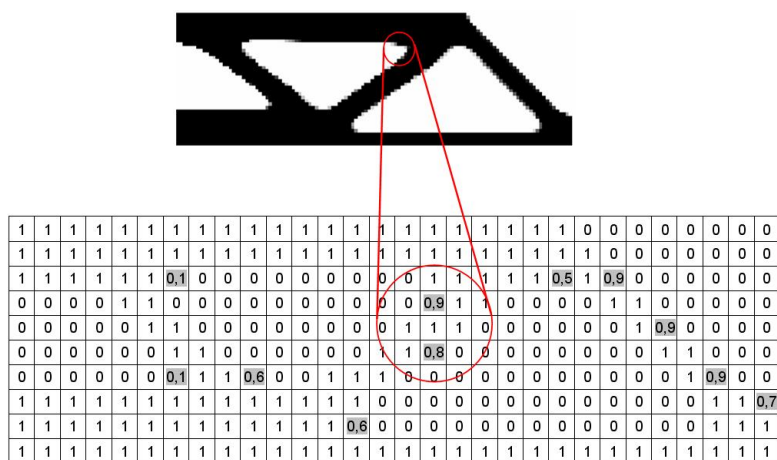


Figure 3: Density variation with the improved filter.

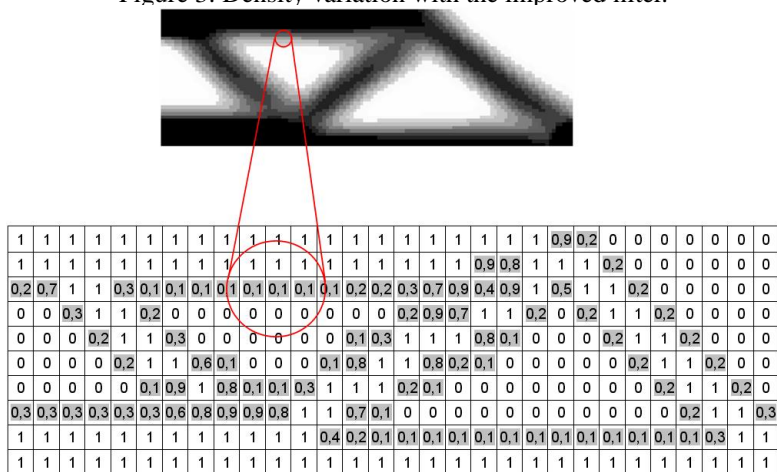


Figure 4: Density variation with Sigmund’s filter.

optimization analysis to the CAD-programs. The conclusion is based on a simple linear problem where the optimization is easy, and work on more complicated problems is in progress.

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