Curupira's Walk

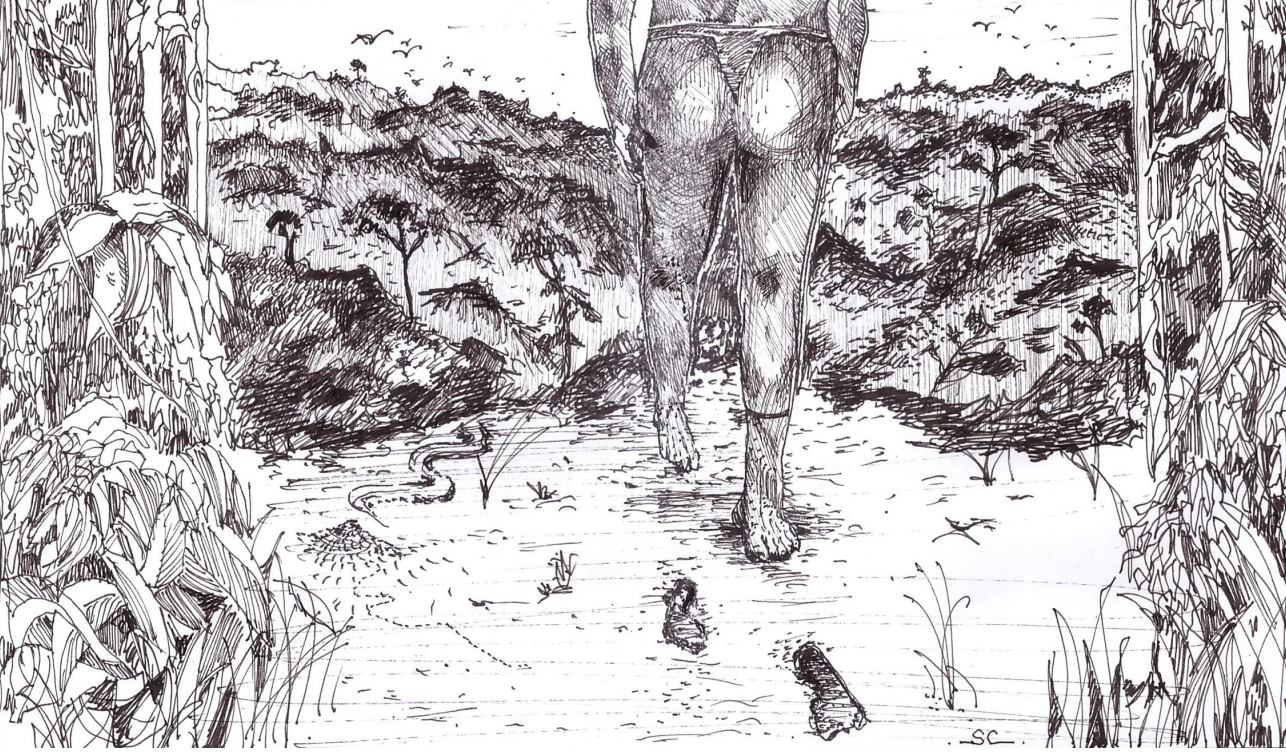
prowling ethnomathematics theory through decoloniality

Parra , Aldo Ivan

DOI (link to publication from Publisher):
10.5278/vbn.phd.eng.00050

Publication date:
2018

Document Version
Publisher's PDF, also known as Version of record



CURUPIRA'S WALK

PROWLING ETHNOMATHEMATICS THEORY THROUGH DECOLONIALITY

BY
ALDO IVAN PARRA SÁNCHEZ

DISSERTATION SUBMITTED 2018

AALBORG UNIVERSITY
DENMARK

Curupira's Walk: Prowling Ethnomathematics Theory Through Decoloniality

Ph.D. Dissertation

Aldo Ivan Parra Sánchez

Dissertation submitted June, 2018

Dissertation submitted: June , 2018

PhD supervisor: Prof. Paola Valero
Aalborg University
Stockholm University

PhD committee: Professor Ole Skovsmose (chairman)
Aalborg University

Professor Wilfredo Vidal Alangui
University of the Philippines

Professor David Wagner
University of New Brunswick

PhD Series: Faculty of Engineering and Science, Aalborg University

Department: Department of Planning

ISSN (online): 2446-1636
ISBN (online): 978-87-7210-217-7

Published by:
Aalborg University Press
Langagervej 2
DK – 9220 Aalborg Ø
Phone: +45 99407140
aauf@forlag.aau.dk
forlag.aau.dk

Printed in Denmark by Rosendahls, 2018

Curriculum Vitae

Aldo Parra

I research mathematics teacher education in context of cultural and linguistic diversity. This type of work demands the use of ethnographic methods and cultural theories to investigate issues like cultural identity, subjectivity and development. My main interest is to research how people from different cultural world-views and backgrounds interact with academic and scientific disciplines, specifically mathematics. This involves a deeper understanding of the conflicts, expectations and challenges that students face (or try to avoid) when scientific disciplines appear as important for their future.

I hold a B.Sc. in Mathematics from the National University of Colombia, Bogotá, and a M. Sc. in Mathematics Education from São Paulo State University at Rio Claro, Brazil. For the last 12 years I have been working in educational research projects with Nasa Indigenous people in Colombia. I was lecturer for 6 years, in faculties of engineering and education of colombian universities.

Additionally, I am member of Edutopia, a research group of mathematics educators in Colombia, and also I belong to an international network of research in cultural studies in mathematics education called "Latin American Network in ethnomathematics", I am member of the editorial board of the international journal: *Revista de la Red Latinoamericana de Etnomatematica.*

I like pataphysics and play with words to create bizarre and distorted meanings, but in this text I will behave properly...

Curriculum Vitae

Summary

This thesis adopts a decolonial standpoint to introduce a new theoretical framework for ethnomathematics. There is an emerging trend in ethnomathematics, that considers challenges posed by socio-political movements worldwide. That trend deserves a solid foundation and this thesis is aiming to provide it. The proposed framework calls into question the current theorization (and critique) of ethnomathematics, by claiming a performative, non-essentialist, and interactional approach that assumes the inseparability of socio-political concerns and philosophical questions within the field.

In order to argue for the robustness and contributions of the proposed theorisation, the thesis elaborates further some of the well-established debates on ethnomathematics: the problem of representation in the relation researcher/researched, the coherence of methods employed in research, the sensitivity towards predicaments of Indigenous education, the challenges of multilingualism and linguistic diversity, the political consequences of ethnomathematical research, and the positioning of ethnomathematics on the nature of mathematics.

The thesis works theory in a twofold way: one that develops theory as a structure defining objects and procedures of interest for the field; and other way that introduces and refines analytical notions that function as a toolbox for practice. In the first sense, a theory of interactions and relations among cultures and mathematics is produced. In the second sense, two original theoretical notions that can guide the practice of the working ethnomathematicians are proposed: symmetry and barter. Other notions coming from decolonial studies are adapted and reworked for the specificity of ethnomathematics: locus of enunciation, intellectual property, and *propio*.

As an article-based thesis, it is structured with one introduction, seven chapters containing independent articles, and a conclusion. The introduction states the research questions on the need of new theorisations that can comprise jointly political and philosophical concerns on the foundation of ethnomathematics. The seven chapters are devoted to explain how particular problems of ethnomathematics theory and practice can be addressed under new perspectives that (dis)solve the tensions that framed the debates until now. The

conclusion shows how those perspectives can be articulated within a decolonial approach.

As a result of the investigation, it is argued that a decolonial, performative, non-essentialist, and interactional approach has the potential of providing a robust foundation for the current practices of etnomathematics. It also allows introducing new concerns, scenarios and actors that can lead the theory and practice of ethnomathematics into a new era.

Resumé

Denne afhandling introducerer et nyt teoretisk begrebsapparat, som ud fra et dekolonialiseringsperspektiv bidrager til at forstå de seneste udfordringer indenfor forskningstraditionen om etnomatematik. Disse udfordringer er formuleret af socio-politiske bevægelser fra hele verden. Begrebsapparatet problematiserer eksisterende teorier (og kritik) af etnomatematikken ved at foreslå en performativ, ikke-essentialistisk og interaktiv tilgang, hvor det ikke er muligt at adskille socio-politiske og filosofiske spørgsmål.

For at opbygge et robust bidrag bygges videre på nogle af de velkendte debatter indenfor feltet etnomatematik: problemet vedrørende repræsentation i forholdet mellem forsker og det forskede, sammenhængen i metoder, sensitiviteten i forståelsen af problemer i forhold til uddannelsen af den oprindelige (indigenous) befolkning, problemerne vedrørende flersproglighed og sproglig diversitet, de politiske konsekvenser af etnomatematisk forskning og positioneringen af etnomatematik i relation til matematikken.

Afhandlingen arbejder med teori på to forskellige måder. Den første udvikler teori som en struktur, som definerer objekter og procedurer af interesse for feltet. Der udvikles en teori om interaktioner og relationer mellem kulturer og matematik. Den anden måde introducerer og raffinerer analytiske begreber, som fungerer som en værktøjskasse for praksis. Der foreslås to originale begreber, som kan anvendes i forhold til at lede etnomatematikeres arbejde: *symmetri* og *byttehandel*. Andre begreber, som kommer fra studier af dekolonialisering tilpasses og omarbejdes med henblik på deres anvendelse i etnomatematikkens praksis: Udtalens lokalisering [lokus of ennunciation], intellektuel ejendom [intellectual property] og tilegnelse/ejerskab/appropiering [propio].

Afhandlingen er artikel-baseret. Den er struktureret på følgende måde: En introduktion, syv kapitler, som inderholder selvstændige artikler, og en konklusion. I introduktionen formuleres forskningsspørgsmålet omkring behovet for en ny teoriskabelse, som kan favne politiske og filosofiske spørgsmål i selve fundamentet for etnomatematik. De syv kapitler er dedikeret til at forklare, hvordan forskellige problemer indenfor etnomatematikkens teori og praksis kan addresseres i nye perspektiver, som opløser de spændinger, som har præget

debatten indtil i dag. Konklusionen viser, hvordan disse perspektiver kan artikuleres indenfor en dekolonial tilgang.

Som et resultat fremhæves, at en dekolonial, performativ, ikke-essentialistisk og interaktionel tilgang har potentiale til at skabe et robust fundament for nuværende etnomatematiske praksisser. Desuden kan denne tilgang bidrage med at introducere nye fokusområder, scenarier og aktører, som kan lede udviklingen af teori og praksis indenfor etnomatematikken i en ny retning.

Dedication

-HM, 4-5-P. ¿me escuchas?

A Joaquín, Lina y mis padres, quienes llevaron pacientemente la carga más pesada de esta experiencia. Suplieron tanta ausencia con una alegría y amor infinito que me impidió desfallecer.

-uh, uh, u pa lá.

Dedication

Acknowledgments

Primero que todo, le debo un gran reconocimiento a las comunidades Nasa de Tierradentro en el Cauca colombiano, por haberme permitido estar cerca de su proceso de resistencia en los últimos 12 años. Esta investigación es un intento por compartir con colegas de otras partes del mundo lo mucho que he aprendido de las formas Nasa de trabajo, y que es muy poco frente a la sabiduría que ellos poseen, crean y multiplican. PAY.

First of all I am in great debt to the Nasa communities from Tierradentro, Cauca, for allowing me come close to their struggle in the last 12 years. This research is an attempt to share with colleagues from other parts of the world what I have learned from the Nasa' s ways of working. My learning is very modest in front of the wisdom that they possess, create, and multiply. Thanks.

It is a tremendous privilege to have been a PhD student of Paola Valero, and I am deeply thankful to her for giving me the opportunity to work under her supervision. She believed in my work more than I did and gave me constant advice of how to be an academic in the international arena. She treated me with respect, sharing her doubts, strengths and concerns. I appreciate the way in which she establishes an attitude of constant searching and critical perspective. Thanks for the academic, personal and emotional support. Thanks for the trust and complicity.

I also want to thank the families Trinick and Fairhall who literally adopted me in New Zealand, forgiving my multiple ineptitudes and sharing with me their wisdom and perseverance. It was a dream come true to meet the teachers and students of Te Kura O Te Koutu, the impressive indigenous school that shows how high quality in indigenous education can be real. My foreground was expanded with all the amazing work that Te Puna Wananga In Auckland University make, I really miss their "cup of tea meetings".

Thanks also to my disfunctional and distopic academic troupe in Aalborg: Alex, Melissa, and Lars, who performed a real paradox of continuous absence and presence. Thanks to Giselle Valerde, Kenneth Jørgensen, Ole Ravn, Ole Skovsmose, and Miriam Godoy Penteado, who gave us a relief in the middle of the hard moments of these 4 years.

A constelation of mavericks helped me in this experience, I appreciate

each conversation with Peter Gøtze, Tamsin Meaney, Monica Mesquita, Munir Fasheh, Jehad Alshwaikh, Lisa Lunney Borden, Ulrika Ryan, Veronica Albanese, Natalia Caicedo, Libia Tattay, Carolina Tamayo, and Hilbert Blanco. These friends share with me part of their kindness, professionalism, consideration, intellectual curiosity and openness. They came to my life in the right moment, just when I was lost in the abyss, victim of the pernicious influence of people like *los EdUtópicos* and *os Corticeiros*: Magda, José, Francisco, Gabriel, Camilo, Jorge, Renato, Gustavo, Anderson, Sinval, Luana, Adailton, and Marcos. That dangerous people only provided me their kindness, professionalism, consideration, intellectual curiosity and openness. Oh, wait a minute...

Finally, I would like to the national department of science, technology and innovation (COLCIENCIAS) in Colombia. This institution was the main funder of my doctoral research through the call for loan-scholarship 617. I also want to thank the Department of Learning and Philosophy of AAU and the Department of Planning of AAU for the stipends that they gave me. Without that stipend it would be impossible to complete my the doctoral study.

Contents

List of Figures

Thesis Details

This thesis has been submitted for assessment in partial fulfillment of the PhD degree. The thesis is structured as an article based thesis. As part of the assessment, co-author statements have been made available to the assessment committee and are also available at the Faculty. In part II the reader will find enclosed the research papers that have been written, some of them already published. This is the list of papers:

- Parra, A. (2015). Intellectual property in ethnomathematics. Revista Latinoamericana de Etnomatemática, 8(2), p. 398-414.

  Status: Published as Journal article

- Parra A., Mendes J.R., Valero P., Villavicencio Ubillús M. (2016). Mathematics Education in Multilingual Contexts for the Indigenous Population in Latin America. In: Barwell R. et al. (eds) *Mathematics Education and Language Diversity.* New ICMI Study Series. Springer, Cham. p.67-84.

  Status: Published as book chapter

- Parra-Sanchez A. (2017). Ethnomathematical Barters. In: Straehler-Pohl H., Bohlmann N., Pais A. (eds) *The Disorder of Mathematics Education.* Springer, Cham. p. 89-105

  Status: Published as book chapter

- Parra, A., & Trinick, T. (2017). Multilingualism in indigenous mathematics education: an epistemic matter. Mathematics Education Research Journal, p. 1-21.

  Status: Published as journal article.

- Parra, A., & Valero, P. (forthcomming). Propio as a a decolonizing tool for mathematics education. In A. Andersson & R. Barwell (Eds.), *Applying critical perspectives in mathematics education.* Rotterdam: Sense Publishers.

  Status: Submitted as book chapter

- Parra A. (2018). When and Why Do You Turn on the Mic? Tracing Symmetry in Ethnomathematics Research.

  Status: Unpublished manuscript

Preface: A reader's guide

This doctoral study is a collection of incursions in ethnomathematics theory and practice, independent among them at some extent, but accomplished with the same impulse to explore the future of the field, with the same aim to reinvigorate the passion and engagement that the long-standing contributions done by Ubiratan D'Ambrosio and Paulus Gerdes provided to the field. These incursions connect disparate (and seemingly peripheral) issues like indigenous education, multilingualism, fieldwork methodology and intellectual property, through the common thread of a decolonial inquiry about the politics of mathematical knowledge.

Those issues constitute thematic lines of the thesis, useful to organize the reading. According to that, chapters 1, 2, 4 and 5 are devoted to indigenous education; 2 and 4 to multilingualism; 2, 4, and 5 to decoloniality. Chapters 1, 3 and 6 address ethnomathematics research by making a review that inquiries about collaborative methodologies. Explicit concerns about epistemology are included in chapters 4 and 5. There is also a narrative line along the text, consisting in a research experience of the Nasa people in Colombia since 2006, that is described and analysed in chapters 1, 4 and 5. Being constant source of reflections, such experience receives a theoretical extrapolation in chapters 2, 3 and specially 6, when other research experiences are contrasted.

Therefore, this text is structured with one entry, multiple paths, and one coda. Reader can organize his/her reading according the thematic or narrative lines already described, or even, she/he can also proceed as usually, following successive chapters, that were organized resembling the most the chronology of writing. To aid the reading, graphs will be presented after each chapter, summing up the conceptual entanglement being built. It is up to the reader the creation of her/his own hopscotch.

Aldo Parra
Aalborg University, June 22, 2018

Part I

If you are Between Scylla and Charybdis...

I want to begin by explaining the motivation for this thesis before entering fully in the discussion. I was invited in 2006 to join an indigenous research on mathematics by the Nasa people in the Cauca State of Colombia. As I was previously involved in ethnomathematics, I contributed with several approaches and resources from the field to that research experience.

When the first results appeared in 2008, my indigenous colleagues and I began to share the experience in regional events of mathematics education and ethnomathematics. It did not take too much time for an uncomfortable feeling to surround us. Our experience was very different to the standard research in ethnomathematics, many times the relevant issues (for us) were not considered by reviewers and non-indigenous colleagues, and conversely, we were asked for things that we barely consider. It is fair to say that our ideas were never banned, rejected or diminished, but we were not classifiable under the frameworks used by the colleagues.

I was disappointed because I could not find experiences to take as reference, to share and improve our own work. When such loneliness was very clear, we met Gelsa Knijnik in a congress, she saw our work and invited us to follow up the research and encouraged me to pursue graduate studies. I completed my masters in Brazil, continuing the experience of joint work with Nasa people and contrasting educational Nasa initiatives with ethnomathematics theory. While undertaking this I noticed the existence of several other experiences, including some developed in non-indigenous contexts that share some characteristics, interests and results with the Nasa research.

As far as my experience with the Nasa continued evolving, I became more

conscious of the conditions of possibility that constituted that research process and its results, and I began to wonder if the feeling of incommensurability was due to the available theorisations of ethnomathematics (even the most avant-garde ones), that provided few, if any, visibility to a set of tensions and theoretic-methodological concerns present in the Nasa process and similar ones. That led me to consider a bigger picture, inquiring for the type of theorisation (and critique) that ethnomathematics research was receiving.

To deal with this increasing concern I decided to accomplish this doctoral study, which is structured by two research questions:

i. What set of tensions and concerns for ethnomathematics are identifiable within the Nasa research experience on mathematics and similar ones?

ii. Which kind of theoretical framework for ethnomathematics can address those tensions and concerns?

Thus, this thesis is an effort to make sense of my research experience with Nasa people and to reflect about its theoretical implications for a future research and practice of ethnomathematics.

To summarise, I am doing this contextualization to warn (and warm) the reader, highlighting that the analytical movement of this thesis is not about a theoretical idea looking for an empirical experience that can confirm its statements. Instead, it is about a series of empirical experiences looking for a theorisation that can express consistently and coherently its results, tensions and procedures.

Once I tried to relate the research questions with the development of ethnomathematics, the Greek image of Scylla and Charybdis emerged strongly as an illustrative metaphor for such development. In one of the Odysseus' adventures, the mythical hero and his sailors had to face a double threat when passing for Sicily. They found two beasts on opposite sides of the Strait of Messina: Scylla and Charybdis. The two beasts were located dangerously close enough to each other that they posed an inescapable threat: to avoid Charybdis meant pass too close to Scylla and vice versa.

My perspective is that ethnomathematics is inherently caught in a similar double trap, consisting in a combination of socio-political demands and philosophical questions. Since its origins, ethnomathematics has been interested in both concerns, as was explicitly stated in the last sentence of the seminal paper written by Ubiratan D'Ambrosio: "We should not forget that colonialism grew together in a symbiotic relationship with modern science, in particular with mathematics, and technology" (D'Ambrosio, 1985, p. 47) and in the latter definition of "Ethnomathematics is a research program about the history and philosophy of mathematics, with obvious implications for teaching" (D'Ambrosio, 2006, p. 17).

The debate in ethnomathematics has framed socio-political demands about the ways of empowering and dignify populations being discriminated and minoritized through mathematics (Pais, 2013; Vithal and Skovsmose, 1997) and operated those demands within the dichotomy between the minorities right to not be marginalized and the right to be treated differentially. Conversely, the philosophical questions comprised discussions on positioning about the ontology and epistemology of mathematics (Rowlands and Carson, 2002) and formulated those discussions within the dichotomy between the universality of mathematical statements and the locality of the ways to express and signify those statements.

Both in philosophical or socio-political concerns, tensions around essentialisms have been raised and appears as insurmountable. Concerning the political demands: Is it possible to refuse an essentialist account of cultural groups, without endorsing a homogenizing and neoliberal educational project? To what extent does the ethnomathematical call for differentiated education limit and re-discriminate populations? Similar tensions emerge when considering philosophical questions: is it possible to refuse an essentialist account of mathematical knowledge without adhering to cultural relativism? To what extent does the recognition of multiple and incommensurable forms of mathematical knowledge entail a "particularism that precludes the possibility of construction of translocal relations" (Savransky, 2012, p. 358)?

The importance of this double challenge became evident in the criticisms that ethnomathematics had been suffering. When the research program of ethnomathematics is not accused of increasing the disadvantages of minoritized people by ghettoizing them (Pais, 2013; Setati, 2002), it is accused of not having a clear account of the mathematical knowledge, falling into a cultural relativism (Horsthemke and Schafer, 2007; Rowlands and Carson, 2002). My reading on these debates is that attempts to theorise ethnomathematics have often addressed only one of the challenges, and whenever the existence of both demands is perceived, they are treated separately, as if they were independent or separable. (CONTCEPI, 2013)

This landscape on the development of the ethnomathematics theorisation revealed to me with clarity the importance and utility of the research questions that structure this thesis. The experience lived with the Nasa people has let me to conceive a different theoretical approach, which attempts to address jointly and simultaneously the political and philosophical challenges, in a way that new work fronts for practice and research in ethnomathematics are created.

In the next chapters I will explain how and why I can conceive such a different theoretical approach. An open debate is about the specificity and singularity of ethnomathematics. It is certainly not a subset of mathematics education, neither of cultural anthropology or philosophy of mathematics, although it is also imbricated in those areas. This thesis joins the efforts to build such singularity.

References

CONTCEPI (2013). Perfil del sistema educativo indígena propio, seip.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics*, 5(1):44–48.

D'Ambrosio, U. (2006). *Ethnomathematics. Link between traditions and modernity.* Rotterdam: Sense Publishers.

Horsthemke, K. and Schafer, M. (2007). Does' african mathematics' facilitate access to mathematics? towards an ongoing critical analysis of ethnomathematics in a south african context. *Pythagoras*, 2007(65):2–9.

Pais, A. (2013). Ethnomathematics and the limits of culture. *For the learning of mathematics*, 33(3):2–6.

Rowlands, S. and Carson, R. (2002). Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? a critical review of ethnomathematics. *Educational Studies in Mathematics*, 50(1):79–102.

Savransky, M. (2012). Worlds in the making: Social sciences and the ontopolitics of knowledge. *Postcolonial Studies*, 15(3):351–368.

Setati, M. (2002). Is ethnomathematics= mathematics= antiracism. In Valero, P. and Skovsmose, O., editors, *Mathematics Education and Society. Proceedings of the Third International Mathematics Education and Society Conference MES3,(sec. ed.)*, volume 2, pages 31–33. Centre for Research in Learning Mathematics.

Vithal, R. and Skovsmose, O. (1997). The end of innocence: A critique of 'ethnomathematics'. *Educational Studies in Mathematics*, 34(2):131–157.

Part II

...take a journey...

At a molar level

Chapter 0

Rebooting theory

This chapter introduces the molar level of a performative theory of ethnomathematics. I will not proceed as usual in doctoral theses about ethnomathematics, that is, to present a history of ethnomathematics progression and a theoretical framework about the philosophy of mathematics, to finally launch the approach itself (v.gr. Rohrer (2010)) supported by some fieldwork. Instead, I will get straight to the proposal, presenting it with the aid of a mathematical metaphor and then I explain some of its implications for theory and comment its differences and convergences with previous approaches. As this thesis is long and demands time, I will take it for granted that you, my dear reader, know what happened since the plenary conference of ICME-5 in Adelaide. If it is not the case, I recommend you read a couple of complete accounts: (Gerdes, 1996) and D'Ambrosio (2016)).

0.1 A mathematical image

For the sake of the metaphorical game, I propose considering that the vast amount of knowledge and practices recognized as belonging to disciplinary mathematics (and its institutionalized education) constitutes a set[1]:

$M(t)$:= {algebra, geometry, graph theory, games theory, Godel's theorems, modus tollens, aristotelian logic, fuzzy logic. Induction's proofs, negation's proofs, scientific notations, Matlab, trivium, PISA, TIMMS, common core standards, etc., at certain time t}.

[1]This idea is inspired by a scheme proposed by Barton (1996b).

This is a way to schematically represent the "near-universal, conventional mathematics" (NUC Mathematics) proposed by Barton (2008, p. 10) and part of its institutionalized ways of propagation trough school systems, but locating it at certain time t in human history. In the same way, I also propose considering the amount of knowledge and practices recognized as constitutive of the indigenous Nasa culture:

$\mathcal{C}_N(t) :=$ {myths, rituals, costumes, gastronomy, skills, language, procedures, political organisation, QRS systems[2], educational practices, ... belonging to the Nasa indigenous people at certain moment t}.

This is also a representation of the Nasa indigenous knowledge system and, as any representation, it reduces the complexity. Having defined those representations, basic concepts of set theory can be used to create other representations. For instance, with two sets A and B the set of relations going from A to B can be defined as follows: $B^A := \{f \mid f : A \to B\}$.

It is important to stress that B^A constitutes a different space, and its elements are not members of the original sets A or B. Just not to make unnecessarily hard the notation, for the subsequent discussion I will write the t only in special cases to stress its action, but temporality is always included in the argumentation, as all those sets change over time.

Thus, $M^{\mathcal{C}_N}$ can be imagined as the set of relations connecting elements in Nasa culture with elements in mathematics. This is to say, this set comprises attempts to read cultural practices through the lenses of mathematics. It covers all possible efforts that takes a myth, practice, object, or notion belonging to Nasa culture and relates it with a (disciplinar) mathematical object. For instance, any possible research study on mathematical modelling of Nasa cultural practices can be considered one of the elements of this "set."

Conversely, we can consider ${\mathcal{C}_N}^M$ as the set of relations connecting elements in mathematics (and its education) with elements in Nasa culture. As expected, this set comprises attempts to read mathematical practices through the lenses of Nasa culture. It gathers all possible efforts taking a mathematical object and relating it with cultural practices of Nasa people. For instance, a Nasa culturally rooted explanation of time and space can be considered an element of this "set."

One twist more, we can consider the union of the last two sets as: $\hat{\mathrm{E}}_N(t) := {\mathcal{C}_N}^M \cup M^{\mathcal{C}_N}$. That set depends on one cultural group. As expected consequence, we can go through all possible cultural groups to define this other set:

$$\hat{\mathrm{E}} := \left(\bigcup_{\alpha} \hat{\mathrm{E}}_\alpha \right) : \alpha \text{ is a community or a cultural group.} \tag{1}$$

That is the main equation of this theory. $\hat{\mathrm{E}}$ is my "metaphorically precise"

[2] A QRS-system is "a system for dealing with quantitative, relational, or spatial aspects of human experience" (Barton, 2008, p. 130).

definition of ethnomathematics research program.

To say $f \in \hat{\mathrm{E}}$ means that f is a research project establishing a connection between the mathematics of certain time and the culture of a group at that certain time $\left(\in \hat{\mathrm{E}}_\alpha(t)\right.$ for some α at the time $\left.t\right)$. Such a connection can be from the cultural knowledge system of a group to the realm of mathematics $\left(f \in M^{\mathcal{C}_\alpha}\right)$, or the other way around, from the realm of mathematics to the cultural knowledge system of a group $\left(f \in {\mathcal{C}_\alpha}^{M}\right)$.

This representational game is not an arcane way to express that ethnomathematics does "create a bridge between mathematics and the ideas (and concepts and practices) of other cultures" (Barton, 1996a, p. 203). It presents ethnomathematics as composed of a series of contingent and purposefully constructed relations between mathematics and culture. Such an image comprises a reconceptualization of the ethnomathematical object of study and the way in which the object is studied. Consequently, notions like interpretation, creation, intelligibility, and interaction gain a centrality that they did not have before in the research program. At the end of this chapter I will comment some of the consequences and challenges emerging from such reconceptualization. Chapters 1 and 3 will re-elaborate this relational approach to ethnomathematics, establishing some methodological consequences and introducing tools to address associated challenges.

Concerning theoretical issues, this relational idea reevaluates simplistic essentialist views about culture or mathematics, as far as they are conceived as historical constructs, capable of changing (M and $\mathcal{C}_\alpha$ depends on t). I consider Luis Radford's ideas about Foucauldian and Marxist contribution on the nature of reason (a kernel for mathematics) and culture as illustrative of such a stance:

> There is no universal regulatory reason. The reason is historical and cultural. Those proper forms that Foucault called epistemes are conditioned, without being so in a causal and mechanical sense, by their imbrications in the social and political practice of individuals. (Radford, 2014, p. 54, my translation)

> The concept of culture that Marx elaborates indirectly in his writings is, in fact, profoundly historical and transformative. Individuals create culture and, in a reverse or dialectical movement, culture offers the conditions for individuals to create systems of thought whether scientific, aesthetic, legal, etc., and create themselves. That is why, from a materialist dialectical perspective, human cultures are much more than reified and static entities. (Radford, 2014, p. 56, my translation)

The image of a relation connecting different realms retakes previous contributions made in the field, like "ethnomathematics is not mathematics, but is *about* mathematics" (Alangui, 2010, p. 84, italics given) or Knijnik's argument to the question of why cultural practices, appearing on different times

and spaces cannot be called as enactments of the same universal/underlying mathematical concept. Several works (Knijnik, 2012; Knijnik and Wanderer, 2010; Vilela, 2010), appeal to the Wittgensteinian idea of "family resemblances" between language games to explain such commonality in the practices, and to highlight the presence of different forms of life producing different language games. I will return to those issues later, but it is enough to say here that, in our framework, to show a "family resemblance" means to build an $f \in \hat{\mathrm{E}}_\alpha$. In the next section I will continue the characterization of the approach by describing its antecedents within ethnomathematics.

0.2 Standing on the shoulders of giants: a glimpse of a theoretical lineage

In this section I make a selection of previous works on ethnomathematics, focusing on contributions from four authors that have inspired this approach, by commenting which parts of their production are integrated to this theory and which ideas were extended or developed to create a different proposal. This means, I do not intend to criticize or point shortcomings of any of those approaches, but to take them as point of departure.

Cauty: Hybrid knowledge, language and the *Kwibi Urraga* experience

André Cauty is a French ethnolinguist and mathematician who visited Nasa indigenous communities in Colombia during the 1990's, obtaining a perspective on the colonization process that occurs through the language imposition and how indigenous people develop strategies of resistance (Cauty and Ramos, 1990). He also conducted a research experience with *Wayuu* indigenous people and linguists at Universidad de la Guajira in the north of Colombia. That work was developed within the *Kwibi Urraga*[3] *Laboratory*, a centre for research on cognitive linguistics, applied to ethno-education. That centre worked from 1995 to 1997 on different translation projects from the Wayuunaiki (the Wayuu Indigenous language) into Spanish, one of them on mathematics. In that project, Cauty promotes the creation of new and hybrid knowledge, through the constant interaction of three types of authority: linguistic, mathematical and cultural creating chains of interpreters within intercultural and interdisciplinary groups. For this author the collective debate is a condition for cognition and the generation of knowledge. Cauty was very aware of the impossibility to achieve term-to-term translations and the political project of assimilation that sustains those forms of translation. Instead, he conceives translation as deeply

[3] *Kwibi Urraga* is an expression in Kagaba, the language of the Kogui indigenous people; it means "house of wisdom".

related to invention, motivating an expansion and rearrangement of cultural, linguistic, and scientific knowledge systems, as it is clear when he stated:

> The founding principle of the Kwibi Urraga program is that it is less about translating and adapting Western texts, aimed at [a type of] education within the traditional school framework, rather than about the creation of conditions that allow Amerindians to invent their mathematical varieties, and this in their languages and within the framework of their specific conceptualization systems". (Cauty, 2001, p. 84, my translation)

Cauty did not continue his work on knowledge creation with indigenous communities. He decided to research more on the on Amerindian numeration, specially the Mayan systems in Central American (Cauty and Hoppan, 2005),(Cauty and Hoppan, 2007; Cauty et al., 2001). Nonetheless, Kwibi Urraga laboratory principles are a cornerstone for the approach that this thesis proposes. In his research experience, the author pointed two things of interest for the present discussion: one is the intertwining of language with mathematics and the other is a philosophical stance.

The first contribution aims to consider the act of translation as an inherent part of the mathematical practice. For Cauty, the mathematical language is basically a writing which has a series of registers and genres that goes back and forth between the abstract and the concrete, the material and the immaterial, using particular codes and notations sometimes, that cannot be fully disconnected from the natural languages (Cauty, 2001). Within that frame, Cauty conceives a mathematician as a specialist (more precisely, a juggler) of representations, an intellectual that can create, re-interpret, and transform signals, symbols, and signs. Such a role makes the mathematician basically a translator between languages of diverse kind. Within this insight "there is no thought in a pure state, independently of a form that expresses it, represents it, and (above all) allows its communication to others" (Cauty, 2001, p. 75, my translation); accordingly, comprehension and representation fecundate each other.

> To translate is not to put or change a label; it is to achieve an understanding of how to say the same thing differently, under certain perspectives. (Cauty, 2009, p. 31, my translation)

The important role that Cauty recognizes to intended translations, by stressing their new and different nature, is central in the molar level of my theorisation and inspired my idea of relations between sets. This intertwined condition of language and mathematics is also sustained when looking at the history. Cauty (2009) emphasized how big changes in the development of mathematics occurred when different languages collide and translations are required. He pointed particularly in the transition from the Middle Ages to the Renaissance:

> The Arab mathematical treasure was eventually translated into Latin, especially by the Jewish sages, due to the increasing flows of goods, men, and ideas according to all sorts of voyages, settlements, wars, conquests, and other crusades. These translations into Latin were incessantly copied and later retranslated into the languages of European states. Such texts generated several critiques and comments, stimulating European thinking from the Middle Ages to the Renaissance and brought about an undeniable and increasingly international mathematical development. [...] The scientific heritage of the Europeans of the Renaissance was the result of the assimilation efforts of various secular corporations (*e.g.* university students and professors) who, since the Middle Ages, received and could achieve the immense work of translators and copyists of Arabic or Greek texts. (Cauty, 2009, p. 33, my translation)

The second contribution of interest for our discussion provided by Cauty is a position on the ontology of mathematical objects, very related to his ideas about language and close to Radford's ideas on the historical development of mathematical knowledge:

> That is, it is to understand that the independence of notations and representations with respect to languages is neither absolute nor given a priori, but relative and acquired along the history, the individual history of learning of each person, and the millennial collective history of the discipline. [...]
>
> We defend a thesis based on the observation of the historical construction of mathematics, as well as on the observation of the epigenetic time of the formation of a mathematician. This thesis prevents us from fully adhering to the most extreme doctrines: idealism and positivism. Therefore, neither do we believe only in the reality of ideas (Conceptus), like the too much idealistic doctrines do, nor believe only in the reality of things (Res), as the too much materialistic doctrines do. A classic solution consists in considering a third order of reality, the one of signs (Vox) and representations. That is, to address entities that are neither things nor ideas, but substitutes for references, both imaginary and real (Cauty, 2001, p. 77, my translation)

With those statements, Cauty refuses to adhere to any of the traditional poles in the philosophical discussion about the nature of mathematical entities. With such refusal, not only does Cauty prefigure my concerns about the philosophical dilemma faced by ethnomathematics; he also foresees a strategic use of language to surpass such a dilemma. Unfortunately, this philosophical stance is a marginal digression on Cauty's work and I did not find any posterior work in which he explored further this interesting insight into ethnomathematics.

Barton: Philosophy, language and the necessity of a theory

Bill Barton is a New Zealander scholar, considered one of the main researchers on ethnomathematics. Although his academic production has a wide range, covering teacher education (Barton, 2009), mathematics (Barton et al., 2005), language (Barton, 2008), linguistic diversity (Barton et al., 1998; Barwell et al., 2007) among others, I want to consider here only his contributions to ethnomathematics theory. He has explored the nature of ethnomathematics and its complex relation to mathematics when proposing several redefinitions of ethnomathematics:

> Ethnomathematics is a research programme of the way, in which cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics. (Barton, 1996b, p. 214)

> In the mathematics braid some strands are bigger than others, some strands merge with each other or split apart, some strands are disguised within non-mathematical coverings. But if we regard mathematics as QRS-systems[4], I argue that mathematics consists of parallel systems, not one consistent body. Ethnomathematics can be regarded as the study of the different fibres of mathematical knowledge. (Barton, 2008, p. 107)

When analysing existing conceptions of ethnomathematics at the time, Barton (1996b) introduced a series of diagrams in which culture and mathematics were modelled as sets. Those diagrams inspired the mathematical presentation of the molar level of my theorisation. In addition, Barton was one of the scholars that started the quest for a philosophical foundation for ethnomathematics (Barton, 1998), (Barton, 1999) , and he was the first to highlight the benefit that the post-metaphysical approach of Ludwig Wittgenstein can bring to explain and sustain the ethnomathematical work. Guided by the contributions of this philosopher, Barton refuses Platonist and cultural relativistic approaches and stands for a more radical version of relativism, that conceives mathematics evolving together with language and resulting from social practices. His non-essentialist stance is evident in statements like: "So mathematics is not about anything, it is a way of talking" (Barton, 1999, p. 56) or "we bring mathematics into existence by talking about it, and the way we talk about it changes the questions we can ask" (Barton, 2012, p. 227).

Concerning the classic discussion on the universality of mathematical knowledge, an initial Barton's theorisation of ethnomathematics proposed a flexible consideration: for some matters mathematics became universal and for others relative (Barton, 1996b). Later, he rephrased his stance and conceived the

[4] A QRS system is a system of meanings by which a group of people make sense of *Q*uantity, *R*elationships, and *S*pace. So the model of mathematics being proposed here is one where each cultural group has its own QRS system" (Barton, 1999, p. 56).

verb mathematising and the adjective mathematical as derivatives of the noun mathematics. He invite us to think in the verb mathematising as the primary expression of the field, focusing on it as the social practice (culturally and linguistically situated), and leaving the noun mathematics as the result of such a practice (Barton, 2012). Such a change has strong consequences because it "dissolves" the discussion on the universal existence of mathematical objects. As the author says:

> There would be no question about whether they [the mathematical objects] exist independently or about how we come to know them. We mathematise, and therefore we create the objects by our thought, and attempt to communicate them to one another. The ontology and epistemology of mathematics simply is not a problem anymore. (Barton, 2012, p. 228)

To my view, with this insight Barton meets Cauty on the need to overcome dichotomies on the philosophy of mathematics, but they differ on the purposes and ways of accomplishing such an endeavour. Although Barton gave more importance to this philosophical issue, as he considered it central to a theorisation of ethnomathematics, both authors made me aware of the importance of addressing the philosophical dichotomy in other terms. I consider that Barton's contributions about mathematising as a process and his considerations of QRS systems pave the way for a new entire terrain for ethnomathematics. The theorisation that I attempt with this thesis operates in such a new terrain. However, I do not adhere in full to the relativistic approach that Barton promotes (for reasons that will be explained in the final chapter).

Another element provided by this author is central to my concerns. Barton warned about the inconvenience of labelling a series of cultural practices in social groups that do not have the mathematics category for themselves as *mathematical* (Barton, 1996b). However, he was fully aware of the possibilities that cultural encounters generate:

> What happens when different mathematical systems meet? Wittgenstein's answer is that there are no 'gaps' in mathematics. Each system is complete at any moment. It is not waiting to be added to with new mathematics. Thus (Shanker, 1987, p. 329), any connection between two worlds is not in the same space as either of the worlds. The interconnections are not waiting to be discovered. We choose whether or not to make connections between systems, and if we do then the connections create a new system. (Barton, 2008, p. 130)

> Mathematicians from different worlds can look at their own practices and conceptions in the light of the practices and conceptions of other worlds; modify, reinterpret, discard, or adopt particular practices; and retain the knowledge of how and why this was done as part of their mathematical understanding. In the same way that we do not reject as

> wrong historical practices and conceptions (only see them as consistent within their historical context and use that knowledge to inform the present), so too could mathematicians from each world acknowledge the other mathematics within their context and use the knowledge to reflect on their own. (Barton, 2008, p. 133)

My view on these excerpts is that inexistence of the "mathematics" category in certain cultures does not imply the impossibility to create rooted meanings, explanations and positions on the westernized category of mathematics, otherwise one would be adhering to a subtle form of essentialism, and that is precisely a thing that Barton would reject.

Knijnik: Philosophy and politics coming from social movements

Gelsa Knijnik is a Brazilian scholar who started her career in ethnomathematics working with the Brazilian Landless Movement (in Portuguese, Movimento Sem Terra [MST]), deeply inspired by Paulo Freire's ideas. She is also a very renowned researcher in ethnomathematics because her long-term experience with MST (more than 20 years) allowed her to formulate and analyse the political consequences of ethnomathematics (Knijnik, 1996). However, Knijnik research interests go far beyond the level of activism or militancy; her work consists in analysing the relation of power and knowledge that ethnomathematics can reveal. In addition, she is also interested in the philosophical discussion on the diversity of mathematical practices, aiming to provide a foundation for ethnomathematical practices.

Therefore, Knijnik's work is influential and relevant to this thesis because she addressed the two challenges for ethnomathematics theory that this work highlights: the political and the philosophical questions. This is not to say that the other authors referred to in this chapter had no political awareness, commitment to communities, or philosophical insights. Not at all. However, Knijnik was the first one that included both concerns in a theorisation of ethnomathematics. She taught us that ethnomathematics must face Scylla and Charybdis. I will give a very brief (and incomplete) account of the developments that this author provided to the field and how I want to elaborate on them.

To cope with the political challenge, Knijnik developed the Freirean defence of the popular knowledge, through a discussion on the relationship between legitimate and popular knowledge, based on ideas coming from the sociology of education, provided by authors like Pierre Bourdieu, Claude Grignon, and Claude Passeron (Knijnik, 1996). Some years later she re-evaluated her own readings about ethnomathematics and incorporated the theoretical framework of Michel Foucault (Knijnik, 2006). At that stage of her career, she stated:

> The basis of the ethnomathematics thought I have been elaborating is based on a Post-Modern perspective in its connections with Post-Structuralist theorisations, more specifically, those associated with the work of Michel Foucault. According to such theorisations, I have considered that Ethnomathematics may consist of a toolbox which allows analysing: a) the Eurocentric discourses that institute academic mathematics and school mathematics; b) the effects of truth produced by the discourses of academic mathematics and school mathematics; c) issues of difference in mathematics education, considering the centrality of culture and the power relations that institute it. (Knijnik, 2007, p. 8)

The approach elaborated by this author on the study of the politics of mathematical knowledge is decisive and central to my interests in this thesis. Gelsa Knijnik's insights were very revealing for the experiences that I am trying to articulate:

> Introducing power into the ethnomathematical discussions avoided a naïve understanding of the mathematical diversity. Making power explicit in ethnomathematics could allow us to analyse how the politics of knowledge operates in schooling processes and, in particular, in the mathematics curricula. What is at stake here is to consider school mathematics not as a set of fixed subjects whose higher level of abstraction would allow students to cope with the multiple dimensions of their lives or, in the words of Wittgenstein (2004), in the multiple forms of life to which they belong, but as an arena marked by struggles for the imposition of meaning. (Knijnik, 2012, p. 89)

This excerpt is illustrative of Knijnik's approach, because in it the political concerns became connected with a philosophical discussion on the nature of mathematics. The way in which she elaborated such discussion and faced the philosophical challenge for ethnomathematics also deserves our attention. Accepting Barton's invitation about post-metaphysical philosophy, Knijnik interpreted key notions of Wittgenstein's work to explain the existence and intelligibility of different cultural practices that have been studied by ethnomathematics:

> In fact, using it, we can argue that there are language games of non-Western school forms of life which are "mathematical" because we identify family resemblances between them and the language games we were schooled in the West. This is the criterion to be used to decide on the ways language games are "mathematical" or not. (Knijnik, 2012, p. 92)

Gelsa Knijnik is the leading scholar of an entire series of studies (Giongo, 2008; Knijnik, 2007; Knijnik and Duarte, 2010; Knijnik and Wanderer, 2010; Knijnik et al., 2012; Vilela, 2010; Wanderer, 2007), that explore and develop further Wittgenstein's ideas for the research program of ethnomathematics. Those

efforts have provided great impulse within ethnomathematics to the study of the multiplicity of rationalities and how such rationalities are constituted by forms of life.

> Here comes a key point: when one abandons the idea of a single, natural, reason-producing structure, it is possible to understand rationality as an "invention," a "construct" (Condé, 2004, p. 29). It is this "construction" that will enable language to articulate itself inside a form of life and establish which rationality will indicate to us what we should accept. (Knijnik, 2012, p. 91)

Knijnik's contributions present a remarkable advance in the theorisation because they allow ethnomathematicians to find similarities and overlaps among cultural practices, without appealing to an essential or primary mathematical source. For that reason, her sophisticated theoretical insight has gained currency on the field (Albanese et al., 2017; François and Vandendriessche, 2016), to the extent that nowadays other researchers claim: "Stressing this last idea of cultural relativism we have summarised the thought of many ethnomathematicians who indicate Wittgenstein's philosophy of a language game as the theoretical foundation of the field" (Albanese et al., 2017, p. 324).

The entire theoretical approach presented in this thesis is deeply inspired by the Foucauldian-Wittgensteinian standpoint that Knijnik developed for ethnomathematics, as far as it shares an interest in problematizing essentialist views of math and culture. There is also a similar concern with agency of people in the defence of their own forms of life, and with power relations that (in)validate and (de)legitimize some sort of knowledge. Notwithstanding that, it is also important to comment on some issues of her theorisation, not with the aim of presenting a critique or a refutation, but of attempting to show what else remains to be done and deserves to be thought.

My first comment is about the strategy of articulating two different approaches, one for the political challenge (Foucault) and other for the philosophical (Wittgenstein). Knijnik perceives the need to provide answers to both challenges, but she uses separate tools to cope with each one. Such a separation is almost mandatory; if one considers that although Wittgenstein recognised the social nature of knowledge and focused on how mathematical meaning is constituted by its use within practices, he was not interested in the power struggles that operate in such constitution. Conversely, Foucault was aware of the localized and rooted character of discursive practices, but he did not elaborate on the intelligibility among such practices. Therefore, a question that emerges is about the possibility to provide a simultaneous answer to both challenges. In the conclusions of this thesis I will elaborate on this comment.

A second comment is about the cultural relativism that is usually sustained on Wittgensteinian stances by ethnomathematicians like Albanese et al. (2017), Barton (1999) and Knijnik (2012) to a lesser extent. Different cultural practices

are conceived as different language games that came from different forms of life. However, the fact that people do not play the same language games does not deny the universal capacity of playing games that allow us to learn new games. Therefore, there is something that escapes from an overarching cultural relativism. In the same sense, the notion of family resemblances is used to deny a common essence or zero point for mathematical knowledge, and therefore, to support the relativist account. However, there is a paradox: the more reported family resemblances, the more stressed the pre-existence of a family. It seems to me that non-essentialism cannot be equated with cultural relativism.

A third comment is also drawn from the idea of family resemblances, a notion that can be called into question. How can a family resemblance be noticed? Who can claim the existence of a resemblance between two practices? How did that thing become observable? I presume that the late–Wittgenstein would refuse the idea that resemblance can be discovered. I realized that a resemblance demands to be actualized. Family resemblances can be observed only if there is someone interested in making such comparisons. This means, someone is needed to examine, find the resemblances, and argue to affirm or discard the commonality in a determined aspect. A similarity emerges not because one is lucky to notice the resemblance, but because one has the will to find it, or the need to use it, or even more, the need to create it. This also supposes a purpose: why are ethnomathematicians looking for such resemblances? An obvious addition can be made: to claim a connection demands to know both realms of practice.

The mathematical image that introduces my molar theorisation of ethnomathematics shows an elaboration on the last two comments, by fostering the creation of hybrids, specifically designed to have family resemblances (again, $f \notin M(t)$ and $f \notin \mathcal{C}_N(t)$) and demanding to have a deep understanding of the language games considered. Those hybrids mix elements coming from different realms of practice; they mix the grammars of language games on purpose. Through those hybrids, it is possible to play one language game in a different way, resulting in changes of the entire game. Summarising, for this theory the similarities between practices (family resemblances) are created, not discovered. Such a strategy avoids the requirement of a pre-existent family; instead, it attempts to increase the family. Then, the task is not to find relatives but to create them.

Alangui: A methodological way

Wilfredo V. Alangui is a Philippine scholar who conducted doctoral studies under the supervision of Bill Barton[5]. Based on a comment made by Mendoza (2001) about the need for mutuality within studies of Indigenous Knowledge, in his doctoral thesis Alangui proposed a methodology for ethnomathematics

[5] A more detailed biographical account of this author will be provided in Chapter 6.

called mutual interrogation. Mutual interrogation was defined as the "process of setting up two systems of knowledge in parallel to each other in order to illuminate their similarities and differences, and explore the potential of enhancing and transforming each other" (Alangui, 2010, p. 86).

This interpellation of knowledge domains is expressed in the metaphor of relation between sets. My formulation of mathematics as time dependant ($M(t)$) and sensitive of the changes coming from ethnomathematics ($f \in \hat{\mathrm{E}}_\alpha(t) \to f \in M(t+\epsilon)$ for some ϵ) also echoes Alangui's insights like the following:

> Hence the importance of having to reiterate that ethnomathematics is not mathematics, but is about mathematics. [...] [Ethnomathematics] agenda is directed mainly at mathematics, to change our assumptions and underlying beliefs about the subject, to broaden our conceptions of it, and to allow for its transformation. (Alangui, 2010, p. 84)

> Ethnomathematics offers an arena where indigenous peoples can assert their alternative views and knowledge about the world. (Alangui, 2010, p. 25)

Alangui's work is a major pillar of this thesis, as his notion is concerned with the bilateral exercise of enquiry into two different knowledge domains (cultural and mathematical), and how such an enquiry can be done with integrity. For this author, the main point is the act to relate two knowledge domains that are not usually related; with Alangui I learned that the action of calling/recognising one mathematical issue as cultural-rooted issue or vice-versa is more relevant than the truly issue's membership of the intended corpus. Such an action is fully related to political intentions and epistemological beliefs. However, despite the many convergences, there are also some aspects that can be elaborated upon, mainly with the role of facilitators that his theory foresaw for ethnomathematicians. The research experience reported by Alangui seems insufficient when considering the ways in which the interrogation was conducted, as the same author recognises:

> In this case, the interrogation between the two knowledge systems happened internally – within me as the ethnomathematician who studied these two cultural practices through ethnography. (Alangui, 2010, p. 191)

> At a very personal level, the dialogues, which have been central to this thesis, have been dialogues with myself. (Alangui, 2010, p. 193)

> [...] mutual interrogation can take place internally in the mind of the researcher. As a consequence, the success of the methodology, the extent to which either mathematical or cultural practice conceptions may be altered, depends upon the state of mind of the researcher. (Adam et al., 2010, p. 15)

Such mental and personal development of the interrogation process arose as a serious issue that jeopardises the entire methodological proposal. This issue converge with the reflection done by Miarka (2011) when considering mutual interrogation: "I wonder, however, to what extent the dialogue between both groups is equitable. After all, production within the cultural community of practice is triggered by the speech of a member of the group of mathematicians, while production in the group of mathematicians is triggered by cultural practice, not by the practitioner's talk about mathematics" (Miarka, 2011, p. 366, my translation).

Alternatively, this thesis contends that a transformative and critical dialogue between cultures and systems of knowledge can happen through open, public and collective interactive processes of meaning-negotiation and sense-making. Along this doctoral study, I will review different cases that indicate how such processes can be performed. It is also my intention to elucidate which constraints and practices are common to those cases, to envision how creative processes can be conducted. Even Alangui himself has expressed the need to work in this direction and identified a series of refinements necessary to be explored in the future:

> If mutual interrogation is to become a significant methodology within ethnomathematics, we need to find ways of addressing both sides of this issue. We need to find a way to ensure that practitioners of two systems face each other with mutual respect, understanding, and ignorance of each other's' practices and systems. [...]
>
> The refinements we see as needing work are:
>
> - Understanding the different levels on which mutual interrogation operates.
> - Understanding the role of ethnography in the process of mutual interrogation.
> - Finding a way to get deeper into the thinking behind the cultural practice, and this includes identifying the characteristics of mathematical thinking as a backdrop.
> - Linking cultural practices with wider aspects of the host culture.
> - Finding ways to more deeply involve the mathematical community in this type of thinking. (Adam et al., 2010, p. 16)

Several of the subsequent chapters of this thesis address tensions comprised in those refinements and, at some extent, this doctoral study as a whole can be described as an attempt to enhance the notion of mutual interrogation, transforming it from a methodological tool (a molecular level) to a foundation for ethnomathematics theory (the molar level).

I conclude this account of theoretical contributions to ethnomathematics that inspire my approach by saying that all the four authors referenced share

some sensitivity towards political struggles and the ongoing effects of colonialism. At the same time, all of them provided some elaborations on the philosophy of mathematics. And finally, all of them were committed to going beyond contemplative views on ethnomathematics, locating their practice within the here and now of the communities they were working with and looking for the generation of new knowledge and the resignification of cultural practices. In the next and final section of this chapter I will present some consequences of this image of ethnomathematics as the building of cultural connections.

0.3 Implications for theory

The theorisation of ethnomathematics introduced in this chapter entails important changes to the study objects and the corresponding work with those objects. Traditionally, ethnomathematics focuses on finding cultural practices that can contain elements belonging to mathematical knowledge, and the aim of those practices is to establish that they possess certain mathematical characteristic (a proof that $M \cap \mathcal{C}_N$ is empty or not). Instead, within the intended approach, the focus is on the creation of connections between cultural practices and mathematics, and the related work is to argue for the plausibility of the connection (that is to say, to show a possible f in $\hat{\mathrm{E}}_N$) and its power to change cultural or mathematical practices.

An unfolding of this idea adheres to the Wittgensteinian understanding of mathematics as a social practice and the assumption that the meaning of a word/concept is given by the use of such word/concept within the social practice (Knijnik, 2012). However, I do not use such an insight only as an analytical tool to describe or interpret mathematical knowledge, but also as a performative tool, emphasising that people can intervene and operate within the social practices; people can therefore impact on what is accepted by mathematics.

In that sense, this theorisation makes it clear how ethnomathematics can contribute to and participate in cultural and mathematical meaning-making processes, by the production of new types of knowledge that contrast, hybridize, and evolve different realms. Coming back to the metaphor, when a connection f is stated, I consider f as a new piece of knowledge, $f \notin M(t)$ and $f \notin \mathcal{C}_N(t)$ although the creation of one f can change mathematics or cultural systems after certain amount of time. This means $f \in M(t+\epsilon)$ or $f \in \mathcal{C}_N(t+\sigma)$ for some ϵ or σ. Examples of this situation can be found in Gerdes' work (2007; 2013) with the African heritage and mathematical matrix theory, and also with the cultural change resulting from educational initiatives in New Zealand (Barton et al., 1998; Meaney et al., 2011).

Research validation processes also became different due to this proposal for a paradigm shift in ethnomathematics. How is a connection launched? What is necessary to build one? How can it be assessed? At what extent does a par-

ticular relation provides resources for mathematics education (or not)? Considering that connections are not among static entities but among vibrant and knowledge systems conditioned by social, historical, and political constraints, one must need to acknowledge human agency and will, and assume that any research experience in ethnomathematics comprises a process of negotiation, creation, and learning about mathematics and culture that demands the participation of experts, not only on the mathematical/academic side but also on the cultural one.

Although it is not my expectation to provide any final solution on this matter of validation, I can foresee two alternatives to conduct ethnomathematical practice in order to be sensitive to the agency that cultural groups may have: through individual initiatives in which the ethnomathematician behaves as a medium, consulting experts and locating the dialogue within himself/herself, as Alangui (2010) proposed; a second alternative, to face the challenges of alterity is to perform a collective and interactive process through practices of bartering, as explored in Chapter 3 of this thesis.

Summarising this chapter, I present a molar level of a performative theory for ethnomathematics. This theory reconceptualizes ethnomathematical work by stressing the acts of interpretation and interaction. The intended reconceptualization abandons any essentialist view about culture or math; at the same time, it does not avoid academic mathematics, or school mathematics. Consequently, a political dimension appears with the study of the interactions, and ethical and methodological concerns about the ethnomathematical research and its intellectual property deserve to be addressed.

I consider that the molar level is sufficiently explained so far. Therefore, the first step of the journey was completed. Now I can move on, to explore and discuss how this theory can be enacted and developed through a series of concepts informing the ethnomathematics research practice. Those concepts are introduced in the subsequent chapters and constitute the molecular level of the theory.

References

Adam, A., Alangui, W., and Barton, B. (2010). Bright lights and questions: using mutual interrogation. *For the Learning of Mathematics*, 30(3):10–16.

Alangui, W. (2010). *Stone walls and water flows: Interrogating cultural practice and mathematics*. Phd thesis, University of Auckland, New Zealand.

Albanese, V., Adamuz-Povedano, N., and Bracho-López, R. (2017). *The Evolution of Ethnomathematics: Two Theoretical Views and Two Approaches to Education*, pages 307–328. Springer International Publishing, Cham.

Barton, B. (1996a). *Ethnomathematics: Exploring cultural diversity in mathematics*. Thesis, University of Auckland, New Zealand.

Barton, B. (1996b). Making sense of ethnomathematics: Ethnomathematics is making sense. *Educational Studies in Mathematics*, 31(1-2):201–233.

Barton, B. (1998). The philosophical background to ethnomathematics: Where is it taking us. In *Proceedings of First International Conference on Ethnomathematics (ICEM1)*, Granada. Universidad de Granada.

Barton, B. (1999). Ethnomathematics and philosophy. *ZDM*, 31(2):54–58.

Barton, B. (2008). *The Language of Mathematics: Telling Mathematical Tales*. Mathematics Education Library. Springer, New York.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In Hunter, R., Bicknell, B., and Burgess, T., editors, *Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia*, volume 1, pages 3–9. Palmerston North.

Barton, B. (2012). *Preface to "Ethnomathematics and philosophy"*, pages 227–229. Springer, Berlin.

Barton, B., Fairhall, U., and Trinick, T. (1998). Tikanga reo tātai: Issues in the development of a māori mathematics register. *For the learning of mathematics*, 18(1):3–9.

Barton, B., Lichtenberk, F., and Reilly, I. (2005). The language of topology: a turkish case study. In *Applied general topology*, volume 6, pages 107–117. Universitat Politècnica de València.

Barwell, R., Barton, B., and Setati, M. (2007). Multilingual issues in mathematics education: introduction. *Educational Studies in Mathematics*, 64(2):113–119.

Cauty, A. (2001). *Matemática y Lenguajes: Como seguir siendo amerindio y aprender la matemáticas que necesitara?*, page 49–87. Ediciones Morata, Madrid.

Cauty, A. (2009). *Como nascem e se desenvolvem as tradições escritas matemáticas. Exemplos mesoamericanos*, pages 29–52. Editora da Universidade Federal Fluminense, Niteroi.

Cauty, A. and Hoppan, J. (2005). L'arithmétique maya'. *Pour la science*, pages 12–17.

Cauty, A. and Hoppan, J. (2007). Les écritures mayas du nombre. *HAL*, pages 1–38.

Cauty, A., Hoppan, J., and Trelut, E. (2001). Numération et action. le cas des numérations maya. *Journal des anthropologues*, (85-86):115–155.

Cauty, A. and Ramos, A. (1990). Vigilancia etnocultural: el caso de la numeración tradicional nasayuwe. *Boletín de Lingüística Aborigen*, 2:3–15.

D'Ambrosio, U. (2016). *An Overview of the History of Ethnomathematics*, pages 5–10. Springer, Cham.

François, K. and Vandendriessche, E. (2016). Reassembling mathematical prac-

tices: a philosophical anthropological approach. *Revista Latinoamericana de Etnomatemática*, 9(2):144–167.

Gerdes, P. (1996). *Ethnomathematics and Mathematics Education*, pages 909–943. Springer Netherlands, Dordrecht.

Gerdes, P. (2007). *Adventures in the world of matrices*. Contemporary mathematical studies. Nova Science Publishers, New York.

Gerdes, P. (2013). *Da etnomatemática a arte-design e matrizes cíclicas*. Autentica, Belo Horizonte.

Giongo, I. (2008). *Disciplinamento e resistência dos corpos e dos saberes: um estudo sobre a educação matemática da Escola Estadual Técnica Agrícola Guaporé*. Phd thesis, Universidad de Vale do Rio dos Sinos-UNISINOS, Brazil.

Knijnik, G. (1996). *Exclusão e resistência: educação matemática e legitimidade cultural*. Artes Médicas, Porto Alegre.

Knijnik, G. (2006). *Educação matemática, culturas e cohecimento na luta pela terra*. Edunisc, Santa Cruz do Sul.

Knijnik, G. (2007). Mathematics education and the brazilian landsless movement: Three different mathematics in the context of the struggle for social justice. *Philosophy of Mathematics Education Journal*, 21:1–18.

Knijnik, G. (2012). Differentially positioned language games: ethnomathematics from a philosophical perspective. *Educational Studies in Mathematics*, 80(1-2):87–100.

Knijnik, G. and Duarte, C. (2010). Interweavings and dispersions of statements in the discourse of school mathematics education: A study about the importance of bringing the student's "reality" to mathematics classes. *Bolema - Mathematics Education Bulletin*, 23(37):863–886.

Knijnik, G. and Wanderer, F. (2010). Mathematics education and differential inclusion: a study about two brazilian time–space forms of life. *ZDM*, 42(3-4):349–360.

Knijnik, G., Wanderer, F., Giongo, I., and Duarte, C. (2012). *Etnomatemática em movimento*. Autêntica, Belo Horizonte.

Meaney, T., Trinick, T., and Fairhall, U. (2011). *Collaborating to meet language challenges in indigenous mathematics classrooms*. Mathematics Education Library. Springer, New York.

Mendoza, J. (2001). *The condition of Indigenous Knowledge (IK) from a structurationist perspective*, volume 2. Cordillera Studies Center, Michigan.

Miarka, R. (2011). *Etnomatemática : do ôntico ao ontológico*. Thesis, Universidade Estadual Paulista "Júlio De Mesquita Filho"-UNESP/Rio Claro, Brazil.

Radford, L. (2014). *Cultura e historia: dos conceptos difíciles y controversiales en aproximaciones contemporáneas en la educación matemática*, pages 49–68. Livraria da Física, São Paulo.

Rohrer, U. A. B. V. (2010). *Ethnomathematics: New approaches to its theory*

and application. Phd thesis, Universität Bielefeld, Germany.

Vilela, D. (2010). Discussing a philosophical background for the ethnomathematical program. *Educational Studies in Mathematics*, 75(3):345–358.

Wanderer, F. (2007). *Escola e Matemática Escolar: mecanismos de regulação sobre sujeitos escolares de uma localidade rural de colonização alemã no Rio Grande do Sul*. Phd thesis, Universidad de Vale do Rio dos Sinos-UNISINOS, Brazil.

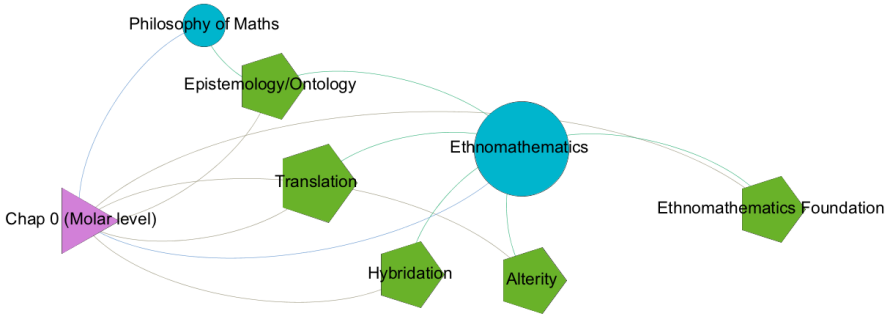


Fig. 1: Conceptual entanglement 0. This is the first image of a series of graphs that will appear after each chapter, explaining the development of the conceptual entanglement. It can be assumed as a visual guide to locate the reader in the territory being constructed. In this image triangle represent chapter, circles represent areas of study, and pentagons represent topics of interest

At a molecular level

Chapter 1

Intellectual property in ethnomathematics

Aldo Parra

The paper has been published in
Revista Latinoamericana de Etnomatemática, Vol. 8 (2), pp. 398–414, 2015.

Parra, A. (2015). Intellectual property in ethnomathematics. *Revista Latinoamericana de Etnomatemática*, 8(2), 398-414.

Artículo recibido el 30 de octubre de 2014; Aceptado para publicación el 5 de junio de 2015

Intellectual property in ethnomathematics

Propiedad intelectual en etnomatemática

Aldo Parra[1]

Abstract

Beginning from the reflections about a methodology used in a research project with an indigenous Colombian community, this paper outlines some possibilities for ethnomathematical research. Issues like intellectual property and social relevance are discussed in order to propose a broader concept of "academic instance", trough the acknowledgement and legitimation of alternative scenarios of generation, transmission and transformation for mathematical knowledge. This paper has five sections: a) preliminaries about the indigenous community, b) description of the research process and its products (for their very nature, it will be written in a first-person plural voice), c) individual thoughts, treating the harmony between the ethnomathematical methodology and its theoretical, humanistic and political foundations, d) final remarks, sharing insights for further development, e)an epilogue or a review about the experience, to discuss the spirit which aims the analysis made.

Keywords: Ethnomathematics Research; Indigenous Education; Intellectual Property; Authorship.

Resumen

Partiendo de reflexiones sobre la metodología empleada en una experiencia con una comunidad indígena colombiana, este artículo propone posibilidades para la investigación en etnomatemática, que tienen implicaciones en aspectos como la propiedad intelectual y la pertinencia social, así como en el reconocimiento y legitimación de ámbitos alternativos de generación, difusión y transformación del conocimiento matemático. El texto tiene cinco secciones: a) información sobre la comunidad indígena, b) descripción del proceso investigativo (por su carácter colectivo y comunitario es narrado en primera persona del plural), y de la elaboración de sus productos bilingües, así como de la dinámica actual del grupo investigador, c) consideraciones individuales del autor, discutiendo la consonancia de la metodología en investigaciones etnomatemáticas con los presupuestos teóricos, humanistas y políticos del campo disciplinar. d) consideraciones finales, compartiendo elementos para un desarrollo posterior, e) epílogo o lectura de la experiencia desde otra mirada, donde se discute el espíritu que anima los análisis hechos.

Palabras Clave: Etnomatemática; Educación Indígena; Propiedad Intelectual; Autoria.

[1] PhD student. Aalborg University. Denmark. Email: aldo@plan.aau.dk

INTRODUCTION

The Nasa indigenous community is located mainly in the Cauca state, in Colombia. It consists of more than 100,000 persons. They are recognized among other Colombian indigenous nations, by their powerful cohesion, inner organization, and their political participation with national impact. Although their negotiations with the national state began in 1650, since 1971 Nasa people have adopted an organizational scheme grounded in communitarian participation. They have created an indigenous regional Council, the *Consejo Regional Indígena del Cauca* (CRIC), which allowed them to be engaged in a consistent process of right's assertion, setting several lines of work such as health, judicial autonomy, land recovery and education. Nasa's educational practices have been developing for the last 30 years, sketching and developing their own autonomous educational system, recognized by the Colombian government.

In order to achieve their goals, they have established as priorities the reinforcement of their mother tongue (*Nasayuwe*), the defense of their land, and the development of alternative pedagogies. All of these initiatives are linked to a political project of resistance against their extinction as a culture. (CRIC Consejo Regional Indígena del Cauca 2004).

The Indigenous Center of Intercultural Research in Tierradentro[2] (CIIIT) was created in 2003 by the indigenous movement as one of several endeavors. In that center the Nasa people have been doing several and diverse projects, such as risk prevention plans; sociolinguistic studies; communitarian development; agricultural and ritual calendars, all of these addressing strategic and alternative responses to several indigenous problems. Embedded in culturally diverse environments, the projects were developed with a communitarian approach. In a general meeting[3] for education in 2006, the community ordered CIIIT to create a team of bilingual indigenous researchers, coming from several villages of Tierradentro's zone, with the aim of conducting a research on their mathematics.

[2] Tierradentro is the sacred region for Nasa people, and the place where *Nasayuwe* language is more used; UNESCO recognized that zone as a cultural heritage for humankind. Tierradentro also allocates most indigenous *Cabildos* than any other area in the Country. *Cabildo* is the political and organizational unit that rules into an indigenous territory area called *Resguardo*. In this paper *Resguardo* is translated as "Village".

[3] That assembly was held in the anniversary of Benjamín Dindicué´s death, an indigenous leader who was leading the educational processes inside the region. Dindicué was murdered in 1979 by paramilitary forces.

They sent me an invitation to be part of the team, as a collaborator and adviser, considering my Bachelor degree in Mathematics. Another non-indigenous professional, who had previously collaborated with the CIIIT educational issues, was also engaged.

The aim of this paper is to reflect theoretically about intellectual property in ethnomathematics, through an analysis of the theoretical standpoint and methodological way in which the CIIIT´s team conducted their research. The specific empirical research results were already presented in a book Caicedo (2009), that provides an interpretation of the Nasa mathematical thinking.

DESCRIPTION OF THE COLLECTIVE EXPERIENCE

In this section, the pronoun "*we"* will be used to describe the things developed into the research, regarding the collective spirit that drove the process. For further methodological considerations the singular "*I*" will be used.

To start the process, we considered useful to sensibilize on the social and dynamic self of mathematical knowledge. A set of previous ethnomathematical experiences located in Colombia was reviewed in the first meetings: (Cauty, 1999), (Albis, 1989) and (Parra, 2003). Also we include some "classic literature" like (Carraher, Carraher, & Schliemann, 1987), (Soto, 1995) and (Ascher, 1981), which were motivating in this team's departure and served to create a background.

Considering Alan Bishop's work (1988) about the existence of (at least) six "universal" mathematical activities (to measure, to play, to count, to explain, to locate and to design), we started a description process of several practices that the Nasa people have noticed inside their villages, stimulating the collection and interpretation of data directly by ten indigenous researchers. Every month, for a period of one and a half years, we arranged meetings to contrast and socialize the data collected by the researchers along the month. These encounters lasted two days and happened in a different place each time, helping to know (and own) the territory, showing and sharing the job among the several communities. A great deal of those meetings were organized based on the indigenous scheme of assembly, demanding participation of all different types of members of the community (local governors, ancient healers, elders, children, teachers and adults), approving,

complementing and correcting the assertions about the practices, that had been collected by the research team.

It was remarkable the main role of the mother tongue in every single reunion or assembly, as well as in the oral testimonies which were collected by the indigenous researchers. In several instances the analysis were constructed in *Nasayuwe*, because it facilitates them to conceptualize (despite being all of them bilingual). At the end of that stage, some members tried to make an abstract in Spanish for the two non-indigenous members who did not know the language, in order to understand the resulting ideas. Far from forbid or limit those situations of apparently lack of communication (in behalf of the feeling "losing control of the research"), we wanted to promote them, as a gesture of trust and interdependence. Today we believe that this was crucial to strengthen the whole research process.

With the set of the collected data in several villages, it was assigned to each team member the responsibility to organize, in a written and bilingual way, all the issues related with one of the selected activities (as we said before: to measure, to play, to count, to explain, to locate and to design) in order to go further in the knowledge founded. For this new stage three members left the group and those who stayed assumed a new research role, more personal but still collective, which lead them to create an explanatory discourse, coherent with the information obtained collectively and diachronically along the process. This writing process demanded seven months. We thought those findings should not be reduced to a mere ethnocentric accounts, prone to be labeled as ethnic folklore (and because of that, unable to interact) and we started to contrast that knowledge with theoretical stances from the official academy, read some specific papers related with these subjects. We studied materials from (Huizinga, 2000) (Chamorro & Belmonte, 1991), (Chávez & Puerto, 1998), (Rojas, 1998), and the curricular guidelines for mathematics, from the Colombian ministry of education (COLOMBIA. Ministerio de Educación, 1998). That stage was not easy but it was very satisfactory because we could see how our indigenous colleagues managed to contrast, evaluate, reformulate and look for evidence to prove or disapprove those explanatory discourses that have come from different and strange contexts. In every case it was evidenced an appropriation and a conceptual re-elaboration of the academic discourse, impossible to be achieved by a researcher who does not belong to the Nasa community.

Complementary to the facts related above, our indigenous colleagues attended several national and regional conferences in mathematics education, disseminating and presenting our work and being in touch with the dynamics of those other instances of knowledge's circulation. Furthermore, the CIIIT presented this experience to the Colombian Ministry of Culture, and won the national fellowship for indigenous languages research. This prize constituted the only economic aid received along 3 years and allowed the printing of an entirely bilingual book, which describes all the results, aiming to raise concerns, questions and possibilities about mathematics and cultural heritage. The editorial process demanded eight months of work and was itself another stage in the research. Every single chapter was submitted to an inner system of group's filters.

First, every researcher wrote a bilingual draft with an overview about a particular activity entrusted to him/her. This version was complemented by them, after a work session with me, as a mathematical advisor, in those meetings we consensuated assertions and concepts, both in Spanish and in *Nasayuwe*. In all those meetings arose clearly a text's main feature for us: it was not a mere translation from a text thought in Spanish to *Nasayuwe*. The changes were mutual in both languages and it was looked for an easy expression but without trivialize the concepts and practices related. Keeping that in mind, each author should like the manuscripts that they have in charge. That second version was read by a couple of members of the team, doing orthographical, grammatical and style comments and suggestions. A third version was submitted to a main review for the whole research group. That moment was critical: we had to consider approaches and make decisions about the writing style and how to manage the writing as an act because three different but related items emerged: a) the necessity to maintain expressions in a familiar style within *Nasayuwe*, which allow us "to come in" easier to our target reader (indigenous teachers and parents, b) the notice of the presence of dialectal variations in the *Nasayuwe* language, plenty of sayings, idioms, jargons, contractions and particular intonations[4], c) the

[4] Although some members proposed to keep one unified and neutral style, to be "official" for the *Nasayuwe* language, other members want to make evident the accents and idioms distinctive of each author, urging the readers to identify and recognize those differences between the several indigenous areas. The last option prevailed because we considered it strategic, using sociolinguistic issues about the importance to revitalize the indigenous languages.

introduction, or not, of neologisms to describe mathematical terms, needed in the texts, such as “unit of measure ” or “discrete”[5].

Addressing all the related issues above, we wrote papers that relates and describes mathematical objects within a cultural view, specifically Nasa, including interpretations and metaphors coming from spiritual dimensions. It is very interesting to note how concepts typically assumed as mathematical were being re-created within Nasa indigenous worldview, e.g. the continuous and the discrete, ordinality, cardinality, unit of measure for measurable magnitudes, etc… even notions related to physics, such as velocity, motion and inertia. Mathematical language of logic was equally considered, expressions used for equivalences, implications, negations, disjunctions and other logical conjunctions, which are used to shape and format discourses inside the culture.

It is important to note that no single stage of research related until now had escaped to strong debates inside the research group, neither to unsolved questions. A deadline was defined to deliver the material, and one thousand copies of the book were printed, seven hundred and fifty were delivered to the indigenous teachers and their villages. All the authors agreed that the book became copyrighted by the CIIIT, as a way to keep the intellectual property in the indigenous community as a whole To stress that commitment, in the very copyright page, were listed the names of the elders, cabildos’ authorities, former members of the team and teachers who provides, check and comment the information and guide and assess the research process. Villages’ assemblies were also included as sources in the copyright page because the Nasa community assumes those meetings as actors with agency and legitimacy. However, we did not want to deny the work of the research team members. So we decided to detach authorship from intellectual property, the former was recognized to the nine members of the team, and the latter remain in the community.

[5] In this topic, the question about the utility and reception of the terms by the community becomes a very strong element of judgment. We inquired if within *Nasayuwe* there are words that could account for what is intended, and also if the previous words created to express mathematical concepts were naturals or forced. As much as were possible, we look for not using neologisms, but we could not make it totally. We also proposed to extend the meaning of existing words in the *Nasayuwe*, to express metaphorically the idea we wanted. For every proposal that we made we included explanatory notes.

The book was (and still is) disseminated and assessed in the Tierradentro's zone, as well in others Nasa's villages in Cauca. In that new process all the authors continued playing a fundamental role, commenting, explaining and sharing their experience with teachers and general public, through workshops and assemblies. They were collecting impressions, suggestions and mistakes of several types. Despite our initial efforts, some of the chapters were severely criticized in their style in *Nasayuwe*, but welcomed in their content.

We want to highlight here that we did not take those critique as a comment from the readers (who act as "product final users"), rather than that, we assumed their intervention as a natural step in the editorial process. Parents, teachers and authorities are considered agents in the research, and because of that, they became authors, in an extended sense. We decided to make an enhanced second edition, with entire new *Nasayuwe* versions of some chapters and one additional chapter, including some advices and suggestions for parents, to help their kids with the development of mathematical skills outside the school in cultural spaces and events. That new chapter gives some tools about how parents could not promote negative images about kids' school performance, based in their earlier experiences, but to focus in their current skills as grown-ups out of school. That was a suggestion and a reflection made by some parents. We took Brazilian booklets (BRASIL,2002) with tips and suggestions as an inspiration, and shared some personal experiences as teachers to propose our own version for the chapter.

External to this process, indigenous movement obtained the legal permission to manage locally part of the educational administration state funds. This implied some autonomy in the economical investment for schools, books and teachers salary. So, the local association that gathers cabildos in Tierradentro gave to the CIIIT in 2012, some funds to print the second edition and distribute it to the villages.

One of our current challenges is to create new teams with teachers who take the data as input, and develop didactical tools for the classroom. To do these, it's crucial to engage a former group member, as an advisor to the new teams.

In the last four years we diversified our work, in one hand we monitor the use and reception of the book; in the other hand we study and research new ideas. We are currently developing with the group another process of "owned" investigation on Nasa's

mathematical thinking, outside the frame of Bishop's work, following another path to create new kinds of hybrid knowledge, becoming appropriate and more specific to the Nasa community. We like to think this research as a collective creation, in which three agents learn, interact and add, from their diverse past, knowledge and expectations: the Nasa community, the research group and the non-indigenous collaborators. Each element of this triad has autonomous but related and convergent actuation, possessing non-transferable areas of decision and creation, in which its authority is respected and the necessity of their presence is validated. This idea of collective creation refers to a multilateral structure, which allows the research to avoid the trapdoor of "everybody does all" or "everybody knows everything", as well as a taylorist division of the work.

Several stages of the research and the preeminence of each agent could be represented using the spiral image in Figure 1. This symbol had served Nasa people (and another indigenous people across Latin America) to conceptualize their own ways to build knowledge, and ultimately, to survive as a culture. So, this image that often belongs to mathematics, is present not only into Nasa's conceptualization, but depicts fairly well our research.

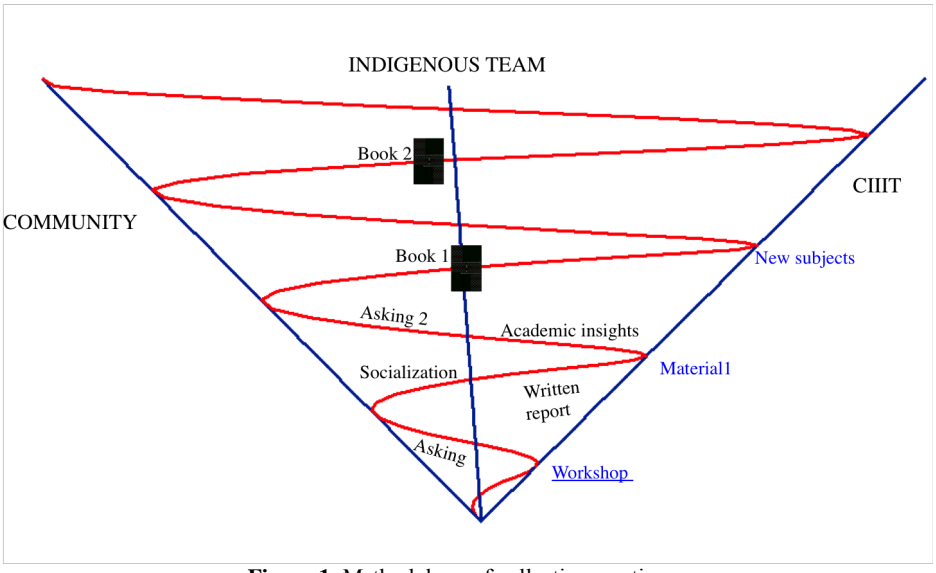


Figure 1. Methodology of collective creation

It is perfectly possible inside the ethnomathematical field to disagree with, or to put in doubt, the interpretation about the Nasa mathematical thinking that we outlined, saying some of our results could be improved or even discarded. As authors, we will be very pleased if such a thing happens because it would imply a growth in the population interested in the study of Nasa mathematics. Due to the collective dynamic implemented, any change in the data will be received as a consubstantial an inevitable part of the methodological process, which turns notions like "success" or "fail" inapplicable

INDIVIDUAL REFLECTIONS

In this section I want to share some reflections from this experience that I believe that can be useful for ethnomathematics researchers. Some of them, and many others, were discussed with the indigenous people at Tierradentro.

Ethnomathematics has been recognized for its interest to vindicate and legitimate skills, knowledge and practices that have served several groups and nations to survive and transcend in time and space. If we understand that those issues do not exist in a vacuum, but they are manifested inside normed contexts of socialization, in which such they are disseminated, evaluated and transformed, we can appreciate that ethnomatematical research does not culminate to understanding/sharing several knowledge, but it also would have to do with understanding/sharing these ways of generation and dissemination.

Certainly this assertion is not new in a theoretical level, (D'Ambrosio, 1994, 1998), but it highlights a current methodological void/oversight because while most research work register mathematical practices of a group of indigenous people, artisans, workers, etc., only few experiences have taken into account the practices of socialization and research used in the groups.

Most research in ethnomathematics have been conducted under the same invariable style/canon: findings belong to a researcher, which is external to the researched community, and decides what is published, in which format, when and where. Although the knowledge and practices belong to a researched "other", they are showed under the style and criteria of the researcher, which even boasts about to have "entered" into the community and to have "cracked/decoded" their mathematical knowledge, which (obviously) had not been revealed

until the researcher came. Those "others" do not have any participation in the released information. They do not obtain royalties, profits or non-monetary benefits from the research on which they were subjected. "Others" are not more than raw research material. Sometimes, even their real names are not registered in papers and other publications. The most they can expect is to be mentioned in the acknowledgements because they answered the researcher's questions.

It is very symptomatic fact that Eduardo Sebastiani Ferreira, a well-known researcher of the *mother mathematics*[6] of several Brazilian indigenous groups, had to draw attention on this point, proposing in (Sebastiani-Ferreira, 1994) something as basic as to present the research's final results to the studied community/group. Such proposal bares a conception of ethnomathematics in which the researches are made *about* groups, and not *with* groups.

The presence of that methodological pattern, and their underlying conception, could be rooted in the location of ethnomathematics between mathematics and cultural anthropology (D'Ambrosio, 1998), and also in the initial claims to link ethnomatematical research with ethnography (Millroy, 1992; Sebastiani-Ferreira, 1994), particularly in its more classical view. That view had been already questioned within anthropology itself, by authors like (Rappaport, 1998, 2008), (Wielewicki, 2001) and (Montero, 2006). The first one problematizes the subject/object dichotomy and the historicity of the very ethnographical report. The other two understand ethnography as a discursive construction, tied to the political-religious and philosophical paradigms of the time/space to which the research belongs to.

With the emergence of new styles of ethnographic research (Denzin & Lincoln, 2011), as well as other emerging methodologies of de-colonial and postmodern studies, it would be worthwhile to explore the potential that these tools have to build new answers for the specific dilemmas facing in ethnomathematics. However, this ethnographical intend is hard to maintain if one considers some aims, which are explicitly or implicitly related with ethnomathematics.

[6] In portuguese: *matemática materna*.

For instance, when the mutual enrichment is promoted, a question arises: in what ways the people researched can be enriched, despite the fact that some part of their knowledge circulates among us? Certainly, it is a great step forward that we can learn from them, but: what do they learn? It is also used largely the concept of *dialogue*, but that actually means that we can talk about one "other" (indigenous, black, farmer, etc…) then more questions are raised: when that other can talk? Where? To whom? With what purpose? This issue becomes particularly sensitive if the community to be studied has consciously developed a process of social-political reivindication about their own knowledge[7]. In this kind of situations is very problematic to arrogate the right to be the "other's voice", when those others are struggling for their empowerment, and for building their own discourse.

Within this discussion arise problems with authorship and intellectual property: on the one hand the researcher appears as a *knowledgeable/erudite voice*, who can call certain set of practices as mathematical, and has the preparation to produce a paper. On the other hand is the community, as an *authorial* voice because it generates and performs the practices. We can imagine the former as a reporter and the latter having the intellectual property of that knowledge; however, it does not resolve the social and legal implications. Especially if we take into account that the idea of "copyright" is individual and has always had in its very nature a mercantilist function, which collides with initiatives of collective appropriation of knowledge.

This dilemma had appeared in other fields and is far from being solved. Using the definitions that UNESCO has launched about traditional knowledge and intangible cultural heritage, Wanda George has pointed a clever question: *Does a community really own its distinctive intangible cultural heritage?* Although this scholar exemplifies the discussion

[7] I am referring here to the indigenous peoples of Latin America, the Maori people in New Zealand, the peasants of the "Movimento Sem Terra" (Brazilian Landless movement) and many others. Enlightening the words of the Nasa leader Gentil Guegia (Caicedo & Parra, 2009) "In our language we call research *ûus atxah*, and this involves thinking, planning, reflecting, listening and understanding, demands to share and produce new knowledge. For us, to investigate is a pedagogical strategy that we have deployed to know ourselves in relation to our culture, hence the spaces provided by the community at home, school, barters, congresses, assemblies and ritual, are those which facilitates best this learning" (p.7, my translation)

using tourism and non-indigenous communities, she derives some useful reflections for our question in ethnomathematics:

> "While certain facets of a culture can be copyrighted, such as pieces of written music, artworks or other visible manifestations, the intangibles – ideas, meanings, collective identity attributes, oral and unwritten expressions, and the symbolism attached to these – cannot be easily protected. Intangible and tangible cultural aspects are accumulated and reproduced during the evolution of a local community in building its collective and social value system – a social construct." (George, 2010)

Naturally the academic context is relevant for the professional researcher and not necessarily (but probably) members of cultural groups are interested in getting into those spaces in which research traditionally circulates (classrooms, congresses and books). We do not advocate for belittling or ignoring the complex preparation possessed for the scholars, neither the specific, own and natural questions, which arise from them. But if the ethnomathematical claims to respect and share knowledge are taken seriously, the scenarios to circulate knowledge should be diversified. Such idea could imply two-way movements: in one direction stimulate the presence and participation of *knowledge-holders* in meetings and other traditional context of the academic community, and in the other direction, to ensure that the investigations will be developed, presented and assessed also in the traditional instances that sociocultural groups have established to gather and produce its knowledge. This second direction demands from us a positioning about a question: do we think that those instances do not or should not exist? Even worse: do we think that those instances do not have the capability to understand the motivations and procedures of an ethnomatematical research?

While it is difficult to identify the public scenarios of transmission/generation of knowledge in some labor groups (nurses, craftsmen, peddlers), for ethnical groups those scenarios are visible and vigorous: markets, house of knowledge, malokas, mingas, roads, and of course, rituals. In all of them, peoples have never stopped to conceptualize, to think, to feel and to act. Right there, people have taken, and still take, the crucial decisions that have allowed them to survive across time and space, without dissolving unconsciously in the mainstream. Under-estimate the power of conceptualization and prompting that those context have, weakens the explicative and transformative capability of the research, leaving it confined in

a sort of church in where is adored an unique type of academy, that is, this oversight condemns the ethnomatematical research exactly to the same type of ivory tower against which the ethnomathematics was initially raised: a sad paradox.

The questions and tensions raised in the previous paragraphs were addressed in the collective experience, although not in a predetermined and calculated way. Instead of trying to "give voice" to the others, I was just "listening their voices" and that facilitated many decisions. In that sense, to adopt a figure as "main researcher" did not seem to be consistent with the Nasa communitarian scheme of work, and to circulate the results only for a restricted audience would not have been fitting with the aim of bringing new meanings of academy neither with the claim to vindicate indigenous knowledge as mathematical (in an extended sense). Writing in a bilingual way harmonized with the ideas of matheracy and literacy D'Ambrosio (2006); hold meetings with the communities was in tune with an anthropological sensitivity approach to the teaching of mathematics, the work of conceptualize mathematics using *Nasayuwe* language fits completely with the creation of new knowledge. In short, we were doing an exercise in composition, trying to "give life to our artwork".

FINAL REMARKS

I hope within ethnomathematics could be combined[8] the presence of the voices mentioned above, erudite and authorial, facing the research from multiple views, with diverse theorization models, taking advantage of the several ways of communication and validation that any organizational process generates, and considering the objective that peoples and groups have made for themselves[9]. I mean, by exploring unreleased and timely possibilities, methodological strategies could be strengthened at the junction of several

[8] This combination is not without its problems: although it seems possible to recognize collective authorship of research in journals, conferences, projects and books, what can we do with the thesis and dissertations, which universities demands to perform individually?

[9] In addition to the work reported here, we know experiences like Cauty (1999) with the Wayuu people in the Colombian Guajira, which also explores these possibilities.

research agendas. Is this not exactly the hallmark of a truly alive academy, to be exploring possibilities?

A new path for the ethnomathematics would be to undertake investigations which recognizes that individuals and people often foreign to centers of power (power of every nature), do not only have various types of knowledge, but also have the capability to disseminate them, broaden them and contrast them with the knowledge of others[10]. Indeed, people and individuals have the power to define how, when and where their knowledge can circulate. This, to my view, besides fulfilling by far Ferreira's proposal, helps to concrete the ethnomathematical desire to recognize and promote a character of intellectual and political subject for the marginalized sectors, reinterpreting the role of academy and turning society a little less violent and discriminatory.

This approach is a result of awareness that several authors have made on the need to deepen and complement the ethnomathematical criticisms to imperial models of submission, due to a fervent exaltation of the knowledge of a sociocultural group could generate a rejection of "hegemonic mathematics", which paradoxically does not provide any help to a diminished sociocultural group in its own journey to transcend and survive. At this point it can be considered the character of *undetermined* that Ole Skovsmose (2011, pp. 15-16) gives to mathematics education, as well as the power relations which Knijnik (2006) reveals around the mathematical knowledge, and it can be extended for every group the question of André Cauty (2001) "How to remain Amerindian and learn mathematics needed for today and for the future?".

In short, this approach tries to highlight how futile and cumbersome can become the dichotomies of subject/object, self /others, pure/impure, local/global or academy/life, when put in terms of a stigmatization of the "other", only serving to hinder the construction of the human society proposed by D'Ambrosio(1998) .

[10] Wendy Millroy conducted an ethnography on the mathematics of carpenters, five months living and working as an apprentice to a carpenter's shop, making herself some furniture, etc. Pooling her previous knowledge in mathematics with her recent skills of woodworking she identified some elements of mathematics in this group (Milroy, 1990). We could imagine that one member of the carpentry shop had gone to a college math for 5 months and working as an apprentice, doing some work, etc.. Pooling his previous carpentry knowledge with his recent math skills, what elements of mathematics of their own group he could have identified?

EPILOGUE

In an academic event where I related this experience at Cauca state, some colleagues praised the work, saying that it highlights an ethic factor inside ethnomathematical research. Maybe it does so, but not intentionally, since the concern that prompted me to develop this experience was one of aesthetic nature. I will explain it. When a musician seeks stay on the scale of the piece of music he plays, or when an actor cares for not entering or leaving the scene at the wrong time, they pursue the same thing: to preserve the coherence of the artwork that are helping to build and achieve the desired aesthetic effect. I assumed that I should participate in the related research in the same way. Caring to be consistent with the deep motivations that I assume underpin ethnomathematics.

Therefore, I ask the reader to keep out from the text any attempt to prescribe rules from which someone would settle who does or does not research “rightly" or what is good or bad in Ethnomathematics. Nothing can be more against my will. I'm just sharing some thoughts on an experience, in order to be useful to others in their specific crossroads. They are nothing more than paths, possibilities....

ACKNOWLEDGEMENTS

This is an enhanced and updated English-version of a previous work published in Spanish, presented in the XIII INTER-AMERICAN CONFERENCE ON MATHEMATICS EDUCATION. I want to thank Lina Téllez, Magda González, Vagi Bino and Natalia Caicedo by the contributions of substance and form that gave drafts of this text. This new version was possible for the funds of Departamento Administrativo de Ciencias, Tecnología e Innovación (COLCIENCIAS) in Colombia and Aalborg University in Denmark. This research also makes part of the NordForsk Center of Excellence “JustEd”.

REFERENCES

Albis, V. (1989). *Temas de etnomatemática*. Paper presented at the VI Coloquio distrital de matemáticas, Bogotá, Colombia.

Ascher, M. (1981). *Mathematics of the Incas*. *Code of de Quipu*. New York: Dover Publications Inc.

Bishop, A. J. (1988). *Mathematical Enculturation: A Cultural Perspective on Mathematics Education*. Dordrecht: Kluwer.

BRASIL, Ministerio de Educação e Cultura (2002). *Educar É Uma Tarefa de Todos Nós: Guia para a Família Participar no Dia-a-Dia da Educação de Nossas Crianças*. Brasilia: Assessoria Nacional do Programa Parâmetros em Ação.

Caicedo, N. & Parra, A. (Eds.). *Matemáticas en el Mundo Nasa*. Bogotá: CIIIT.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1987). Written and Oral Mathematics. *Journal for Research in Mathematics Education, 18*(2), 83-97. doi: 10.2307/749244

Cauty, A. (1999). *Etnomatemáticas: El Laboratorio Kwibi Urraga de la Universidad de la Guajira*. Paper presented at the Congreso de Antropología – Simposio de Etnoeducación, 7, Barranquilla, Colombia.

Cauty, A. (2001). Como seguir siendo amerindio y aprender la matemáticas que necesitará? In G. Zapata (Ed.), *Pluriculturalidad y aprendizaje de la matemática en américa latina*, (pp. 49-87). Madrid: Editorial Morata.

Chamorro, M., & Belmonte, M. (1991). *El problema de la medida*. Madrid: Síntesis.

Chávez, A., & Puerto, M. (1998). *Vivienda precolombina e indígena actual en Tierradentro*. . Bogotá: Banco de la República.

Colombia. Ministerio de Educación Nacional. (1998). *Lineamientos Curriculares: Matemáticas*. Magisterio, Bogotá, Colombia.

CRIC Consejo Regional Indígena del Cauca (2004). *¿Qué pasaría si la escuela…? Treinta años de construcción de una educación propia*. Popayán, Colombia.

D'Ambrosio, U. (1994). A etnomatemática no processo de construcão de uma escola indígena. *Em Aberto, 14*(63), 93–99.

D'Ambrosio, U. (1998). *Etnomatemática, raizes socio-culturais da arte ou tecnica de explicar e conhecer* (5th ed.). São Paulo: Editora Ática.

D'Ambrosio, U. (2006). *Ethnomathematics. Link between traditions and modernity.*: Rotterdam: Sense Publishers.

Denzin, N. K., & Lincoln, Y. S. (2011). *The Sage handbook of qualitative research* (Vol. 4. ed.): Thousand Oaks : Sage

George, E. W. (2010). Intangible cultural heritage, ownership, copyrights, and tourism. *International Journal of Culture, Tourism and Hospitality Research, 4*(4), 376-388. doi: 10.1108/17506181011081541

Huizinga, J. (2000). *Homo ludens*. Madrid: Alianza.

Knijnik, G. (2006). *Educação matemática, culturas e cohecimento na luta pela terra*. Santa Cruz do Sul: Edunisc.

Millroy, W. L. (1992). An Ethnographic Study of the Mathematical Ideas of a Group of Carpenters. *Journal for Research in Mathematics Education. Monograph, 5*, i, 1-210.

Montero, P. (2006). *Deus na aldeia - Missionarios, indios e mediação cultural*. São Paulo: Globo.

Parra, A. (2003). *Acercamiento a la etnomatemática*. (Bachelor Monography), Universidad Nacional de Colombia, Bogotá. Colombia

Rappaport, J. (1998). *The Politics of Memory: Native Historical Interpretation in the Colombian Andes*. Durham, N.C.: Duke University Press.

Rappaport, J. (2008). Beyond Participant Observation: Collaborative Ethnography as Theoretical Innovation. *Collaborative Anthropologies, 1*(1), 1-31. doi: 10.1353/cla.0.0014

Rojas, T. (1998). *La lengua paez, una visión de su gramática*. Bogotá: Editorial Ministerio de Cultura.

Sebastiani-Ferreira, E. (1994). A importância do conhecimento etnomatemático indígena na escola dos não-índios. *Em Aberto, 14*(62), 89 - 95.

Skovsmose, O. (2011). *An invitation to critical mathematics education*. Rotterdam, The Netherlands: Sense Publishers.

Soto, I. (1995). Problemas de proporcionalidad resueltos por campesinos chilenos. *Educación matemática, 7*(1), 77-95.

Wielewicki, V. (2001). A pesquisa etnografica como contrução discursiva. *Acta scientiarum, 23*(1), 27-32.

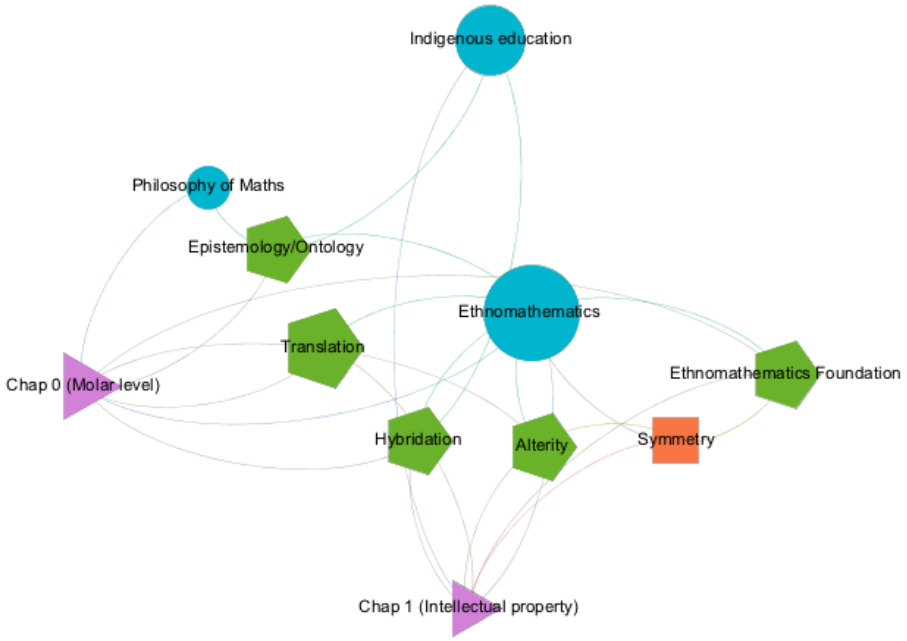


Fig. 1.1: Conceptual entanglement 1. In this image triangles represent chapters, circles represent areas of study, pentagons represent topics of interest

Chapter 2

Mathematics Education in Multilingual Contexts for the Indigenous Population in Latin America

Aldo Parra
Jackeline Rodrigues Mendes
Paola Valero
Martha Villavicencio Ubillús

The paper has been published in
Parra A., Mendes J.R., Valero P., Villavicencio Ubillús M. (2016)
Mathematics Education in Multilingual Contexts for the Indigenous
Population in Latin America. In: Barwell R. et al. (eds) *Mathematics Education and Language Diversity.* New ICMI Study Series. Springer, Cham
Print ISBN 978 – 3 – 319 – 14510 – 5

The layout has been revised.

Chapter 4
Mathematics Education in Multilingual Contexts for the Indigenous Population in Latin America

Aldo Parra, Jackeline Rodrigues Mendes, Paola Valero, and Martha Villavicencio Ubillús

4.1 Introduction

Within internationally published research in mathematics education dealing with multilingualism, little is known about the challenges faced in the education of the Indigenous peoples of Latin America. This chapter has a double contribution. On the one hand, it will present trends in the mathematics education of Indigenous people in Brazil, Colombia, and Peru. This presentation intends to inform an international mathematics education audience about the common traits as well as particular developments of Indigenous mathematics education in the continent.

On the other hand, and based on the information presented, we discuss three interrelated issues. First, the education of Indigenous populations in Latin America needs to be understood in the framework of Spanish and Portuguese colonization. Within colonization, the project of religious conversion and evangelization is a constitutive element of the models of education, multilingualism, and mathematics. Through history, the struggle between different Indigenous communities and state policies and programs have made it evident that (mathematics) education is a terrain of cultural politics for indigenous communities.

Second, in the construction, negotiation, and implementation of different models for Indigenous education, varying from monocultural monolingualism, to bilingualism, and more recently cultural and linguistic diversity, conflicting claims about

A. Parra • P. Valero
Aalborg University, Aalborg, Denmark
e-mail: aiparras@unal.edu.co; paola@learning.aau.dk

J.R. Mendes
Universidade Estadual de Campinas, São Paulo, Brazil
e-mail: rodrigues@mpc.com.br

M. Villavicencio Ubillús (✉)
Ministerio de Educación, Lima, Perú
e-mail: MVILLAVICENCIO@minedu.gob.pe

R. Barwell et al. (eds.), *Mathematics Education and Language Diversity*,
New ICMI Study Series, DOI 10.1007/978-3-319-14511-2_4

what counts as Indigenous languages and cultures have been present. Indigenous education in Latin America is a space of struggle for the recognition of Indigenous worldviews and knowledge; as such, it is a space where language, knowledge, and identity go hand in hand. In this context, issues of language revitalization as well as tension between oral traditions and written registers are important discussions that shape education and mathematics education.

Third, the current move towards the recognition of cultural and linguistic diversity also poses challenges as to what counts as the mathematics of the Indigenous people and the mathematics of the mainstream cultures in the context of the life of communities and also in schooling. Different state policies of Indigenous mathematics education have resulted in diverse meanings and appropriations of ethnomathematics. Here research in mathematics education has provided tools to deal with these challenges.

In the three main sections of this chapter, we address each one of these issues. Our strategy is intentionally descriptive of the historical development and current situation in three countries: Brazil, Colombia, and Peru. We use the descriptions of these national contexts to depict both commonalities and differences in the discussion about how mathematics education in the cultural and linguistic diversity of the Latin American Indigenous population is highly political.

4.2 Models of Indigenous Education in History

Latin American countries share similar histories around the treatment of their Indigenous communities. The intersection points can be traced back to the Spanish and Portuguese incursions into the continent in the sixteenth century. Colonization represented a process of extermination and domination of Indigenous communities. The natives in this period were treated as *nonhuman, wild savages*. At that time, the definition of "wild" was structured around three components: "unaware of the Castilian language, the Christian religion, and the larger society model" (Roldán & Gómez, 1994, p. 65). From the very beginning of the contact between European and native South American civilizations, language and religion appeared as a decisive factor structuring relations with Indigenous peoples. The Iberian monarchies entrusted to the Catholic Church the tutelage of Indigenous peoples, to carry out their spiritual salvation through evangelization, with the work of missions being central in such task. Religious conversion and with it, cultural assimilation, led to the loss of cultural traditions like written codices and languages. The hard and extenuating work for the economic exploitation of the natural richness of the Americas was also associated with the systematic extermination of large proportions of the native population. In some cases, religious conversion and the exoticization of the "wild" native resulted in the annihilation of Indigenous people. The most extreme case of the eradication of an Indigenous people is that of the Charrúas, the aboriginal settlers of the Uruguayan territory.

In the first quarter of the nineteenth century, independence from the European empires took place. The creation of national states and republics produced a change in philosophy towards the few native populations left. Indigenous people were not

seen any more as belonging to an inferior species, but rather as *potential citizens*. The work to integrate them was once again left under the tutelage of the Catholic Church. The policy of the Church to control the education and the language of the natives remained alive in Brazil, Peru, and Colombia until the last decade of the twentieth century. Even some of the early efforts to register Indigenous languages were associated with religious movements. For example, the Summer Institute of Linguistics (SIL), an organization supported by Christian factions in the USA, created alphabets for various Indigenous languages in the continent and advanced the recording and study of native languages, for the purpose of evangelizing the natives in their indigenous language. Educational initiatives during this period correspond to a transition model (Larson, Davis, & Ballena, 1979, p. 57), in which it is hoped that indigenous students will move from their cultural and traditional practices to become part of the national society. The instrumental use of language and of religion was fundamental in taking that step.

In the course of the 1960s and 1970s alternative institutional educational programs emerged around *bilingualism* in schools. In Peru, the Universidad Nacional Mayor de San Marcos was the first to start a pilot program of research and experimentation of bilingual education with the Quechua-speaking population in the area of Quinua, Ayacucho Department. Since then and within the Peruvian government's Language Development Plan, studies on bilingualism were strengthened. Years later, that university was able to influence the idea of a National Policy for Bilingual Education in the framework of the 1972 Peruvian Education Reform (López and Küper, 1999).

In the 1970s in Brazil and Colombia a series of political processes and social movements started to discuss and promote changes in public policies for Indigenous peoples. There was a critique of the transitional bilingual model, which had prevailed for more than two centuries. In Colombia, the Regional Indigenous Council of Cauca and the Tayrona Indigenous Confederation were founded. These organizations demanded the presence of bilingual Indigenous teachers in schools to respect the history and traditions of the communities. In Brazil, the Indigenous Missionary Council was created in 1972. Two years later, this Council organized the first of a long series of "Indigenous assemblies." The first Indigenous organization in Brazil, the National Indigenous Union, was created in the 1980s and it made alliances with non-Indigenous NGOs and with the Alliance of the People of the Forest in Amazonia, which included non-Indigenous rubber gatherers in Amazonia.

The first experiences of bilingual education for Indigenous peoples were characterized by their linguistic bias, downplaying the enculturating role of education. In the early 1980s, and as a reaction to the focus on language, the concept of *interculturality* emerged strongly. It involved the search for educational models that are specific, differentiated and bilingual and that recognize the importance of Indigenous cultures and languages. Such a view is expressed in educational policy documents and programs in Peru (Bilingual Education Experimental Project of Puno and General Directorate of Bilingual Education) and Colombia (Office of Ethnic Education). It also permeated the production of bilingual teaching materials in mathematics. Elements of the sociocultural practices of the communities were incorporated. Although these initiatives were not sufficiently grounded in indigenous worldviews, such programs played a role in the discussions that took place in Brazil

about Indigenous teacher education and preparation of materials for bilingual Indigenous schools.

During the 1980s, many countries entered a process of change in their political constitutions, which had a huge influence on educational development. Leaving a narrative of national unification, the new constitutions became pluralistic and acknowledged the *cultural and linguistic diversity* of the population. This move facilitated reclaiming the rights of Indigenous nations and safeguarding their worldviews, cultural practices and languages. In Peru, the Political Constitution of 1979 promoted the study and knowledge of Indigenous languages and guaranteed the right of the Quechua, Aymara and other communities to receive primary education in their own languages. It declared that Indigenous peoples have the right to preserve their ethnic and cultural identity and that the state should recognize and protect the ethnic and cultural plurality of the nation. In Brazil, the Constitution of 1988 included the rights of Indigenous peoples to the preservation and maintenance of their values, languages, and cultures. The Political Constitution of 1991 in Colombia declared a multiethnic and multicultural state. In the three countries, after the new political constitutions, several indigenous languages were recognized as official, and many initiatives were developed aimed at including the cultural specificities of each group in the educational system. This context drew more attention to the need for indigenous teachers in schools and led to the development of Indigenous teacher education programs to prepare these teachers.

The attempt to create and establish bilingual, intercultural, and culturally specific Indigenous schools resulted in curricular changes in the three countries during the 1990s. It also increased the participation of Indigenous teachers in writing bilingual materials. Since 2000, the development of Indigenous, bilingual education in each of our countries has a sociocultural orientation that makes education relevant and increases quality. This may be due to several reasons. First, there is a record of 30 years of implementation of experiences in bilingual education. Second, educational initiatives are decentralized and organized around each Indigenous people, as an attempt to take into account their views and wishes. Finally, sociocultural research in mathematics teaching and learning has advanced progressively in the region. This has motivated a search to explicitly include in local curricula the mathematical knowledge of each people.

Nevertheless, interest in Indigenous worldviews has not only produced better conditions for dealing with bilingualism and mathematics teaching but also generated new challenges and emerging tensions. We elaborate on them in the rest of the chapter.

4.3 Dynamics and Tensions Between Languages and Cultures

The historical changes in models of education of Indigenous peoples in Latin America are not unique to the continent. Meaney, Trinick, and Fairhall (2012) discuss the political tensions experienced in the (mathematics) education of Māori

people in New Zealand, and also review similar discussions in Australia, Africa, and North America. Their research highlights the two main topics of this section, namely language revitalization and the creation of written registers on the basis of an oral tradition of knowledge. These two dynamics of language are of particular interest given how Indigenous language, worldviews, and identity intersect in one of the powerful tools of the colonizer, namely, mathematics.

4.3.1 Language Extinction, Revitalization, and Development

In Brazil, Colombia, and Peru there is a great diversity of cultures and languages. Notwithstanding that, sociolinguistic studies conducted in the three countries depict a critical scenario for the vitality of Indigenous languages. Table 4.1 summarizes the situation.

The study carried out by UNICEF and FUNPROEIB (2009) discussed the status of preservation of Indigenous languages in Latin America. In general, Indigenous communities with better organization and more efficient and consistent adaptive strategies have coexisted for centuries with the national society and its dominant language. Their strategies result in the vitality of Indigenous languages and their articulation with the Spanish language. Language vitality is also strong in communities living in isolation, whereas in communities with recent and increasing contact with the national community there will be a tendency to lose language use, leading to a decrease in vitality.

More open political constitutions in the countries around the 1990s promoted changes in the colonial model of a monolingual language policy. Indigenous communities were granted rights over their land, cultures, and languages. This change went together with the development of language policies decreasing the speed of language extinction and preserving and maintaining Indigenous languages. One of the results of such a trend was the formulation of educational policies for preservation, aimed at generating special professional development of Indigenous bilingual teachers, as well as the creation of alphabets and written registers for these indigenous languages.

These new settings have greatly impacted the possibilities of claiming an Indigenous cultural identity, not only from a legal and institutional point of view but also from the point of view of Indigenous peoples themselves. We are taking here the notion of identity as a discursive concept. Identity is seen as a construct that happens in and through language. According to Woodward (1997), identity construction processes are always built in relation to and maintained by history and culture and are dependent upon the nature of social relationships established over time.

The inseparability of identity, language, and culture has resulted in the politicization of research on Indigenous languages and education. From the denial of the Indigenous in colonial policies to the awareness of multilingualism and multiculturalism, many Indigenous education policies have adopted a naïve, but still colonizing, approach to the "development" of Indigenous languages and cultures.

Table 4.1 Indigenous groups, their languages and their language vitality

	Basic statistics				Vitality		Languages by number of speakers			
Country	Indigenous groups	National population (%)	Indigenous languages	Linguistic families	Endangered languages	Severely endangered	1–99	10–999	1,000–50,000	>50,000
Brazil	305	0.47	184	42	24	10	34	106	44	1
Colombia	91	3.3	66	13	19	5	25	33	33	2
Peru	60	15.9	47	19	4	17	6	20	17	4

Sources: For Brazil (IBGE. Instituto Brasileiro de Geografia e Estatística, 2010); for Colombia (Landaburu, 2004; UNICEF and FUNPROEIB, 2009); for Peru (INEI. Instituto Nacional de Estadística e Informática, 2009; Ministerio de Educación del Peru, 2013, pp. 13–16)

Simultaneously political discourses have evidenced tensions in the meeting between Indigenous identities and the dominant cultures, their languages and related forms of identity represented in schooling. For example, the development of Indigenous languages leads to their normalization, as a prerequisite to achieve a standard style of writing. In Peru in 1985, the Quechua and Aymara alphabets were normalized, and at present 21 more Indigenous languages have been normalized. However, the processes of negotiation for settling alphabets and rules of writing have not ended.

There have been integrationist policies or policies oriented to the revitalization and maintenance of the cultural and language practices of minority groups. Concerning the latter, various educational programs have been developed with a focus on language and culture. These programs have generated the production of alphabets and written registers for some Indigenous languages. The categorization of these languages as "written" or "unwritten," or having an alphabetic system or not is a politically highly contentious issue which is part of the history of the power relationship between the colonial power and dominant groups and Indigenous communities. Indigenous worldviews conceive as legitimate texts registers of various kinds, such as inscriptions, fabrics, or pictograms, which are part of their cultural artefacts. The fact that the linguistic structures of the Indigenous languages have been recognized as extremely complex (UNICEF and FUNPROEIB, 2009) is of importance when moving into the discussion of the relationship between language and mathematics (Parra, 2013). This is of particular relevance for understanding how different national histories have dealt with the "lack" of alphabetical registers in Indigenous languages.

4.3.2 Tensions Between Oral and Written Registers

Indigenous teachers and communities have argued for the need to raise awareness of the political dimension and impact of introducing schooling and writing in their communities. Criticism has been raised of the inclusion of local cultural knowledge in the curricula. In discussions about mathematics education for Indigenous people, a major point is the relationship between orality and writing, because the introduction of alphabetical writing in contexts shaped by an oral tradition creates many tensions. We can see these tensions in standardization processes, technical problems in the production of alphabets, teaching materials, loanwords, neologisms, and processes of re-semantization. Moreover, these tensions are related to the ultimate purpose of writing oral languages to affirm cultural identity. Written languages find a role in the school, and they create the necessity of exploring social uses for that writing outside the school.

When addressing the relationships between orality and writing in Indigenous contexts of mathematics education, a source of tension is the fact that Indigenous languages have strong oral traditions, while Portuguese and Spanish have a strong written tradition. With respect to Indigenous students' development of mathematical thinking, local mathematical knowledge and Indigenous worldviews appear close to each other. This is evidenced in oral expression and graphic representations that are valid only in the limited scenario of one culture. School mathematical knowledge, by contrast, demands a different type of link with the students'

sociocultural context. It implies the use of expressions not only at the oral, concrete, and graphic level but also at a symbolic one.

The oral-written tension can also be discussed in situations where oral patterns and other forms of symbolic representation that make part of the language practices of these communities appear in written texts, such as, for instance, when aspects of orality influence the writing of mathematical problems, as discussed by Mendes (2007). The writing of these problems in Indigenous languages and in Portuguese by Indigenous teachers have a narrative form. The problems are presented without question(s), which is the expected form. They preserve the discursive characteristics that could be associated with language use to solve problems in daily life. The narrative form incorporates alphabetical and numerical writing and visual representation, i.e., drawings are part of the narrative. They transfer oral narrative characteristics of an everyday problem situation into writing and give a narrative function to drawings that appear in the problem. This type of writing of mathematical problems in a narrative form brings to light the difference between oral and written traditions. "Narrative is a strong trait in the process of production, incorporation and maintenance of social and cultural knowledge and norms among indigenous groups" (Mendes, 2007, p. 224).

The development of a bilingual and intercultural mathematics education for Indigenous students entails another challenge related with expressions in Indigenous languages, since their structure may sometimes complicate the understanding of symbolic expressions. For instance, the expression "7>4," in Spanish is read "7 es mayor que 4" [7 is greater than 4], while in the Amazonian language Shipibo Conibo is expressed as "7 riki 4 bebon iká," an expression that has a different syntactic structure. In Shipibo Conibo this expression means "7 is in front of 4," appealing to the order of natural numbers, instead of their cardinality. In the same way, there are tensions generated by the mismatch between some Indigenous concepts and those used in mathematics. In some Andean cultures, time and space form a conceptual unit, termed in Quechua as *pacha,* where there is no distinction between the two (Yáñez Cossío, 1990, p. 4). Another example is that in some Indigenous numeration systems, the terms for integer numbers in the Indigenous language could be a useful resource from a pedagogical point of view. For example, "in Quechua, Aimara and in some Amazonian languages the numbers up to ten have each one a name. From ten on, they are named "ten and one," "ten and two," "ten and three"… "two tens," "two tens and one," etc. This structure facilitates both understanding of the decimal positional numbers system and the construction of algorithms for the basic operations" (Villavicencio, 2013, p. 36). All these tensions can be seen either as obstacles to overcome or as opportunities to enrich the cultural dialogue of different mathematical knowledge.

4.4 Relevant Mathematics Education in Indigenous Contexts

The changes towards diversity in linguistic and educational policies took place at the heart of cultural, social, and political claims of a national nature. Due to this, the dynamics of multilingual education are not restricted to bilingualism and they are

not separated from conceptions, policies, and projects based on interculturality in each country. With regard to mathematics education, and in order to contribute to the cultural reaffirmation of Indigenous peoples, we understand mathematics as a cultural phenomenon and thus recognize the mathematics of each Indigenous people.

Although we use distinct perspectives for the concept of *ethnomathematics*, we find common ground in our countries with respect to the importance of research on connections between language, culture, and mathematics. Drawing on D'Ambrosio (2011, pp. 111–112), ethnomathematics is not only the study of the mathematics of various ethnicities but also more than that; it is a spatially and temporally differentiated study of the various *technes* or *ticas* (ways, techniques, abilities) of *matema* (to explain, to understand, to deal, to coexist) in different *ethnos* (natural cultural and socioeconomic contexts). Since cultural encounters take place in a network of power relations, these relations set in operation a hierarchy of knowledge that declares what counts as official, valid, or even invalid (Knijnik, 1996).

The cultural forms particularly associated with mathematics are produced in the weaving of cultural understanding as interactive processes or as a web of meanings, using Geertz's (1973) idea of culture as a web of significance. If culture is produced like a web of meanings in interaction, there is a close connection between language and culture. This is because we do not understand language only in a verbal sense, as a vehicle of expression of culture, but as a symbolic system. Thus language can be seen as the stage of cultural production.

Ethnomathematics in the mathematics curriculum of basic education for Indigenous peoples in Brazil, Colombia, and Peru has the aim of contributing to cultural reaffirmation of Indigenous students, through their language and in the context of their worldviews. The inclusion of mathematical aspects of local cultures in the school curriculum is not planned to build a bridge that serves to facilitate students' learning of Western mathematics. Rather, it is a strategy for building cultural identity. This idea is aligned with the ethnomathematical aim of understanding and making visible the cultural forms in which mathematical thinking is produced. The routes that each country has taken into formulating relevant mathematics education for Indigenous are diverse.

4.4.1 Officializing Ethnomathematics in Peru

In Peru, current mathematics education in Indigenous contexts comes from the systematization of accumulated experiences. The documentation on this matter dates from the 1950s. Arithmetic texts produced by SIL were used by students and teachers in Amazonian bilingual schools (Larson et al., 1979). In the 1980s, the study "Numeration, algorithms and application of numerical and geometric relations in the rural communities of Puno" identified the mathematical knowledge of Quechua and Aymara communities (Villavicencio, 1983, pp. 135–141). The results were taken into account in the Experimental Project of Bilingual Education in

Puno. In this pilot project and other similar ones, as well as in the creation in 1989 of a section in the Ministry of Education responsible for Bilingual Education, a new attitude towards and awareness of Indigenous mathematics started.

Since 2003, educational law establishes interculturalism as a mainstream trend in the entire education system. This includes an intercultural mathematical education for all, i.e., for Spanish speakers and speakers of Indigenous language. This law also contributes to the recognition of ethnomathematics as part of the official discourse. For the purpose of implementing and developing an intercultural and bilingual education, ethnomathematics is operationally understood as the knowledge of a cultural group, identifiable as part of their worldview, manifested through the activities of counting, measuring, locating, designing, playing, and explaining.

For basic mathematics education and monolingual Spanish speakers, learning is planned through school activities that enable students to acknowledge the presence of mathematics in Indigenous cultures. In the mathematics education for students whose culture and language are Indigenous, the Peruvian government promotes and implements a pedagogical proposal in the context of the Bilingual Intercultural Education program (Dirección General de Educación Intercultural Bilingüe y Rural [DIGEIBIR] 2012). This proposal has been developed in consultation with Indigenous representatives and the participation of teachers and wise men from the involved communities. The pedagogical approach for mathematics in this educational model is problem solving, in addition to the focus on democratic, intercultural, and welfare rights.

When a school year begins, an elementary teacher in the above mentioned program identifies the students' language proficiency using a psycholinguistic diagnosis tool. There are four levels of proficiency. In classrooms where children speak mainly Indigenous language, the intercultural approach starts with learning-oriented activities of ethnomathematical knowledge, using strategies from their cultural practices, complemented later with school mathematics. While school mathematics has its own codes that can be verbalized in different languages, ethnomathematical non-Western knowledge is inseparable from the corresponding native language and culture. Hence, in multilingual Indigenous contexts, an intercultural bilingual mathematics education is necessary. With respect to the teaching and learning of school mathematics, the language of instruction is chosen, taking into account the students' proficiency in the indigenous languages and Spanish.

For several reasons, mathematics education in Indigenous Peruvian schools has not encountered adequate technical support. Since 2012, the Ministry of Education started prioritizing the education of Indigenous peoples when it was made evident that Indigenous students' achievement in mathematics was low. However, bilingual education strategies may have a positive impact on these students' achievement. López (1998) found that "children served by the PEEB-P [bilingual education program in Puno province] outperformed their peers in the control schools in terms of reading comprehension, oral proficiency of Spanish and mathematical problem solving" (p. 70). Results indicate that learning achievements are better when mathematics education for indigenous children is bilingual, in Andean languages (Quechua or Aymara) and Spanish.

In a study in rural schools in Puno within the bilingual and intercultural approach, Cueto and Secada (2003) did not find statistically significant differences between students' performance in monolingual Spanish and students in these schools. Rather than accepting these results as evidence that the introduced program does not work (or just does not "harm"), the authors argue that the program is not running adequately. They suggest that:

> Any effort to create a truly operational program should take into consideration the language teacher and his mastery of reading and writing, the values and beliefs of the students and their parents about the importance of indigenous languages, and the support given to the program inside and outside classroom (Cueto & Secada, 2003, p. 19)

Studies providing more evidence about the quality of mathematics education in Indigenous contexts are still needed.

4.4.2 Using Language to De-colonialize Indigenous Education in Colombia

During the last 20 years in Colombia, there has been a growing development of multilingualism in mathematics education, emerging from Indigenous communities and researchers. Such development corresponds to a wider context, in which a bottom-up process has now reached policy levels. This section shows important elements of that process for mathematics education as the political changes in the country since 1991 have helped Indigenous communities to legitimately claim the right to defend their cultural identity.

In the 1980s, Indigenous peoples of Cauca were working on creating school readiness booklets on arithmetic. As a result of ethnolinguistic work, Queixalos (1988) created "neonumerations" for the Sikuani language, and Cauty and Ramos (1990) for the Nasayuwe. These first attempts had the intention of creating a basic register for arithmetic and numerals in these languages. It was even intended that these new registers would enter schools. They were introduced in schools, but the dominant use of Spanish in out-of-school situations was difficult to challenge.

By tracking the evolution of curricular proposals for Indigenous education, whether governmental or initiated by Indigenous people, a vacuum on the subject of mathematics can be seen. More often than not, mathematics is considered to be an autonomous differentiated discourse that cannot be addressed using Indigenous knowledge. There has been a tendency, therefore, to deal with mathematics in Spanish. In the few cases where mathematics is connected with Indigenous cultures, it is reduced to arithmetic, with numeral translation, and implemented in school problems of trading. This situation can be related to the fact that during the 1980s, Indigenous movements lacked theoretical tools to associate elements of mathematics with cultural practices.

In the mid-1990s, Ochoa and Peláez (1995), supported by an Indigenous association, presented the mathematical knowledge of the Tule people and a mathematics that

they called "Western." They explained the idea of number in the Tule worldview, topological notions, some classifiers and operations with numbers. They also exposed basic school mathematics tied to arithmetic and its operations. In their text, we find together, but separately, two conceptions of mathematics. Issues of language are made evident in the bilingual writing in both Spanish and Tule language.

Cauty and Tovar developed a project with the Wayuu people, reported in Cauty (1998). The project provided a new approach to structure the fieldwork dynamics for articulating Indigenous claims with advances in mathematics education and ethnomathematics. Cauty engaged in the difficult task of translating an algebra textbook into the Wayuunaiki language. He convened an interdisciplinary and intercultural team of experts: traditional knowledge-holders, linguists, and mathematicians. They worked on creating new knowledge, generating explanations of different concepts from cultural legacies, academic mathematical knowledge, and the structure of Wayuunaiki. It became clear that such an endeavor was not about making literal translations, or equivalences word to word, but about building networks of explanations and representations around concepts and practices. The process could not only be steered by the mathematical knowledge of the dominant society. The product was not a translated text, but a process that brought together different peoples and knowledge. They proposed new words and alternative conceptualizations in mathematics, which expanded the original fields of knowledge of each culture. One contribution of this work was to overcome the idea that arithmetic is the only content to be worked in Indigenous education. Another element was to suggest a multidirectional approach that evidenced how this type of encounter cannot be a translation from academic mathematical knowledge to words used in the native language, tacitly maintaining a relationship of hierarchy between the two cultures. It was also necessary to describe with Western mathematical representations some topics and features of the Wayuu worldview, which were conceptualized by the Wayuu as hallmarks of their rationality.

Since then, there has been an increase in research on Indigenous knowledge associated with mathematics in different Colombian communities. Such work privileges a strategy of dialogue of knowledge which emphasizes cultural and cosmological issues, while it downplays a linguistic focus (Aroca, 2007; Parra, 2003).

In 2006, some of the Caucan communities expressed interest in researching their worldview, forming an Indigenous intercultural research center, and within it a research team on Nasa people's mathematics. Such a team was formed by Indigenous teachers who had been involved in the Nasa-alphabet unification as well as in the standardization of the writing process. The team also involved experts in mathematics and language, with experience in Indigenous education. The work process reflected some elements of Cauty's proposal, but coordinated with the Nasa people's ways of producing knowledge. The work was collectively developed in different shelters, with the participation of knowledge-holders, elders, children, teachers, and educational authorities.

Previous published information about the mathematical practices of the Nasa people was refuted, complemented, and increased. The discussions during the exploration took place in the Nasayuwe language and the teachers recorded,

transcribed, and summarized the elements found in them. When the community elders expressed mathematical concepts, unexpected words that had fallen into disuse or were not known to newer generations appeared. Some common words also gained a new meaning. Considering issues of localization, there emerged old expressions in Nasayuwe for movement and stillness. Words indicating specific lapses of time during the day were commonly used as time markers. Also the existence of some local measure units for weight, length, and volume led the researchers to propose a word for the concept of "measure unit." When conceptual issues appeared, e.g., velocity or continuity, common words gained a new meaning, as the constant and uninterrupted presence of the territory and its spirits has a particular word in Nasayuwe. That word was proposed to express the concept of continuity. The road, that sometimes disappears, was used as an image for "the discrete" as a mathematical concept. Research findings were published in a bilingual book, in which Indigenous teachers were the authors (Caicedo et al., 2009). The book includes several stories of the oral tradition and rescues what the community considers as mathematics.

The book also raises conceptual elements to be developed by other teachers in their process of *Educación Propia. Educación propia* (in Spanish) is an educational approach resulting from 35 years of political negotiation between Indigenous organizations and the Colombian government. This approach encapsulates three meanings of the word *propia*: propia as *their own* or belonging to them, propia as *adequate* for their needs, and propia as other forms of knowledge and education that get *appropriated* by the community for their political struggles (Parra, 2011). In the same way, attuned to government regulations on Indigenous education, Viluche and Yujo (2006) and Tamayo (2012) have initiated studies on mathematics education in other communities, discussing the Western disciplinary classification and displaying its failure to deal with cultural practices. Appreciation of the integrity and holism of Indigenous thought generates resistance to the act of isolating elements of cultural practices and to typifying them as mathematical.

4.4.3 Problematizing Schooling and Mathematics Education in Brazil

In Brazil, issues related to mathematics education for Indigenous peoples emerged linked to discussions about Indigenous schools. Although school was understood as a historical space of value imposition and assimilation for incorporation into the market economy, Indigenous communities began to claim schools as a place to build intercultural relationships based on political autonomy (Ministerio de Educação do Brasil, 2007). Therefore, Indigenous education, particularly mathematics education, cannot be divorced from Indigenous students' realities in seeking to meet the Indigenous communities' aspirations and respect for group cultural issues (Correa, 2002).

The discussions about mathematics education have been influenced by ethnomathematics research in contexts of Indigenous communities and also by processes of Indigenous teacher education in different projects in the country, such as those by Costa (2008) and by Mendes (2001). As stated by Sebastiani Ferreira (2004), the movement of teacher education with teachers as researchers in Indigenous cultures is crucial. Indigenous teachers assume a key role in research practices around language and cultural knowledge in their communities with the purpose of incorporating and articulating the community's knowledge in relation to school mathematics practices. Many Indigenous teachers have this perspective when questioned about why mathematics should be taught in Indigenous schools. One indigenous Xacriabá teacher asserted:

> We have to know the math also, since the activities are not just from textbooks, but in relation to our people. Mathematics in Indian schools is very important to our day-to-day. We are living in it. (Mendonça, 2006, p. 5)

The rights claimed by Indigenous communities call for an Indigenous School Education characterized by the "assertion of ethnic identities, the recovery of historical memory, the appreciation of languages, and knowledge of indigenous peoples and the revitalized association between school/society/identity, in accordance to societal projects defined autonomously by each indigenous people" (Ministerio de Educação do Brasil, 2007, p. 21). Indigenous schools are being proposed as intercultural spaces where situations of teaching and learning are related to the political and cultural identity of each Indigenous people. As said by a Guarani teacher:

> I see that this school should have all conditions of an Indigenous people, the Guarani-Kaiowá people, Terena, Xingu, Xavante […] whatever. But it has to be a school of that group, that nation, which teaches the language, dances, rituals, ceremonies, which is a school with proper autonomy of the local community. (Rodrigues, Ferreira, & Domite, 2009, p. 9)

In the same way, Indigenous peoples have linguistic rights to participate in school learning processes conducted in students' mother tongues. Those rights direct attention towards the community's sociolinguistic reality and language use in the community space as well as in school. Indigenous school education in an educational program that has no connection with the reality of Indigenous students can generate a weakening of a people's identity, making harder the struggle for survival. Therefore, mathematics education courses for Indigenous teachers have been anchored on issues related to Indigenous communities' aspirations and respect for cultural identity, to meet the needs of Indigenous peoples for building their own educational curriculum, according to their reality and consistent with the new demands that post-contact situations imposed (Correa, 2002).

Among studies with a focus on cultural practices of Indigenous groups, Indigenous languages have a key role in knowledge constitution. Ribeiro and Ferreira (2004) show that the Xerente people have an organization that is based on a dual logical thinking process. Numbers expressed in that Indigenous language have a dual construction because the number one is not a complete entity. A unity, for this community, occurs in the encounter between one half and the part that is

lacking: "The whole number, reflecting the knowledge that comprises it, consists of the junction of two halves, which form an ethnomathematical dual system" Ribeiro and Ferreira (2004, p. 152). As this conception has no possible translation from the Xerente language into Portuguese, the authors underscore the need for vitalizing language to maintain the Xerente culture.

The case above exemplifies the fact that the encounter between Indigenous and non-Indigenous mathematics have generated tensions of a political and symbolic nature in relation to the uses of Indigenous languages and Portuguese. Such tensions have resulted in the development of new numerical terms, particularly in producing mathematical literacy materials in Indigenous languages, as a form of affirmation and maintenance of an ethnic identity as discussed by Mendes (2004, 2011). Following this trend, Indigenous schools are conceived of as intercultural spaces that can be conceptualized as "border schools"; that is, public spaces in which teaching and learning situations are related to the political and cultural identity of each Indigenous people. More generally, it can be argued that discussions of Indigenous school education in Brazil have questioned the relationship between society, culture, and school. Thus, intercultural and multilingual mathematics education draws on the social life of these groups in order to establish new meanings and functions for the school contexts.

4.5 Concluding Remarks

Since the 1980s, there has been a favorable advance of state policies towards the participation of Indigenous peoples regarding the recognition of their right to an appropriate education, according to their worldviews, cultures, and languages. In contrast to the colonial view of assimilation of Indigenous people through evangelization and monolingual/monocultural education, the advance towards the acknowledgement and respect of Indigenous cultures has been closely linked to the struggle of Indigenous organizations. In Brazil, Colombia, and Peru there are different routes for the implementation and development of educational models in the context of Indigenous peoples, which depend on the autonomy and dialogue that they establish with the state. In Peru, there is a diversified bilingual intercultural education provided by the government, with the participation of the Indigenous peoples. Education and professional development for bilingual and intercultural teachers are offered in pedagogic institutes and some universities. In Colombia, the communities are developing an entire educational system, following the principles of the *Educacion Propia* approach. In Brazil there is a differentiated, intercultural, and bilingual proposal for Indigenous schooling with emphasis on the initial education of Indigenous teachers in undergraduate programs at universities.

To attend to the needs of Indigenous populations, the three countries have diversified curricula, finding in different interpretations of ethnomathematics the theoretical and methodological elements necessary to support mathematics education in situations of bilingualism involving meetings of different cultures. Nevertheless, the implementation and development of mathematics education for Indigenous

populations have generated tensions derived from the diverse worldviews, cultures and languages involved. Indeed, Indigenous peoples generally agree to establish a harmonious relationship with nature, as opposed to those who see nature as an object of exploitation. American Indigenous cultures seek the good life of all beings, including humans. Also, cultural practices associated with mathematics are present in the field of daily life and social relations and are interwoven with spirituality and cosmologies. Such ways of understanding themselves and the environment usually clash with dominant views of school mathematics belonging to the main culture, where school mathematics is seen as a knowledge that stands on its own, decontextualized of its origin and use. All this brings to the fore the epistemological debate about what counts as mathematical knowledge in the domain of schooling, as well as the criteria to select what deserves to be addressed in Indigenous education.

The production, dissemination, and use of bilingual educational materials in mathematics may lead to solving linguistic, social, and epistemological issues. In fact, creating neologisms and determining loanwords suitable for the intended objectives of mathematics teaching requires an in-depth study of the linguistic structures of Indigenous languages. It also demands the active participation of the community and institutional support to ensure the use of the terminologies proposed. A factor of complexity is the fact that these languages are predominantly oral, with all that this implies for the understanding of the symbolic language of traditional school mathematics, particularly when symbolic expressions of mathematical relations do not match the syntactic structure of the corresponding oral expressions in Indigenous languages.

Research on the mathematics education of Latin American Indigenous peoples is recent compared to the advances in other similar situations (e.g., Meaney, Trinick, & Fairhall, 2012). Research on the systematically low levels of achievement of Indigenous students in mathematics alerts us to the urgency of considering educational approaches that build on Indigenous traditional knowledge, ensuring basic conditions to enable a *pertinent* and *quality* mathematics education. It is also important to reconsider the relevance of applying standardized tests to Indigenous peoples or the weighting of their results to the policies to be implemented, since the exercise of citizenship is differentiated in these peoples. Not everyone wants to enter into the schemes of economic production that mainstream society builds, with its standardization in school mathematics. Multilingualism in Latin American schools is engaged in a broader political project for respect and protection of cultural diversity and Indigenous heritage.

References

Aroca, A. (2007). *Una propuesta de enseñanza de geometría desde una perspectiva cultural. Caso de estudio: Comunidad indígena Ika-Sierra Nevada de Santa Marta.* Unpublished master's dissertation, Universidad del Valle, Colombia.

Caicedo, N., Guegia, G., Parra, A., Guegia, A., Guegia, C., Calambas, L., Castro, H., Pacho, C., & Diaz, E. (2009). *Matemáticas en el mundo Nasa* (N. Caicedo & A. Parra, Eds.). Bogotá, Colombia: CIIIT.

Cauty, A. (1998). *Etnomatemáticas: el Laboratorio Kwibi Urraga de la Universidad de la Guajira*. Retrieved from http://etnomatematica.org/articulos/cauty4.pdf

Cauty, A., & Ramos, A. (1990). Vigilancia etnocultural: el caso de la numeración tradicional Nasayuwe. *Boletín de Lingüística Aborigen, 2*, 3–15.

Correa, R. A. (2002). A educação matemática nos cursos de licenciatura e a formação de professores indígenas. *Cadernos de Educação Escolar Indígena-3º grau indígena* (pp. 117). Barra do Bugres, Brazil: UNEMAT.

Costa, W. N. G. (2008). *A etnomatemática da alma A'uwe-xavante em suas relações com os mitos*. Unpublished doctoral dissertation, Universidade de São Paulo, Brazil.

Cueto, S., & Secada, W. (2003). Eficacia escolar en escuelas bilingües en Puno. *Revista Electrónica Iberoamericana sobre Calidad, Eficacia y Cambio en Educación, 1*(1), 19–20.

D'Ambrosio, U. (2011). *Educação para uma sociedade em transição* (2nd ed.). Natal, Brazil: EDUFRN.

Dirección General de Educación Intercultural Bilingüe y Rural (DIGEIBIR). (2012). *Hacia una educación intercultural bilingüe de calidad*. Lima, Peru: Ministerio de Educación de Perú.

Geertz, C. (1973). *The interpretation of cultures*. New York: Basic Books.

Instituto Brasileiro de Geografia e Estatística (IBGE). (2010). *Censo*. Brasilia, Brazil: Instituto Brasileiro de Geografia e Estatística. Retrieved from http://www.censo2010.ibge.gov.br/

Instituto Nacional de Estadística e Informática (INEI). (2009). *Censos Nacionales 2007: XI de Población y VI de Vivienda.Resumen Ejecutivo*. Lima, Peru: Dirección Nacional de Censos y Encuestas. Retrieved from http://www.inei.gob.pe/media/MenuRecursivo/publicaciones_digitales/Est/Lib0789/Libro.pdf

Knijnik, G. (1996). *Exclusão e resistência: Educação matemática e legitimidade cultural*. Porto Alegre, Brazil: Artes Médicas.

Landaburu, J. (2004). La situación de las lenguas indígenas de Colombia: Prolegómenos para una política lingüística viable. *Amérique Latine Histoire et Mémoire. Les Cahiers ALHIM, 10*. Retrieved from http://alhim.revues.org/125

Larson, M., Davis, P., & Ballena, M. (1979). *Educación bilingüe: Una experiencia en la amazonía peruana*. Lima, Peru: Ignacio Prado Pastor.

López, L. E. (1998). La eficacia y validez de lo obvio: Lecciones aprendidas desde la evaluación de procesos educativos bilingües. *Revista Iberoamericana de Educación, 17*, 51–89.

López, L. E., & Küper, W. (1999). La educación intercultural bilingüe en América Latina: Balance y perspectivas. *Revista Iberoamericana de Educación, 20*, 17–85.

Meaney, T., Trinick, T., & Fairhall, U. (2012). *Collaborating to meet language challenges in indigenous mathematics classrooms*. New York: Springer.

Mendes, J. R. (2001). *Ler, escrever e contar: práticas de numeramento-letramento dos Kaiabi no contexto de formação de professores Índios do Parque Indígena do Xingu*. Unpublished doctoral dissertation, Universidade Estadual de Campinas-UNICAMP, Brazil.

Mendes, J. R. (2004). Aspectos políticos e simbólicos na apropriação do discurso da etnomatemática: O caso dos professores Kaiabi do Parque Indígena do Xingu. In G. Knijnik, F. Wanderer, & C. J. de Oliveira (Eds.), *Etnomatemática, currículo e formação de professores* (pp. 364–376). Santa Cruz do Sul, Brazil: EDUNISC.

Mendes, J. R. (2007). Numeracy and literacy in a bilingual context: Indigenous teachers education in Brazil. *Educational Studies in Mathematics, 64*(2), 217–230.

Mendes, J. R. (2011). The meaning of ´number´ in Kaiabi: Identity and language in the context of Indigenous teacher education. In M. Setati, T. Nkambule & L. Goosen (Eds.), *Proceedings of the Study 21 Conference: Mathematics Education and Language Diversity* (pp. 218–223). São Paulo, Brazil.

Mendonça, A. N. (2006). *Investigando as práticas educativas dos professores Indígenas que ensinam Matemática na escola Xacriabá. Anais do EBRAPEM*. Belo Horizonte, Brazil: UFMG.

Ministerio de Educação do Brasil. (2007). *Educação Escolar Indígena: diversidade sociocultural indígena ressignificando a escola*. Brasilia, Brazil: Secretaria de Educação Continuada, Alfabetização e Diversidade. Retrieved from http://www.dominiopublico.gov.br/

Ministerio de Educación del Peru. (2013). *Documento nacional de lenguas originarias del Perú*. Lima, Peru: Ministerio de Educación.

Ochoa, R., & Peláez, J. (1995). *La matemática como elemento de reflexión comunitaria del pueblo Tule*. Medellín, Colombia: Lealon.

Parra, A. (2003). *Acercamiento a la etnomatemática*. Unpublished monograph, Universidad Nacional de Colombia Sede Bogotá, Colombia.

Parra, A. (2011). *Etnomatemática e educação própria*. Unpublished master's dissertation, São Paulo State University at Rio Claro, Brazil.

Parra, A. (2013). Linguagem escrita e matemática: Um viés etnomatemático. *Revista Latinoamericana de Etnomatemática, 6*(2), 24–34.

Queixalos, F. (1988). Numeración tradicional Sikuani. *Glotta, 3*, 28–31.

Ribeiro, J. P., & Ferreira, R. (2004). Educação escolar indígena e etnomatemática: um diálogo necessário. In M. C. S. Domite, J. P. Ribeiro, & R. Ferreira (Eds.), *Etnomatemática: Papel, valor e significado* (pp. 149–160). São Paulo, Brazil: Editora Zouk.

Rodrigues, M., Ferreira, R., & Domite, M. C. S. (2009). A formação de professores de professores e suas relações com cultura e sociedade: A educação escolar indígena no centro das atenções. *Bolema: Boletim de Educação Matemática, 22*(34), 263–282.

Roldán, R., & Gómez, J. (1994). *Fuero indígena colombiano*. Bogotá, Colombia: Gente Nueva Editorial.

Sebastiani Ferreira, E. (2004). Os Índios Waimiri-Atroari e a etnomatemática. In G. Knijnik, F. Wanderer, & C. J. de Oliveira (Eds.), *Etnomatemática, currículo e formação de professores* (pp. 70–88). Santa Cruz do Sul, Brazil: EDUNISC.

Tamayo, C. (2012). *(Re)significación del currículo escolar indígena, relativo al concimiento matemático, desde y para las prácticas sociales de la cestería y el cultivo del plátano: el caso de la comunidad Tule de Alto Caimán*. Unpublished master's dissertation, Universidad de Antioquia, Colombia.

UNICEF & FUNPROEIB. (2009). *Atlas sociolingüístico de pueblos indígenas en América Latina* (I. Sichra Ed.). Quito, Ecuador: FUNPROEIB Andes.

Villavicencio, M. (1983). *Numeración, algoritmos y aplicación de relaciones numéricas y geométricas en las comunidades rurales de Puno*. Puno, Perú: Ministerio de Educación. INIDE. DDE.

Villavicencio, M. (2013). *Matemáticas en educación intercultural bilingüe. Serie Matemáticas en EIB*. Lima, Peru: JERGIMPRESS E.I.R.L.

Viluche, J., & Yujo, S. (2006). *Nasawe'sx kiwaka fxi'zenxi êen*. Popayán, Colombia: Consejo Regional Indígena del Cauca, Programa de Educación Bilingüe; Asociación de Cabildos Ukawe Cxhab.

Woodward, K. (1997). *Identity and difference*. London: Sage.

Yáñez Cossío, C. (1990). *Representaciones y conceptos estructurantes*. Quito, Ecuador: Editorial Abya-Yala.

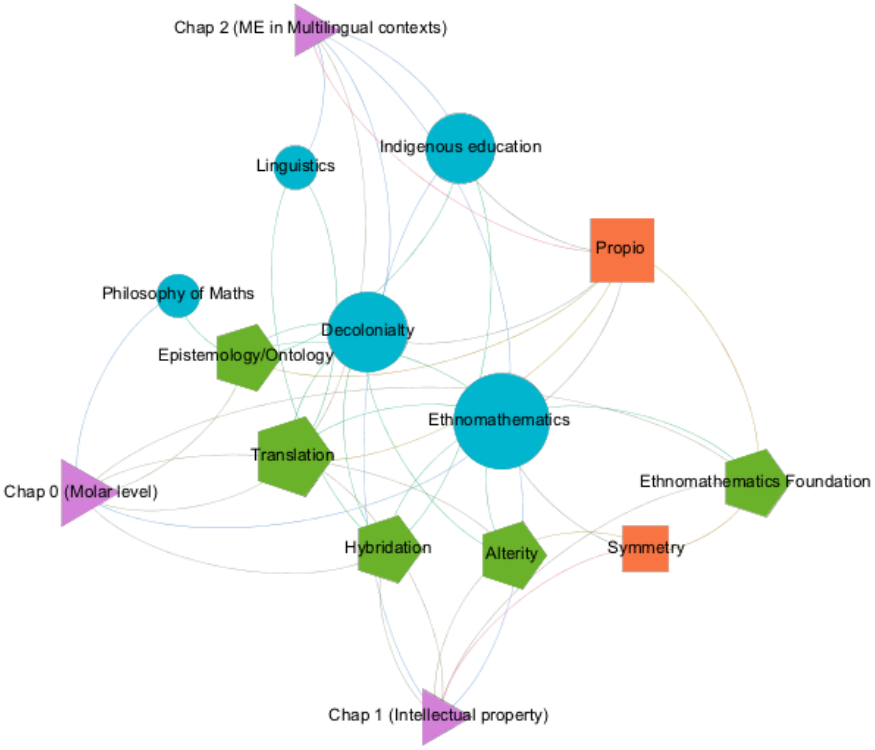


Fig. 2.1: Conceptual entanglement 2. Triangles represent chapters, circles represent areas of study, pentagons represent topics of interest, and squares represent new conceptual tools.

Chapter 3

Ethnomathematical Barters

Aldo Parra-Sanchez

The paper has been published in
Parra-Sanchez A. (2017) Ethnomathematical Barters. In: Straehler-Pohl H., Bohlmann N., Pais A. (eds) *The Disorder of Mathematics Education*.
Springer, Cham.
Print ISBN 978 – 3 – 319 – 34005 – 0

The layout has been revised.

Chapter 6
Ethnomathematical Barters

Aldo Parra-Sanchez

Abstract This chapter identifies and criticizes one assumption of the ethnomathematical research field, regarding the ways in which the relationship between mathematics and culture is addressed. Many developments and theoretical conflicts within this field can be traced to that assumption, which has been widespread indistinctly by practitioners and critics of ethnomathematics. Looking for a new understanding of this field, an alternative approach is proposed, trying to respond to some theoretical critiques and prompting new horizons. This intended approach privileges non-colonialist interactions among stakeholders, recognizing their different interests, their different ways to conceptualize and their interdependence. It is discussed how interactions can be conducted to hybridize different kinds of knowledge, constituting political and epistemological endeavors. The essay concludes observing which types of problems would appear due to the new approach.

Now Ethnomathematics…

Since ethnomathematics emerged as a research field within mathematics education, a proper theorization and definition has been sought and almost every researcher has attempted to give his/her personal view regarding its definition and intend. Although this is common to new and growing fields of research, this diversity of methods and approaches might be seen as a sign of disorder, non-cohesion, or absence of a shared horizon. Most of the researchers identifying themselves with this field share a common conception of ethnomathematics as a research program in the history of ideas, that seeks to understand the "generation, organization, institutionalization and propagation of knowledge" (D'Ambrosio, 1993) throughout the history of humanity, in the contexts of different interest groups, communities, peoples, and nations. However, this conception is developed in different directions. Even D'Ambrosio, who introduced first such conception (D'Ambrosio, 1985), has been modifying parts of his seminal theoretical statements about ethnomathematics over the last 15 years.

A. Parra-Sanchez (✉)
UNESCO Center of PBL in Engineering and Science, Department of Planning and Development, Aalborg University, Aalborg, Denmark
e-mail: aldo@plan.aau.dk

H. Straehler-Pohl et al. (eds.), *The Disorder of Mathematics Education*,
DOI 10.1007/978-3-319-34006-7_6

He proposes to embrace a more holistic and transdisciplinary approach, not only regarding mathematics but knowledge in general (D'Ambrosio, 2001).

In the last 5 years, new attempts have been made to produce theories that allow room for such variety of comprehensions. Miarka and Bicudo (2012) studied the relationship between mathematics and ethnomathematics through a phenomenological explanation of academic fieldwork. Rohrer and Schubring (2013, p. 78) proposed an overarching conceptualization of ethnomathematics by claiming that this theory "needs to be regarded as an interdisciplinary discipline that covers theories from both the exact and social sciences."

Parallel to such theorizations, critiques of ethnomathematics—pointing to assumptions that research in this field make when using terms such as "culture" and "mathematics"—have been published. Rowlands and Carson (2002) warn about the uses of ethnomathematics in education, by stressing the fact that western mathematics encompasses and formalizes all previous cultural systems that humans had developed. If this particular critique were to be accepted, a study of previous systems would result in a throwback that education cannot allow. Other critiques question the effectiveness of ethnomathematics in achieving its own intended political goals (Pais, 2011, Vithal & Skovsmose, 1997). While the first of these critiques attempts to dismiss ethnomathematics as a whole, the other two invite to sharpen the core ideas within them. Nevertheless, all of them share an "external" positioning with respect to ethnomathematics, since the authors do not consider themselves as researchers in this area. Only a few "internal" critiques—this is to say, critiques made by scholars who consider themselves involved in ethnomathematics research—have been published. For example, Alangui (2010) warned about the very old fashioned concept of culture that is commonly used in research. Knijnik, Wanderer, Giongo, and Duarte (2012) also problematized how some assumptions about students' realities and the use of concrete materials for teaching became naturalized in ethnomathematical research.

…Has an "Intersection Approach"…

A common feature of inner trends and external critiques within the diversity of approaches and purposes, is the intention of addressing the existence/absence of shared objects between mathematics and culture (despite the diverse definitions of those terms). By considering the particular culture of a group as one set and mathematics as another set, ethnomathematics as a research field might relate to examine the *intersection* of these two sets. Such intersection can be called the ethnomathematics of that group or even the mathematics of that group. Whatever the chosen name is, and without considering neither the possible methodological procedures to perform this examination nor the theoretical considerations that would make impossible comparing those sets, the underlying assumption is such an intersection matters.

It is not difficult to show in the growth of ethnomathematics as a research field, how the intersection of mathematics and cultures has been considered as relevant.

For instance, the North American Study Group on Ethnomathematics sponsors a journal which on its online version states that: "the journal's contents examine the intersections between mathematics and culture in both western and non-western societies, and among both math professionals and non-professionals" (NASGEM).

It is common in research articles to find expressions such as "every culture has mathematics" (Selin & D'Ambrosio, 2001, p. xvii); "ethnomathematics seeks to revive mathematics *living* in different traditions and cultures, not by considering them to be exotic, but by including them in the new historiography of mathematics" (Rohrer & Schubring, 2013, p. 84, emphasis added); and even the dilemma pointed out by Bishop (1994, p. 15, emphasis added): "is there one mathematics *appearing in* different manifestations and symbolizations, or *are there* different mathematics being practiced which have certain similarities?". As we can see, these references share the common ground of assuming first a distinction between mathematics and culture and assuming then that there is *something* in the intersection of these two entities. Accordingly, this *intersection* becomes the main—though not the sole—object of study for ethnomathematics.

Even historically it seems to exist a continuity regarding the importance of such intersection. This can be already seen in early approaches, such as that of Ascher and Ascher (1986) who define ethnomathematics as mathematical practices of non-literate people; it continues with a reconceptualization by D'Ambrosio (2006, p. 1, emphasis added) who states that "ethnomathematics *is* the mathematics *practiced* by cultural groups" and it can be also seen in contemporary work like that of Furuto (2014, p. 122) who assumes that ethnomathematics is "the intersection of culture, historical traditions, sociocultural roots and mathematics."

Barton (1996) explicitly pointed to the intersectional approach in a visual way, using Venn-diagrams. When trying to analyze the comprehension of the field by three influential researchers, he drew a diagram for each of the approaches to exaggerate the differences existing among them (Fig. 6.1):

Through all those excerpts, it can be seen how the assumption regarding the importance of the intersection has been central to conceptualizations about ethnomathematics, providing a way of thinking and talking about the field as well as framing the debate. In the next section, I show the limitations of such approach and I point out some consequences of these limitations.

…Problem…

There are different ways in which critics and followers of ethnomathematics position their arguments within the intersection approach and according to their personal alignment with the ethnomathematics program. In this section, four possible types of positioning for researchers are considered.

The first possible situation is to simultaneously support the ethnomathematical research program and also claim that the aforementioned intersection is not empty. From this starting point, a researcher will try to show the presence of some mathematical notion, skill, or concept within cultural artifacts and practices. Roughly speaking,

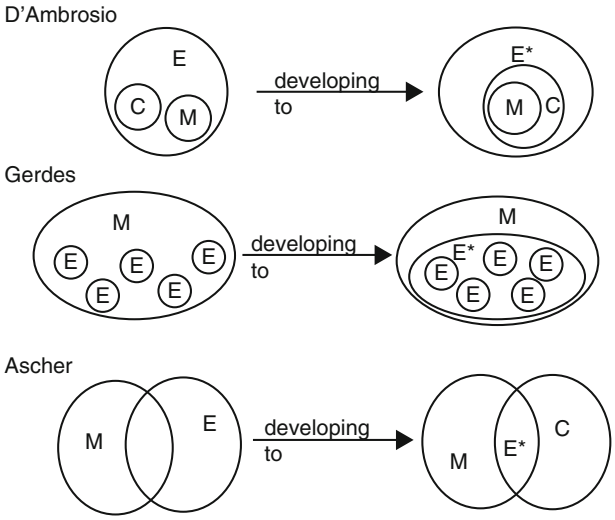


Fig. 6.1 Barton's diagram for ethnomathematics. (Barton, 1996, p. 213) (*E* ethnomathematics, *E** ethnomathematics as a research program, *M* mathematics, *C* culture)

the aim is about to "reveal the culture's hidden math." The ethnomathematician's goal becomes uncovering that presence and mathematical modelling appears as a natural complement in this endeavor. Therefore, there is a necessity of developing proper methodological tools to do that uncovering, as Ferreira (1994) and more recently Albertí Palmer (2007) have intended. If ethnomathematics is theorized using these arguments, the paradox of Millroy (1992) quickly arises as problematic. This paradox points out that it is not possible to find any knowledge other than the academic because researchers will be acting with their academically trained mathematical gaze. This is an echo of the anthropological "reflexivity problem" (Woolgar, 1988) resulting from considering ethnography as the proper methodology for ethnomathematics.

A second possibility is to be sympathetic with ethnomathematics as a research program, but to consider the intersection being empty, due to the nonexistence of the category "mathematics" in some cultures. This posture focuses on how knowledge is developed in different cultural groups by recognizing how it affects and is affected by educational discourses. A key question to this possibility was raised by Lizcano (2002, p. 1, own translation): "what can we see if, instead of looking at popular practices through 'mathematics,' we look at mathematics through popular practices?" This positioning puts in doubt the preeminence of mathematics as a superior knowledge over others. Conversely, efforts to embed holistic knowledge into the restricted boundaries of mathematical discipline are rejected. It seems to have a subtle essentialist view of culture as it leaves unexplained why a cultural group that does not have the category of mathematics would not be able to understand the existence of that category in the cultures of other groups, or at least to create an inner explanation about mathematics.

A third option presents similar reasons to consider the intersection as empty by definition, but differs from the second option as it does not follow ethnomathematics

as a research program to expand the social understanding of mathematics as part of a culture. In this posture, it is common to use an argument of authority: cultural practices are not mathematics because they are not developed within a scholarly context. They have not been refined by one legitimate institution (universities, journals, the academic community of teachers, mathematicians, and so on), and because of that, they lack a "warranty" certificate. This attempt to create an essence of mathematical knowledge using the hegemony of one particular group was summarized by Rômulo Lins (2004, p. 99) with irony: "mathematics is the thing made by mathematicians when they said that they are doing mathematics." Although such a circular definition cannot be contested, its self-sufficiency is at the same time its big weakness as it implies an unacceptable omission: those institutions are cultural and historical. Lins reminds us how the professionalization of mathematics appears only in the nineteenth century and many of the literature recognized as mathematics before that time could barely satisfy current standards.

The last position does not follow ethnomathematics as a program neither, but is less radical; it considers the intersection as non-empty (i.e., it recognizes that mathematics can be present in several cultures). A hierarchical model drives this account of the development of the discipline, believing that the world has adopted conventions of mathematics, "because they have been sifted and tested and refined within the crucible of practical experience, which yields neither to passion nor to ideological persuasion" (Rowlands & Carson, 2002, p. 86). In such a model, mathematical knowledge has constantly been evolving in a universal process of improvement that transcends civilizations. If such an approach is accepted, any strong review of the history and epistemology of mathematics is impossible and accordingly ethnomathematics has nothing worthy to offer. Hence, the goal would be to fill in minor details of how the only possible rationality was achieved across space and time until now as an inevitable fate.

All these positions refer mainly to what is (or what has been) mathematical instead of what could be mathematical. I consider that the intersection problem conduces to a false dilemma, which is responsible for the critiques received and also for the growing "domestification" ethnomathematics has been the object to in the last decade as attested by Pais (2012).

…That Can Be Changed…

With this essay, I try to develop an alternative approach to theorize ethnomathematics that goes beyond the "intersection" problem by building on several research projects conducted in multilingual environments (Barton, 2008; Caicedo et al., 2009; Cauty, 2001; Meaney, Trinick, & Fairhall, 2011). These projects serve as an impulse to reflect upon the possibilities of developing dialogic processes within cultural groups around the concept of mathematics, its educational implications and its political uses. The political views of Knijnik et al. (2012) and Alangui's methodological contribution about "mutual interrogation" (Alangui, 2010) deserve to be considered because they give the interactions a central role, despite not working directly with linguistic issues.

The task is not to discover or find elements within the intersection of mathematics and culture, but to create links between them. When the Māori Language Commission proposed terms to be used within schools (Meaney et al., 2011) in a way that took care of sensible features of the Māori language and their cultural heritage, the research experience resulted in a collaborative and multilateral process that expanded boundaries in mathematics, culture, and language simultaneously. The new lexicon entailed new knowledge and new sorts of relations about that knowledge. Language provided a backdrop for interactions through translations and negotiations of meaning.

However, one can find the same type of movement in cases without that linguistic issue; Eglash (2000, p. 17) for instance, recognizes how Gerdes (2007) proposed new mathematical ideas inspired by cultural practices from Africa:

> Ethnomathematics of indigenous societies is not limited to direct translations of western forms, but rather can be open to any mathematical pattern discernable to the researcher. In fact, even that description might to be too restrictive: previous to Gerdes' study there was no western category of "recursively generated Eulerian paths"; it was only in the act of applying a western analysis to the Lusona that Gerdes (and the Tchokwe) created that hybrid.

The basic idea is to provide an interpretation of ethnomathematical research practice as intentional and deliberated processes. These processes generate connections between mathematics and culture in a non-essentialist understanding of both constructs.

This approach assumes a different role of the researcher: from that of one who looks for something hidden and preestablished to one who creates representations and meanings. With such consideration, researchers can be found on both sides, not only on the academic side. Practitioners and knowledge-holders become researchers as well, following their own agenda as proposed in post-colonial studies by Chambers (1996) and in ethnomathematics by Cauty (2001). Therefore, the intended connections produced are not unidirectional, as they do not create mathematical interpretations of cultural practices only, but they also provide culturally grounded explanations of mathematical practices. Another consequence is that the web of those multiple links creates a space for cultural encounter where unforeseen actions and situations can happen.

The features of this approach resemble the practice of *barter* in the manner that is done by some indigenous communities in South America (Townsend, 2012). In this type of barter, people from different villages participate in a meeting contributing in her/his own way with food, tools, and workforce. Participants bring food, tools, and elements that other participants do not produce. Sometimes these elements are either exchanged, or given as a gift. During the meeting, jokes and stories are told and people dance and sing songs. Barters are arranged to build a house or to help families through agricultural labor. Every person involved in the barter returns home with something new gained in the barter. Roughly speaking, when people are engaged in a barter, tasks that only can be done with a joint effort are accomplished and the interdependence of the agents is emphasized, benefiting everyone. A barter is therefore more than a mere exchange but an opportunity to share, create, and learn.

Ethnomathematical practice can assume the underlying principles of bartering as a way of addressing cultural encounters. These principles would put forward the relationship between academic researcher and communities far away from the realm of ethnography and demand unforeseen alternatives in each particular encounter.

…To an Interactional, Hybrid…

I explain the idea of creating connections that is central for bartering, using an example that Alexandre Pais (2013) proposed to illustrate his critique of ethnomathematics. He invites the reader to imagine a group of indigenous people observing students in a mathematics classroom where the topic of the day is the Pythagoras Theorem. After some time watching the students,

> They [the group of indigenous people] realize that what the students are doing while seated at tables with pens in their hands solving exercises on a sheet of paper *is actually* the construction of a house. Why does this sound absurd? Why is the direction of research always one of going to the local communities to recognize as mathematics what these people are doing? (Pais, 2013, p. 3, emphasis added)

As it is argued, it is irrelevant if this mathematical practice "is actually" one house or not. Certainly, it could be less problematic if the group says "this exercise *looks like* the way that we build a house" appealing to some *family resemblance* with mathematics (Knijnik, 2012). Nonetheless, the important aspect is the very act of the group claiming a connection between one system and the other. Pais found his story absurd because it underlies the colonial relationship on which classic ethnography relies. In such story, facts have no consequences and there are no interactions between people. Pais, like many of the followers and critics of ethnomathematics, does not conceive ethnomathematics as a form of barter.

Let us imagine a continuation of the story, a second part that decreases the colonial bias, by making relevant the diversity of voices and agents that are present in the situation, and involving those voices in a common goal. Let us imagine that one of the indigenous says that those equations on the chalkboard remind her of the building of a house.

Indigenous A: This is a house
Student 1: No, it is a theorem
Indigenous A: Well, it seems a house to me
Student 2: How so?
Student 3: I do not understand
Indigenous B: Of course, it is not the same, but when we construct, we put rows and pillars in a cross
Student 1: Oh, perpendiculars?
Indigenous A: Whatever, if you want to call it like that, it is ok. But we say "in cross." With that cross we put all the tiles, caring that the water rain falls easy.
Student 3: Teacher, that one would be the hypotenuse?
Teacher: Not one, many of them, because there are many tiles in different directions
Student 2: But Pythagoras has only one hypotenuse!
-…

This endless hypothetical discussion starts involving different worldviews with explanations from multiple sources. This conversation sketches a process that requires collaboration among the agents. Such agents contrast, translate, criticize and appropriate ideas from the practice observed, constituting a barter of insights. This interaction is in itself an educational process that does not intend to arrive at a shared *happy end* by destroying differences in a common, unified knowledge.

This second part of the story does not sound absurd to me for a simple reason: it has already happened. There have been several experiences where the direction of research goes from the local communities to recognize cultural and political concepts and practices that are usually related with mathematics. Meaney et al. (2011) reported the challenging process of one Māori community in New Zealand trying to educate their children in a Māori-immersion school, highlighting how collaboration becomes central to confront the different challenges that arise in every stage. Gelsa Knijnik enquired how Brazilian farmers in a settlement discuss the different ways in which the land is measured, contrasting farmers' techniques with official ones used by banks and the state (Knijnik, 1996). Caicedo et al. (2009) registered the experience of an indigenous community in Colombia trying to appropriate mathematical knowledge for their political process of cultural resistance, applying their own idea of collective research along the way.

This new story is very close to the idea of mutual interrogation proposed by Alangui (2010) because the involved groups were engaged in processes that created new knowledge, instead of just more effectively and more equitably reproducing existing knowledge. Based on this image I would like to propose Fig. 6.2 as a first metaphor:

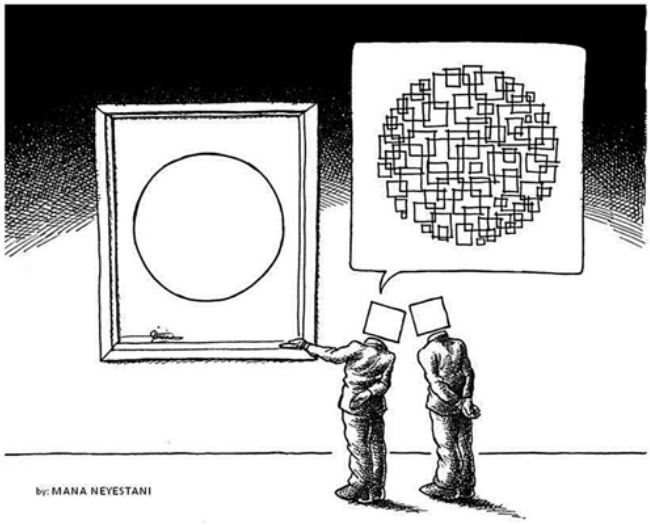


Fig. 6.2 "Square Heads 10" by Mana Neyestani

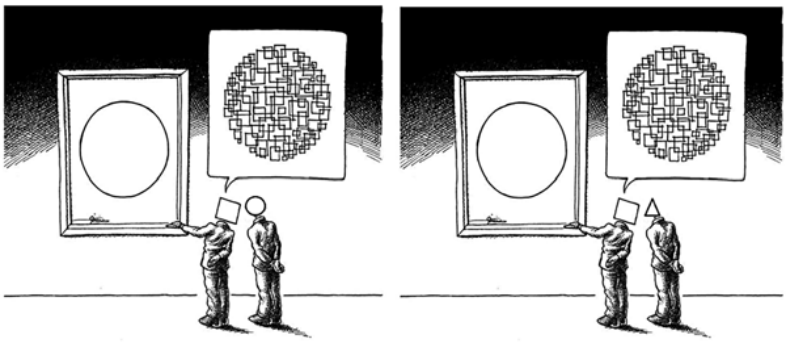


Fig. 6.3 Illustration by Aldo Parra-Sanchez, inspired in Square Heads

The effort to describe the other's practices in one's own terms demands a rearrangement of the own knowledge. Such reorganization implies the creation of *hybrid knowledge* which expands the "core" of the discipline and the culture. Those hybrids are particular, localized, and multiple at the same time. The conceptual movement from identification to creation demands that research projects in ethnomathematics become political processes of negotiation of meaning where differences are exposed and fostered, like in the imagined conversation of Fig. 6.2 or like in these other possible encounters presented in Fig. 6.3.

The situations proposed in Fig. 6.3 highlight the fact that the intended hybrid knowledge is by definition an attempt to communicate with others and requiring their response. It is the aftermath of a translation process that from the beginning knew the impossibility to achieve a final consensus. Indeed, the translation process was undertaken due to such impossibility. Success is not the result but a process of mutual adjustments that may never end.

…And Political…

Once the posture to consider interactions as barters is adopted, some of the critique developed on ethnomathematics can be employed as resource for the research field, in order to cast a new light on knowledge as a changeable social and historical practice. The self-referential definition of mathematics, provided by Lins (2004) entails an invitation to challenge authoritarian efforts, since what is assumed as mathematics cannot be predetermined. Every particular appropriation of a concept trough practice expands the concept's limits, transforming its meaning with the unavoidable presence of social life. By using a non-colonial perspective, one that understands power and knowledge as mutually constituted, it is no longer acceptable to be passive to the arguments of authority. Those appropriations of mathematical knowledge can be seen as acts of sovereignty and resistance against domination. For the case of indigenous communities Brayboy (2008, p. 342) asserts:

> Indigenous peoples engage in survivance through survival and resistance, and we are talking back. More than simply talking back, however, we are moving forward, claiming spaces and demanding acknowledgment of sovereignty that has existed since time immemorial. (…) If we as scholars are to consider the connection between Indigenous knowledge and sovereignty, then we must realize that our knowledge systems serve as a place of power and a source of continuance of our groups.

This idea of "talking back" reflects a bold trend among oppressed and marginalized communities understanding and assuming their agency in the struggles for power, not only at the concrete level of material needs, but also in an ideological dimension, breaking with the self-reinforcing cycle of hegemony-power-knowledge that Fasheh (1990, p. 24) explained:

> Hegemony does not simply provide knowledge; rather, it substitutes one kind of knowledge for another, in the context of a power relationship. Power in this sense, is almost defined by what is excluded. (…) To recognize my mother's activity as math was for me to recognize that education and knowledge are not only about facts but also about the inner logic of society, both within itself and in relation to outside forces (…) Hegemony is characterized not only by what it includes, but also by what it excludes: by what it renders marginal, deems inferior, and makes invisible.

This contention allows us to understand how political is the ethnomathematical activity involving and enhancing knowledge systems that have been dismissed by Eurocentric domination. As far as power and knowledge are imbricated, the struggle of marginalized groups and cultures to make their knowledge survive in time and space, emerges as a political action of resistance.

> Meaningful education, or community education, thus reclaims people's lives, their sense of self-worth, and their ways of thinking from the hegemonic structures, and facilitates their ability to articulate what they do and think about in order to provide a foundation for autonomous action. (Fasheh, 1990, p. 26)

It is fair to say that these ideas about the interaction between academic disciplines and nonacademic knowledge are not entirely new. Barton addressed something similar, when he conceived ethnomathematics as "a process of the social construction of knowledge at a cultural level" (Barton, 1996, p. 216) and claimed:

> Ethnomathematics does create a bridge between mathematics and the ideas (and concepts and practices) of other cultures. Part of an ethnomathematical study will elucidate why those other ideas are regarded as mathematical, and therefore why they might be of interest to mathematicians. Such a study creates the possibility both of mathematics providing a new perspective on the concepts or practices for those within the other culture, and of mathematicians gaining a new perspective on, (and possibly new material for), their own subject. (Barton, 1996)

…Approach…

I intend to add a second metaphor for these ideas, reworking the initial image of mathematics and culture as sets, and observing that it is possible to establish relations between the sets (see Fig. 6.4), instead of studying their intersection.

The first metaphor was that of an aesthetical interpretation, which highlights the presence of members (*knowledge-holders*) of different communities working together to produce, negotiate and interpret meanings. Complementary, this second metaphor of mathematical relation stresses the rise of those meanings, as new types of knowledge that do not belong to any established cultural (or mathematical) tradition.

These resultant meanings are plural and they do not unify, merge or melt worldviews; they perform a connection instead. Like any mathematical relation relating two domains without being part of them, those new meanings belong to a third domain and have another regime. Therefore, they cannot be treated within any essentialist approach because they are hybrids, anomalies, resulting in a process of interaction.

The mathematical diagram of Fig. 6.4 resulted from developing the analogy of culture and mathematics as sets: what could their elements be? How does ethnomathematics operate within that image? Following the intersection approach corresponding to Barton's Venn diagrams, there is only one possible intersection between two sets. Each member of a set is examined with the principle of excluded third: It must belong or must not belong to the other set. Alternatively, in the second metaphor there are multiple possible relations between sets; a relation is defined as a bundle of associations among the elements of two sets. One element in a set can be associated with (i.e., translated as) another element in the other set, associated with more than one element, or even associated with no element. If a connection seems "unsatisfactory," another relation is chosen, i.e., another bundle of associations is built. The relations are "customizable" while the intersection is not.

It is noticeable that the change from intersections to relations is not a small one at all. Such change entails an entirely different role for an ethnomathematics researcher. In the intersection approach, the researcher behaves as a detective looking for, uncovering, and trying to prove facts based on evidence. Researcher pursues a factual truth (timeless and univocal), which requires *proof*. In contrast, within the relational approach, the ethnomathematician acts like an artist: creating, proposing, and performing interactions; researcher tries to make sense through translations of meanings. The truth that the researcher aims at is a poetical one (ephemeral and polysemic) that deserves to be *experienced*.

This displacement does not intend to replace one image by another two, which simply describe better the same thing. It is rather an invitation to change the objects

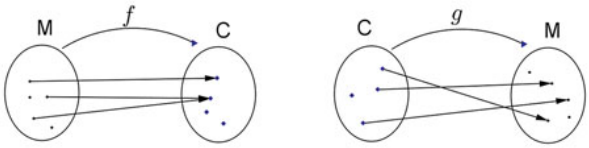


Fig. 6.4 *f* and *g* represent possible ethnomathematical research projects, comprised of multiple associations among objects of both sides (mathematics and culture); Therefore *f* and *g* are members of E* (*M* mathematics, *C* culture, *E** ethnomathematics as a research program)

of study and the ways in which practitioners can develop the field. For instance, a central role is proposed for the awareness and political intentionality of making conceptual connections. Consequently, a requirement to involve different voices gains preeminence to make possible the links between systems of knowledge. It is also critical to notice that collective processes of sense-making can be considered educational rather than merely curricular or school-bounded.

To conclude this section I want to make two comments about this new approach; First, I think it as an original idea, not because it is a novelty, but because it can be traced back to the origin of ethnomathematics. D'Ambrosio (1985, p. 47, emphasis added) stated in his breakthrough paper:

> We are collecting examples and data on the practices of culturally differentiated groups which *are identifiable* as mathematical practices, hence ethnomathematics, *and trying to link* these practices into a pattern of reasoning, a mode of thought.

Second, I emphasize the importance of grounding conceptual images in empirical research. Theoretical standpoints shape and condition the empirical research, by establishing what is thinkable, what deserves to be studied and what procedures of study can be applied. Theories are particular arrangements of concepts, agents, relations, hierarchies, and taxonomies. By using analogies, metaphors, diagrams, pictures, and other conceptual images, we can make explicit our understanding of those arrangements, and move through them to produce new insights.[1] Hence, the current effort to conceptualize ethnomathematics using diagrams and drawings is more than a flirtation with images.

At the same time, the theoretical reflection through images can have concrete consequences. For instance, if the intersection approach is assumed, and cultural practices are researched with the only purpose of finding examples of cultural differences, then ethnomathematics research would be reduced to collecting exotic cultural practices. If researchers, on the other hand, problematize the encounter of cultural practices with academic practices through a relational image, then ethnomathematics can be conceived as a barter, a kick-starter of new understandings and new types of hybrid knowledge that will defy hegemonic forms of power.

However, it is important to acknowledge that any conceptual image implies—by its nature—a particular delimitation of the problem. Such delimitation is both a reduction of complexity as well as a possibility to generate fruitful insights that cannot be achieved otherwise. Thus, I have presented several images, hoping that their overlap can minimize the reduction of complexity. The rationale of this strategy is explained further at the end of this essay.

[1] For instance, the term most used to describe the theoretical interplay is precisely the image of field, as a place where objects of a theory coexist and can be gathered in particular ways. Accordingly, researchers declare which is their position in the field and how they see the objects, when they state their personal understandings of the theory.

…Which Is Also Problematic!

The aim of ethnomathematics to expand the social understanding of concepts like mathematics or knowledge becomes clearer in this approach. It states that the object of study of ethnomathematics is no longer the intersection of mathematics with culture, but the multiple connections that people can build among them. Academic researchers, local communities and other stakeholders, attempt to appropriate and hybridize knowledge from various sources in their particular ways and for different purposes. Instead of previous and preestablished elements to be uncovered, we might consider the multiple and unexpected possibilities to be developed.

If ethnomathematics uses this approach to (dis)solve the "reflexivity problem" posed by the Millroy paradox, then a "symmetry problem" arises. Those negotiations and new meanings imply an active participation of different stakeholders in long-term processes. How can this participation be guaranteed? How can cultural groups be interested in establishing such processes? All the examples provided here were embedded in broad political projects of organized communities, which started earlier than these particular ethnomathematics research projects. These communities understand the relationship between mathematics and culture as a field crossed and impacted by politic and economic forces, where issues of identity, heritage and survival with dignity are at stake. Accordingly, these communities could conceive research in ethnomathematics as a relevant and strategic part of their political struggle for self-determination. However, not all groups have articulated their demands or concerns through an organizational strategy. How can dialogic processes be undertaken by non-organized groups? How can dialogue be established in the constrained space of a school system?

More considerations can be raised about the interactions. How much time does a process of dialogue require? It seems that it is not compatible with the increasingly short times imposed by universities for academic research, suggesting that interaction might need additional scenarios to be undertaken. What types of instances could they be? This challenge meets another natural tension, if dealing with dialogue and interaction, issues of mutuality and co-responsibility become central. Therefore, it is natural to ask how can "the others" do research on their own terms. Particularly, how can ethnomathematics be engaged in a political/epistemological level with other systems of knowledge, and in a way that respects self-determination and sovereignty? Although a first guess may be to follow the path of the mutual interrogation proposed by Alangui (2010), some issues remain regarding the risk to fail in a sort of tokenism. For instance, by prompting local communities to formulate questions and developments through a mimicry of traditional academic research not expressing their sincere concerns. New types of research results and new validation procedures emerge when the agency and insights of the local communities are a constitutive part of the theoretical tools deployed in research, and not simply the data to be collected.

Many of these questions cannot be answered directly or require an analysis that goes beyond the scope of this text. Nevertheless, they configure a promising landscape for the approach as long as they emphasize a condition of inherent uncertainty for every piece of ethnomathematical research. The vision of ethnomathematics as

a process of barter could restore the seminal impulse to reveal the historical and cultural grounding of mathematics and could also reinforce its critical position in the relationship among power and mathematical knowledge.

Ethnomathematics should not only observe the past but also look towards the future. The field should not be concerned only with a better understanding of current western knowledge, through the study of how others cultural groups build their knowledge. Rather, ethnomathematics can also engage those other groups in the change of the accepted body of knowledge. That "broader vision of knowledge" claimed by D'Ambrosio (2012) cannot be static, but dynamic. I think this is the central contribution of the barter that I am fostering.

This chapter brings forward an old idea that considers mathematics as central to ethnomathematics. Certainly, I could have deployed more or less the same argument by substituting "mathematics" for "western/academic knowledge." However, I preferred it this way because by including mathematics in comparisons, links and images, those connections turn to be heretical. If mathematics are left out of the focus, the field of ethnomathematics subsumes in a general discussion, losing its strongest feature: the problematization of mathematics education and mathematics epistemology.

In the recurrent discussion about the role of mathematics in ethnomathematics, this approach rejects dichotomies. It assumes that ethnomathematics must and can reject the accusation to superficially empower people because their culture would be "one step up, closer to the divine conventions of mathematics," as some critiques suggest, i.e., Pais (2013) or Rowlands and Carson (2002).

On the contrary, by assuming that mathematics is not above other knowledge, and that it has its specificities like any other knowledge, it is not a problem if we refer to some of its objects in yet unfamiliar ways to gain new insights. Paradoxically, to forbid or to avoid the use of mathematics embodies a new way to enthrone the discipline by reinforcing its supposed untouchable character. It is important to realize that ethnomathematics does not wish to break or discard mathematics; it wants to break its sacredness. Accordingly, within this approach, mathematics is demystified when bartering reveals it as being mundane, just as any other affair.

I attempt such demystification not only with the arguments deployed so far but also with the way in which these arguments were deployed. I decided to play with metaphors throughout this text, intending that the overlap of images constitute a hybrid. A connection can be seen as a translation, which is like an aesthetical interpretation of a painting, resembling a mathematical relation that works like a bunch of arrows, which suggests a connection, etc. I drew all these family resemblances on purpose. By combining a variety of analogies in a network with multiple agents and voices (see Fig. 6.5), ethnomathematics could contribute to change mathematics at an epistemological level.

In this chapter I argued:

> Now ethnomathematics has an "intersection approach" problem that can be changed to an interactional, hybrid and political approach, which is also problematic!

This content is a way to invite interplay with the dynamic condition of culture and mathematics, instead of merely watching it. As mathematics are shaped by our efforts, the main problem is not to perceive the difference, but what to do with it. How can we live with, and through, the difference?

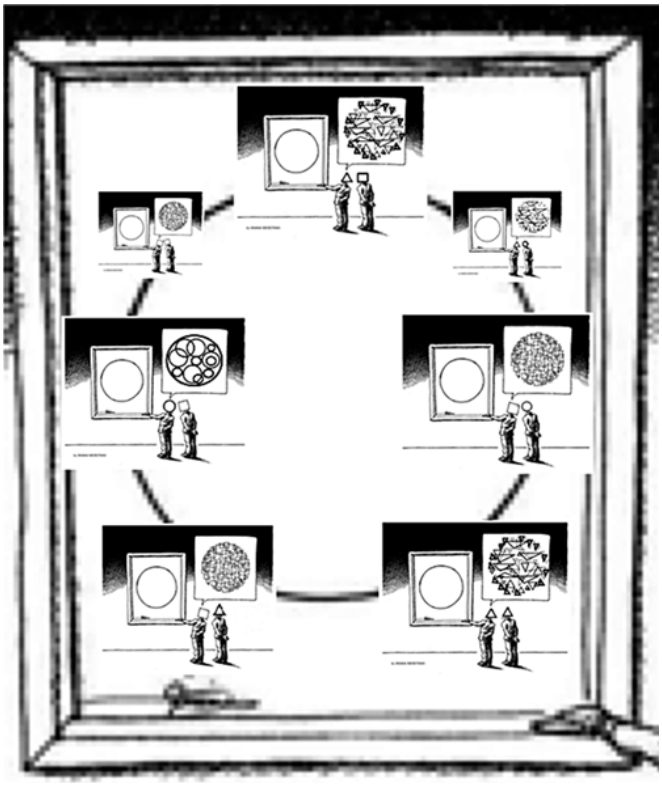


Fig. 6.5 A multiplicity of intentioned interpretations of a practice becomes the practice. Illustration by Aldo Parra-Sanchez, inspired in Square Heads

Acknowledgments A brief and early version of this chapter was published in the Proceeding of the Eighth International Mathematics Education and Society Conference, in 2015. The research leading to this chapter was funded by Departamento Administrativo de Ciencias, Tecnología e Innovación (COLCIENCIAS) and Aalborg University in Denmark. I would like to thank the members of the Science and Mathematics Education Research Group (SMERG) at Aalborg University for their comments on previous drafts of this chapter. This research is also part of the NordForsk Center of Excellence "JustEd."

References

Alangui, W. (2010). *Stone walls and water flows: Interrogating cultural practice and mathematics*. Auckland, New Zealand: University of Auckland.

Albertí Palmer, M. (2007). *Interpretación matemática situada de una práctica artesanal*. Barcelona: Universitat Autónoma de Barcelona.

Ascher, M., & Ascher, R. (1986). Ethnomathematics. *History of Science, 24*, 125–144.

Barton, B. (1996). Making sense of ethnomathematics: Ethnomathematics is making sense. *Educational Studies in Mathematics, 31*(1–2), 201–233.

Barton, B. (2008). *The language of mathematics: Telling mathematical tales (Mathematics Education Library)*. New York: Springer.

Bishop, A. J. (1994). Cultural conflicts in mathematics education: Developing a research agenda. *For the Learning of Mathematics, 14*(2), 15–18.

Brayboy, B. M. J. (2008). Yakkity yak" and "talking back": An examination of sites of survivance in Indigenous knowledge. In M. Villegas, S. Neugebauer, & K. Venegas (Eds.), *Indigenous knowledge and education: Sites of struggle, strength, and survivance* (pp. 339–346). Cambridge, MA: Harvard Education Press.

Caicedo, N., Guegia, G., Parra, A., Guegia, A., Guegia, C., Calambas, L., et al. (2009). *Matemáticas en el mundo Nasa*. Bogotá, Colombia: CIIIT.

Cauty, A. (2001). Matemática y Lenguajes: Como seguir siendo amerindio y aprender la matemáticas que necesitara? In A. L. G. Zapata (Ed.), *Pluriculturalidad y aprendizaje de la matemática en américa latina: Experiencias y desafíos* (pp. 49–87). Madrid: Ediciones Morata.

Chambers, I. (1996). Signs of silence, lines of listening. In I. Chambers & L. Curti (Eds.), *The post-colonial question: Common skies, divided horizons* (pp. 47–62). London: Routledge.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44–48.

D'Ambrosio, U. (1993). Etnomatemática: Um Programa. *Educação Matemática em Revista: Revista da Sociedade Brasileira de Educação Matemática–SBEM, 1*(1), 5–11.

D'Ambrosio, U. (2001). *Etnomatemática: elo entre as tradições e a modernidade*. Belo Horizonte: Autêntica Editora.

D'Ambrosio, U. (2006). *Ethnomathematics. Link between traditions and modernity*. Rotterdam: Sense Publishers.

D'Ambrosio, U. (2012). The Program Ethnomathematics: the theoretical basis and dynamics of cultural encounters *Cosmopolis. A Journal of Cosmopolitics/Revue de cosmopolitique, 3–4*, 13–41.

Eglash, R. (2000). Anthropological perspectives on ethnomathematics. In H. Selin & U. D'Ambrosio (Eds.), *Mathematics Across Cultures* (pp. 13–22). Dordrecht: Springer.

Fasheh, M. (1990). Community education: To reclaim and transform what has been made invisible. *Harvard Educational Review, 60*(1), 19–35.

Ferreira, E. S. (1994). A importância do conhecimento etnomatemático indígena na escola dos não-índios. *Em Aberto, 14*(62), 89–95.

Furuto, L. H. L. (2014). Pacific ethnomathematics: Pedagogy and practices in mathematics education. *Teaching Mathematics and Its Applications, 33*(2), 110–121. doi:10.1093/teamat/hru009.

Gerdes, P. (2007). *Etnomatemática—Reflexões sobre Matemática e Diversidade Cultura*. Ribeirão: Edições Húmus.

Knijnik, G. (1996). *Exclusão e resistência: educação matemática e legitimidade cultural*. Porto Alegre, Brazil: Artes Médicas.

Knijnik, G. (2012). Differentially positioned language games: Ethnomathematics from a philosophical perspective. *Educational Studies in Mathematics, 80*(1–2), 87–100. doi:10.1007/s10649-012-9396-8.

Knijnik, G., Wanderer, F., Giongo, I., & Duarte, C. G. (2012). *Etnomatemática em movimento*. Belo Horizonte: Autêntica.

Lins, R. C. (2004). Matemática, monstros, significados e educação matemática. In M. A. V. Bicudo & M. C. O. Borba (Eds.), *Educação matemática: pesquisa em movimento* (pp. 92–120). São Paulo: Cortez.

Lizcano, E. (2002). Las matemáticas de la tribu europea: un estudio de caso. In *II International Congress on Ethnomathematics, Ouro Preto, Brasil* (pp. 5–8).

Meaney, T., Trinick, T., & Fairhall, U. (2011). *Collaborating to meet language challenges in indigenous mathematics classrooms (Mathematics Education Library* (Vol. 52)). New York: Springer.

Miarka, R., & Bicudo, M. A. V. (2012). Matemática e/na/ou Etnomatemática? *Revista Latinoamericana de Etnomatemática: Perspectivas Socioculturales de la Educación Matemática, 5*(1), 149–158.

Millroy, W. L. (1992). An ethnographic study of the mathematical ideas of a group of carpenters. *Journal for Research in Mathematics Education. Monograph, 5*. doi: 10.2307/749904.

NASGEM. *Journal of Mathematics and Culture*. Retrieved October 23, from http://nasgem.rpi.edu/pl/journal-mathematics-culture.

Pais, A. (2011). Criticisms and contradictions of ethnomathematics. *Educational Studies in Mathematics, 76*(2), 209–230. doi:10.1007/s10649-010-9289-7.

Pais, A. (2012). A investigação em etnomatemática e os limites da cultura. *Reflexão e Ação, 20*(2), 32–48.

Pais, A. (2013). Ethnomathematics and the limits of culture. *For the Learning of Mathematics, 33*(3), 2–6.

Rohrer, A. V., & Schubring, G. (2013). The interdisciplinarity of ethnomathematics: Challenges of ethnomathematics to mathematics and its education. *Revista Latinoamericana de Etnomatemática: Perspectivas Socioculturales de la Educación Matemática, 6*(3), 78–87.

Rowlands, S., & Carson, R. (2002). Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? A critical review of ethnomathematics. *Educational Studies in Mathematics, 50*(1), 79–102. doi:10.1023/A:1020532926983.

Selin, H., & D'Ambrosio, U. (2001). *Mathematics across cultures: The history of non-Western mathematics* (Vol. 2). Dordrecht: Springer.

Townsend, S. (2012). Minga: The communal work tradition of Bolivia. *Americas, 64*(3), 14–17.

Vithal, R., & Skovsmose, O. (1997). The end of innocence: A critique of "ethnomathematics". *Educational Studies in Mathematics, 34*(2), 131–157. doi:10.1023/A:1002971922833.

Woolgar, S. (1988). *Knowledge and reflexivity: New frontiers in the sociology of knowledge*. London: Sage.

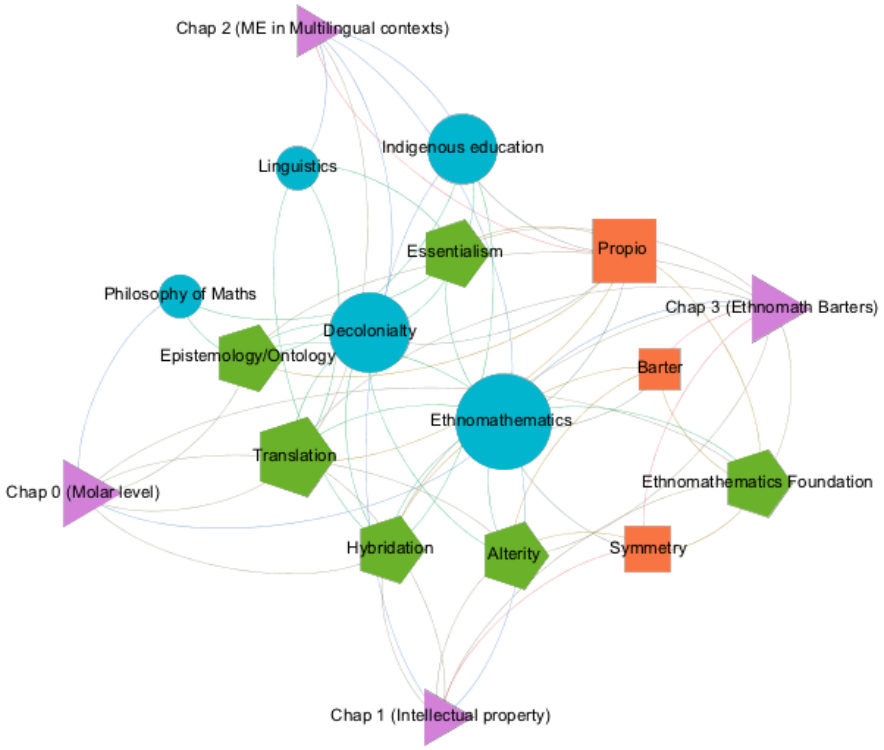


Fig. 3.1: Conceptual entanglement 3. Triangles represent chapters, circles represent areas of study, pentagons represent topics of interest, and squares represent new conceptual tools.

Chapter 4

Multilingualism in indigenous mathematics education: an epistemic matter

Aldo Parra
Tony Trinick

The paper has been published in
Mathematics Education Research Journal (2017).
https://doi.org/10.1007/s13394-017-0231-5
Print ISBN 1033 – 2170

The layout has been revised.

Math Ed Res J
https://doi.org/10.1007/s13394-017-0231-5

ORIGINAL ARTICLE

Multilingualism in indigenous mathematics education: an epistemic matter

Aldo Parra[1] · **Tony Trinick**[2]

Received: 17 July 2017 / Revised: 30 November 2017 / Accepted: 5 December 2017

Abstract An investigation into an aspect of indigenous education provides the opportunity to forefront an epistemological discussion about mathematical knowledge. This paper analyses indigenous peoples' educational experiences in Colombia and Aotearoa/New Zealand of mathematics education, focusing on, among other things, sociolinguistic issues such as language planning. In these experiences, researchers, teachers and local communities, working together, elaborated their respective languages to create a corpus of lexicon that has enabled the teaching of Western mathematics. An analysis using decolonial theory is made, showing how this corpus development works to enable the teaching of [Western] mathematics resulted in investigations into culture, language and mathematics that revealed an interplay among knowledge and power. Such analysis raises issues about the epistemology of mathematics and the politics of knowledge, analogous with current discussions on multilingualism in mathematics education and in ethnomathematics. The paper concludes that mathematics educators can explore and take advantage of the sociolinguistic and epistemological issues that arise when an indigenous language is elaborated in a short period of time in comparison to other languages which have been developed incrementally over hundreds of years and thus much more difficult to critique.

Keywords Multilingualism · Decolonial theory · Indigenous education · Ethnomathematics · Language planning

✉ Aldo Parra
aldo@plan.aau.dk

[1] Department of Planning, Aalborg University, Aalborg, Denmark

[2] Te Puna Wananga, Faculty of Education and Social Work, The University of Auckland, Auckland, New Zealand

Published online: 21 December 2017

Introduction

Much of the research into indigenous students in mathematics education has focused on students' mathematics performance. This is because the success or otherwise of mathematics programs for schools around the world has long been determined by student achievement, and in many countries, students from marginalised and/or indigenous communities have generally fared the worst. Indigenous student performance in mathematics is often manifested in low performances in standardised tests and other forms of external examinations, and the low numbers of indigenous students participating in mathematics-related tertiary level courses. As noted by Anthony and Walshaw (2007), much of the earlier research into this issue attributed indigenous students' failure to characteristics rooted in their cultures and communities, for example, an ambivalence towards learning mathematics and a lack of resilience within academic study. To explain these differences, a cultural deficit model was created that generally positioned indigenous students as being "the other", because of their low academic achievement (including in mathematics) and cultural differences. However, during the 1980s and 1990s, several critiques of, and resistance to, the deficit-based interpretation of indigenous students' schooling performance emerged, highlighting the Eurocentric, (neo)colonial prejudices embedded in such an interpretation (Bishop 1990; Meaney et al. 2008; Joseph 1987, 1991).

Research has also emerged that focuses on indigenous groups' endeavours to elaborate their own indigenous languages to teach and learn mathematics within the frame of wider language revitalisation efforts (Barton et al. 1998; Kimura and Counceller 2009). Drawing on cases of the Nasa people in Colombia and Māori people in Aotearoa/New Zealand (NZ), this paper examines the approaches of these indigenous groups to mathematics education that challenge aspects of conventional educational theory and existing assumptions about cultural and linguistic rights. We also discuss how perceptions of linguistic and cultural diversity in education around the world have changed in response to a range of social, political and linguistic factors. More recently, a paradigm of recognition and honouring of the linguistic rights of different communities has emerged, which has supported many groups to elaborate their languages to teach subjects at various levels of schooling (May 2012), including mathematics (Barwell 2012). Concomitant with the emergence of this new paradigm in sociolinguistics, ethnomathematics, a relatively new idea in mathematics education, has theorised cultural diversity as a positive (D'Ambrosio 1985) rather than a negative condition, and has acknowledged the role of culture in mathematics education as one of its cornerstones (Bishop 1991). While these much more positive views of indigenous and multilingual attempts are to be celebrated, there still lingers a number of unresolved tensions associated with issues of colonialism and power, which will be discussed in this paper.

Thus, our research journey examining the intertwined terrain of sociolinguistics and ethnomathematics is theoretically driven by decolonial theory. Decoloniality studies show how imperial power is still being exercised well after colonialism, in an interconnected matrix of power called *coloniality*, which entangles hierarchies of "sexual, political, epistemic, economic, spiritual, linguistic and racial forms of domination and exploitation where the racial/ethnic hierarchy of the European/non-European divide transversally reconfigures all of the other global power structures" (Grosfoguel 2011,

p. 11). This theoretical framework gathers and refines critiques of colonial power relationships, raised previously by anticolonial (Dei and Kempf 2006) and postcolonial studies (Young 2003; Harding 2008).

Decoloniality also aims at transforming the referred matrix of power, through a strong effort to recognise and strengthen alternative ways of theorisation produced by the oppressed side of the colonial difference, in this case, the indigenous population. Decoloniality problematises how knowledge is being built across time and space, by focusing on questions such as "Who and when, why and where is constructing knowledges(sic)?" (Mignolo 1999). Writers drawing on decoloniality have also formulated concepts such as "coloniality of power" that relate to the practices and legacies of European colonialism in social orders and forms of knowledge that live on in contemporary societies (Quijano 2000). The coloniality of power identifies the racial, political, linguistic and social hierarchical orders imposed by European colonialism that prescribed value to certain peoples and their knowledge, while disenfranchising others (Mignolo 2000).

Those interests connect decolonial struggles with the realm of epistemology, which we understand here as a theory of knowledge that studies how humans access knowledge (by discovery, creation or other ways) and the multiple forms in which knowledge is transmitted and transformed; epistemology also studies the procedures through which cultures accept something as valid and proper knowledge. When applied to the specificity of mathematical knowledge, epistemology has been recognised as a central concern for mathematics education (Ernest 1991; Hersh 1997; Sierpinska and Lerman 1996). Thus, we will examine epistemological issues associated with advancing multilingualism and indigenous languages in colonised contexts. We argue that the challenge of elaborating an indigenous language to teach mathematics is not merely a technical process of creating a lexicon that should be treated instrumentally. Rather, it is a challenge in which the politics of knowledge is at stake, since it emerges from social and cultural tensions about knowledge, and any linguistic elaboration produces new tensions.

We contend that mathematics educators and researchers should pay attention to sociolinguistic and epistemological issues when an indigenous language development for schooling is being developed, particularly so if the language is endangered. A decolonial stance can illuminate the issues, inquiring into such things as how mathematics is created, taught and learned most effectively in indigenous education contexts, and opening up a new range of possibilities of research and practice.

To provide a background to this paper, we first examine literature to highlight relevant trends and tensions to do with language diversity that help us articulate our argument within a critique of the coloniality of power in postcolonial times. Second, we present a historical contextualisation of the sociolinguistic situation of Māori, the indigenous people of Aotearoa/NZ, and Nasa, an indigenous people of Colombia, who are attempting to save their indigenous language(s). Third, we report language planning initiatives related to mathematical education that have been undertaken by these two indigenous groups. Fourth, we analyse these initiatives using decolonial theory to highlight some linguistic and epistemological tensions. The paper concludes by exploring the potentialities of these considerations regarding epistemology with the aim of validating the educational practices valued in indigenous environments rather than those values embedded in the coloniality of power (e.g. student achievement in mathematics).

Language as a (problem/resource/right): the impact on indigenous education

While there is no universally accepted definition of "indigenous", there are common characteristics among indigenous peoples. We use the following characteristics to define indigenous for the purposes of this paper. These include indigenous people as distinct populations relative to the dominant postcolonial culture of their country. They are often minority populations and have (or had) their own language (often endangered), cultures and traditions influenced by living relationships with their ancestral homelands. They have (or had) their own land and territory, to which they are tied in myriad ways (Trinick 2015).

These indigenous people and groups from around the world and throughout their histories have practised some forms of mathematics in their everyday cultures to ensure their survival (Trinick 2015). Concomitantly, they have developed, within their own languages, registers to describe and perform these mathematical activities. However, the current matrix of entangled hierarchies, established since colonial times (Grosfoguel 2011), has fostered a systematic denial and elimination of these knowledges and activities. This can be evidenced by the fact that implicit (e.g. beliefs about cultural knowledge) and explicit (e.g. language policies of instruction for schooling) approaches in many indigenous contexts around the world have privileged the coloniser. Such policies have been part of a political ideology to assimilate indigenous peoples to the coloniser's language and culture (Parra et al. 2016; Trinick 2015).

Consequently, for many decades indigenous language use has been restricted to a few non-formal schooling domains, which lamentably has impinged on the ability of the language users to incrementally develop their languages in areas such as mathematics. Cumulatively, various linguistic and educational policies have often led (explicitly or implicitly) to language endangerment or, in some cases, language extinction (Krauss 1992). In some contexts, indigenous language use has been permitted in various levels of education, as a result of the indigenous people's often prolonged struggle against linguistic imperialism. However, indigenous language use in higher education has been frequently restricted to areas of knowledge such as the humanities, and has precluded disciplines such as mathematics, as was the situation for the Māori language up until the 1980s (Trinick 2015) and is the situation nowadays for the Nasa language. In these contexts, the content and language used to teach and learn mathematics belong to the oppressor side of the colonial difference, generally Western European (hence, the title Western maths), thus enacting the epistemic hierarchy identified by Mignolo (2000) in which the Eurocentric system of knowledge assigned itself superiority over the traditional knowledge of indigenous conquered people or denial of its existence altogether (Grosfoguel 2011; Quijano 2000).

In the 1960s, language planning (LP) emerged as a subdomain in sociolinguistics to solve language problems in new, developing and postcolonial nations (Ricento 2000). Initially, LP research was considered the work of technical experts who mastered efficient and objective standardisation techniques (Nekvapil 2006). As Ricento (2000) suggested, LP was initially developed by Westernised sociolinguists who assumed that linguistic diversity impeded national development and consequently that linguistic homogeneity was necessary to achieve national unity, modernisation and Westernisation.

Although LP was launched as a new subdomain in the 1960s, it resembled previous initiatives of linguistic hegemony. For example, as noted by Trinick (2015):

> One of the imperatives that underpinned the 1867 Native Schools Act in New Zealand, which decreed that English should be the only language used in the education of children, was based on similar linguistic hegemonies of "rationalising" languages so as to select a national language for the purposes of nation building (Simon 1998). (p. 26)

Despite its origins in the positivist traditions, where multilingualism was considered a problem, LP research has evolved since the mid-1980s, with the development of two alternative orientations for considering language diversity in LP—"language-as-resource" and "language-as-right" (Ruiz 1984).

Language-as-resource is considered pragmatic, since it conceives language diversity as a resource for development, rather than an impediment, and highlights the benefits of linguistic diversity. Those who favour this perspective promote the bridge-building value of language in terms of its economic, social and cultural roles (Harrison 2008). It is not surprising then that this approach has been very well received in mathematics education research, promoting the development of linguistic diversity as a resource to provide mathematical learning opportunities (Planas 2014), to foster mathematical participation (Planas and Civil 2013) or to promote students' identification with (school) mathematics (Davis and Williams 2009, p. 137).

However, language-as-resource discourses have been contested in sociolinguistics for their bonds to nationalistic projects that celebrate linguistic diversity only in as much as they focus on its "projected relative value in a particular sector of economic or military activity rather than on locally determined interests of the language minority communities themselves" (Ricento 2005, p. 363). Research into the application of economic models in linguistic policies has found serious limitations in their aim to promote linguistic diversity, due to their scarce "recognition of intrinsic non-quantifiable resources associated with languages, including psychological, cultural, affiliational, aesthetic, and historical aspects, among others" (Ricento 2005, p. 362). Such omissions belong to the classic repertoire of colonial political relationships, and because of that, we do not endorse their use in mathematics education for indigenous peoples.

In contrast, since the 1990s, several researchers, such as Skutnabb-Kangas (2000), Phillipson (1992) and May (2001, 2005), have introduced a language-as-right orientation to LP, by drawing on human rights frameworks. These rights primarily pertain to the right of individuals to identify with their mother tongue and the right of minority groups to maintain their languages. A rights discourse has similarly been applied to issues such as the protection of indigenous languages (see Skutnabb-Kangas 2008 for discussion on the draft Universal Declaration of Linguistic Rights). For language rights approaches, processes of establishing a minority/majority language hierarchisation are neither natural nor merely linguistic. Instead, they are historically, socially and politically located, and particularly "deeply imbued in wider (unequal) power relationships" (May 2012, p. 2). For that reason, the language rights paradigm argues that speakers of minority languages such as Māori or Nasa should have the same rights and protections that speakers of majority languages enjoy (Skutnabb-Kangas 2000). However, the language rights paradigm

does not necessarily address the epistemological considerations associated with the unequal power relationships—this issue will be considered in this paper.

Treasuring language diversity: indigenous contexts

This section contextualises the linguistic and educational struggles of two very different marginalised indigenous groups who have had to elaborate their languages to teach mathematics—the Māori in Aotearoa/NZ and the Nasa in Colombia. Both groups share a common goal: to maintain their indigenous language and culture, notwithstanding the ongoing pressure from policies that continue to privilege the language and culture(s) of the old colonial empires (English and Spanish, respectively). A key component of their struggle is the desire to teach and learn in their own indigenous language, including mathematics. This is somewhat of a paradox considering the role Western mathematics has had in suppressing indigenous cultures (Bishop 1990; Gerdes 1985). However, both the Nasa and Māori are cognisant of this issue. We examine some initiatives in mathematics education developed by these two indigenous nations that reveal how a concern about epistemology arose when language was considered, especially in the creation of a lexicon. We argue that Māori and Nasa people have enacted a decolonising attitude in their LP initiatives—to what degree is open to question.

Māori language loss: an account of socio-political forces

The Māori language (called *te reo Māori*), an Eastern Polynesian language, is the only indigenous language of Aotearoa/NZ. Like its sister languages throughout Polynesia, the Māori language was confronted with European languages and cultures in the late eighteenth and early nineteenth centuries (Harlow 2005). At the time the first missionaries and settlers arrived, Māori had a robust system for educating their children to ensure the survival of their communities in Aotearoa/NZ (Riini and Riini 1993). After 1840, with more and more European settlers arriving and a British colony being created, European forms of government and missionary schooling were established. Simon (1998) argued that the hegemonic function of the missionary schools in the early 1800s was to provide a formalised context to assimilate Māori communities into European beliefs, attitudes and practices, with the intent to "civilise" the Māori population.

By the turn of the twentieth century, the state had adopted an even harder line towards the assimilation of Māori, introducing overt policies to ban Māori language speaking even on school grounds (the use of the Māori language was later completely excluded in schools), and children were, in some cases, physically punished for speaking Māori right up until the 1960s (Simon and Smith 2001). The imposition of the English language on the local language corresponds to a hegemonic model that can be traced to ideologies and beliefs that influenced Eurocentric education at the time. McWhorter (2003) argued that "urgencies of capitalism require governments to exact as much work and allegiance from their populations as possible, and the imposition of a single language has traditionally been seen as critical to this goal" (p. 261). Aotearoa/NZ was an emerging country during this early colonisation period, and English linguistic hegemony was also utilised as a means to build a nation-state, and to gain

political power and control from Māori (May 2012). As a result, the English language gained prestige, quickly taking over as a language of wider communication and as the dominant language of Aotearoa/NZ, while the Māori language was still the predominant language spoken by Māori people in the rural communities (as opposed to the schools) (Benton 1991).

However, the health of the Māori language was significantly compromised in the 1950s, when Māori people shifted from socially isolated Māori-speaking rural communities into English-language-dominated urban areas, and into English-language-only schooling systems and workplaces (Spolsky 2005) that were structured under clear assimilationist education policies. Consequently, by 1960, it was estimated that only 26% of Māori people spoke *te reo Māori* (May 2012), and by the 1970s, the Māori language was recognised as an endangered language (Fishman 1991), threatened with possible extinction (Spolsky 2003). It was in such a context of rapid and significant language loss that the Māori community initiated Māori-medium education in Aotearoa/NZ in the 1980s (May and Hill 2005). The following section examines the development of initiatives to revitalise the Māori language through Māori-medium schooling.

Māori language gain: Māori-medium schooling

For a doctoral thesis, Trinick (2015) interviewed three of the key developers involved in the elaboration of the Māori language to create a lexicon and mathematics register. In this section, we will draw on their voices and views already collected in the thesis. Additionally, we will draw on the seminal work of Barton et al. (1998). We will compare and contrast this development work with the experience of the Nasa people from Colombia.

In Aotearoa/NZ, the effort to teach all subjects bilingually in the modern era began in rural areas in the late 1970s and early 1980s with the bottom-up demands of teachers, schools and communities (Benton 1981). This created an urgent need for Māori language mathematics vocabulary, resulting in, as Trinick (2015) says, "an ad hoc coining of words by teachers and elders, using whatever word creation strategy was available to each group including transliterations (giving the maths word a Māori phonology and spelling)" (p. 151). As one interviewee noted:

> This mode of classroom imperative, rather than any formal language planning approach, has formed the basis for much of the early development of the *pāngarau* (mathematics) corpus of terms. (Informant 3 in Trinick 2015, p. 152)

This form of unplanned LP was characterised by teachers actively creating terms to meet their lexical needs, not by any formal committee (Barton et al. 1998). Trinick (2015) states, "By the early 1990s, the proliferation of localised mathematics word lists from the regions and the publication of different word lists raised educational and Māori language change issues on a national scale" (p. 144). By this time, Māori-medium schools had become state-funded, and thus state agencies, including the Māori Language Commission and their agents, assumed responsibility for lexicon creation.

> I was asked to collate the terms in current use in schools, and to identify the gaps and to present the terms to the Language Commission for discussion. We had no theoretical base to guide the creation of terms and no linguistic brief, and I think we were aware of it. I organised a number of meetings around the country with people I knew who were involved in teaching mathematics in bilingual classes. (…) We wrote a report to the Language Commission and said "Here's what we have found, now what?" This led to a meeting between Language Commission and us to discuss the (many different) terms. (Informant 3 in Trinick 2015, p. 144)

The state agencies responsible for language planning and education, along with a group of mathematics educators (including one of the authors of this paper), agreed to standardise a set of principles governing the creation of new terms (Trinick 2015).

> We set up five principles: *pāngarau* (mathematics) terms should be consistent with each other, terms to be as short as possible, words should sound correct to a native speaker, usage to be grammatically correct that was done at one of Language Commission meetings, probably the first one. (Informant 3 in Trinick 2015, p. 145).

New problems and tensions emerged with the process of standardising the terms. Trinick (2015) identified some of these related to the insufficiency of the mere lexical expansion in the development of a mathematics register. Other tensions appeared with the importance given by teachers to language acquisition, sometimes at the expense of the mathematical learning. And last but not least, there were tensions to do with language purism and authenticity. We will explain these tensions in more detail in the next section.

Authentic, elaborated and language purity: building a pragmatic chimera

The development of new terminology, especially the issue of lexication following the indigenous roots approach versus the borrowing (transliterating) approach, can be emotive and contentious. For example, for over 100 years *meiha*, a term borrowed from English, was used in the Māori language for measure. *Meiha* was purged from the vernacular in the 1990s and replaced with *ine*, the traditional Māori term for measure which had long fallen out of use (Barton et al. 1998). While on one level Māori-medium schools may not have universally supported standardisation, they have supported, for the most part, the strategy to create terms from native stock, such as *ine* for measure. The desire to purge Māori of the phonologically adapted terms during the corpus development process was related to the language purism standpoint and linked to epistemological issues, for example, how language is created. In Aotearoa/NZ, purism became important during the time of lexical modernisation and language elaboration in the modern era, providing criteria for the choice of a new lexicon and codification (Harlow 1993). Purism is frequently considered from sociolinguistic rather than epistemological perspectives. This paper considers both dimensions. Linguistically, purism has been defined in several different ways, but often involves cultural fundamentalism and a return to (or a search for) linguistic authenticity (Annamalai 1979).

Annamalai (1979) suggested that the situation of accepting non-native vocabulary, as was the case with Māori, during a certain period (pre-1980s in the case of Aotearoa/NZ), then rejecting it later (post-1980s), arises under certain social conditions, and is often the corollary to policy development of an LP agency. When the Māori Language Commission was set up in 1987, as an outcome of the Māori Language Act, this is precisely what materialised (Trinick 2015). According to Trinick (2015):

> Pragmatically, it would have been more efficient to transliterate (borrow) the terms from English, as was the practice previously. However, led primarily by the Māori Language Commission and mathematics educators, instead of borrowing terms, it was decided to create new terms by changing the meaning and or function of existing Māori language terms—all of which takes time and negotiation. Corpus development work in the 1980s was concerned to either resurrect Māori terms that had fallen out of use, or expand the meaning of Māori terms in use in everyday language by giving them an explicit mathematical meaning. (p. 224)

The development of a Māori language lexicon and curriculum in the 1990s was highly politicised and contested. Many involved in the language revitalisation movement, particularly in schooling, believed the goal of Māori-medium schooling was not just to revitalise the language but also to explicitly promote Māori knowledge. Drawing on native words and not using borrowed words from English partly satisfied the epistemological demand from those involved in the growing Māori-medium schooling movement to also resurrect Māori knowledge.

The attitude of some mainstream English-medium teachers, by far the dominant group in mathematics and mathematics education in Aotearoa/NZ, was also a factor that influenced the decision to resurrect and then give traditional terms a mathematical meaning. During the early corpus development work in the 1980s, there was a perception in English mathematics education in Aotearoa/NZ that it was simply not possible to develop a legitimate Māori-medium mathematics register—a challenge familiar to other indigenous groups adapting their languages to teach Western mathematics (see Schindler and Davison 1985 for issues to do with adapting the Navajo language). So, the adoption of purist beliefs was in some way a reaction to the lack of support of a range of groups—or their outright hostility—towards efforts to elaborate Māori and the thought of teaching mathematics in the medium of Māori. A key factor, therefore, behind the ideologies of purism that underpin the elaboration of the mathematics language is primarily non-linguistic and more concerned with the status of the Māori language and people's attitudes to its status, during the period of codification of the linguistic norms (Harlow 1993):

> To preserve the language as a living means of communication entails preserving it in opposition to and distinct from English. If in order to fit Māori for the Māori world, we borrow from English, this looks like a sort of admission of defeat, an admission that in fact Māori is not capable of handling new ideas and topics with its own resources. (p. 129)

While enabling the teaching of mathematics in the Māori language was a cause for celebration, and progress towards empowerment, there was a view that like the Trojan horse it had the potential to "destroy the conceptual basis of Māori culture and language" (Barton et al. 1995, p. 1). The issue of language and cultural change as an outcome of the elaboration of the Māori language to teach schooling subjects such as mathematics was a contentious topic of debate in the wider Māori community throughout the 1980s and 1990s (Harlow 1993) and continues to be so to this day, particularly in regard to cultural knowledge. It is not surprising, therefore, to hear that the informants involved in the development of the *pāngarau* lexicon were confronted with similar concerns:

> Looking back, I think of some of the tensions highlighted in the meetings with the Language Commission was that some of the words that were coined were leading to the encouragement of incorrect use of grammar outside of the maths class. (Informant 1 in Trinick 2015, p. 165)

One of the ironies facing the revival of the Māori language is that to save the language; it would seem the language has had to become more like English. Despite the emphasis on "authentic" vocabulary, developing Western mathematics in the indigenous language can still become an unwitting vehicle for transforming the phonology, syntax and semantics of a language. While research has shown that the pronunciation of *te reo Māori* has been greatly influenced by English (Keegan et al. 2008), there is growing evidence to suggest that the syntax of *te reo Māori* generally is also changing to be more like English (Harlow 2001). A type of hybrid language is evolving—a grammatical mixture of Māori and English. Research is now showing that in many cases the way *te reo Māori* is being articulated in the mathematics classroom approximates the structure of English (Trinick 2015). For example, for the syntactic structure for algorithms, students and teachers follow the English-language structure rather than a more traditional Māori grammatical form (Trinick 2015). For Māori, the concern is linked to the issue of cultural assimilation and maintenance of ethnic identity. For many Māori, *te reo Māori* is an important symbol of their ethnic identity: "it's what makes us Māori distinct from European!"

The series of works on mathematics education in the Māori language discussed here have involved more than the creation of a lexicon but also new styles of meaning (e.g. representations and graphs), new ways to tell stories (e.g. mathematical number stories), new syntactic structures (e.g. for multiplication) and combining existing grammatical features into new combinations (Barton et al. 1998). All these forms of expression are integral parts of what Halliday (1978) calls a mathematical register. The development of these forms demands long-term social interactions, and it is a process with inevitable linguistic and cultural consequences for the language communities and for the language itself. We are not suggesting that mathematics education initiatives on language can alone be responsible for potential negative effects on the local language use and culture; there are many other possible variables to consider, such as the influence of other curriculum areas and the fact that, in many homes, parents are second-language learners of the Māori language and/or monolingual speakers of English.

Nasa language: constantly threatened

The Nasa language is one of the 65 languages officially recognised as indigenous in Colombia, being one of the three languages spoken by more than 50,000 people (Landaburu 2004). This multilingual linguistic context is significantly different from the language context in Aotearoa/NZ, where the Māori language is the only indigenous language. The Nasa language is considered by linguists to be an independent language, without a known linguistic family. Although the first contact of Nasa people with the Spanish Crown can be traced back to 1562, it was not until 1755 that the Nasa language was first identified and studied by a European missionary. Nasa people have been resisting, with relative success, the interference of foreign state powers for more than 250 years.

As Triana y Antorveza (1997) relates, during the second half of the sixteenth century, the Spanish Crown was inconsistent in the languages used for catechising, allowing some experiences in indigenous languages and others in Spanish, until the late seventeenth century, when the latter option was chosen. Because of this period of ambivalence, indigenous languages began their phase of sliding into extinction. Independence from the Spanish Crown in the nineteenth century did not result in better conditions for the indigenous people or their languages in Colombia. Coloniality of power continued working in the republican period, this time under the control of the Catholic Church, which was commissioned to conduct the savages to the "civilised life" and turn them into citizens. Unsurprisingly, knowing Spanish was considered central to the process of cultural subjugation (Pineda-Camacho 1997).

The Colombian government signed several agreements with the Catholic Church, renewing the mandate to manage the education in isolated areas. Because of the disdain towards indigenous languages that the Catholic Church showed, those renewals implied a continuity of the language extinction. In 1962, however, the government made a seismic change in their linguistic policy, through an agreement with SIL (formerly known as the Summer Institute of Linguistics). Although SIL maintained the same missionary idea of "civilising" indigenous people and had equal disrespect towards indigenous heritage and culture, their works produced an extraordinary advance in the recognition of Colombian linguistic diversity. Up until 1980, they worked with 38 different indigenous peoples, creating alphabets and writings, and training Colombian missionary linguists. It was during this period, in 1964, that SIL begun to work with the Nasa people. According to Pachon (1997), just 2 years later they proposed a way of writing in the Nasa language, taking the gospel of Mark as an example. During the 1970s, SIL researchers produced booklets to teach how to write and read according to their proposal for the Nasa language. The quality and intentions of their proposals were much criticised by indigenous people and linguists in subsequent years, as Rojas Curieux (1998) and Pineda-Camacho (1997) summarised.

A grass-rooted defence of the Nasa language

The 1970s mark a turn in the linguistic dynamics in Colombia because of the rise of indigenous rights' movements that gained visibility and forced the nation-state to include some changes to linguistic and educational policy. In the Cauca state, the Nasa people and six other indigenous peoples launched the Regional Indigenous Council of

the Cauca (CRIC).[1] With this new organisation operating, the linguistic imperialism ideology lost some strength. The CRIC and the Catholic Church developed their own alphabet proposals, promoting an active use through their channels of communication. This gave the Nasa language a central role in the ideological debate. Despite the diversity of alphabet proposals during the 1980s, Nasa language use was still decreasing. Pachon (1997) identified three main factors contributing to diminishing the vitality of the Nasa language: school, miscegenation[2] and trade of labour.

In 1991, a new constitution was signed acknowledging Colombia as a multicultural and multilingual nation; indigenous languages were included as official in the indigenous territories. This was the beginning of a new era for linguistic policies, which now acknowledged the linguistic rights of indigenous communities. This acknowledgement resulted in more investment and interest in linguistic studies. In 1998, an alphabet unification process ended successfully (Rojas Curieux 2005), naming the language as *Nasayuwe* and opening a stage of joint efforts to implement the new proposal. In the next decade, (UNICEF and FUNPROEIB 2009) estimated that half of the Nasa population of 138,000 would be monolingual in *Nasayuwe*, one-quarter bilingual in *Nasayuwe* and Spanish, and one-quarter monolingual in Spanish. UNESCO studies considered the Nasa language endangered in 2001 (Wurm 2001) and definitely endangered in 2010 (Moseley 2010).

Within the frame of this historical account of *Nasayuwe* vitality, it is important to provide a contextualisation, although basic and incomplete, of the mathematics education initiatives involving LP that were developed by the Nasa people. The first documented effort of the Nasa people to work with mathematics education as it is considered in contemporary times is a set of booklets for preschool children, created in the early 1980s, that tried to minimise the use of Spanish and introduce some basic words in their language, *Nasayuwe* (CRIC Consejo Regional Indígena del Cauca 2004). After that, the program of indigenous bilingual education researched their own number system and launched a proposal of a neonumeration, coping with the international 10-base system (Cauty and Ramos 1990). However, the proposal was not adopted by the people, because they did not find any practical advantages in it for daily life. Another factor that might have contributed to the low acceptance was that, at that time, few people understood the written grammar of the *Nasayuwe* language (alphabet and grammar were not unified then).

Learning from Nasa community research experiences

We report on two experiences in mathematics education from a Nasa study (Caicedo et al. 2009, 2012), which have not been analysed from this perspective before now. Since one of the authors (Parra) participated as a research collaborator, the following description is presented in the first person. It is important to note that these experiences

[1] This council presented an agenda of negotiation with the Colombian authorities, including "Defense of Indigenous history, language, culture and traditions", and "Training of Indigenous teachers to educate according to the indigenous situation and in their mother tongue" (CRIC Consejo Regional Indígena del Cauca 2004, p. 28, our translation).

[2] Pachon (1997) reported that when non-indigenous settlers arrived to some indigenous areas and married Indigenous women, many often forbade their children to speak *Nasayuwe*, with the result that in some cases, children refused even to be considered Indigenous, preferring to be called *mestizos* (half-breeds).

are different from the Māori case because they did not occur in classrooms, but in the cultural space of assemblies, a well-known and legitimate space for collective knowledge construction and education in Nasa communities.

Experience 1

In the middle of a workshop organised by the CRIC within an indigenous Nasa settlement, we were trying to find or create a word in *Nasayuwe* that can be used as a translation of "unit of measure". I mentioned properties about that concept and provided some examples. In response, a team of indigenous researchers (all of them were bilingual in *Nasayuwe* and Spanish) explored and discussed possible words in their own language. When they reached a consensus on one term, I asked them if the meaning of that selected word included another property of measure unit, and they realised that the word did not have such connotations. Therefore, another round of discussions in *Nasayuwe* was made, and they changed the word to another: *kxteeçxah.* One of the researchers explained to me that both words have different original meanings (last term is the expression for "a bunch"), but they estimated that with the last word they could explain better to other Nasa the meaning of "unit of measure". I realised then that they had achieved not an equivalent word, but a suitable point of departure to recreate the discussion in the future with other indigenous people, grasping the concept in their own ways.

Experience 2

In another Nasa settlement, elders, parents and children were discussing the practices of localisation that Nasa people use often. As part of the exercise, I was asked to describe the way in which Western rationality understood the concept of space. I decided to mention the Cartesian space, and therefore, the assumption of a space that can be divided infinitely came up. That led me to describe the difference between continuous and discrete. Considering my audience, I tried to explain without using technical terms, but at the same time, in the most precise way possible. I saw some people's expressions and heard them make comments in *Nasayuwe* (I do not speak that language) that I (mis)interpreted as a signal that they had not understood anything. So, I decided to do analogies with nature and explained that finite is "as rocks in the surface of a river, among them there are no rocks"; in contrast, continuous is "like the water in the river". I was tempted to propose another image, but an elder stopped me and said, "Can we move to another topic, please? We are wasting time here, the idea that you are trying to explain is a basic one, we can grasp it in *Nasayuwe* as *nes*, that is a very well-known word." Thereafter, my colleagues explained that *nes* refers to the permanence in time and space, without breaks or end. Several entities can have *nes* as a property, for instance, the cycle of day and night, the territory, or even the spirit of the legendary leader and deity Juan Tama. Nasa people say that those entities accompany them everywhere they go. Probably such an image of continuity did not embrace all the mathematical properties that I was intending to express, but provided to the audience (me included) a deeper reflection about the Nasa worldview and the possibility of conceptualising mathematical knowledge in a different way than the school system does.

Reading the experiences with decolonial lenses

As noted earlier, the process of terminology creation for marginalised indigenous languages is not simply a technical process. The above episodes illustrate how sometimes a concept (e.g. unit of measure) cannot be translated simply with a word: the translation process is often underpinned by linguistic ideologies and beliefs of what mathematical knowledge is, that influence the strategies used to elaborate indigenous languages to teach subjects such as mathematics. Even if a word in a native language is found for a concept (e.g. *nes* for continuity, as was the case for Nasa), that single term can emerge only within a process of interaction, which provides additional non-mathematical meanings and expands the scope of connotations for the concept. Every technique used for creating terminology (for strategies in Māori-medium mathematics see Barton et al. 1998; Meaney et al. 2011) implies a negotiation that includes the influence of beliefs and perceptions, and different types of knowledge, and the status and interests of key stakeholder groups. To embrace such complexity, a round of circumlocutions and paraphrases is always needed to solve discrepancies and achieve a consensus, even if temporary. Considering that those negotiations are accomplished over several meetings (changing time and space) and agreements can be reconsidered every time, we identify the negotiations of meaning as part of a process. Furthermore, it is a sort of process that happens through the language and culture, is educational in its results and is political in its dynamics of struggle among relations of power.

We can see in this interplay between discrepancies and consensus the fluidity and indeterminacy of languages (Makoni 2011) that is considered central to communication within integrationist perspectives in sociolinguistics (Harris 2009). The process of interaction deployed by Nasa and Māori people share the idea that "meanings of words are indeterminate and individuals can initiate changes and create their own idiosyncratic meanings" (Makoni 2011, p. 684). By trying to solve the misunderstandings of a purposely confused situation, Nasa and Māori produced not only words or lexicons, but also several contexts of use in which words can have expanded meanings.

The related episodes on developments of a mathematical register and creation of contexts of use also give support to our standpoint in mathematics education research on multilingualism, by illustrating how it is possible to overcome the orientation of language-as-right, and at the same time, problematise the orientation of language-as-resource.

Within the approach of language-as-resource, Planas (2012) uses the idea of orchestration attempting to combine in a productive way the uses of the student's first language (aiming for conceptual understanding) with the uses of the student's second language (looking for the gain of procedural skills, and for the "access to social goods such as higher education and qualified employment") (p. 337). However, it is natural to ask how such a combination can be done. How can teachers pass from one language to another without a discrepancy, without a moment in which terms of several languages are considered simultaneously? Those moments of language-switching require diverse clarifications, explanations in both languages and partial agreements. In those moments, we find an opportunity to understand deeply language, culture and mathematics.

The approach of language-as-resource has been used widely in the subfield of mathematics education research devoted to multilingual environments, as can be seen in Barwell et al. (2016) and Planas and Civil (2013). This use can be located within the

frame of the improvement of school achievement. However, it is important to note that often those environments are multilingual but not necessarily indigenous. The achievement referred to is with respect to a national system, in which migrants or indigenous students are struggling to fit in. This is not a type of achievement within an indigenous-medium education such as *Kura Kaupapa* for the Māori or *Educación Propia* for the Nasa. Language-as-resource generally leads to one kind of improvement that is to do with the mathematical school content, or political concerns to do with classroom participation and disempowerment. The idea of language as a creative space or as an identity marker is given minimal consideration in this approach, whereas in such indigenous experiences discussed here for Māori and Nasa, language, identity and rights are of central concern. For these reasons, Ricento (2005) criticised the language-as-resource approach for its tendency to understate the socio-political dimensions of language acquisition and revitalisation. We support and expand such a critique, by observing the instrumental conception of mathematics education that this approach entails.

We consider that mathematics education is not related just to the learning of procedures and use of concepts (aiming to develop technical skills to solve problems, as a sort of expertise) but it also encompasses a knowledge about what mathematical thinking is, what kinds of problems can (or cannot) be addressed with mathematics, and what is meant by the kind of treatment that mathematics gives to problems. Therefore, mathematics education developments such as those experienced by the Nasa and Māori also involve epistemology of mathematics considerations. Thus, in the elaboration of the Nasa and Māori languages, beliefs and understandings about mathematical knowledge are in themselves an important factor in the process of developing mathematical competence. When Nasa people argue that "the unit of measure" cannot be simply translated but can be explained in *Nasayuwe* using the word *kxteeçxah*, they show a critical distance to a Eurocentric epistemology of mathematics and they reveal that a particular conceptual understanding of measurement practice was achieved. So, Nasa people are not falling into the *banality of the expertise* denounced by Skovsmose (2016) when he discussed the politics of meaning.

In the same vein of problematising the epistemology, we can reflect on the episode about continuity. Instead of labelling it as a wrong and bizarre misinterpretation of the essence of real numbers, we can observe the episode as a displacement of what can count as mathematics. Why can the cosmogonic notion of *nes* resonate with continuity? What sorts of things do they have in common? To reflect on these questions is to reflect on epistemology. A culturally intended relocation of mathematics has happened, in which the subject is being connected with something different than shape and number. We are twisting the ontological question of *is this a mathematical object?* to a performative one of *what if we operate with this?*

The change of question also entails a strong change in the *locus of enunciation*, allowing the participation of several stakeholders that were previously discarded as invalid interlocutors by practices of research permeated by colonial relationships. As was visible in the examples presented, the usual colonial division of labour was altered. Indigenous peoples were not acting as informants, and nor were researchers alien individuals for the communities. These examples overcome the ethnographical idea *of being there, reflecting and writing here*, where the "there" means the space of communities and the "here" the academic sphere, outside communities. As a

characteristic feature of decolonial studies, our examples enter into the problematisation of "Who and when, why and where is constructing knowledges(sic)?" (Mignolo 1999) by acknowledging indigenous people as co-researchers, and indigenous communities as the main validators of the research results.

To consider more interlocutors implies more scenarios: although the episodes presented gravitated towards school practices, we highlight the fact that discussions about discrepancies in meaning had occurred in different spaces—in the assemblies among elders and researchers, in classrooms with teachers and students, and at home, when children request help and further explanations from their parents. This fact allows us to stress how involved epistemology is in the type of work that linguistic diversity can create because it shows how spaces other than the school can be permeated to discuss what mathematical knowledge is.

In this reading of the experiences we have focused on how a change in the locus of enunciation implies different places and actors, an alternative conception of mathematics knowledge and a different role for mathematics education. Such a reading does not attempt to merely verify the well-known incommensurability among worldviews. Instead, the goal is to stress the contemporary character of languages, cultures and types of knowledge, and their power to generate explanations and interpretations that aim to respond to the drive for survival and transcendence that groups have. This goal is inspired by a decolonial claim: non-European cultures and societies are coeval with European cultures and societies (Grosfoguel 2011; Quijano 2000). Often, non-mainstream cultures are considered capable of coping with the past, but not with the present or the future. In contrast, this approach is non-essentialist when it assumes language can accommodate and change.

Moving from linguistic instrumentalisation to epistemology

We want to raise some concerns about epistemology (of mathematics) that arise when cultural and linguistic diversity are addressed by teaching and learning in indigenous languages, particularly for minority indigenous languages that have been systematically excluded from schooling for many decades, thus disrupting the incremental development that has occurred in languages such as English and Spanish. It is important to note that the process of elaborating indigenous languages to teach mathematical content is not without considerable challenge. Considering that cultural, linguistic and pedagogical aspects have been thoroughly commented on elsewhere (see Bishop 1990; Barton et al. 1995, 1998; Meaney et al. 2011), we focus primarily on the issues of epistemology implicated in the elaboration of indigenous languages to teach school mathematics.

An integral part of our contention is that language can be a resource for achieving an enriched understanding of mathematics, that is, the mathematics conceived in a plural sense, such as in ethnomathematics theory (D'Ambrosio 1985; Gerdes 1991; Barton 1996), namely as a historical and cultural construct, with associated limits and possibilities. Therefore, language diversity would come to be used in a critical and reflexive way, rather than a utilitarian one. Some research (Caicedo et al. 2012; Meaney et al. 2011) has tried not only to improve mathematics learning processes through language, but also to problematise the very concept of mathematics and its associated rationalities. Within such an approach, language reveals its ideological nature (Radford 2012) since

it conveys particular rationalities and subjectivities. This means that language can be useful for the cultural encounter because it can provide opportunities for the students not just for learning specific mathematical content, but also for questioning what mathematics is about and how diverse worldviews address problems in different ways.

Current discussions on multilingualism in mathematics education are focused mainly on the technical aspects, to achieve expertise in the mathematical content, without considering the associated cultural values and practices, nor to problematise mathematics as a cultural and historical production. Therefore, mathematics and its rationality are not questioned. As Barwell (2012) has noted, there is an assumption that mathematics is unified, above any language or culture, and consequently students "do not bring different mathematics, only different languages" (p. 319), which justifies strategies of *many voices for one (mathematical) goal* such as language-as-resource. This assumption is undoubtedly connected to the hierarchising of knowledge denounced by many critics of colonial thinking, such as Thésée (2006), who argues that "scientific knowledge is an epistemological tool, or weapon, used to develop, dominate, and shape minds. What has been the global impact of scientific knowledge on the different expressions of cultural knowledge?" (p. 25).

Sometimes mathematics is conceived of as a tool for solving problems, but what does it mean to have mastery of that tool? We suggest that it is not only a matter of the skilful use of the tool, but also of acquiring an understanding of the tool's nature, history, applicability and limitations. Barwell (2012) suggests that "greater recognition of the heteroglossia of human communication in mathematics classrooms would better reflect the lives and experiences of students and teachers" (p. 329). A question then arises as to how to deal with this multiplicity of voices. Radford (2012) commented on Barwell's suggestion, highlighting two main concepts (ideology and alterity) that move the discussion of linguistic diversity to the grounds of epistemology.

Drawing on Radford's (2012) contribution, a strategy of *many voices for many goals*, exemplified here with Māori and Nasa cases, could be claimed as taking a decolonising stance, with multi-directed outcomes and perspectives. Accordingly, languages and worldviews are recognised as capable of producing knowledge and learning in particular ways. Rather than looking for a perfect and finalised translation that maintains the (univocal and fixed) meaning, this different approach to multilingualism takes as its point of departure the impossibility of a successful translation, and focuses on the analysis of particularities, similarities and differences in meaning. It means to take advantage of the multiplicity and perceive the polysemy as something fruitful for language diversity, mathematical learning and cultural resistance.

Conclusion

To build our disquisition, we have delineated two different perspectives of multilingualism in mathematics education for indigenous populations. On the one hand, the goal is to improve indigenous students' performance in the usual mainstream mathematics education with its associated external assessments. On the other, the goal is to enhance indigenous languages and cultures, through teaching in indigenous languages and inclusion of indigenous cultural knowledge and worldviews in the mathematics curriculum. Although these two perspectives

have been considered unrelated, or even in opposition, we argue from a decolonial stance that they have the potential to be complementary rather than contradictory. Experiences such as the Māori and Nasa ones reported here illustrate that both perspectives can be developed jointly, as part of the same decolonial effort to overcome the dichotomy between antimodern fundamentalism and Eurocentric colonial imposition (Grosfoguel 2011).

When arguing for such complementarity, this paper contributes to politicising the research and practice of multilingualism in (mathematics) indigenous education in a different way from the usual appeal to empowerment, because it intends to make visible how within the multilingual situation the well-known relation between knowledge and power is operating, and questions whether such relations should continue being managed in a way that exercises a colonial hierarchisation, by privileging just one locus of enunciation (the Eurocentric one) about mathematics knowledge. In other words, multilingualism politics are at stake, not just because more learning opportunities for indigenous students can be provided, empowering this population to interact with the majoritarian society, but mainly because it unfolds a discussion about epistemology, inquiring which knowledge is being recognised as mathematical, and where, why, when and through which language.

In addition, such discussion about epistemology intersects LP politics, since indigenous languages can be released, as we saw in the Nasa and Māori cases, from the role of mere instruments to carry information; instead, their power to create words and expand meanings can be reinstated. This means re-establishing the vitality of indigenous languages, and abandoning conservational approaches to multilingualism whereby multilingualism was seen as a problem.

Looking for that re-establishment, teachers and educators could "deviate" from their mathematical content goals for the classroom and address cultural issues, reflecting about mathematics and language. Their teaching could be more holistic if they paid attention to those episodes of misunderstanding and adjustment in the sense-making process, unfolding a sort of translanguaging practice (García and Sylvan 2011). The mathematics teaching extends beyond mere proficiency in the subject and problematises mathematics itself. This can be done because language conflicts configure a proper scenario to discuss and reflect with the students about what mathematics knowledge is about.

So, our main concern is not about a discrepancy between mathematics and culture, but about how such a discrepancy is addressed in the classroom. We need to explore the extent to which those episodes are conceived as opportunities to be used proactively to gain conceptual understanding, rather than situations to be prevented, fixed or avoided as something wrong or negative—in brief, to enjoy the conflict instead of suffering with it.

To conclude our argument about the role that epistemology plays in multilingualism, we observe that our experiences with the Nasa and Māori people illustrate a set of tensions and possible responses, but these experiences are far from being exemplary solutions. We considered these cases more for the questions and dilemmas that they raise than for the specific answers they have provided. We stress this point because we understand that each indigenous context is different and demands particular considerations when exploring epistemological matters.

Acknowledgements The authors want to express their gratitude to Uenuku Fairhall, Paola Valero and the colleagues of Te Puna Wananga in the University of Auckland for their support in the development of these ideas.

References

Annamalai, E. (1979). Movement for linguistic purism: the case of Tamil. In E. Annamalai (Ed.), *Language movements in India* (pp. 35–59). Mysore: Central Institute of Indian Languages.

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/Pāngarau: best evidence synthesis*. Wellington: Ministry of Education.

Barton, B. (1996). Ethnomathematics: exploring cultural diversity in mathematics (Unpublished PhD thesis). University of Auckland, New Zealand.

Barton, B., Fairhall, U., & Trinick, T. (1995). He kōrero kupu tātai: word stories in Māori mathematics vocabulary development. In B. Barton & U. Fairhall (Eds.), *Mathematics in Māori education* (pp. 9–13). Auckland: Auckland College of Auckland.

Barton, B., Fairhall, U., & Trinick, T. (1998). Tikanga reo tatai: issues in the development of a Māori mathematics register. *For the Learning of Mathematics, 18*(1), 3–9.

Barwell, R. (2012). Heteroglossia in multilingual mathematics classrooms. In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education* (pp. 315–332). Berlin: Springer.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J. N., Planas, N., et al. (Eds.). (2016). *Mathematics education and language diversity*. Cham: Springer.

Benton, R. (1981). *The flight of the Amokura: oceanic languages and formal education in the South Pacific* (Vol. 63). Wellington: New Zealand Council for Educational Research.

Benton, R. (1991). The history and development of the Māori language. In G. McGregor, M. Williams, & R. Harlow (Eds.), *Dirty silence: aspects of language and literature in New Zealand* (pp. 187–199). Cambridge: Cambridge University Press.

Bishop, A. J. (1990). Western mathematics: the secret weapon of cultural imperialism. *Race & Class, 32*(2), 51–65.

Bishop, A. (1991). *Mathematics enculturation: a cultural perspective on mathematics education*. Dordrecht: Kluwer Academic.

Caicedo, N., Guegia, G., Parra, A., Guegia, A., Guegia, C., Calambas, L., et al. (2009). *Matemáticas en el mundo Nasa*. Bogotá: CIIIT.

Caicedo, N., Guegia, G., Parra, A., Guegia, A., Guegia, C., Calambas, L., et al. (2012). *Matemáticas en el mundo Nasa* (2nd ed.). Bogotá: CIIIT.

Cauty, A., & Ramos, A. (1990). Vigilancia etnocultural: el caso de la numeración tradicional Nasayuwe. *Boletín de Lingüística Aborigen, 2*, 3–15.

CRIC Consejo Regional Indígena del Cauca. (2004). *¿Qué pasaría si la escuela…? Treinta años de construcción de una educación propia*. Popayán: Author.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44–48.

Davis, P., & Williams, J. (2009). Hybridity of maths and peer talk: crazy maths. In L. Black, H. Mendick, & Y. Solomon (Eds.), *Mathematical relationships: identities and participation* (pp. 136–146). London: Routledge.

Dei, G. J. S., & Kempf, A. (2006). *Anti-colonialism and education* (Vol. 7). Rotterdam: Sense.

Ernest, P. (1991). *The philosophy of mathematics education*. London: RoutledgeFalmer.

Fishman, J. A. (1991). *Reversing language shift: theoretical and empirical foundations of assistance to threatened languages* (Vol. 76). Clevedon: Multilingual Matters.

García, O., & Sylvan, C. E. (2011). Pedagogies and practices in multilingual classrooms: singularities in pluralities. *Modern Language Journal, 95*(3), 385–400.

Gerdes, P. (1985). Conditions and strategies for emancipatory mathematics education in underdeveloped countries. *For the Learning of Mathematics, 5*(3), 15–20.

Gerdes, P. (1991). *Etnomatemática: Cultura, matemática, educação: colectânea de textos*. Maputo: Instituto Superior Pedagógico.

Grosfoguel, R. (2011). Decolonizing post-colonial studies and paradigms of political-economy: transmodernity, decolonial thinking, and global coloniality. *Transmodernity: Journal of Peripheral Cultural Production of the Luso-Hispanic World, 1*(1), 1–37.

Halliday, M. A. K. (1978). *Language as social semiotic*. Baltimore: University Park Press.

Harding, S. (2008). *Sciences from below: feminisms, postcolonialities, and modernities*. Durham: Duke University Press.

Harlow, R. (1993). A science and mathematics terminology for Maori. *SAMEpapers, 1993*, 124–137.

Harlow, R. (2001). *A Maori reference grammar*. Auckland: Pearson Education.

Harlow, R. (2005). Māori: introduction. In A. Bell, R. Harlow, & D. Starks (Eds.), *Languages of New Zealand*. Wellington: Victoria University Press.

Harris, R. (2009). *Rationality and the literate mind*. New York: Routledge.

Harrison, K. D. (2008). *When languages die: the extinction of the world's languages and the erosion of human knowledge*. Oxford: University Press.

Hersh, R. (1997). *What is mathematics really?* London: Jonathan Cape.

Joseph, G. G. (1987). Foundations of Eurocentrism in mathematics. *Race & Class, 28*(3), 13–28.

Joseph, G. G. (1991). *The crest of the peacock: non-European roots of mathematics*. London: IB Tauris.

Keegan, P., King, J., Harlow, R., Maclagan, M., & Watson, C. (2008). Ngā nekehanga o te whakahua i te reo Māori i roto i te rautau kua hipa nei. *AlterNative: An International Journal of Indigenous Peoples, 4*(2), 180–197.

Kimura, L., & Counceller, I. (2009). Indigenous new words creation: perspectives from Alaska and Hawai'i. In J. Reyher & L. Lockard (Eds.), *Indigenous language revitalisation: encouragement, guidance & lessons learned* (pp. 121–139). Flagstaff: Northern Arizona University.

Krauss, M. (1992). The world's languages in crisis. *Language, 68*(1), 4–10.

Landaburu, J. (2004). La situación de las lenguas indígenas de Colombia: prolegómenos para una política lingüística viable. *Amérique Latine: Histoire et Mémoire. Les Cahiers ALHIM, 10,* http://alhim.revues.org/125

Makoni, S. B. (2011). Sociolinguistics, colonial and postcolonial: an integrationist perspective. *Language Sciences, 33*(4), 680–688.

May, S. (2001). *Language and minority rights: ethnicity, nationalism and the politics of language*. London: Longman.

May, S. (2005). Language rights: moving the debate forward. *Journal of SocioLinguistics, 9*(3), 319–347.

May, S. (2012). *Language and minority rights: ethnicity, nationalism and the politics of language* (2nd ed.). New York: Routledge.

May, S., & Hill, R. (2005). Māori-medium education: current issues and challenges. *International Journal of Bilingual Education and Bilingualism, 8*(5), 377–403.

McWhorter, J. (2003). *The power of Babel: a natural history of language*. New York: Perennial.

Meaney, T., McMurchy-Pilkington, C., & Trinick, T. (2008). Mathematics education and Indigenous students. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W.-T. Seah, et al. (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 119–139). Rotterdam: Sense.

Meaney, T., Trinick, T., & Fairhall, U. (2011). *Collaborating to meet language challenges in indigenous mathematics classrooms* (Vol. 52). New York: Springer.

Mignolo, W. D. (1999). I am where I think: epistemology and the colonial difference. *Journal of Latin American Cultural Studies, 8*(2), 235–245.

Mignolo, W. (2000). *Local histories/global designs: coloniality, subaltern knowledges, and border thinking*. Princeton: Princeton University.

Moseley, C. (Ed.). (2010). *Atlas of the world's languages in danger* (3rd ed.). Paris: UNESCO.

Nekvapil, J. (2006). From language planning to language management. *Sociolinguistica, 20*, 92–104.

Pachon, X. (1997). El Nasa Yuwe, o la lucha por la supervivencia de una lengua dominada. In X. Pachon, F. Correa, & E. Benavides (Eds.), *Lenguas amerindias: condiciones sociolingüísticas en Colombia* (pp. 269–319). Santafe de Bogotá: Instituto Caro y Cuervo.

Parra, A., Mendes, J. R., Valero, P., & Ubillús, M. V. (2016). Mathematics education in multilingual contexts for the indigenous population in Latin America. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Plana, et al. (Eds.), *Mathematics education and language diversity* (pp. 67–84). Cham: Springer.

Phillipson, R. (1992). *Linguistic imperialism*. Oxford: Oxford University Press.

Pineda-Camacho, R. (1997). La política lingüistica en Colombia. In X. Pachon, F. Correa, & E. Benavides (Eds.), *Lenguas amerindias: Condiciones sociolingüísticas en Colombia* (pp. 155–176). Santafe de Bogotá: Instituto Caro y Cuervo.

Planas, N. (2012). Commentary on the chapter by Richard Barwell, "Heteroglossia in multilingual mathematics classrooms". In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education* (pp. 333–338). Heidelberg, Springer.

Planas, N. (2014). Hacia una noción situada de lengua para la educación matemática. *Revista Latinoamericana de Etnomatemática, 7*(2), 151–169.

Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: tensions in the bilingual mathematics classroom. *Mathematics Education Research Journal, 25*(3), 361–378. https://doi.org/10.1007/s13394-013-0075-6.

Quijano, A. (2000). Coloniality of power and Eurocentrism in Latin America. *International Sociology, 15*(2), 215–232.

Radford, L. (2012). Commentary on the chapter by Richard Barwell, "Heteroglossia in multilingual mathematics classrooms". In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education* (pp. 339–342). Heidelberg: Springer.

Ricento, T. (2000). Historical and theoretical perspectives in language policy and planning. *Journal of SocioLinguistics, 4*(2), 196–213.

Ricento, T. (2005). Problems with the "language-as-resource" discourse in the promotion of heritage languages in the USA. *Journal of SocioLinguistics, 9*(3), 348–368.

Riini, M., & Riini, S. (1993). Historical perspectives of Māori and mathematics. In *Pāngarau–Māori mathematics and education* (pp. 16–20). Wellington: Ministry of Māori Development.

Rojas Curieux, T. (1998). *La lengua paez, una visión de su gramática*. Bogotá: Editorial Ministerio de Cultura.

Rojas Curieux, T. (2005). *En la reflexión sobre lo oral y lo escrito: Educación escolar y práctica en pueblos indígenas. Cuadernos de trabajo, Jigra de Letras*. Popayán: Editorial Universidad del Cauca.

Ruiz, R. (1984). Orientations in language planning. *NABE Journal, 8*(2), 15–34.

Schindler, D. E., & Davison, D. M. (1985). Language, culture and the mathematics concepts of American Indian learners. *Journal of American Indian Education, 24*(3), 27–34.

Sierpinska, A., & Lerman, S. (1996). Epistemologies of mathematics and of mathematics education. In A. Bishop, M. A. K. Clements, C. Keitel-Kreidt, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 827–876). New York: Springer.

Simon, J. (1998). *Nga kura Maori: the native schools system 1867–1969*. Auckland: Auckland University Press.

Simon, J. A., & Smith, L. T. (2001). *A civilising mission? Perceptions and representations of the Native Schools system*. Auckland: Auckland University Press.

Skovsmose, O. (2016). Politics of meaning in mathematics education. *Philosophy of Mathematics Education Journal, 31*, 1–15.

Skutnabb-Kangas, T. (2000). *Linguistic genocide in education—or worldwide diversity and human rights?* Mahwah: Lawrence Erlbaum.

Skutnabb-Kangas, T. (2008). Human Rights and Language Policy in Education. In. M. Stephen & N. Hornberger (Eds.), *Language policy and political issues in education, Volume 1 of Encyclopedia of Language and Education, 2nd edition* (pp 107–119). New York: Springer.

Spolsky, B. (2003). Reassessing Maori regeneration. *Language in Society, 32*(4), 553–578.

Spolsky, B. (2005). Māori lost and regained. In A. Bell, R. Harlow, & D. Starks (Eds.), *Languages of New Zealand* (pp. 67–85). Wellington: Victoria University Press.

Thésée, G. (2006). A tool of massive erosion: scientific knowledge in the neo-colonial enterprise. In G. J. S. Dei & A. Kempf (Eds.), *Anti-colonialism and education: the politics of resistance* (Vol. 7, pp. 25–42). Rotterdam: Sense.

Triana y Antorveza, H. (1997). Factores políticos y sociales que contribuyeron a la desaparición de lenguas indígenas. In X. Pachon, F. Correa, & E. Benavides (Eds.), *Lenguas amerindias: Condiciones sociolingüísticas en Colombia* (pp. 85–153). Santafe de Bogotá: Instituto Caro y Cuervo.

Trinick, A. (2015). *Te reo Tātai: the development of a mathematics register for Māori-medium schooling* (Unpublished PhD thesis). University of Waikato, New Zealand.

UNICEF, & FUNPROEIB. (2009). *Atlas sociolingüístico de pueblos indígenas en América Latina*. Quito: FUNPROEIB Andes.

Wurm, S. A. (Ed.). (2001). *Atlas of the world's languages in danger of disappearing*. Paris: UNESCO.

Young, R. J. (2003). *Postcolonialism—a very short introduction*. Oxford: Oxford University Press.

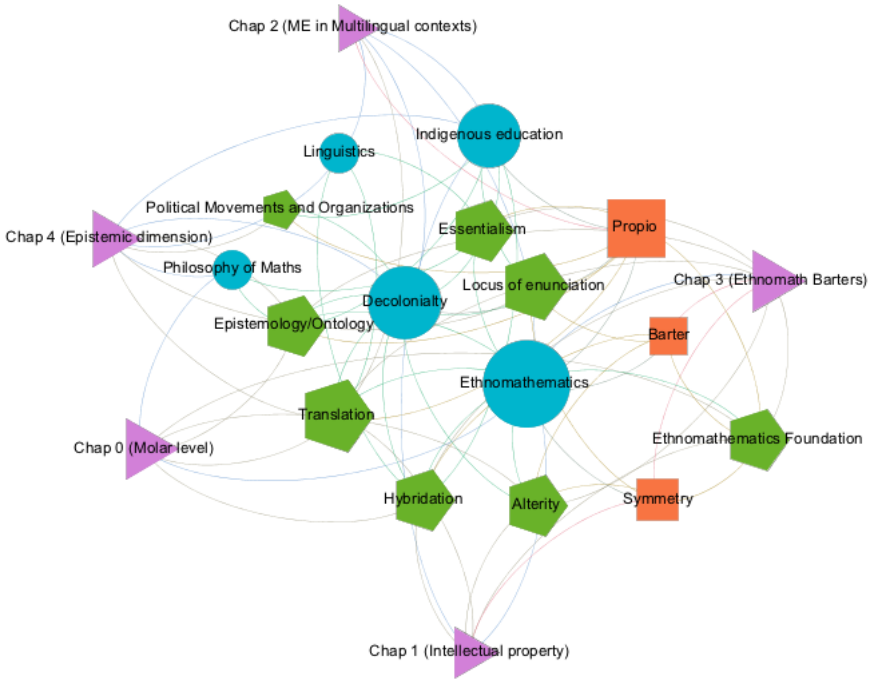


Fig. 4.1: Conceptual entanglement 4. Triangles represent chapters, circles represent areas of study, pentagons represent topics of interest, and squares represent new conceptual tools.

Chapter 5

Propio as a decolonizing tool for mathematics education

Abstract

The notion of *propio* used in indigenous *educación propia* guides a form of education articulated around a triple meaning: *propio* as pertinence/utility, as property/belonging, and as appropriation/transformation. This conceptualization results from a decolonial stance towards education as a central element in the political struggle for indigenous identity affirmation and self-determination. The examination of the notion through an educational experience with the Nasa Indigenous people in Colombia allows exploring the three meanings of *propio* and differentiating it from current existing approaches to differentiated indigenous education. Furthermore, the question of whether the notion of *propio* can be of relevance in non-indigenous contexts and of what are its contributions to political approaches in mathematics education are addressed.

5.1 Introduction

As education has increasingly become a key mechanism of social, economic and political in(ex)clusion, the predicaments of education for Indigenous populations have been intensely debated in both Indigenous communities and communities of scholars who work for and with the struggle of Indigenous self-determination (Dei and Kempf, 2006; Tattay, 2011). As a result, different proposals for Indigenous education and Indigenous mathematics education have

emerged, that challenge in different degrees the colonial desire of states to assimilate Indigenous people into the dominant nations. What is common to different proposals is a political reading of the effects of knowledge, school knowledge and school pedagogies in generating not only in(exclusion), but also assimilation or possible routes to identity affirmation and self-determination.

This paper explores the conceptualization of *propio*[1], a notion that emerged in Colombian Indigenous education to think about, characterize and purposefully manage the relationship between community, knowledge and action in face of the life and political struggles of an Indigenous group. The instantiation of *propio* in an educational project, *educación propia*, is explored as a tool that unfolds decolonizing standpoints in and for education. In particular, we focus on what such an idea may mean in the realm of mathematics education. We argue that *propio* and *educación propia* in mathematics resonates with the political turn in mathematics education (Gutiérrez, 2013; Valero, 2004). It allows to sharpen the critique of the epistemic superiority of Western, dominant mathematics in front of different forms of mathematical knowledge, and, at the same time, offers a performative capacity for engaging in transforming educational practices.

We start providing an account of the emergence of the *propio* concept as part of the political struggle of the Nasa Indigenous people in Colombia. We then illustrate the main features of the notion as it unfolded in a concrete educational practice involving mathematics. Resemblances and differences with other proposals of Indigenous mathematics education are discussed. Finally, we ponder the ways and the extent in which the notion can be unfolded in non-Indigenous settings, constituting a useful political tool for mathematics educators.

5.2 Traces of one Colombian Indigenous education

The Andes mountains, what today is recognized as South America, is the land called *Abya-yala*, "the land in full maturity". This was the name that several Indigenous tribes gave to their territory when the Spanish colonizers declared it to be the "the new world of the Americas". Abya-yala in its contemporary use signals the political resistance of Indigenous tribes in front of the now national —still colonizing— government forces. Particularly, since 1971, several Indigenous nations raised their voices against the ethnocide that they were suffering in the state of Cauca, a predominantly rural area at the south west

[1] *Propio* in Spanish is an adjective which is mainly used in the sense of belonging to someone, or owned. When qualifying the noun *educación* [education], the adjective changes to propia for grammatical reasons of agreement between noun and adjective. *Educación propia* mean literally "own education".

of Colombia (see Figure 5.1). They created the Regional Indigenous Council of Cauca [In Spanish, Consejo Regional Indígena del Cauca, from now on CRIC] to represent their people's interests towards the national government. This council presented a seven-point agenda of negotiation with the Colombian authorities, one of those points being the "defense of Indigenous history, language, culture and traditions", and another being the "training of Indigenous teachers to educate according to the Indigenous situation and in their mother tongue" (CRIC, 2004, p.28, our translation).

Fig. 5.1: Geopolitical division of Colombia. In red, the State of Cauca.

From the beginning of the struggle, education has been at the center of the resistance, and Indigenous people have tried to consolidate a specific and differentiated educational model. Gradually, during the last 40 years, they have forged the right to have Indigenous teachers, to teach bilingually, to introduce culturally relevant contents in the curriculum, and to create Indigenous teacher education programs. These changes have been the product of long debates with, protests against and proposals to the regional and national authorities, always aiming at gaining more autonomy in the processes of schooling. Nowadays, the Colombian national educational policy acknowledges the right of each Indigenous nation to have a "Differentiated Indigenous Educational System" (In Spanish, Sistema Educativo Indígena Propio, from now on SEIP). Recently, the Indigenous educational model has been called *educación propia*, which is the result of a long process of refinement, rearrangement, and differentiation

of several other educational models proposed before, such as ethno-education (in the 1970–80's) and bilingual intercultural education (in the 1990–00's)[2]. *Educación propia* can be understood as one stage in a continuous quest for autonomy, now guided and structured by the notion of *propio*.

In the negotiation process, internal debates have been carried out, about the necessity of the very existence of the *school* as an institution, and of its intended effects of subjectivation through the individualization and the universalization of the self, and the detachment from the community and from home. These are effects of power which alter in fundamental ways the inseparability of person, community and territory in the Indigenous worldview. The debates have resulted in an educational model that goes beyond the boundaries of a school and covers the whole community and the inhabited territory. *Educación propia* addressed the demand for school-based education imposed by the Colombian government. By re-structuring the "guidelines, foundations, approaches, methods, contents, ways of administration, assessment and control" (CRIC, 2008, our translation), the curriculum organisation allowed to structure school space in continuity and interrelationship with family and community spaces. *Educación propia* aims at "strengthening authority, autonomy, territory, self-esteem and cultural identity, to promote the knowing and valorization of knowledges and practices that can be own or appropriated, and to promote a critical understanding of intra- and extra-cultural conflicts, as well as a positive interpretation of cultural diversity" (CONTCEPI, 2013, p.20, our translation).

These ideas got concretized in a school that projects community interests, founded on culturally rooted ways to express identity, and on ancestral ways of teaching and learning. Such approach makes possible to transcend the focus on individuality and universality, and to conceive of the educational process as a collective and openly political engagement. The organisation and functioning of *educación propia*, expressed in a Communitarian Educational Project (from now on PEC), covers not only teachers and children of certain age within the school's building, but also comprises all community members as well as their territory. In other words, *educación propia* expresses an understanding and commitment toward education as a process of teaching-learning acquired "before birth and even after death" (Sichra, 2004, p. 65, our translation). The PEC promotes awareness of the subjectivation processes that shape and consolidate Indigenous identities. Furthermore, PECs are defined by each community in a *resguardo*[3], according to the community's traditions, capacities, interests

[2]For a short overview of Indigenous education in Latin America with respect to mathematics education see (Parra et al., 2016).

[3]*Resguardo* is an Indigenous reservation area created by the Spanish Crown in 16th century as the basic unit of Indigenous territory. It has been acknowledged by the republican governments in Colombia, and it remains as a constitutional right of Indigenous people.

and concerns. It is the responsibility of the *cabildo*[4] to guide the formulation and implementation of the PEC. Research and the production of knowledge are considered a connatural activity in the PEC, and they are devoted to address the predicaments that *resguardos* have to deal with. Teachers are often selected among community members, privileging political, linguistic, and cultural capacities over pedagogical or academic skills. Finally, a PEC is continuously assessed and refined by the community, through a communication process that transforms "the very notion of school, the ways in which community appropriates it, the teacher's roles within the community, and the role of education in an Indigenous society" (Tattay, 2011, p. 39, our translation).

Although there are documents that provide details on how indigenous education in Cauca has been structured, we provide an example of the principles of *educación propia* in action. We illustrate some of the characteristics mentioned above and connect them with mathematics education. It is important to note that this experience is just one among many other enactments of *educación propia*, and for that reason the case cannot be considered as a guide of what should/must be done. However, it serves the purpose of highlighting the connections that the notion of *propio* articulates.

5.3 Recognizing Lomitas: a fieldwork experience

This educational experience was conducted in 2011 (Parra, 2011) in the Village of Lomitas, belonging to the Resguardo of San Andres de Pisimbalá, in the region of Tierrradentro, the sacred land of the Nasa people. Lomitas is a village located at the top of a mountain, bordering with the urban settlement of the municipality of Inzá, a town located in a valley. The village is an excellent viewpoint of Inzá (see Figure 5.2), but the village and the town are separated by a river and two sloppy mountains, making the two places distant for more than one hour. Lomitas is only accessible on foot. This geographical condition of isolation and proximity made the village a strategic point in the military confrontation that Colombia suffered for 60 years. In 2010 the school building in the village was (ab)used by guerrilla troops to attack, from the distance, the police station at Inzá. This is just an example of the serious threats to sovereignty and self-determination that the Nasa people in Lomitas confront.

Lomitas school offers pre-school and primary school for students from 4 to 10 years-old. In 2011, it had three teachers in charge of two grades each. The teachers had different backgrounds: one had no experience at all neither teacher education, and he was in his first year of teaching. Another finished secondary

[4] *Cabildo* is the political and organisational council that rules the *resguardo*. Such political and administrative organisation allows some degree of autonomy and self-determination.

Fig. 5.2: Inzá seen from the school at Lomitas

school and had some short training as teacher. The last one had a university degree in ethno-education, which is a bachelor program for Indigenous teachers. The two latter had been working in Lomitas for seven years and have been re-elected by the community.

When Aldo arrived to the community to learn about mathematics education as part of *educación propia*, one of the teachers, called Diego Guegia, explained an idea that he had been considering with the aim of rescuing the memory of the community. The elders in the community had told him stories about some of the special places in the territory, involving mythical and cultural aspects. He noticed that his students were calling some of those places using names in Spanish, because they did not know the names in Nasayuwe, the Nasa language. He realized that the children did not know the stories either. He considered this to be an educational problem.

Diego began to explore ways to integrate such stories and the knowledge involved in them with the school activities. He, the other two teachers and Aldo exchanged ideas and decided to make a project of toponymy, involving seven distinctive places of Lomitas. The project consisted in making a collective research about those places, identifying their stories, and putting around each one an informative plaque for visitors and locals. Each plaque would contain name, history and other relevant information. The goal was to make a

demarcation and re-affirmation of the community's territorial property.

During the subsequent days, Diego explained the idea to the community and invited some elders to tell the stories and assess the project. They arranged to make a journey to visit the places with the students. One week after the first meeting, elders and authorities went to the school and guided a 4 hour walk through the village, telling the stories of the places which were important for diverse reasons: because there is a special plant used for cleaning and health, or a special historical event happened there, or the place is of spiritual and sacred significance. Stories were told in Nasayuwe; kids also asked questions in Nasayuwe. At the end of the journey, everybody ate together, with a lunch prepared by some parents that were waiting for the group to return to the school.

Next days in the school were used in organising the information. Kids wrote and drew each story. If further clarification was needed, the teacher complemented. Diego proposed to create a kind of footpath for visitors that connected the places and where information was provided about the place, the route and the resguardo. Kids recalled signalizations plaques that they had seen near the archaeological park of Tierradentro, and they wanted to do something similar. A new task emerged: the children had to establish what kind of information an informative plaque should contain. They identified the altitude, the distance to other places, a map and sentences as slogans about the place. Also, there should be a red point indicating where the reader is located. To do something similar, the kids had to obtain data, establish a design for the plaque, and decide which information they wanted to put on each one of them along the path. Therefore, students needed to measure the distance between the places.

How to measure became an issue, because the path is full of slopes and curves. It was not clear which instruments students could measure with. Diego and Aldo proposed to combine and contrast several measures. Students in groups should measure how much time it takes to go from one place to the next; one group proposed to measure the number of steps that a person takes. They realized that such measure unit is relative to each person, so they established a sort of protocol for the measurement by selecting the smaller kids and asking them to walk slowly and with regular steps. Diego suggested to use another instrument, a *guasca* that is a rope made with natural fiber, used by almost all Nasa people for different purposes. We prepared ropes of the same length (7 meters). To measure time, we had borrowed five mobile phones and used their chronometers. Kids learned how to use mobile phones on the same day, as these artifacts were not very common at that time because they were expensive and there is no good signal in the mountains where Lomitas is located.

Diego devoted one entire day to do the first measurements. Each team had five kids with different roles: one was the walker, one counted the steps of the first one, two measured the walking path with guascas, and the last one

took the time spent by the walker. Conflicts emerged soon: the number of steps got quickly quite high, and the child counting steps spent a lot of time saying the number and asking the walker to slow down so that he could count. But that was protested by the ones taking the time because slowing down the pace affected their measure. The children took the measures several times, going back and forth, and found them similar in guascas, but very different in time. Many groups explained the difference saying that it is not the same when a person goes down-hill than when climbing up. Some of them believed that the distance is not the same; while others understood that speed could explain the differences. When converting the number of steps to the amount of guascas, teachers perceived a problem in the accuracy and asked the kids to explained how they had obtained the measurements. Some kids revealed that they had lost the count of steps and they were just guessing. A group created a strategy to count the number of steps: each ten steps they took a small rock in the pocket and started again, and when they arrived to the end, they just multiplied the amount of rocks by ten, and added the final number. It became evident that a new day of measurement was necessary to solve the issues that appeared in the first day.

When the data was obtained in homogeneous ways, all groups gathered and explained their result. Diego made a table with the results and a drawing of the situation. Questions appeared about time and distance: Which is the longest section of the route? Which are the closest places? Which segment has sloppier? How much time takes to walk the entire footpath? How long is the proposed route? Trying to address these questions, students needed to add seconds and minutes that are not in a decimal system. Diego also requested answers in meters, so they needed to convert their measures of guascas into meters. We measured in front of the students the length of a guasca (7 meters) and the length of a student step (0.5 meter). So, they had to multiply numbers with decimals and round up. They converted the informal measures of each route section into meters and added all of them. They checked if that number was equal to the conversion of the sum of the whole route, obtained by adding the informal measures. Diego explained that the results must be the same, exemplifying the distributive property of addition and the product. A lot of standard contents of the non-Indigenous mainstream mathematics curricula gained meaning and utility within the project's framework.

With the numerical data analyzed, the next task was to trace a proper map for Lomitas. Students consulted some geography books, to see the usual conventions of representation, but they did not find anything specific about Lomitas. Aldo suggested the use of Google Maps and explained to Diego how to use it. When he was showing the tool to the children, explaining that a satellite can take pictures of the Earth, one student replied that such system is a threat for the territory, because many people can obtain information about them without indigenous permission and without any control of the use of such

information. Diego extended the reflection saying that Indigenous people need to be aware of the possibilities of those technologies. Students used the views that Google Maps offered —at that time it was two-dimensional and not so close— as a reference to create on paper their own representations of Lomitas. Working on drawing a map, students located the different special places that they had previously identified, in such a way that the data obtained could be represented.

A new stage was to define a particular plaque for each place. Students watched a video that Aldo had recorded during the first work session with the elders, and they remembered important details of the stories. Groupwork returned. This time they were responsible for proposing a draft of the final plaques. They had to display in a blank page all the information collected. They were also asked to create small verses about the particular story of each place. Part of the plaque needed to be written in Nasayuwe, other parts in Spanish, and other with the numerical information. Teachers made comments about the draft, until they were approved. After that, another type of work started. Teachers invested part of their salary in paint, brushes, wood tables and sandpaper. Students had to prepare the final plaques. Colors, texts, and proportions were defined. They needed to know how to create some colors, how to prepare a wooden surface, which type of painting they would use, and how to paint. The school turned into a small carpentry workshop for a week.

In parallel, Diego was coordinating the final event in the form of the traditional *minga*[5]. The community cut some wooden poles to hang the plaques; and the school prepared special food for that day. Elders, parents and authorities were invited. This time, it was the kids who were telling back the stories to the community. They recited the verses created, explaining how to read the information contained in the plaque. Kids signed the back of each plaque and adults installed it. When we finished the journey at the school, parents expressed their approval and assessed the work, stressing the importance of knowing the history of Lomitas and the Nasa tradition.

5.4 Delineating the notion of *propio*

The fieldwork experience described help us to characterize the notion of *propio*. The educational activity was not kickstarted to improve students' performance in a curricular pre-stablished topic. The overall aim addressed community's concerns of cultural and political nature. This aspect illustrates the envisioned role of school in *educación propia*: to take distance from discourses about the

[5] *Minga* comes from the Quechan *minka*, and refers to a collective work of common utility, usually agricultural work, but it also can be the building of bridges, roads or even an intellectual work. Minga is the form of work organisation most used by Nasa people (Bolaños et al., 2016).

improvement of the teaching, with a customary focus on the disciplinary contents, in order to improve individual results. Rather, schools and educational activity respond to the agenda of relevance of the community. Thus, a first connotation of *propio* connects with *proper*, *adequate*, *pertinent*, and *useful*. A proper education for the student and the community fosters knowledge and values needed to address the demands of their life in a political struggle for identity and territory. It also legitimates local forms of organisation and communication, and it allows to nurture leaders who can address the perceived educational problems of the community. It can be said that educational practices in the communities are *made for* Indigenous and therefore are attuned to the Indigenous interests.

We can see resemblances between this approach and other experiences in ethnomathematics and in critical mathematics education, such as the mathematics in the Landless People Movement in Brazil (Knijnik, 2004, 2007) , and the conceptualizations of mathematics in action (Skovsmose, 2009, 2011). Concerning the former, "learning in the process" and "knowledge is political" are principles developed in the movement to "allow articulating the local world of each settlement and the immediate demands of the struggle with educational processes" (Knijnik, 2004, p. 128). The articulation resulted in new pedagogies of mathematics that responded to the necessities of the peasants in their struggle, and at the same time gave opportunities for children and youth of continuing further studies. Concerning the latter, the critical distance to the different uses of mathematics favors the recognition of the intentions, motivations, consequences, and risks that each mathematical object or technique comprises. Such critical distance makes possible a critique to the modernist conception of (mathematical) knowledge in which objectivity, certainty, transparency, progress, and neutrality are naturalized values (Skovsmose, 2009).

Furthermore, the experience of Lomitas allows us to problematize the traditional sources and places of knowledge. Who is deciding what is true or false or what deserves to be learned and problematized in the educational process? The answer is clear in this case: the community and the territory. In the PEC, the community defines what is important to be discussed in schools. It is important to mention that the SEIP grants to communities the power of hiring and assessing teachers according to their own criteria. For instance, teachers can be selected by a spiritual test of their energies, made by a *Kiwethê*, a traditional healer of the community, who establishes which candidate is best aligned to the desires of the community. This form of selection is suitable since the teachers' role is to be processes facilitators or catalyzers and not transmitters of previously determined knowledge. Knowledge is held and created by the community members.

As our example illustrated, elders and children were the main characters. They exchanged the role of source and target of knowledge in different stages of the process. They recovered and actualized their knowledge about the commu-

nity and the territory and made it available. This entails a second sense of the term *propio*: Nasa are the *owners* of this education; and values like *property* and *belonging* are a fundamental in the relation between the community and the content of education. As stated by its Intercultural and Bilingual Educational Program [In Spanish, Programa de Educación Bilingüe Intercultural, from now on PEBI], education is their own because the Nasa have "the faculty to orient, direct, organise and construct educational processes and proposals with a critical and purposeful positioning towards the education that we want" (PEBI, 2010, p. 7, our translation). We can summarise that characteristic as an education *made by* Indigenous people.

The third sense of *propio* that we want to explore became evident in the experience when students proposed to make the plaques and used new technological devices to obtain and analyze the information. *Propio* then takes the sense of *appropriated*, or something foreign, not belonging to one's culture, that is taken on board because it is functional to one's needs. For neither the teachers or the parents there was an issue to use and take advantage of foreign technologies such as the mobile phones or computers. They consider important to take advantage of non-Indigenous tools, as far as they contribute to their process of autonomy. The possibility of appropriation of the technologies or knowledge of others moves away from essentialists views of Indigenous education that argue for a purity of traditional community knowledge (Scandiuzzi, 2009). Rather, this view of appropriation constitutes a political conception of interculturality where "based on the own knowledge, we move to integrate knowledges from the outside" (CRIC, 2004, p. 115, our translation).

Tattay (2011) explains that interculturality is conceived as:

> [H]orizontality within the frame of a political relationship. [...] Interculturality implies a relation among diverse agents, conceptions, strategies and tools; it also implies contact among diverse cultural expressions, considering relations of power and domination. And because of that, the permanent effort to create situations of equity among knowledge(s) of diverse natures becomes necessary. (Tattay, 2011, p. 91, our translation)

The comment of the 9 year-old about the information available to other people on the community's territory through geographic information systems such as Google Maps being a threat is framed within such horizontality that allows to use with critical sense the developments of other cultures.

Finally, it is important to comment on the role of research in education, and how it is understood by the Nasa people. Research is central in *educación propia* as it has become a strategy to create and re-create knowledge in relation to their culture, and to circulate the knowledge in the community. In this sense, research is also a pedagogical strategy. Research unfolds "a process of self-recognition, because it allows to discover how much we know ourselves about our culture, our territory and about different ways to communicate in the family,

in the community and in other spaces" (Caicedo et al., 2012, p. 131, our translation). Research is conceived as a tool for strengthening (Indigenous) identity, appropriating the territory, and producing collective knowledge. Therefore, research is "a strategy of hope, dignity and freedom" (Caicedo et al., 2012, p. 131, our translation).

The participatory and intercultural way in which the experience in Lomitas combined diverse types of sources (*e.g.*, oral tradition, internet, the territory) and procedures (*e.g.*, information sharing in the community, data collection, conceptualization in mother tongue, space intervention, minga) allows us to see an intentional re-signification of notions such as research, knowledge and learning. The third meaning of *propio* and the role of research highlighted above are the condition that transforms, adapts and condenses external elements in such way that they can be taken on board according to the Nasa cultural and political conception. In short, Nasa people appropriate non-indigenous concepts and procedures, when trying to do an education *à la* Indigenous Nasa.

5.5 Walls and bridges with other educational proposals

In this section we analyze similarities and differences between *educación propia* and other approaches in Indigenous and in mainstream education that seems to be in line with the characteristics of *educación propia*. Our argument is apparently less specific to mathematics education because alternative educational proposals are usually designed and conducted within wholistic frameworks that do not follow disciplinary scientific divisions. To balance this situation, we discuss the few (but growing) experiences that have tried to bring Indigenous or communitarian education issues to the mathematics education.

Educación propia shares characteristics with place/land-based education in its deep interest for the local environment as source of knowledge (Sobel, 2004), also with inquiry-based or problem-based education for their research driven methods (Nicol et al., 2013), and even with Freirean popular education and participatory action research for their interest in criticizing and attempting to change reality through collaborative action (Zavala, 2013). The centrality of the community and the collective construction of meaning relates *educación propia* with the African philosophical humanist notion of *Ubuntu* that has been identified as a key component of African Indigenous ways of knowing and being (*e.g.*, Swanson (2015)). Meanings of *propio* like belonging and transformation resonate with the decolonizing methodologies proposed by Smith (2013). However, we claim that the notion of *propio* demarcates a different problematization that generates new insights to (mathematics) education.

To frame the differences between *educación propia* and other proposals, we follow Rojas and Castillo (2005) in their study of how educational policies have

dealt with cultural diversity in Colombia. They claim that the recognition of cultural diversity during the late 20th century is a double edge sword. While it is true that the acknowledgement of cultural diversity is the result of demands raised by social movements —and it can be considered an achievement— at the same time, it can be seen as the state cooptation of those demands. Until the middle of the 20th century most states promoted the image of a homogeneous, mono-cultural nation, in which ethnical minorities were positioned below the line of human, their existence was denied and they were treated as the savage "Other" in need of becoming civilized through education. At the end of the century, there was a substantial change in the social imaginary in which "cultural difference" was promoted to the level of human right, and many nations were reframed as multi-cultural and pluri-ethnical. Therefore, discourses, ways of enunciation and representation were modified. However,

> [N]ew representations are still controlled by dominant places and discourses, subjected to their forms and processed in their space [...] For the state, constitutional recognition of ethnic diversity constitutes an opportunity to consolidate a model of state development and action in territories towards previously marginalized populations or towards population to which it had been unable to exercise an effective action (Rojas and Castillo, 2005, p. 54, our translation).

Rojas and Castillo argue that the right to be diverse may also constitute a re-arrangement of hegemonic discourses that maintain the subaltern and *minoritarian* condition of diversity, where ethnic groups and cultures are presented as special groups full of particularities and localities, while the *majoritarian* society is depicted as not particular and universal. Accordingly, education for those minorities is defined as one that respects and recognizes those localities but at the same time gives access to the "real deal" of universal knowledge. Evidence of this type of re-arrangement is that educational proposals for differentiated education make room for inclusion of cultural contents in the curricula, but do not give leadership, control of assessment processes or of economic resources to the Indigenous populations or the grass-roots movements. As a result we face what Walsh has called "neoliberal multiculturalism" (Walsh, 2007). In such frame, the provision of Indigenous education is a right "that the dominant sectors have considered as their own and a way of making the minorities part of (in-corporate into) the project of society" (Rojas and Castillo, 2005, p. 138, our translation).

We contend that *educación propia* attempts to contest this subaltern condition in different ways than other proposals for differentiated education have done so far. For instance, the very same role of school is different. Makoni and Pennycook (2007) rightly stress that:

> While indigenous communities regard schools as sites of contact between indigenous communities and the 'white-man's world,' education being

> understood as taking place at home, Western scholarship takes the opposite view, defining what indigenous communities regard as education to the relegated status of socialization. (Makoni and Pennycook, 2007, p. 30)

Although in *educación propia* school is useful in the process of acquiring non-indigenous tools needed to improve the ways in which communities have interlocution with the state, that is not the main role attributed to the school. Tattay (2011) pointed that "[since the early years, the CRIC] designed school to catalyze communitarian processes, understanding by communitarian not only what requires 'inner' tools, but also what strengthens the culture and the organisation" (Tattay, 2011, p. 43, our translation).

Educational models and associated educational research on indigenous education, such as ethno-education, bilingual intercultural education, culturally responsive/relevant education and place/land-based education propose to implement changes in the curriculum or in pedagogical styles, without doubting or fundamentally questioning the structure and purpose of the school itself. No attention is paid to the integrationist principle that subsumes the diversity of Indigenous knowledges and practices into the traditional hegemonic educational models managed and normalized by the state. For instance, when trying to link school and culture, some of these proposals include cultural knowledge as part of the curriculum subjects to be taught (*e.g.*, bilingual intercultural education in Perú), while others take advantage of cultural heritage to facilitate learning of disciplinary subject-matter (*e.g.*, culturally responsive education). Despite that important difference, the direction of the link is from the school to the culture.

In mathematics education, such proposals are illustrated in the following two approaches. Nicol et al. (2013) observed that "[t]eachers generally did not consider Indigenous knowledges, culture, or the community as resources for mathematics problems, however they did speak to the need to draw upon students' prior knowledge in designing mathematics lessons" (Nicol et al., 2013, p. 79). They developed a model of culturally responsive mathematics in which traditional indigenous stories can be re-worked to achieve the learning of the disciplinary knowledge prescribed in the mainstream curriculum. In the same line, Sobel (2004) noted that "[p]lace-based education is the process of using the local community and environment as a starting point to teach concepts in language arts, mathematics, social studies, science and other subjects across the curriculum" (Sobel, 2004, p. 6).

We can see in these proposals how the school intervenes the communitarian space: first schools and its agents —teachers and/or researchers— find some objects in the community —like social or environmental problems, oral stories, traditional forms of knowing, etc. Then these objects are treated, studied and problematized in school ways to obtain some individual learning in the students.

It is assumed that collective benefit will result from the unfolding of the persons' potential. In brief, the school intervenes the community to achieve its own aims. Conversely, the ideas of *educación propia* explore the relationship school and community the other way around. It is the community that intervenes the school space, finding some objects in it —such as the curriculum, assessment techniques, teacher education, etc.— that the community treats, studies and problematizes in communitarian ways, to obtain some collective learning in the community. Individual benefit results from the achievement of collective goals. In brief, the community enters the school to achieve the community's aims —not the aims of the school–.

At first glance this difference may seem a mere change of order because the educational models use similar means (inquiry-based and participatory methodologies), and because *educación propia* does not refuse academic (disciplinary subject-matter) achievement. Also, each one of the models mentioned have clearly stressed that they aim to "develop and / or maintain cultural competence" (Ladson-Billings, 1995), or to "help students develop stronger ties to their community" (Sobel, 2004). However, they are not pursuing the same aims as *educación propia*. Making school more context-aware generates the risk of scholarizing the culture. Instead, by trying to make the community more school-aware, *educación propia* may communitarize the school. Therefore, these trends face the cooptation force of (neoliberal) multiculturalism in very different ways.

Proposals that start from the school towards the community many times get instrumentalized and in the long run reduced to styles of teaching that end up being assessed in terms of how efficient they are to increase school performance and reduce, for example, the achievement gap in mathematics. This indirectly reinforces the idea that there is certain type of knowledge and way of life that deserve to be pursued. In their strong critique to some North American attempts to incorporate Indigenous perspectives in mathematics education, Stavrou and Miller (2017) pointed that:

> [A] decolonizing and anti-oppressive education mean much more than the common understanding of promoting cultural diversity and non-Western perspectives. It requires identifying and challenging the root causes of oppression, how inequality is reproduced in the classroom, and finding strategies to counter educational discourses that position Western knowledge as superior and other knowledge (such as Indigenous knowledge) as inferior (Stavrou and Miller, 2017, p. 99).

Furthermore, Molina and Tabares (2014) argued that although the concept of "dialogue of knowledge(s)" permeates almost any current proposal of Indigenous education, such concept falls short in recognizing that "more than the convergence of knowledge(s) of diverse type, what is needed for an effective interculturality is the dialogue among human beings, coming from different

spaces, with cultural practices and diverse historic experiences" (Molina and Tabares, 2014, p. 13, our translation). In the same vein, Grosfoguel (2012) has clearly pointed how the logic of neoliberal multiculturalism does not entail any change in the *locus of enunciation*[6] established by the colonial hierarchizations:

> Hegemonic liberal multiculturalism allows each racialized group to have its space and celebrate its identity/culture, as long as they do not question the ethnic/racial hierarchies of white supremacist power and as long as they leave the status quo intact. This privileges certain elites within the racialized/ inferiorized groups, granting them a space and resources as "tokens", "model minority," or "symbolic showcases," thereby giving a cosmetic multicultural tinge to white power, while the majority of these populations victimized by this rampant racism experience the coloniality of power on a daily basis (Grosfoguel, 2012, p. 87).

It is precisely to this point that the triple meaning of *propio* (pertinent, owned, and appropriated) becomes relevant as it enacts a radical displacement in the locus of enunciation and locates the discussions of Indigenous education on a different plane. Through the notion of *propio*, Nasa people are not just claiming that their knowledge deserves to be called knowledge instead of beliefs. They also reject the subaltern position that denies them the intellectual status of knower and condemn them to the role of the known. Therefore, the problem in the midst of education is no longer a struggle for recognition, but a fight for autonomy and self-determination.

5.6 Propio: a decolonizing tool

The Nasa people conceive autonomy in relation to the construction of their own thinking (Levalle, 2017). This became evident in their re-definition of education, research, school, and knowledge. Such constructions counteract an epistemic violence that hierarchizes and classifies knowledges in dichotomies such as theoretical/practical, local/universal, scientific/vernacular, and that operates in the intertwining of knowledge and power, (dis)abling entire populations as ignorant or trustable. The deliberated creation and use of such re-definitions allow us to recognize the potential of the notion of *propio* as a de-colonizing tool for thought.

It is a consensus in the community of researchers of and activists in decolonizing processes that coloniality is not only a matter a political dominance, but it is always accompanied by the establishment of a matrix of hierarchizations

[6]Mignolo (2000) introduced this concept first, but we take it here as "the geo-political and body-political location of the subject that speaks" (Grosfoguel, 2011, p. 5). This notion is crucial to understand not only the epistemic and political struggles of Indigenous nations, but also why well-intended discourses of inclusion are structurally constrained.

of diverse nature: religious, linguistic, ethnic, economical and, of course, epistemic. Colonization simultaneously run a military invasion in the territories that today constitute Latin America and enthroned a geo-politics of knowledge (Mignolo, 2002). The latter vindicated the character of universality and objectivity of Euro-centered knowledge and, accordingly, labelled the knowledge developed in colonized territories as localized and subjective. Such assemblage of hierarchizations was fertile soil for the consolidation of the project of modernity, that naturalized ideas such as the primacy of scientific knowledge, the split body/mind and human/nature, or the individual nature of learning. The correspondent regimes of truth that sustain the modernist project proscribed any doubt about those divisions as a threat to development and as an anchor to the obscure and esoteric past. The decolonial project and Indigenous resistance converge in their rejection to this pretended "God's eye-view" epistemology, and share a disdain to replace one hegemony for another. This is a point of relevance when it comes to mathematics, given its centrality in Western project of modernity and in its entanglement with power for the assimilation of native populations in Latin America (Valero and García, 2014). As Stavrou and Miller (2017) pointed out: "Applying decolonizing strategies to mathematics will make visible that it is a subject developed in time and place, and will expose the imperialistic operation that deprives mathematics of its historical roots and human construction" (Stavrou and Miller, 2017, p. 106).

Very important in the conceptualization of *propio* is its decolonial response to the imposed modernist universality. *Propio* is not calling for an essentialist account that would impose a different center; neither is it supporting a relativist proposal that would deny the advances and achievements of modernist science. Instead, the idea of *propio* appeals to the lack of unified global epistemic center, being interested in the processes of appropriation and creation of knowledge that each culture and community can develop. Thus, the notion of *propio* refers to a complex interplay of the categories inside/outside (of Native social, cultural and political spaces), that are not conceived as mutually excluding but as nested and in continuous mutual reshaping. Rappaport (2008), an anthropologist who has studied the political and cultural resistance of the Nasa people for more than 30 years, reflected about this issue:

> (For indigenous activists), culture is not an existing constellation of practices and meanings located on the "inside" but a projection of how future lifeways should look, driven by a process in which elements of the inside are revitalized through the incorporation of ideas from outside; that is, culture necessarily straddles the frontier. This is not a strategic deployment of essentializing discourses to describe what exists "out there" but a model of what "should be," a blueprint for the future. [...] While ethnographers engage in cultural description with an eye to analyzing it, indigenous autoethnographers study culture to act upon it. (Rappaport, 2008, pp. 20-21)

Rappaport calls our attention to the ways in which "inside" is conceived by Nasa people as *process*, instead of *condition*. When the latter prefigures an immanent and already existing identity that deserves to be preserved, the former is appealing to the journey that communities are doing when interacting with other groups, aiming to transcend and survive without cooptation. Thus, identity is in permanent construction. As Tattay (2011) explains, "the notion of *propio*, underscores that the appropriation, adjustment and reorientation of external knowledge(s) are not exclusive privilege of the others. Internal adjusment and orientation make those elements belong to current indigenous identities. In that sense, *educación propia* does not only deal with the inside" (Tattay, 2011, p. 34, our translation, italics added). As illustrated with the experience in Lomitas, Nasa people do not refuse knowledge and technologies (such as digital geographic information systems) produced by others; instead, they are trying to appropriate them.

With these insights, we locate the notion of *propio* beyond shortsighted identity politics and multicultural approaches, and move it closer to Harding's (2008) standpoint epistemologies and decolonial stances. *Propio* escapes a modernist, "God's eyes-view" perspective. Instead it situates its approach to knowledge geographically, historically, linguistically, and culturally. In *educación propia*, the notion of *propio* expresses an epistemological conceptualization made by Indigenous people in Cauca. Levalle (2017) reported that for the Nasa people every process of knowing is at the same time a process of feeling. Indigenous researchers talk about "feeling the messages of the territory" or to "getting reconnected with the origin's history". This has the aim of reharmonizing with the territory. In that sense, *propio* knowledge is a knowledge that is felt, mainly bodily. The Nasa educational leader Inocencio Ramos explained how the Nasa people understood "to know" as related to the capacity of feeling and dreaming, and this relation is expressed in the Nasayuwe language. The words for reflect, think, analyze and plan are all related with the heart and the spirits. For those reasons, Inocencio Ramos stated that to know is to "think with the heart" or "corazonar"[7] (Levalle, 2017, p. 132, our translation). *Propio* knowledge is also collective, because "it is re-created in the memory of the people and also in the set of presences that inhabit the territory. It is a deeply relational and intersubjective knowledge" (Levalle, 2017, p. 141, our translation).

5.7 How global is the local?

In this final section we address how the political, cultural and epistemological insights gathered in the notion of *propio* and its unfolding in *educación propia*

[7]Corazonar is a metaphoric word game in Spanish, that mixes *corazón* (the heart) with *razonar* (to reason).

resonate with concerns and questions raised in other contexts, Indigenous or not. Although *propio* is resultant of the Nasa strategy of cultural resistance, it is our contention that it may allow us to think other experiences in mathematics education and propose directions for action. In this way, *propio* may also help us addressing the tendency in some socio-political research towards critical contemplation and turn critique into a performative force.

The development of a differentiated education (*Te Aho Matua*) in Aotearoa/ New Zealand by Māori people has been another long process characterized by a quest of autonomy and self-determination, accomplished by the community and invited researchers. The process was also acknowledged by the state after years of negotiation and legitimation of the educational initiatives. The aim to decolonize methodologies and pedagogies looking for a Maōri perspective made evident how *propio* concept is not an endemic construct by Nasa People. Moreover, when examining the role of the language Te Reo Māori in the teaching and learning of mathematics, Meaney et al. (2011) described a variety of experiences in which indigenous identities became strengthened in the complex linking of past with future. Such experiences deployed a series of techniques and strategies in which we can recognize the same intention to appropriate, make pertinent and relevant the mathematics education, that we have highlighted here for the Nasa case.

Barton et al. (1998) described how Māori people created their own mathematical registers in *Te Reo* for Western mathematical concepts, through an ongoing process of appropriation (one sense of *propio*); Meaney et al. (2011) reported the historical development of Te Kura o Te Koutu school, a grassrooted educational initiative in Rotorua, where Māori indigenous community decide, define and manage their own education (the sense of *propio* as property/belonging is evident). McMurchy-Pilkington et al. (2013) and Meaney et al. (2012) have discussed the evolving political controversies associated with the curricular development in Aotearoa/New Zealand in the last 20 years and the views that academic research promotes (or neglected) within such development (making visible a third sense of *propio*: pertinence). Direct resemblances between the Māori and Nasa experiences were explored by Parra and Trinick (2017).

It is relatively easy to establish connections among educational initiatives in Indigenous nations because the shared history of colonization makes evident the desire to displace the locus of enunciation that *propio* proposes. Certainly, it is more difficult to claim for a decolonization in groups and communities that do not recognize themselves as formerly colonized. The relevance of the concept of *propio* for mathematics education resides in the possibility of using it as a tool to think and act in non-indigenous contexts. We acknowledge that the cultural dimension of indigeneity provides a space for reflections, struggles and actions that non-Indigenous contexts do not have.

Non-Indigenous experiences in mathematics education can also have re-

semblances with the conceptualization of *propio*. The case of Brazilian rural communities in the struggle for the ownership of land is such a context. Knijnik and collaborators (Knijnik, 1996, 2002, 2004, 2007), (Knijnik and Wanderer, 2010; Knijnik et al., 2012), have reported how peasant work techniques of importance for their material and economic survival —such as measuring land and agricultural production— were described, used and contrasted with techniques accepted by the state. These mathematical techniques were also used strategically to defend the peasants' interests; and they were also modified by the appropriation of other techniques that allow them to resist and find new ways of working. These communities were part of the Landless Movement, a large NGO with a political project for the rights of land ownership of dispossessed peasants. Education in the landless settlements was the context in which the mathematical activities were inscribed. Although here there is no cultural struggle given by an ethnic condition, this case has a similar political condition of resistance and communitarian re-affirmation that the CRIC provided to the Nasa's education project.

Could other contexts of tension —*e.g.*, the education of refugee or immigrant minorities in Europe, the education of underprivileged, Black urban population in the USA, or the education of large impoverished population in Latin America— be examined with the lenses of *propio* and *educación propia*? Rather than a new model or trend within political approaches of mathematics education, we propose *propio* as a useful theoretical concept to activate the singularity of each context. To be faithful to the meaning of property and pertinence, *propio* is not a concept to be applied but rather to be interpreted and rooted in new situations.

Nowadays there is a growing tendency towards the tight political governing of (mathematics) education for the satisfaction of the macro-economic interests of the state or of the private capital. Indeed, in a post-political time where the government is organised to serve not the interests of the public common good, but the interest of private corporations, education has transformed its character, and has become an arena for the effective making of economically-oriented subjectivities (Valero, 2017). Given the articulation between mathematics, science and technology with the production of new forms of capital, mathematics education features high as a form of transforming the population into the rational, numerically equipped, economically aware and entrepreneurial subjects that are functional to economic production. The notion of human capitals is provocatively proposed by Brown (2015) to grasp the tendency to the reduction of humanity to economic relations of capital generation. In such scenario, education operates classifications and orderings of people according to their mathematical performance; and it does so not only through assessment systems, but also through its distancing from life and people's experiences.

This tendency does not only impact on the educational practices of mathematics, but also affects the theorisation of the field of mathematics education.

The urge to increase achievement promotes a comprehension of the field as the education needed to obtain mathematical knowledge, in other words, education *for* mathematics. This comprehension installs a regime of truth that operates by naturalizing epistemological and political assumptions, and by turning some research questions and methods as possible and desirable, while deeming some others as irrelevant or deviating. For instance, the current interest in "best practices" or in reducing "the gap" are examples of what is taken as desirable, shaping the field as a quest for *efficiency* in teaching and research practices that should have a direct impact on students' achievement. Instead, a problematization of what is the type of society being reified and reproduced through mathematics school practices does not seem to belong to research in mathematics education. Such endeavor is seen as a task for sociology or cultural studies.

Socio-cultural and socio-political approaches in mathematics education are pushing the limits of what mathematics education could be, by presenting experiences that call into question this desire of achievement and its associated regime of truths, even taking the risk to be labelled as ill-founded or primitive. Ethnomathematics targeted the epistemological nature of mathematics, problematizing the pretended story of universality and culture-free that school fostered for decades. Studies of subjectivity in mathematics classrooms focus on how students are subjectified and disciplined in certain ways. That kind of research analyse the ways in which humans are mathematically enculturated, understanding that mathematics education would study also the education that is being done *through* mathematics.

When *educación propia* problematizes daily life and makes knowledge get closer to people's experience, it is not falling into a naive empiricism or an activism that may ultimately be functional to discourses that capitalize on the subjectivation of the person. Instead, *educación propia* may allow "the recovery of the conditions of a self-determined existence based on the recognition of the complex relations of domination and resistance that occur between certain class sectors" (Molina and Tabares, 2014, p.7, our translation).

The concept of *propio* embodies a critical standpoint towards mathematics education, for it raises awareness on the socio-political constrains in the constitution of mathematical knowledge and the power effects associated with such constitution. As Tattay (2011) pointed out, the interculturality promoted by the notion of *propio* "does not address a merely pedagogic issue, it is also a political issue on the dispute of imaginaries about what is the knowledge that matters, the knowledge that has the magic to be recognized inside and outside" (Tattay, 2011, p.91, our translation). The aim of expanding the history and epistemology of mathematics also makes possible to trace connections of *propio* with ethnomathematics, as far as the conceptions of knowledge as collective, attached to bodily perception and with nature as source, contrast and contest the distinctive Western rationalist narrative of mathematical knowledge. For

these reasons, we think that it is plausible that *propio* escapes the locality of Nasa people and meets and enhances global developments in mathematics education.

While we propose the mobilization of the concept from their Nasa cradle to other spaces, we foresee some critical issues in non-indigenous contexts: all the experiences that we can refer by now as enactments of *propio* share the condition of being conducted within the frame of a pre-existent organisational process that make possible the experience, arranging and project the disagreements of a group with a type of school that does not allow their member to live in the terms that they desire. Such processes of organisation provide a social bond that is not always present in all school contexts or situations. Another critical issue is the reference to a cultural heritage as a base on which new possibilities can be build, as response to the hegemonic ones. Contexts of economic, cultural and linguistic dispossession have denied even the chance to remember a different past and imagine a different future. In that matter, we acknowledge that many issues surrounding the concept of *propio* cannot trespass the boundary of the indigeneity. However, there are situations where the need for pertinence, relevance and belonging of education are highly important. In these contexts, the effects of a position of mathematics education as a technical matter of improving "how to teach mathematics" call for a political position that questions "what and why to teach mathematics".

When contrasting *educación propia* with other educational models (indigenous or not), we can call into question the widespread and normalized definition of mathematics education in general as the education needed for (the sake of) mathematics. We can also formulate an alternative definition of an education that is done through (the support of) mathematics. The former locates the student as cognitive subject, defined in terms of a set of knowledge to match the expectations of a curriculum; while the latter understands the student as a human subject, being and becoming part of a culture and society. Therefore, *educación propia* entails a radically different comprehension of what mathematics education is, both as a field of practice and as a field of research.

The main contribution of *educación propia* and *propio* to mathematics education in non-indigenous environments is to explore the ongoing enactment of this different comprehension. The Nasa educational developments are useful because "they are not only ways to experiment, but above all expressions which show not merely how 'things can be otherwise' but that they indeed are in other ways" (Quijano, 2016, p.302, our translation). Our claim is that by paying attention to the possibilities of the appropriation processes that every local community can undertake, mathematics education can develop rooted and meaningful alternatives to the omnipresent economization that homogenizes and standardizes mathematics education worldwide. Through the notion of *propio*, we can articulate local responses at global scale that, paraphrasing Harding (2008), conceives a mathematics education from below. Emulating

the Nasa assumptions about culture as an operative space in which communities build their future, we do not suggest that a mathematics education based on *propio can be* possible (as an analytical chance). Instead, we contend that communities *can make it* possible (as a political challenge).

Finally, the shift of attention towards action and performativity battles socio-political and critical mathematics education (research) that has avoided to formulate explicit ways in which such understanding can be deployed to conduct educational practices. Although this reluctance can be considered as a coherent effort —not to replace one hegemony for another and falling into a new quest for efficiency— it can also be seen as a reductive and immobilizing choice. The critical contemplation enhances the research field with a framework to analyze educational practices but not with a framework to perform or transform them. *Educación propia* and *propio* are useful tools for mathematics education that offer a performative capacity because the political question of pertinence in practice is foregrounded.

References

Barton, B., Fairhall, U., and Trinick, T. (1998). Tikanga reo tātai: Issues in the development of a māori mathematics register. *For the learning of mathematics*, 18(1):3–9.

Bolaños, G., Bonilla, V., Caballero, J., Espinoza, M., García, V., Hernández, J., Peñaranda, D., Tattay, P., Tattay, L., and D., P. (2016). *Nuestra vida ha sido nuestra lucha: resistencia y memoria en el Cauca indígena.* Organización Internacional para las Migraciones (OIM-Misión Colombia).

Brown, W. (2015). *Undoing the demos: Neoliberalism's stealth revolution.* Zone Books, New York, first edition.

Caicedo, N., Guegia, G., Parra, A., Guegia, A., Guegia, C., Calambas, L., Castro, H., Pacho, C., and Díaz, E. (2012). *Matemáticas en el mundo Nasa.* CIIIT, Bogotá, Colombia, 2nd edition.

CONTCEPI (2013). Perfil del sistema educativo indígena propio, seip.

CRIC (2004). *¿Qué pasaría si la escuela...? Treinta años de construcción de una educación propia.* Popayán.

CRIC (2008). Programa de educación.

Dei, G. and Kempf, A. (2006). *Anti-colonialism and education*, volume 7. Sense Publishers, Rotterdam.

Grosfoguel, R. (2011). Decolonizing post-colonial studies and paradigms of political-economy: Transmodernity, decolonial thinking, and global coloniality. *Transmodernity: Journal of Peripheral Cultural Production of the Luso-Hispanic world*, 1(1).

Grosfoguel, R. (2012). The dilemmas of ethnic studies in the united states: Be-

tween liberal multiculturalism, identity politics, disciplinary colonization, and decolonial epistemologies. *Human Architecture*, 10(1):81.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, 44(1):37–68.

Harding, S. (2008). *Sciences from below: Feminisms, postcolonialities, and modernities*. Duke University Press, Durham.

Knijnik, G. (1996). *Exclusão e resistência: educação matemática e legitimidade cultural*. Artes Médicas, Porto Alegre.

Knijnik, G. (2002). Curriculum, culture and ethnomathematics: The practices of 'cubagem of wood' in the brazilian landless movement. *Journal of Intercultural Studies*, 23(2):149–165.

Knijnik, G. (2004). *Lessons from research with a social movement*, pages 125–141. Kluwer Academic Publishers, New York.

Knijnik, G. (2007). Mathematics education and the brazilian landsless movement: Three different mathematics in the context of the struggle for social justice. *Philosophy of Mathematics Education Journal*, 21:1–18.

Knijnik, G. and Wanderer, F. (2010). Mathematics education and differential inclusion: a study about two brazilian time–space forms of life. *ZDM*, 42(3-4):349–360.

Knijnik, G., Wanderer, F., Giongo, I., and Duarte, C. (2012). *Etnomatemática em movimento*. Autêntica, Belo Horizonte.

Ladson-Billings, G. (1995). But that's just good teaching! the case for culturally relevant pedagogy. *Theory into practice*, 34(3):159–165.

Levalle, S. (2017). *Investigacion comunitaria intercultural y resistencia a la violencia política en el Consejo Regional Indígena del Cauca -CRIC-, Tierradentro, Colombia (1994-2016)*. Master thesis, Universidad de Buenos Aires.

Makoni, S. B. and Pennycook, A. (2007). *Disinventing and Reconstituting Languages*. Multilingual Matters, Clevedon.

McMurchy-Pilkington, C., Trinick, T., and Meaney, T. (2013). Mathematics curriculum development and indigenous language revitalisation: Contested spaces. *Mathematics Education Research Journal*, 25(3):341–360.

Meaney, T., McMurchy-Pilkington, C., and Trinick, T. (2012). *Indigenous Students and the Learning of Mathematics*, pages 67–87. Sense Publishers, Rotterdam.

Meaney, T., Trinick, T., and Fairhall, U. (2011). *Collaborating to meet language challenges in indigenous mathematics classrooms*. Mathematics Education Library. Springer, New York.

Mignolo, W. (2000). *Local histories/global designs: Coloniality, subaltern knowledges, and border thinking*. Princeton University Press, Princeton.

Mignolo, W. (2002). The geopolitics of knowledge and the colonial difference. *The South Atlantic Quarterly*, 101(1):57–96.

Molina, V. and Tabares, J. (2014). Educación propia: Resistencia al modelo de

homogeneización de los pueblos indígenas de colombia. *Polis (Santiago)*, 13(38):149–172.

Nicol, C., Archibald, J., and Baker, J. (2013). Designing a model of culturally responsive mathematics education: place, relationships and storywork. *Mathematics Education Research Journal*, 25(1):73–89.

Parra, A. (2011). *Etnomatemática e educação própria*. Master thesis, São Paulo State University at Rio Claro.

Parra, A., Mendes, J., Valero, P., and Villavicencio, M. (2016). *Mathematics education in multilingual contexts for the indigenous population in Latin america*, pages 67–84. Springer International Publishing, Cham.

Parra, A. and Trinick, T. (2017). Multilingualism in indigenous mathematics education: an epistemic matter. *Mathematics Education Research Journal*, pages 1–21.

PEBI (2010). Construyendo el sistema educativo indígena propio. *Çxayu'ce*, (14):4–10.

Quijano, O. (2016). *ecosimías: Visiones y prácticas de diferencia económico cultural en contextos de multiplicidad.* Editorial Universidad del Cauca, Popayán, 2nd edition.

Rappaport, J. (2008). Beyond participant observation: Collaborative ethnography as theoretical innovation. *Collaborative anthropologies*, 1(1):1–31.

Rojas, A. and Castillo, E. (2005). *Educar a los otros: Estado, políticas educativas y diferencia cultural en Colombia.* Universidad del Cauca, Popayán.

Scandiuzzi, P. (2009). *Educação indígena x educação escolar indígena: uma relação etnocida em uma pesquisa etnomatemática.* Editora UNESP, São Paulo.

Sichra, I. (2004). *Género, etnicidad y educación en América Latina*, volume 4. Ediciones Morata, Madrid.

Skovsmose, O. (2009). *In Doubt - About Language, Mathematics, Knowledge and Life-Worlds.* Sense Publishers, Rotterdam.

Skovsmose, O. (2011). *An invitation to critical mathematics education.* Sense Publishers, Rotterdam.

Smith, L. (2013). *Decolonizing methodologies: Research and indigenous peoples.* Zed Books Ltd.

Sobel, D. (2004). Place-based education: Connecting classroom and community. *Nature and Listening*, 4:1–7.

Stavrou, S. and Miller, D. (2017). Miscalculations: Decolonizing and anti-oppressive discourses in indigenous mathematics education. *Canadian Journal of Education/Revue canadienne de l'éducation*, 40(3):92–122.

Swanson, D. (2015). Ubuntu, radical hope, and an onto-epistemology of conscience. *Journal of Critical Southern Studies*, 3:96–118.

Tattay, L. (2011). *La" educación propia" en territorios indígenas caucanos: escenarios de hegemonía y resistencia.* Master thesis, Flacso Sede Ecuador.

Valero, P. (2004). *Socio-political perspectives on mathematics education*, vol-

ume 35, pages 5–23. Springer Science & Business Media.

Valero, P. (2017). *Mathematics for all, economic growth, and the making of the citizen-worker*, pages 117–132. Routledge, New York.

Valero, P. and García, G. (2014). Matemáticas escolares y el gobierno del sujeto moderno. *BOLEMA*, 28(49):491–515.

Walsh, C. (2007). Interculturalidad colonialidad y educación. *Revista Educación y Pedagogía*, 19(48):25–35.

Zavala, M. (2013). What do we mean by decolonizing research strategies? lessons from decolonizing, indigenous research projects in new zealand and latin america. *Decolonization: Indigeneity, Education & Society*, 2(1).

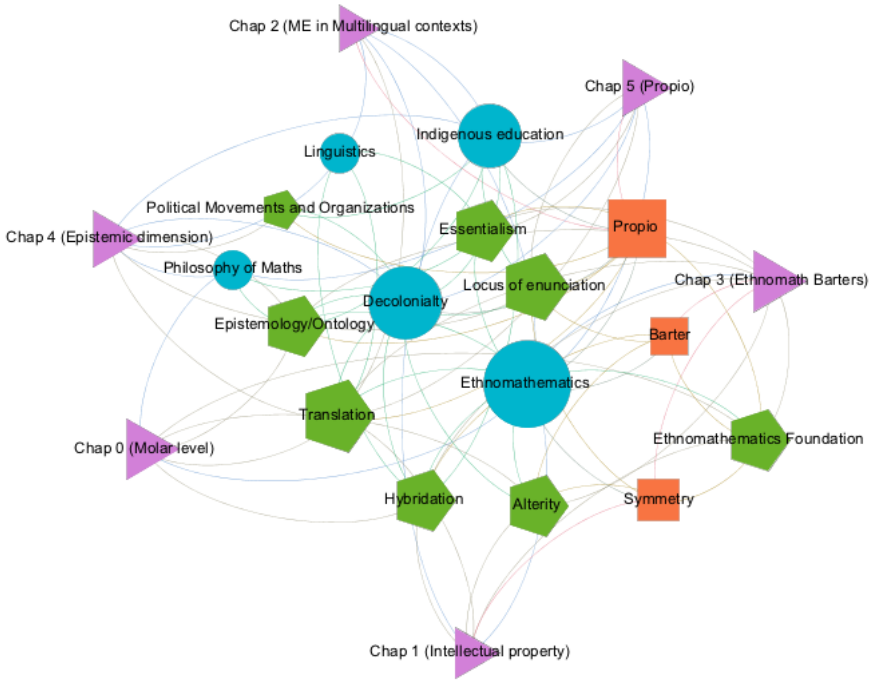


Fig. 5.3: Conceptual entanglement 5. Triangles represent chapters, circles represent areas of study, pentagons represent topics of interest, and squares represent new conceptual tools

Chapter 6

When and Why Do You Turn on the Mic? Tracing Symmetry in Ethnomathematics Research

Abstract

The relationship between researchers and researched communities in the ethnomathematical field has been conceived and developed broadly, calling for a theoretical problematization of such diversity. A notion of symmetry in research is introduced to guide a content analysis of previous ethnomathematical research. Software of qualitative analysis and social networks analysis were combined to explore how theoretical and methodological assumptions unfold leading notions of ethnomathematics such as dialogue, communication, respect, and otherness. Symmetry appeared to be useful to identify some regularities and calls into question the use of ethnographical methods to achieve ethnomathematical aims.

6.1 Introduction

Ethnomathematics emerged in the 1980's within mathematics education, as a program of research into the history and philosophy of mathematics that seeks to understand mathematical knowing and doing. It does so by studying "mathematics practiced by groups, such as urban and rural communities, groups of workers, professional classes, children in a given age group, indigenous societies, and so many other groups identified by objectives and common

traditions" (D'Ambrosio, 2006, p. 1). Since its origins, the field was based on a critical stance on dominant Eurocentric approaches to mathematics and its education, and has advocated the promotion of emancipation and equality of discriminated groups (Powell and Frankenstein, 1997). Nowadays, ethnomathematics is a field of research with a worldwide community of practitioners, most of them exploring implications in education, and using anthropological perspectives.

The task of uncovering mathematical features of cultural practices, at the center of ethnomathematical research, can be done by digging into the past or into the present. The former option paves the way for archeology and historiography, while the latter option is interested in studying current cultural practices performed by communities of diverse kind. Therefore, it is common for ethnomathematics researchers to use tools coming from social anthropology, sociology, and the area of cultural studies. This paper focuses on this last option, due to the complex social and political interactions that occur when living communities, their practices, and forms of knowing are researched by ethnomathematicians. The meeting of cultures is always a potential field of power, particularly when a highly-valued form of knowing in Western culture such as mathematics is at the center.

Ethnomathematical research aims at valorizing new understandings and recognitions about mathematical knowledge, as Gerdes stated: "ethnomathematical studies may broaden the (intercultural) understanding of what are mathematics, of what are mathematical ideas and activities. There cannot be a sole, unified view of mathematics" (Gerdes, 2001). Developing that idea, there is a strong need to formulate ways to relate and interact with communities. Therefore, communication arises as an important issue. Some researchers consider it explicitly, with notions such as dialogue (Vergani, 2007), respect (Powell and Frankenstein, 2006), or mutuality (Adam et al., 2010); while others deal with it in a tacit way. Indeed, all of them make methodological decisions about the participation of the researched community and how that participation will be considered and registered in the communication of the research (inside or outside academia). As D'Ambrosio stated:

> Ethnomathematics fits into reflection about de-colonization and the search for real possibilities of access for the subordinated, the marginalized, and the outcast, or excluded. The most promising strategy for education in societies that are in transition from subordination to autonomy is to restore dignity to their individuals, recognizing and respecting their roots (D'Ambrosio, 2006, p. 30).

Then, certain sensitivity to the researched community agency and needs is of paramount importance. This paper aims to illustrate the potentiality of the notion of *symmetry*, as an analytical tool for ethnomathematics, particularly for the type of research devoted to study (ethno)mathematical practices in commu-

nities of practitioners or socio-cultural groups. Independent of the vast diversity of purposes and theoretical approaches currently existing in ethnomathematics, as evidenced by Alangui and Shirley (2017) and Peña-Rincón et al. (2015), the research with living communities is distinguishable from other types of research, due to the unavoidable challenge that taking a series of methodological choices on the interaction with the communities or groups involved in the study represents. Such choices require considering values, dynamics, participation, and expectations proper to the communities or groups.

This paper focuses on the following research question: *how ethnomathematical research developed in, or with, living communities has devised the roles of communities and researchers*? To answer this question, a theoretical stance is presented, introducing the concept of symmetry. Then, an analytical strategy is proposed for a review of previous ethnomathematical studies, looking for regularities within theoretical or methodological decisions. The use of symmetry allows a characterization of ethnomathematics research that differentiates paradigms about the political and epistemological vindication of non-hegemonic knowledge that ethnomathematics promotes. Such characterization, its contrast with the already referred aims of ethnomathematics, and further implications for ethnomathematics theorisation are presented as conclusions.

6.2 A theoretical motivation for symmetry

Ethnomathematics is recognized as a program of research in the history and epistemology of mathematics (D'Ambrosio, 2006). For this paper, the object of study of epistemology of mathematics is not knowledge as regular entity that exists in a philosophical void, but knowledge as resulting from systems of practices working in concrete places, where actual subjects make assertions. Such systems correspond to historical, geographical, cultural, and political conditions. Accordingly, not only should ethnomathematics cover an examination of the knowledge and practices that count as mathematics, but also encompass the procedures and instances of enunciation that allow some knowledge to be typified as mathematical. Such approach intends to give new insights to the recurrent discussion about the "ethnomathematical paradox" pointed by Millroy (1992): to use the "lenses of a mathematical observer" is problematic to analyse practices that are not originally based on western mathematics. That would imply that the entire ethnomathematical endeavor is circumscribed to find only emulations of western mathematical knowledge. Important critiques and counter-critiques to ethnomathematics have debated that paradoxical issue (Knijnik, 1996; Pais, 2013), expressing concerns about the constraints and contingencies of the academic observation.

Nonetheless, the debate could be reduced to a claim for reflexivity (Salz-

man, 2002; Woolgar, 1988): "check *your* lenses," "research *your own* research," or "be aware of *your* standpoint". Such claim is ultimately solipsistic and does not alter the relation between observer and observed. To expand the epistemology of mathematics requires going beyond reflexivity, especially when living communities are researched. Such move towards a broader series of concerns is what I would call the problem of *symmetry*.

The issue with ethnomathematics might not just be the use of mathematical lenses by outsiders, but also the assumption that insiders do not have their own ways to observe and analyse practices; or even more, the assumption that it is not possible to establish interchanges or interactions between those approaches. Thus, it becomes central to problematize how the involved subjects in the ethnomathematical field (both scholars and practitioners) position themselves with respect to each other through the practices of research, recognizing or neglecting degrees of legitimacy that, therefore, allow or constrain the presence of multiple voices in the research. I am arguing that, instead of limiting the problem to reflexivity, ethnomathematics research can be problematized also around the issue of *symmetry*. Symmetry as a problem of research is the study of how the participation of both researchers and practitioners has been conceived, unfolded, enacted, registered, and assessed in research. In other words, the problem of symmetry is the recognition that the researched subject uses his/her own lenses and that researched people can also have research interests.

Anthropology and sociology have explored alternative ways of doing fieldwork to address similar problems, with approaches such as collaborative ethnography, participatory action research, autoethnography, public ethnography, and others (Denzin and Lincoln, 2011). All of them respond in different ways to the challenge that Woolgar (1988) identified in the work of Clifford about a "dispersal of ethnographic authority in the sense that both researchers and natives be recognized as active creators (authors) of cultural representations" (Woolgar, 1988, p. 25).

A classic assumption of ethnographic fieldwork is encapsulated in the phrase "being there, writing here", demarcating a clear division between places, subjects, and roles. A non-colonial posture about power and knowledge can question such assumption, acknowledging the rights and the capacities to theorise that communities and subjects have. Elsewhere I have pointed out:

> [Individuals and people] do not only have various types of knowledge, but also have the capability to disseminate them, broaden them and contrast them with the knowledge of others. Indeed, people and individuals have the power to define how, when and where their knowledge can circulate. (Parra, 2015, p. 411)

Symmetry is related to notions of infra-reflexivity proposed for studies in sociology of scientific knowledge, trying to achieve "equal status for those who

explain and those who are explained" (Latour, 1988, p. 175) through an engaged interaction in each stage of the research process. I am aware that the word *symmetry* can raise many doubts because it can be associated with a supposedly friendly ideal horizontality that is often not the case. If we consider the complex power relationships involved in the very act of research, "equal status" must be understood as a way to go, instead of a point of departure or arrival. However, I propose the notion of symmetry to stress the presence of practitioners as actors with voice and agency.

Additionally, symmetry can subsume notions like dialogue, respect, and mutuality, which are used by ethnomathematicians with very diverse meanings when they explore possibilities of partnership, collaboration, and reciprocity with practitioners in the ethnomathematical observations and representations. Symmetry also explores the diverse ways in which researchers assume their responsibility towards the people researched and in which ways (and to what extent) researchers became accountable for the experience that they generate when researching.

In this paper, I assume that a singular piece of research does not only express the personal intentions of its direct developers but also a shared set of truths and beliefs, that includes rules to follow, practices to perform, words to say, and ideas to understand within the research field[1]. That set comprises what can be accepted in a specific space-time setting as a research experience.

Consequently, to scrutinize symmetry in an ethnomathematical research project, it is necessary to conceive such project as a collective experience, including the convergence of multiple factors (personal, historical, social, and academic) that made possible and/or constrained the experience, in addition to its inner methodological construction, the theoretical approaches invoked, and the conclusions drawn. Moreover, subsequent activities of researchers and communities after the publication of the experience can also be observed because they allow the study of the positioning of each agent about symmetry. Procedures, subjects, and instruments to conduct a study of symmetry in ethnomathematical research are described in the next sections.

6.3 One strategy to analyse symmetry

To elucidate comprehensions developed in ethnomathematical research about symmetry will allow us to answer the research question. To do so, it is important to characterize the kinds of interplays that have been established among researched communities (insiders) and the ethnomathematics researchers (out-

[1]Of course, the individuals are not helpless operators in that set; they can strength it with their actions, change some components, or eventually they can deflate its importance as paradigm, accordingly to the awareness and consciousness that everyone has about the ways that set operates. Studies like the intended here try to provide such awareness.

siders). Therefore, it is necessary to delimit a corpus of analysis and describe a method that will allow us to produce the intended characterization.

For the matter of this study, the empirical material I considered is a corpus of four doctoral research experiences that had dealt with primary sources, because such condition implies relationship with actual communities of practitioners, allowing the register of real enactments of the practitioner's perspectives; the subjects identified as "we" or "they" are present, having flesh, bones, agency, and interests. The selected reports vary broadly in the date, language, place of development, and place of publication, to stress the fact that the focus is to study the assumptions that circulate within the field and not on specific researchers or research centers. When presenting each case, the criteria for selection will be made evident.

As I am looking for evidences of the enacted comprehensions about symmetry within ethnomathematics research, a content analysis method was developed to study the empirical material. Specifically, elements from Semantic Network Analysis (Krippendorff, 2004) are considered to extract theoretical concepts from the corpus and represent them (and their interrelation) as a network of objects. According to van Atteveldt (2008), this method has advantages over other forms of content analysis because it makes a clear separation between extraction phase (depending on a domain context) and a querying phase (bounded by a research context).

The data extraction was made by reading all the texts[2] and coding their paragraphs and excerpts, according to a list of categories related to the main notion of symmetry. The list and its theoretical relevance are presented in the next section. The coding was made manually and aided by a Qualitative Data Analysis Software[3], following the rule that each excerpt could be linked to several concepts of the list, or to none. This implies that our coding was about occurrences and co-occurrences of theoretical categories, instead of the usual word frequencies, a popular but criticized choice in Content Analysis (Roberts, 1997, p. 147) and data mining. This strategy for the data extraction makes our study a mixture of quantitative and qualitative content analysis that privileges the *latent* content (van Atteveldt, 2008, p. 18) of the text, by allowing the coder to include his/her interpretations. Although such decision can be problematic for other type of data analysis (for instance, studies of grammatical structure, text analysis or studies with a big corpus), I consider it appropriate for the research purposes to capture conceptual assemblages and discourses about symmetry.

[2]In addition, the authors were interviewed to complement the reading, clarify some episodes and decisions in the research experience, to locate the researcher before and after the experience, and mainly to know about *the continuity of the relationship between researchers and groups after a publication of the research.* It was out of my scope to contact the researched communities for a different perspective.

[3]Atlas.ti for Mac, Version 1.6.0.

To decrease the level of subjectivity that the strategy can comprise, I decided to validate the coding process. A document for each category was generated, compiling all the excerpts labeled with the category. Two researchers were asked independently to read those documents and to determine if the excerpts of each document have any resemblance and to establish what it could be. Documents had no title or any specific indication of the category that was attempted to describe. The explanations provided by both scholars met the main idea of all concepts. In addition, scholars identified some excerpts that did not coincide with the others enough in the same document. Those excerpts were eliminated or recoded.

After the coding validation, associations among categories were represented with networks and the querying started. For this phase, software and algorithms for social network analysis (SNA) were used to identify assemblages of concepts and within the text, emulating the analytical methods proposed by Paranyushkin (2011) to identify meaning circulation in texts. Joint analysis of the association was not fruitful to elucidate correlations, due to the diverse number of excerpts selected in each thesis. Therefore, it was decided to do an independent analysis for each case. After running algorithms under different conditions, communities were identified in the network, allowing us to state some correlation patterns among theoretical standpoints and methodological choices about symmetry in ethnomathematics research. Those patterns are presented in the results.

6.4 Cases

In this section, I will briefly present the selected cases, indicating the researcher's career and some relevant details about the relationship with the communities, before and after the research experiences that they conducted. This presentation explains the relevance of each experience to the present study and its research question; therefore, contextualization is not made in the interest of focusing on the researcher as a person but of knowing how her/his decisions concerning symmetry unfold in the research experience.

The first experience (hereafter Case 1) about carpenters in Cape Town, South Africa, was conducted by Wendy Millroy during six months in 1992. Inspired by the framework of everyday cognition presented by Lave (1988), this scholar wanted to "document the valid mathematical ideas that are embedded in the everyday wood working activities of a group carpenters. A secondary objective was to examine and to give a firsthand account of the teaching and learning of mathematical ideas in the context of the researcher's apprenticeship" (Millroy, 1992, p. ix). She did not publish on ethnomathematics after the thesis. This research was selected because it is one of the classic works in ethnomathematics, being the first that problematized the role of the researcher

in ethnomathematics, and the first that took a position about that role, when considering the paradoxical condition of being looking for something essentially academic in non-academic contexts as insurmountable.

The second experience (hereafter Case 2) was conducted by Wilfredo Alangui with a group of Igorot people in the Cordillera region in northern Philippines. This Philippine scholar obtained a B.Sc. and M.Sc. in Mathematics, while he worked as a mathematics teacher in the University of Philippines. His ancestors by paternal line belong to a group of Philippine indigenous people (Igorot), and he located himself prior to the research experience in the process of appreciating and understanding his own identity as an indigenous Igorot. As ethnomathematics offered Alangui a good workspace for his motivations and interests as an indigenous activist and mathematician, he decided to carry out a doctoral study in Auckland University, New Zealand, into "a critical dialogue between mathematical knowledge and knowledge embedded in cultural practice, through the elaboration of mutual interrogation as a methodological approach for ethnomathematics" (Alangui, 2010, p. 11).

To show how such critical dialogue might be done, Alangui researched into the practice of rice terracing agriculture by a group of Igorot people. Therefore, we can see how the author had a personal non-academic interest in the research. Through the notion of *mutual interrogation*, Alangui adopts an overt standpoint in favor of the exploration of twofold relationships among academic researchers (mathematicians) and communities. Such open stance was an important reason to include this research as part of corpus. In addition, this is one of the very few experiences in the ethnomathematics field that goes beyond classic ethnography. Also, in this research the researcher cannot be considered an outsider. Alangui made several fieldwork visits during a period of 8 years, defending his thesis in 2010 and after that he became professor at the University of Philippines and visited the communities after the defense. He published one more paper about mutual interrogation.

The third doctoral research considered (hereafter Case 3) gathers two fieldwork experiences with living communities, one with wood sculptors in Mozambique and other with a Brazilian community of descendants of German settlers, known as *teuto*-Brazilians. The researcher was Ursula Rohrer. This scholar came with a solid background in physics (B.Sc.) and mathematics (M.Sc.), and her first experience in ethnomathematics was her doctoral thesis (Rohrer, 2010), presented to the University of Bielefeld, Germany. The research attempts to provide resources for a theorisation of ethnomathematics as an interdisciplinary theory. To exemplify her proposal, Rohrer made the two experiences. She interviewed sculptors in Mozambique, trying to answer questions like "Which mathematical means and tools do Makonde artists use and which mathematical knowledge can be revealed in their practice?" (Rohrer, 2010, p. 15); "Is it possible to find golden sections within their final sculptures, even though there might be no prior knowledge of this term as such?" (Rohrer, 2010, p.

149). Ursula did not know the communities beforehand and her contact with them was only for the purposes of the thesis. She has published several papers (Rohrer and Schubring, 2011, 2013) about ethnomathematics theory but not more about those communities. This experience was selected because it has two fieldwork experiences and for its attempt to theorise ethnomathematics.

The last experience (hereafter Case 4) was conducted by Adailton Alves da Silva with the indigenous people $A'uw\tilde{e}\backslash$ Xavante in Brazil, published in 2013. da Silva is a Brazilian scholar that has developed a long-term experience with $A'uw\tilde{e}\backslash$ Xavante communities since 1998, when he begun to work as an advisor in educational projects for these Indigenous group. Only after 7 years of interaction and sharing, he decided to carry out an academic study about their practices in relation to mathematics. In 2005 he wrote his master's thesis about the spatial organization of these indigenous people; then he continued working for another four years and decided to start his doctoral thesis in 2009, in the Sao Paulo State University (UNESP) in Rio Claro, Brazil, devoted to studying the ritual ceremonies of Danhono, a set of the rites of these indigenous people. The research aimed to describe "how the mathematical knowledge(s) and practices of the $A'uw\tilde{e}\backslash$ Xavante are sedimented and articulated in the myths, rites and ceremonies, especially those related to Danhono" (da Silva, 2013). Nowadays, da Silva still works with the $A'uw\tilde{e}\backslash$ Xavante, after 19 years of permanent interaction with these people, now collaborating with them in the production of didactic material and in the design and advice on the secondary education curriculum for the Xavante people. This experience was chosen because its long-term fieldwork and the author's knowledge of anthropological theory.

6.5 Instruments

For analytical purposes, selected research was considered in a double perspective. On the one hand, I assume each research study as a lived experience with communities. As such, it is historically, geographically and politically situated, involving antecedents and consequences for the stakeholders. On the other hand, each research study was considered as a written product, with a plot structuring the narrative. This two-fold perspective produced two independent sets of analytical categories that guided the content analysis. The first set was established by reading the corpus in an *interactive-hermeneutic* process (Krippendorff, 2004, p. 303), thereby identifying notions related to symmetry that can comprise more than one study in the corpus (see Table 6.1, Pag.156). Subgroups of categories were also defined, gathering tensions in symmetry.

The second set was easier to define, as far as beginning, middle, and end can be identified in a plot (see Table 6.2, Pag. 165). Following that idea, *Beginning* comprises text sections where important research elements are defined, like introduction, justification, demarcation of research problem, objectives, and

theoretical framework. *Middle* includes sections where tensions arise among the elements explored in the beginning; this covers methodology, fieldwork, data, and data analysis. Consequently, *End* registers the ways in which tensions lose intensity and conflicts are resolved, dissolved, or postponed; this includes conclusion and appendix.

Using both sets, a content analysis was conducted as follows: texts received further readings, in which each paragraph was examined according to the sets of concepts; excerpts were labeled with the categories of the first set that they express or illustrate. Each selected excerpt was also marked depending on the part of the plot where the excerpt was located. Qualitative data analysis software was used to register the coding.

Subgroup	**Analy. Cat.**[4]	**Descriptor**	**Example**
Before fieldwork experience	Input	Account of the research experience antecedents, prompting its conditions of possibility.	(Case 4): "we can also say that the present research is, in a way, the continuity of the experience developed in the masters and, in this sense, a deepening of theoretical and conceptual questions that remained open and that demanded a greater time of investigation and maturation of previous readings" (da Silva, 2013, p. 17, my translation).
	Arrival	In which ways the empirical experience was started? how researchers entered in contact with communities?	(Case 1): "I visited several furniture factories and restoration workshops, but nobody was prepared to accept me as a learner" (Millroy, 1992, p. 72).
	Purposes	Tacit or explicit excerpts about research purposes for the fieldwork experience in/with communities.	(Case 2): "How to conduct ethnomathematical research with integrity is one of the concerns of this study" (Alangui, 2010, p. 4).

[4] Analytical Category.

Posterior to field-work experience	Output	Stating which results were produced by the research, whether academic or not.	(Case 3): "The research pursued in Mozambique has showed that the idealization of beauty given by the mathematical definition of the golden ratio represents an idealization in the Makonde culture as well" (Rohrer, 2010, p. 234).
	Departure	In which ways the empirical experience ended? how researchers finalized their contact with communities?	(Case 1): "He was referring to the fact that I would soon be leaving for the U.S. with my American fiancé. In fact, this paternalistic view made my exit much easier than I had expected. My last day at the workshop was like a pre-wedding celebration" (Millroy, 1992, p. 91).
	Retribution	Excerpts specifying explicitly benefits, reward or retribution of any kind, received by the communities due to the research experience.	(Case 2): "An answer to the question "what kinds of decisions are made for the system to work?" will alert the practitioners to actions and decisions, either individually or collectively, that might ensure the continuation of the practice, or its improvement, even its demise" (Alangui, 2010, p. 181).

Theoretical and methodological concerns on Ethnomathematics that sustain fieldwork	Theoretical Standpoint	Utterances of theoretical nature about ethnomathematics, concerning with the interplay / dialogue / respect with communities.	(Case 3): "Ethnomathematics seeks to revive the mathematics living in different traditions and cultures, not by considering them to be exotic, but by including them in the new historiography of mathematics" (Rohrer, 2010, p. 84) (Case 4): "Ethnomathematics is not only a research program, but also assumes a philosophical stance in which its motivating principle is to establish a bridge of understanding, respect and active tolerance" (da Silva, 2013, p. 22, my translation).
	Methodological Disquisition	Statements of theoretical nature about methodology.	(Case 3): "Gaining the people's trust requires constantly showing honesty, verbally and nonverbally; once this bond of trust has been established, the researcher will be allowed to enter and discover all secret symbols and cultural knowledge, thus augmenting the quality of the collected data" (Rohrer, 2010, p. 96). (Case 2): "An important part of the process of critical dialogue is the reflection on the outcome of the interrogation – what it means to the cultural practices, and what it tells us about mathematics" (Alangui, 2010, p. 163).

	Relational approach	Theoretical standpoint in the ethnomathematical field in which the researcher look for possible or plausible interactions and links among cultural practices and mathematics. A detailed description of this approach can be found in Parra-Sanchez (2017).	(Case 2) "How to relate these QRS concepts and ideas to conventional mathematics (and what they mean to mathematics) is at the heart of the ethnomathematical research. This is a difficult task, especially in the absence of external criteria that can be used to admit a concept or practice as mathematics, or mathematical" (Alangui, 2010, p. 185).
	Intersection approach	Researcher position himself as the one who can identify the mathematics embedded in a cultural practice. A detailed description of this theoretical standpoint can be found in Parra-Sanchez (2017).	(Case 1): "Data are described and analysed to provide evidence that valid mathematizing is embodied in the everyday activities of the group of carpenters with whom I worked" (Millroy, 1992, p. 93). (Case 3): "We were able to recognize many mathematical tools used in the everyday activities that are related to the growing of grapevines." (Rohrer, 2010, p. 230).

Ways of participation and interaction allowed to communities during fieldwork.	Mutuality	Excerpts registering community agency within the research, this includes actions specifically done for sake of the research or reflections about the research itself. This includes common actions and explicit quid-pro-quo strategies.	(Case 2): "Aware of the danger of misrepresentation, I brought the models back to the cultural practitioners. These succeeding dialogues were critical in the interrogation process as they allowed me to re-view the models that were developed" (Alangui, 2010, p. 183). (Case 4): "We took advantage of this space [the Warã] to dialogue with the community about research and, consequently, to "be accountable" for our insertions and, especially, to hear from the community, mainly the elders, suggestions and guidance in order to follow this path" (da Silva, 2013, p. 53, my translation).
	Collaboration	Recognition of forms of joint work outside the research project. they work together in other things.	(Case 4): "This influence/sedimentation was possible, for instance, in a Pedagogical Workshop with the teachers when we were developing an activity related to the concept of "numerical sets" that, consequently, also required to discuss the concept of classification" (da Silva, 2013, p. 312, my translation).

	Respect	Explicit authorial declaration about his/her standpoint towards symmetry issues. This includes definitions of and comments on notions like respect, mutuality, reciprocity, dialogue, valorization, dignification, alterity, otherness.	(Case 1): "To find a way to give a voice to some of those silences, a voice in which the carpenters' own voices could be heard and valued" (Millroy, 1992, p. 13). (Case 2): "Another role of the ethnomathematician is to communicate whatever outcome or reflections that may result from the critical dialogues, especially to the practitioners of both the cultural practice and of mathematics" (Alangui, 2010, p. 191). (Case 3): "To Barton, "ethnomathematics does create a bridge between mathematics and the ideas (and concepts and practices) of other cultures." How effective are these bridges, *i.e.*, how much information is returning to the research subjects? and, how much can they learn from it?" (Rohrer, 2010, p. 72).

Discussions of the problems of representation that a fieldwork entails.	Decentering	Episodes where the researchers make evident the limits of western/academic knowledge to express the cultural practice, appealing to a broadening to the conventional epistemological view of mathematics. Recognizing the existence of diverse ways of knowing the world.	(Case 1): "I claim that the evidence provided in the present study suggests that mathematical knowledge can be implicit in physical actions. Acknowledgement of this claim requires a shift in the accepted epistemology of mathematics" (Millroy, 1992, p. 185). (Case 4): "Within the socio-cultural context of the $A'uw\tilde{e}$\Xavante people, particularly in the act of celebrations of the rites and ceremonies of the Danhono, the conception of "more" and "less" is not mainly quantitative, as it is seen from the Euro-western conception of mathematics" (da Silva, 2013, p. 397, my translation).
	Millroy paradox	Excerpts expressing inherent limitations in researcher's point of view to perceive things that researched people can consider as relevant. Cultural practices that could being labelled as "mathematical" by the communities, but not for the researcher.	(Case 2) "The danger is that something that appears unfamiliar, may simply be misunderstood, or aspects of the subject may have gone unseen. This is a particular hazard when extracting the "mathematical" from something that is a practice embedded within a different knowledge framework" (Alangui, 2010, p. 66). (Case 1): "How can anyone who is schooled in conventional Western mathematics "see" any form of mathematics other than that which resembles the conventional mathematics with which she is familiar?" (Millroy, 1992, p. 11).

	Jourdain effect	Cultural practices being labelled as "mathematical" by the researcher, but not for the communities.	(Case 1): "I recognized from my academic mathematics training that these basic ideas underlie the concepts of vector space" (Millroy, 1992, p. 176). (Case 4): "When the bowstring is flexed and tied at the ends, the hunter is also defining the amount of potential force (Fp) that will be used - after being transformed by elastic force (Fe) into kinetic energy" (da Silva, 2013, p. 269, my translation).
Actions performed by the researcher regarding Symmetry	Limitations/ Shortcomings	Declaration about the limitations in the fieldwork or in the analysis, due to community decisions, researcher identity or unexpected situations.	(Case 1): "It was sometimes difficult for me to work with the other carpenters at a time when they were doing something interesting. I had a responsibility towards MrS as my special teacher and respected his allocation of tasks to me" (Millroy, 1992, p. 193). (Case 3): "I experienced great difficulties to pursue this fieldwork since I had to go to Friburgo and introduce myself on my own." (Rohrer, 2010, p. 231).
	Methodological Decision	Utterances that describe methodological decisions taken in the research, that can be related with symmetry.	(Case 3) "After a long debate, I decided to abandon the idea of working with him because I was not willing to pay what he wanted to receive for having me looking at his work" (Rohrer, 2010, p.155)". (Case 1): "I began to build a social network of people who might be able to introduce me to carpenters" (Millroy, 1992, p. 72).

	Trust Evidences	Evidences of how bonds of trust were built with the researched communities.	(Case 4): "After many invitations and insistence on the part of the Wapté, I decided to take part in the Noni race. On that time, I had to compete with the Wapté of the *Öwawẽ* clan because, according to the elders, I belong to the Poreza'ono clan" (da Silva, 2013, p. 14, my translation). (Case 3) "He invited me to his community to get acquainted with the Makonde women living there. I had the opportunity to spend a whole day with them and it was fascinating. And I am sure that I was able to experience this because I showed so much respect and admiration to him, to his people and to his profession" (Rohrer, 2010, p. 171).
	Time	Excerpts describing the amount of time dedicated to determined tasks in the research experience.	(Case 1): "We arranged that I would work at the workshop on a daily basis for the next five months, for six hours a day, starting the next day. I decided to reserve the latter part of each day for writing field notes at home" (Millroy, 1992, p. 75). (Case 2): "I came back to the village in the last week of April. The following months were spent mostly in the field, except for occasional trips to nearby Bontoc and Sagada, and Baguio City. I stayed in the village until the middle of July 2002, after which I decided I would wind up my fieldwork" (Alangui, 2010, p. 94)

Other issues	School Application	Applicability of research in school environments.	(Case 4): "This is perhaps one of the main curricular adjustments to be discussed in the context of school education, that is, to discuss the temporal/spatial issue of socio-educational processes of culturally distinct peoples." (da Silva, 2013, p. 323, my translation).

Table 6.1: First set of analytical categories.

Category Name	Descriptor
Order-Beginning	Introduction, justification, demarcation of research problem, objectives and theoretical framework.
Order-Middle	Methodology, fieldwork, data and data analysis.
Order-End	Conclusions, final remarks and appendix.

Table 6.2: Second set of analytical categories.

6.6 Data analysis

Once the coding was done, a study of co-occurrences of categories was accomplished to expose the assemblage of antecedents, constraints, theoretical standpoints, methodological decisions, and outcomes of each study. A weighted network was chosen to represent the assemblages in a proper manner, modeling analytical categories as nodes (vertices) and the number of co-occurrences between two categories as the weight of the edge that links them. However, this is not a mere visual representation; concept entanglement can be detected by using tools from social network theory and graph theory, that introduce specific metrics for the collective cohesion and individual relevance of the concepts in the overall assemblage. I will explain those tools in footnotes, when specific inferences need to be raised.

I start presenting the data obtained for each case trying to depict each assemblage of concepts. Different networks were created for each case, a one-mode network (Wasserman and Faust, 1994) using only the first set of categories (Table 6.1), and a two-mode network mixing the first and second set of categories (Table 6.1 and Table 6.2) that can be considered as an affiliation network (Wasserman and Faust, 1994). All networks were plotted as graphs,

using similar parameters: Node sizes representing occurrences, *i.e.* the number of excerpts of each concept within the thesis and the edge width (thin to thick) represents the number (smaller to larger) of co-occurrences between concepts in a thesis. Node colors were distributed according to community detection algorithms[5]. Networks were filtered, leaving the nodes that have lower occurrences and co-occurrences aside. Depictions of filtered versions are in Figures 6.1, 6.3, 6.5 and 6.7. Affiliation networks are in Figures 6.2, 6.4, 6.6 and 6.8.

Case 1

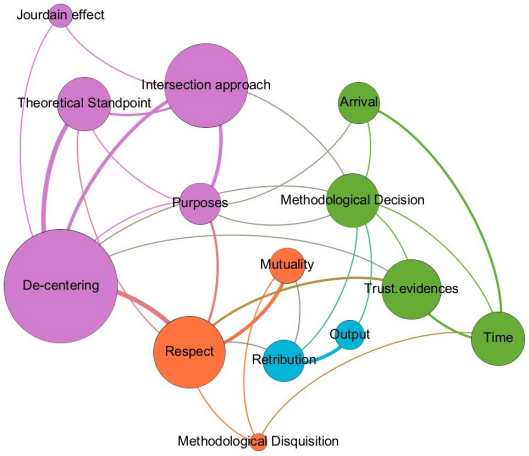


Fig. 6.1: Conceptual network for Case 1.

Contrasting the small network size (14 nodes and 31 edges), its diameter value[6] (3), and its average path length[7] (1.791208791) with a simulation sample of 1,000 random networks with similar characteristics, it can be argued that

[5] Algorithms were Louvain (Blondel et al., 2008) and Spin-glass (Reichardt and Bornholdt, 2006) *m* methods, running separately in *R* and Gephi. In each case the algorithm that produced more communities and higher modularity value was selected (Louvain for Cases 1 and 3, spin-glass for the others). Modularity is a measure that evaluates the goodness of partitions of a graph into clusters. For details about community detection methods in graphs, see (Fortunato, 2010).

[6] A *path* is a route across the network that runs from vertex to vertex along the edges of the network. A *geodesic path* between two nodes is the minimum set of edges one would have to traverse to get from one node to the other. A *geodesic distance* is the length of a geodesic path. Relatedly, the *diameter* of a network is the longest geodesic distance between any pair of nodes for which a path can exist. The diameter of a network is the upper bound of distance between any pair of nodes. This definition and subsequent ones are summarised from Chi (2014), Newman (2010) and Wasserman and Faust (1994).

[7] This measure is the mean of all geodesic distances. It is useful to generate interpretations about the cohesion between the nodes.

this network is a moderately inefficient[8] network in terms of communication among its parts, and it is more degree-centralized[9] than random networks. However, the network (see Fig. 6.1) presents a very good transitivity[10] value, indicating a clusterization among categories that can be neither accidental nor casual. It is also a sparse network, as its low graph density[11] value shows (0.340659341). Four clusters of categories are observed, each one having a hub: *Respect*, *Methodological Decision*, *Decentering*, and *Retribution*.

Decentering is the leading category in this case, not only for its frequency but for its relative importance to the other notions, as can be argued through its high closeness[12] and eigenvector centrality values[13]. Observing the links, it can be argued that *Decentering* is the hub of the main cluster in the conceptual network, gathering other important theoretical categories like *Theoretical standpoint*, *Purposes*, and *Intersectional approach*; this cluster contains two cliques[14] of size four. There is also another cluster, with *Methodological Decision* as a hub, that gathers notions related to the encounter with the researched communities (*Time, Arrival and Trust*). It is also interesting to see the role of the category *Purposes*, acting as a junctor among the clusters; this is

[8] Only 63% of a sample of simulated networks had a smaller average path length.

[9] *Degree centrality* is a node-attribute defined as the number of ties a node has. For our study, *degree centrality* indicates how many categories the author conceives as directly connected with a specific category. Degree-centralization is a network-attribute that measures the percentage of variation of the nodal degree centrality in the network. A centralized network will have many of its links dispersed around one or a few nodes, while a decentralized network is one in which there is little variation between the number of links each node possesses. When running the simulations, 37% of the sample presented a degree centralization equal or greater than the network of reference.

[10] Transitivity (also known as the mean clustering coefficient) is a network-attribute that measures the probability that the adjacent vertices of a vertex are also connected. This measure can also be calculated at node level, then being called *clustering coefficient*; it indicates if there are some parts in the network denser than other parts and suggests an actor's prestige in his/her ego-network. The value of transitivity obtained for this network was 0.560544218, being higher than 99.2% of the simulated networks.

[11] A graph is considered *complete* if each node is connected with all the others (*i.e.* all possible edges are present). Density indicates how close a network is to be complete, by measuring the fraction of the maximum possible number of edges that are actually present. The more size a network has, the more difficult it is to be dense.

[12] *Closeness centrality* is a node-attribute, defined as the inverse of the sum of the geodesic distances between one specific vertex and every other vertex in the network. It measures the extent a node is near all other nodes in a network (directly or indirectly). Closeness reflects the ability to access information through the "grapevine" of network members. For our study, closeness centrality of a specific category indicates how much conceptual elaboration is needed to link that category to the others.

[13] *Eigenvector centrality* is a node-attribute that measures the prestige of a node in terms of the prestige of the nodes that it reaches directly. As this is a recursive definition, the value is obtained with a process that involves eigenvalues and eigenvectors of the adjacency matrix of the network (for our case, the matrix of co-occurrences). In this study, eigenvector centrality of a specific category indicates how related to the most used categories it is.

[14] A *clique* is a subgraph completely connected. It provides information about special groups within the network.

confirmed by its high betweenness centrality value[15]. I interpret this centrality and community detection measures as evidence that concerns about symmetry are of theoretical nature (*Intersection approach* and *Theoretical Standpoint*) and are being articulated about the problems that a researcher can face when attempting to displace the academic perspectives to describe and study cultural practices (*Decentering*). This is consistent with the research goals of "document mathematical ideas embedded in the everyday wood working activities" or "examine the teaching and learning of mathematical ideas during apprenticeship," and it is also related to the famous paradox of the impossibility to achieve a perspective different from the one that privileges disciplinary mathematics.

This interpretation can be reinforced by observing the connections that the category *Respect* has. As stated in the definition of categories, *Respect* traces the direct and explicit mentions to symmetry. It is strongly influenced by *Decentering* (more than 35% of their entries have co-occurrences with this category, and 70% with categories in the theoretical cluster) and has no connections with *Methodological Decisions*. A third element supporting this interpretation appears when looking at the second set of categories and exploring the plotting of the document with a different network.

Fig. 6.2 reveals how *Decentering* is mainly problematized out of the encounter with the community, being initial statements before the fieldwork (assumptions) or after it (reflections and conclusions). Even more, the overall cluster leaded by this concept has the same behavior. The smallest cluster is almost fully deployed around the middle of the plot. Therefore, the two clusters identified in Figure 6.1 are still visible in Figure 6.2.

Interestingly, categories like *Respect*, *Retribution*, *Trust.evidence*, and *Time* appear more relevant in the final sections, suggesting that the researcher, after the contact with the community, appreciates more the issues of symmetry. As part of the conclusions, she considers it necessary to reflect specifically on those issues. Thus, *Respect* emerges as a theoretical conclusion for this author in the experience and a kind of response to the initial purposes of problematizing the decentering in ethnomathematics. This is not a minor thing, if we put the experience in the context of its time, 1992, when ethnomathematics was just starting its development as a research field.

Case 2

This is the most connected and dense network of the overall study (14 nodes and 55 edges) with a diameter value of 3. The density value 0.604395604

[15] *Betweenness centrality* is another node-attribute, defined to be the number of geodesic paths that pass through a specific node. It measures how often a node appears on the geodesic paths between the other network nodes. Betweenness centrality of a vertex indicates its influence over the flow of information between others. For this paper, I interpret this measure as an index of intermediation, or how much a category is needed to articulate categories that are not connected themselves.

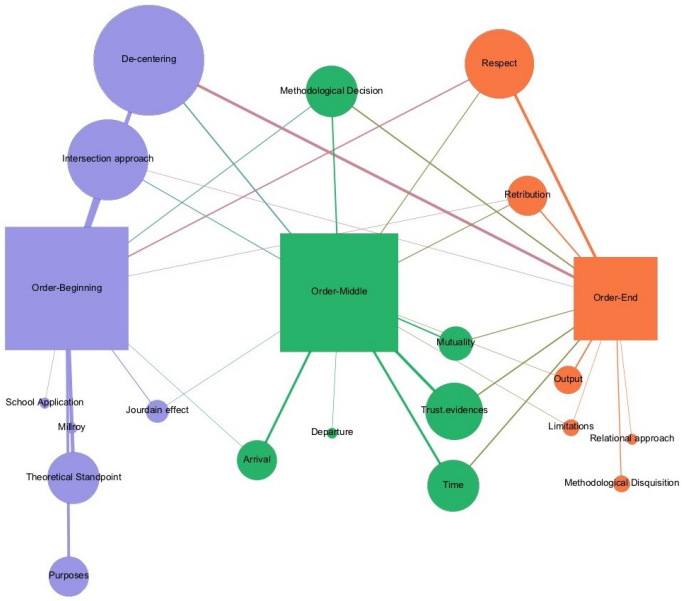


Fig. 6.2: Affiliation network for Case 1. In this conceptual network, nodes were colored according to the part of the text in which have more entries. Width of the edges is proportional to the amount of entries.

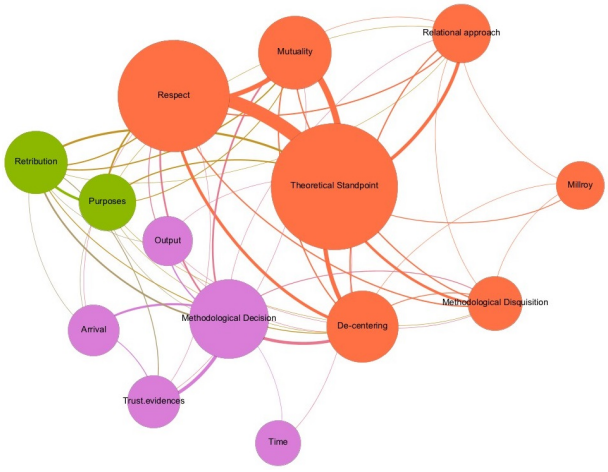


Fig. 6.3: Conceptual network for Case 2.

is high. Those values mean that the network is dense, and almost complete among the most used concepts. Looking at the network efficiency to connect its parts, an average path length of 1.417582418 and degree-centralization value of 0.241758 are very typical values, if compared with networks of similar dimensions[16]. Clustering coefficients are high[17], reinforcing the idea of density and the presence of a defined pattern in the assemblage that does not correspond to a random behavior.

Three clusters can be identified, being *Respect* a hub for one cluster and *Methodological Decision* for another cluster. Very similar to the first case, the biggest cluster gathers theoretical categories (*Theoretical standpoint, Relational approach*, and *Methodological Disquisiton*) with concerns about interaction (*Respect*, *Mutuality*) and the locus of enunciation (*Milroy* and *De-centering*). The second cluster gathers a set of actions (*Methodological Decision, Trust.Evidences,* and *Time*) with a set of preparatory and posterior concerns (*Arrival* and *Output*). It is also interesting to note how *Purposes* and *Retribution* are tightly connected, forming a third cluster that indicates a clear intentionality of the fieldwork experience. However, it is important to complement these clusters identified with the presence of four cliques of size seven, and the fact that the most used categories almost form a clique of size nine. This means that concepts used in this experience are constantly and consistently interrelated.

Due to the high density of the network, the values for betweenness centrality turn very homogeneous and almost zero. However, other centrality measures can be used to identify key concepts in the experience. In this network, two categories stand out, *Respect*, *Methodological Decision* and *Theoretical Standpoint* because they reach the highest values in some centrality measures (degree and eigenvector centrality). That is not surprising, if their number of entries in the text is considered. Besides the expected relevance of such categories, there are another two that gain relevance: *De-centering* and *Mutuality.*

With the identification of hubs and key players within the network, an interpretation of the standpoint deployed in the experience can be proposed. Symmetry issues are at the core of the research experience, determining the theoretical standpoint of the research itself and the methodological decisions as well. The interconnectedness of categories evidence how carefully articulated the experience was. Symmetry is understood in terms of the mutuality of the actions carried out and it is developed through a series of decentered analytical moves in the methodology. It is important to note how such mutuality in the interaction is part of the purposes of the fieldwork and of the whole research. In

[16]A sample of 1,000 random networks was simulated. Each network has the same size of nodes, edges and, diameter as the network of reference. Although the results obtained show that 91% of the sample has a smaller average path length, the difference is in the order of the third decimal digit; 47% of the sample has the same degree centralization value.

[17]The value of transitivity obtained for this network was 0.723737374, being higher than 99.5% of the simulated networks. It is also the highest value in the corpus studied.

this case, there is not an *a priori* or *a posteriori* recognition of the importance of local perspectives only, but the recognition happens in the making.

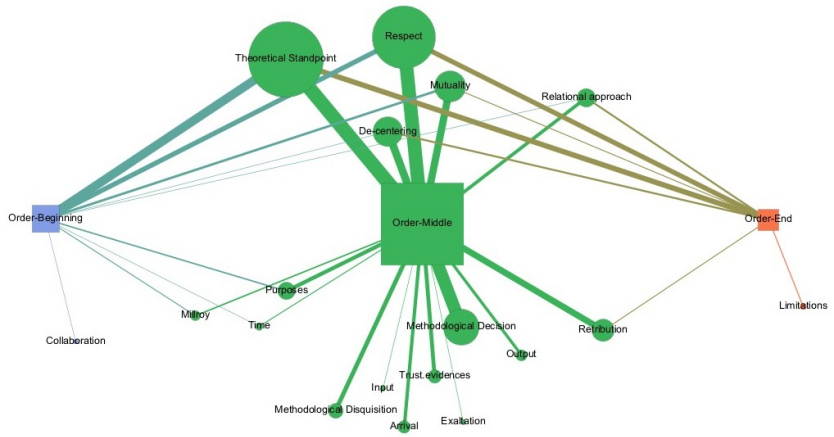


Fig. 6.4: Affiliation network for Case 2. Nodes were colored according to the part of the text in which have more entries. Width of the edges is proportional to the amount of entries.

When looking at the written plot of the experience (Fig. 6.4), our interpretation receives support. Transversal categories in the document, (*e.g.* problematized in every part of the plot) are precisely the five highlighted before. Distinctive of this experience is the saturation of elements in the middle part of the plot, only two categories have not entries in the middle part, and 66% of the text entries were done in that part. This singular pattern shows us that research was purposefully developed to stimulate, enhance, and study community participation. This evident effort to describe the interaction with the community in detail stresses how the theoretical standpoint towards symmetry was unfolded in the fieldwork. The pattern also evidences how organic for the experience was the concern for the symmetry. In this case, *Respect* is mainly in the middle, meaning that for this experience respect is linked to concrete methodological actions more than reflections, an insight that reinforces our interpretation.

Case 3

This is the smallest network of our study (15 nodes and 30 edges) with a diameter value of 3. It also has the lowest density value (0.285714286) consistently; the average path length (1.99047619) is higher than the other networks and higher than 99.9% of a sample of 1,000 random networks with similar characteristics. This combination of values clearly indicates some inefficiency in the communication between nodes. Such insight is reinforced with the compar-

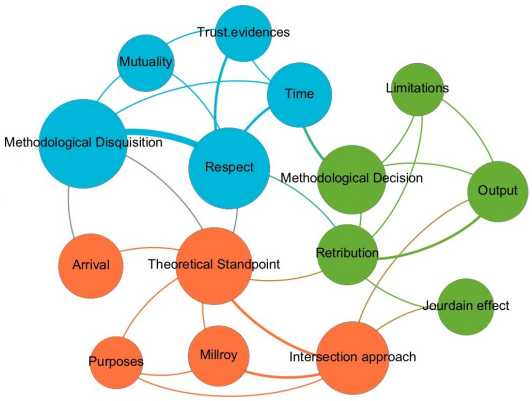


Fig. 6.5: Conceptual network for Case 3.

atively low degree centralization value[18]. Although the transitivity value for this network is the lowest in the corpus, it still exceeds the clusterization that a random network can present by far[19] .

A very important characteristic in this network is that only one of their 30 edges has weight greater than 2, meaning that the relationship between categories is homogeneously weak; this makes it very difficult to claim any clustering or correlations. That is visible in 6.5 when connections between nodes of different clusters became more intense than connections between nodes of the same cluster. In those conditions, the network behavior became determined mainly by the number of connections that a category has (*i.e.* its degree distribution). Considering that, we cannot claim that this network has a main category that articulates the others; instead, we can say that there are certain categories that stand out from the other concepts, when we look at centrality measures. For instance, considering degree, eigenvector, closeness, and betweenness centrality, the categories *Theoretical Standpoint*, *Retribution*, *Respect*, and *Methodological Disquisiton* take the highest scores in both measures. This last result is not surprising, if we consider that the amount of co-ocurrences between *Respect* and *Methodological Disquisiton* are more than double of any other connection in the graph, they are also the most used categories. Further evidence of the (comparatively) lack of cohesion in this network is that the biggest cliques have size 4 and there are only two.

Being aware of the lack of clear leadership in the network, I propose an interpretation about which understanding about symmetry was deployed in the experience. The concern about symmetry was mainly theoretical, aiming to

[18]Almost 69% of the sample presented a degree centralization equal or greater that the network of reference.

[19]The value of transitivity obtained for this network was 0.554603175, being higher than 99.7% of the simulated networks.

find accurate ways to describe cultural practices in mathematical terms. The effort to show how mathematical a cultural practice is became a respectful action, because that effort attempts to validate and appreciate the knowledge of the communities. That explains the importance to determine if the sculptures made by artisans in Mozambique have the golden ratio. The ones that should validate are scholars. As Rohrer states "Thus, ethnomathematics studies ethnomathematical knowledge, but this interpretation is clearly based on what has been agreed upon academic mathematics, its history and education" (Rohrer, 2010, p. 234). This also makes it evident how this author is sympathetic to an intersectional approach to ethnomathematics.

Naturally, in the long run, community members will be benefited for such recognition. Their cultures and their knowledge will not be considered primitive or insufficient, due to the ethnomatematician's effort. Summing up, respect happens when there is accuracy in the representation; respect is assumed as validation and valorization. Such comprehension about symmetry can explain why categories of *Respect* and *Methodological Disquisition* are almost interchangeable (they have the highest number of co-occurrences, and all the main concepts associated with *Respect* were also associated with *Methodological Disquisition*) and why the category of *Methodological Decision* is detached from *Respect* or *Theoretical standpoint*.

Although the researcher in this third case registered some limitations in the fieldwork and recognized the necessity of spending time to inspire trust and confidence with the researched community, the communities that she researched are not her target audience but the academy (as the low frequency of *Retribution* and inexistence of categories like *De-centering* and *Collaboration* can show). This makes her approach theoretical, but in a way different from Case 1.

We can find more evidences for our interpretation within the experience plot (see Fig. 6.6): *Intersection approach* and *Theoretical standpoint* are the only main categories with entries in the three sections of the text. *Methodological Disquisition* entries were not made in the middle, but mainly in the introductory sections. *Respect* is not mentioned in the final section, meaning that the author was conscious about the notions that this category gathers (*Dialogue, Respect, Otherness*), but they were not the focus of their interest for the research conclusions or for the reflections about the fieldwork.

Case 4

In this case, we can observe in Fig. 6.7 four clusters of concepts, three of them having a hub: *Respect*, *Theoretical Standpoint*, and *Trust.evidence*. As in the first case, we can consider this network as inefficient in terms of communication when their main indicators (an average path length of 1.79166667, a diameter value of 3, for 16 nodes and 45 edges) are compared with networks of similar

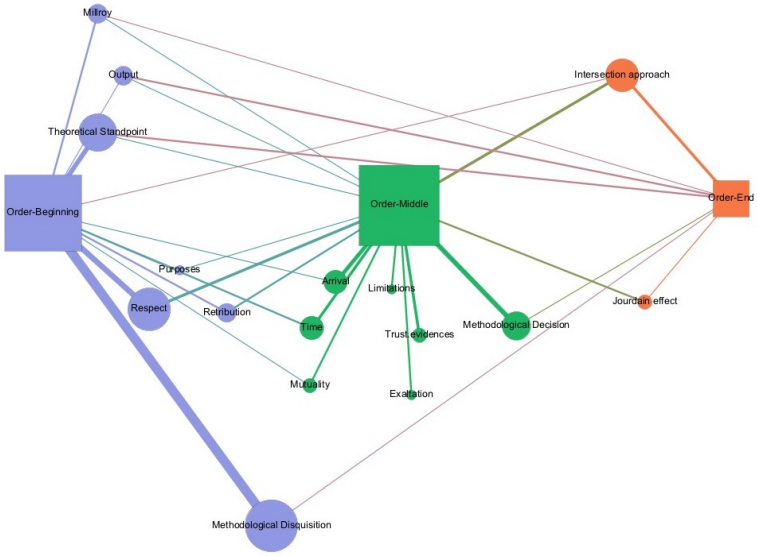


Fig. 6.6: Affiliation network for Case 3. Nodes were colored according to the part of the text in which have more entries. Width of the edges is proportional to the amount of entries.

dimensions[20]. This network is also more degree-centralized than 81% of the sampled random networks; in fact, it is the more centralized network of the corpus, suggesting that concepts gravitate around some special ones, as we will see below. Like the other networks presented, its transitivity value[21] indicates a clear motivation and articulation among the nodes that is not accidental. Although their density value (0.352941176) appears to be not especially high in comparison with the other cases presented, it cannot be considered a sparse network due to its greater number of nodes and the number of cliques of size greater than 3 that can be found (9 cliques).

Although *Theoretical Standpoint* is the most used category, *Respect* has the main role in the network, as it reaches the highest centrality measures (Closseness, Degree, and Eigenvector centrality) and leads a cluster that gathers *Respect*, *Methodological Decision*, *Methodological Disquisition* and *Mutuality*. A second cluster, led by *Theoretical Standpoint* and *Decentering*, gathers categories of theoretical concern (*Relational approach* and *Jourdain Effect*). There are two more clusters, including categories like *Trust.evidence* with *Arrival*, *In-*

[20] More than 98% of a sample of simulated networks had a smaller average path length.

[21] *Transitivity* (also known as the mean clustering coefficient) is a network-attribute that measures the probability that the adjacent vertices of a vertex are also connected. This measure can also be calculated at node level, then being called *clustering coefficient*; it indicates if there are some parts in the network denser than other parts and suggests an actor's prestige in his/her ego-network. The value of transitivity obtained for this network was 0.560544218, being higher than 99.2% of the simulated networks.

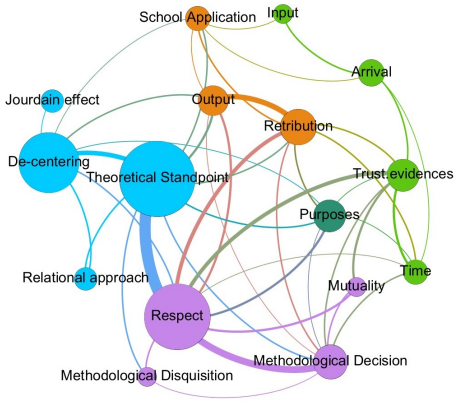


Fig. 6.7: Conceptual network for Case 4.

put Time, *Output*, *Retribution*, and *School application*. Although these clusters have not a clear category as hub, they indicate a clear interest in the community's involvement in the research, whether in the input, the collaboration, and mainly in the resulting retribution.

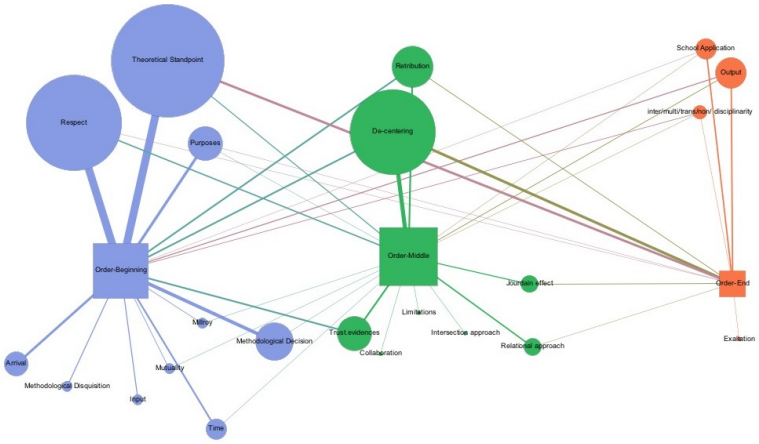


Fig. 6.8: Affiliation network for Case 4, nodes were colored according to the part of the text in which they have more entries. Width of the edges is proportional to the amount of entries

In this fourth case, the conceptual assemblage reveals a great interest in the community world-views, needs and desires, and a conscious and deliberate intention to structure the whole research in a way that community has a prominent role in different stages of the research experience, not only in the data collection acting as informants; this was visible when part of the purposes for the fieldwork experience was established to produce a retribution or a result

that the community members can consider as beneficial for themselves. Such sensitivity means to assume respect through the agency of communities, by using strategies in research that calls for engagement, collaboration, mutuality, and sustaining trust bonds raised through time. Several reasons sustain our interpretation: a) the robust connections that *Retribution* and *Methodological Decisions* have with *Respect*, not only for their high number of co-occurrences, but also because they form the biggest cliques (sizes 5 and 6) with concepts like *Purposes*, *Time*, *Output*, and *Trust.evidences*; b) the presence of categories like *Output*, *School application*, *Retribution*, and *Trust.evidences* as the most used concepts; c) the co-occurrences of categories *Retribution* and *Purposes*; d) the articulating role performed by *Retribution* and *Purposes:* these categories connect all the clusters. Summing up, this case assumed symmetry as the basis for the research experience, by making a series of methodological decisions that aimed to achieve a deep decentering in the analysis and to produce some retribution to the community.

More arguments to defend the plausibility of our interpretation can be found with the second set of categories, especially when analyzing the location of the categories *Decentering*, *Respect*, *Trust.evidences*, and *Methodological decision* in the plot (see Fig. 6.8). Although *Decentering* appears along all the document, it is mainly reported in the middle of the text (12 of 25 entries), this is to say, when the author described fieldwork and provided an account of his interlocution with the community being researched. This fact supports the idea that decentering is not only a theoretical idea to be defended, but also a methodological tool to be implemented when conducting research. While in Case 1 *Decentering* was raised as an open question, in Case 4 it was stated as an answer.

Likewise, we can observe the location of *Methodological Decision*, with 11 entries in the whole text, 10 were made in the initial part; this means that this way of interacting with the community was not accidental or a kind of lucky strike, but intentionally designed by the researcher. In similar manner, it is very illustrative that the category of *Trust.evidence* has half of its entries in the beginning of the text, as evidence of author's engagement and sensitivity in the community dynamics. To finish, the category of *Respect* has 23 of its 28 entries in the initial part; such high proportion allows us to say that Symmetry in this research is not a finding, but a given that explains, motivates, and organizes the whole research enterprise.

6.7 Discussion of results

Although a comparison of the most used categories in the different cases (*v.gr.* Case 1 and 4) reveals that they share almost all the concepts, one cannot imply that they have the same stance on symmetry. Instead, when paying attention to the supposedly slight mismatch a very different interpretation can be drawn,

mainly because of the different intensity of connection between categories. This means, the different ways in which the same elements are intertwined are crucial to understand the positioning of each experience about symmetry.

To make the differences among conceptual assemblages explicit, we will look for a transversal sight of the data, focusing on the categories that surfaced in most of the cases. Due to the different number of entries in each case, we do not attempt to produce a numerical measure; instead we will proceed based on the relation among categories.

Respect

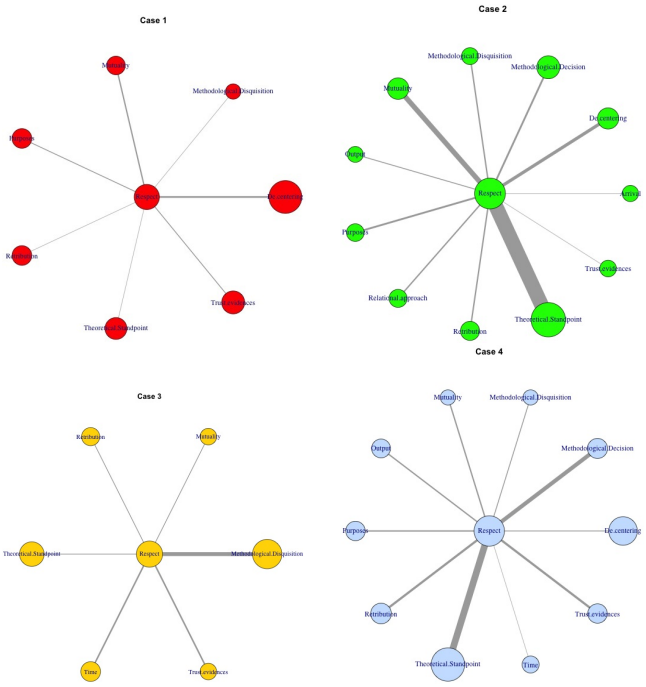


Fig. 6.9: Conceptual network of the category *Respect* for all cases. (Red is for Case 1, green for Case 2, yellow for Case 3, and blue for Case 4). Width of the edges is proportional to the amount of entries.

While each experience makes different explicit mentions of the *Respect*, there is certain commonality in the associations that they made: besides the expected link with *Theoretical standpoint*, all cases relate *Respect* with *Methodological Disquisition*, *Mutuality*, *Retribution*, and *Trust.evidences* (see Fig. 6.9). Therefore, it is generally agreed that community participation is important and deserves to be rewarded; however, there are differences in the degree of importance and the kind of things that are considered as a legitimate reward.

Consistent with previous comments made, we can see how in experiences like Cases 2 and 4, *Respect* relates to many other categories, influence them and is influenced by them; this behavior means for us that these authors share a particular awareness of concerns about symmetry. Such sensitivity allows them to explore and develop multiple possibilities of respect in several aspects of the research process.

Although all authors accept that ethnomathematics needs to develop specific and respectful answers to the problems of cultural encounters (all relate theoretical standpoint with respect); only Case 2 linked *Respect* with a particular approach to ethnomathematics as a field (the relational one). This connection makes the second research experience "special" in the terms of symmetry, because the researcher proposed (and illustrated how) to include symmetry as constitutive part of a renewed ethnomathematics theory.

Methodological decision

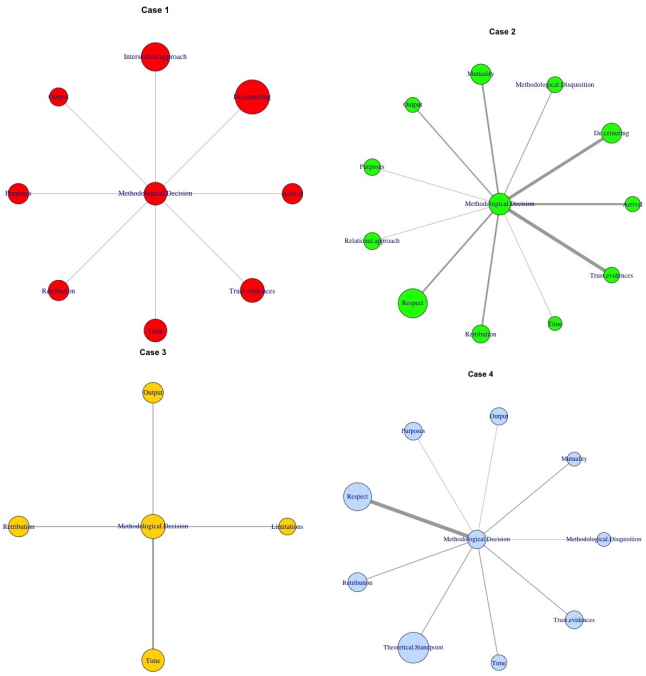


Fig. 6.10: Conceptual network of the category *Methodological decision* for all cases. Colors of cases are distributed as before. Width of the edges is proportional to the amount of entries.

This contrast of the experiences evidences that concerns about retribution for the communities, research results, and time constraints were recognized as influential in the methodological decision of all authors. Notwithstanding that,

other factors were more influential in particular cases, as Fig. 6.10 reveals. Case 4 shows how strong the standpoint was to symmetry (respect) for the decision-making. Instead, in Case 2, the effort to achieve a decentering and trust building are the leading concepts. Other cases do not show a specific tendency; interestingly, Case 3 is the only one that links the methodological decisions neither with the purposes of the fieldwork nor with the building of trust bonds.

Theoretical standpoint

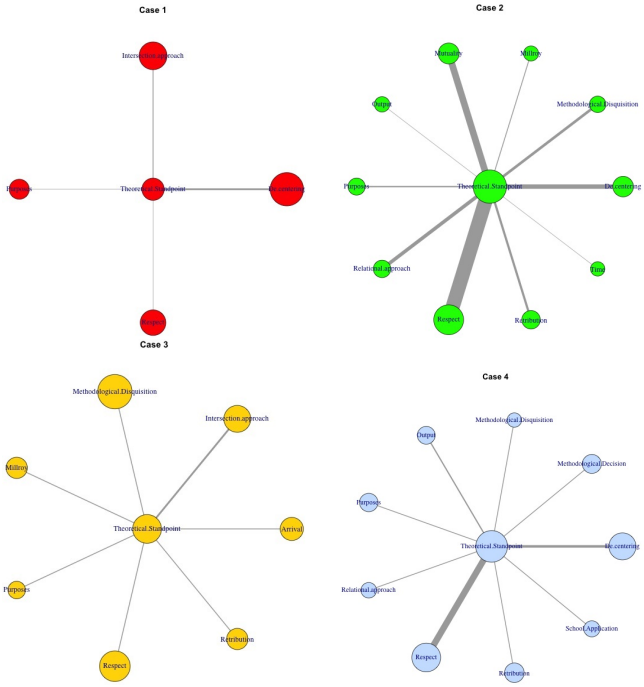


Fig. 6.11: Conceptual network of the category *Theoretical Standpoint* for all cases. Colors of cases are distributed as before. Width of the edges is proportional to the amount of entries.

As expected, this category was connected for all authors with *Respect*, *Purposes*, and *Milroy paradox*, expressing the awareness of all authors of the importance to transcend the referred paradox and at the same time the difficulty of surpassing it(see Fig. 6.11). When comparing this category of *Theoretical Standpoint* (about symmetry) with the categories inquiring about a theoretical approach (on ethnomathematics field), we discovered that standpoints of Cases 2 and 4 are linked exclusively with a relational approach, while Cases 1 and 3 use an intersectional approach.

Decentering

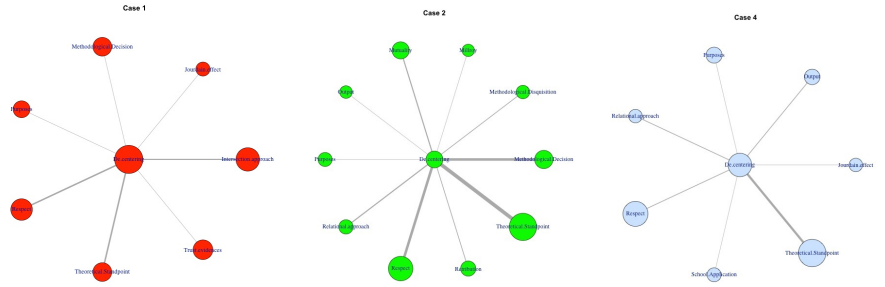


Fig. 6.12: Conceptual network of the category *Decentering* for all cases. Colors of cases are distributed as before. Width of the edges is proportional to the amount of entries. Case 3 did not used this category.

Although it is evident in Fig. 6.12 how this category was driven by the *Theoretical Standpoint* and *Respect* categories, it is interesting to see that *Decentering* was also related to the fieldwork purposes for all authors that used the category.

Purposes

I found in Fig. 6.13 an interesting pattern when trying to establish the relation between symmetry concerns and the reasons to do some fieldwork for research: all the authors adduce theoretical interest and they make explicit mentions of the theoretical approach that they prefer for ethnomathematics. It is also important to note that *Retribution* was not related to *Purposes* for those cases which stand for an intersectional approach; in addition, they were also the cases with sparse networks.

Retribution

Analysis of this category is twofold; it is not only useful to dig into the types of thing that are assumed as retribution, but also for the instances in which a retribution for the communities is considered. A common feature in all cases is to conceive retribution as resulting from *Methodological decision*, that expresses *Respect* for the communities and it can also be counted as a research *Output* (see Fig. 6.14). However, some differences drew our attention; for instance, to produce some *retribution* was considered a way to build *Trust* only in two experiences (Cases 2 and 4). Interestingly, to produce a *Jourdain effect* is considered in Case 3 as *Retribution*, being very consistent and coherent with author's understanding of symmetry and respect. Case 2 also confirms author's stance when a relational approach of ethnomathematics is associated with acts

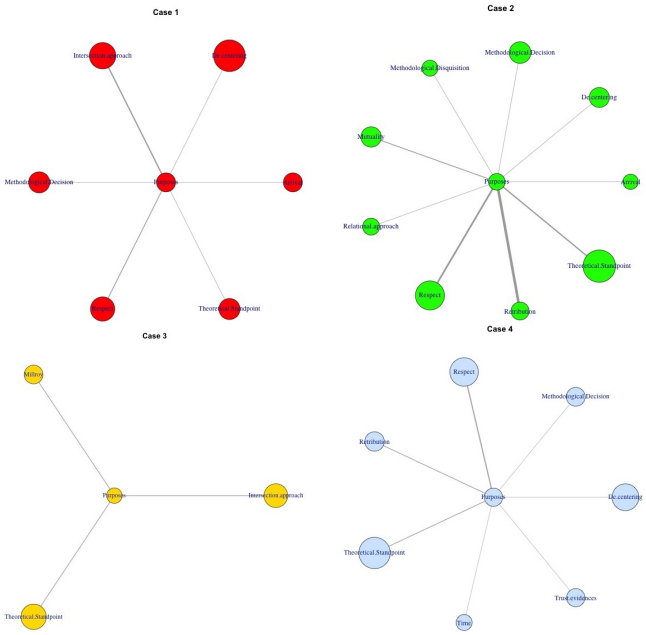


Fig. 6.13: Conceptual network of the category *Purposes* for all cases. Colors of cases are distributed as before. Width of the edges is proportional to the amount of entries.

of retribution to the community. Finally, we can trace the passage of time in the assumptions that gain currency in the field, by noting that the oldest case did not relate *Retribution* with a *Theoretical Standpoint* of ethnomathematics towards symmetry, while for all the newer cases, such connection is strongly present.

By summarising the observations on specific cases and specific notions, we can document a clear cut among the corpus studied in two groups of experiences: one group in which the networks are dense (Cases 2 and 4) and other in which the networks are sparse (Cases 1 and 3). However, density is not the only feature that makes these experiences distinguishable. To end this section, I will try to mention the most relevant issues.

When inquiring about the research motivation, collaboration and partnership appear strongly connected to the (in)existence of a previous relation between researcher and researched communities, that provides a non-academic motivation for the researcher that can be appreciated for the community. In Case 2, the researcher had approached the communities before the experience, developing basic knowledge and posing a research question that can interact and dialogue with practices that are a matter of concern within the communities. Evidence of this correlation can be found by observing the number of co-occurrences of category *Purposes* with the category *Retribution*. Cases like

this contrast with experiences conducted for the pure sake of theoretical interest and without previous contact with the communities, like Case 1, which presents zero co-occurrences among the referred categories. This result suggests that the lack of previous contact makes it more difficult for the researcher to trace a connection or to deploy a result that communities can consider as beneficial for themselves.

It is important to note that fieldworks in Cases 1 and 3 are devoted to register some specific practice or notion (present within the community) that can be identified by the researcher as mathematical. These experiences aim to expand the idea of mathematics, from a culturally-free practice formalized by western rationality to a human and contextualized one, as is stressed when they "argue for a broader conception of what counts as valid mathematics, drawing on the literature to support my claims that mathematics is culturally based" (Millroy, 1992, p. 4) and when they state "the procedures used to solve problems in the everyday life and activities may not be immediately understood or interpreted as mathematics, but they are indeed" (Rohrer, 2010, p. 235). However, according to their standpoint about symmetry, such expansions about what mathematics is should happen within a western academic context, in the spirit of "Nothing better to show a new view of this Science [Mathematics] than observe how other societies are building it" (Sebastiani-Ferreira, 1994, p. 94, my translation).

Another key issue to characterize stances on symmetry appears when considering the amount of time invested in the fieldwork with the communities. Usually, when a research project for a PhD thesis adopts ethnography as a methodology, the common option is to give a short full immersion for a couple of months to collect data, like the research experience in Case 1. In those cases, the researcher did not know the members of the community before she entered the field and she had to elucidate which member could provide the best information. Then she tried to establish some communication with those special members as soon as possible. Therefore, inferences are built using only one data collection; other resources come from (usually previous) academic literature. Case 2 proceeded differently: the researcher made a sequence of visits for an extended period; this allowed him to interact with communities in other ways, to participate in community's special events, and to know a wider range of members of the community, obtaining contradictory and complementary data in each visit.

A third aspect that allows us to differentiate the stances is the conception that authors have of ethnomathematics. In Cases 1 and 3, the intersectional approach is openly declared, devising certain roles for the researcher. In contrast, Cases 2 and 4 are related to a relational approach. In fact, Case 4 can be considered an exemplar case of this approach, since the category representing that approach influences all the main categories, differently from the previous cases, in which the chosen approach contacted only a few categories.

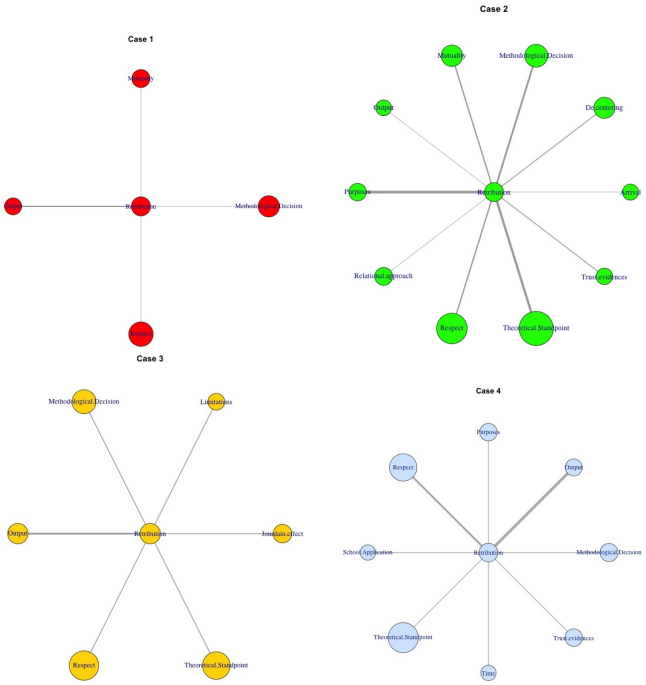


Fig. 6.14: Conceptual network of the category *Retribution* for all cases. Colors of cases are distributed as before. Width of the edges is proportional to the amount of entries.

6.8 Final remarks

The results provide a characterization of the corpus of four research experiences in ethnomathematics, concerning values like dialogue, collaboration and other concepts associated to symmetry. One style of experience can be characterized for a) employing methodologies in which community participation is minor, b) (comparatively) short[22] amount of time invested in fieldwork, c) relation with the community was only for the lapse of time of the research and it was not continued, d) retribution to the community is not considered as part of the purposes of fieldwork, e) low articulation among notions used (conceptual networks are dispersed), f) an intersectional approach to ethnomathematics field.

In contrast, a second style deployed: a) the use of participatory approaches with researched communities, b) (comparatively) large amount of time invested in fieldwork, c) relation with the community was established before the research, d) retribution to the community is central part of the fieldwork purposes, e) strong articulation among notions used (conceptual networks are dense), f) a relational approach to ethnomathematics field.

Such list of differences does not intend to formulate any criteria to predict, control or evaluate the quality of ethnomathematical research. The goal is to pay attention to the existence of one dimension usually unnoticed in the field, that I have referred to as *symmetry*. This dimension expresses concerns about the consistence and effectiveness of the researcher's choices and actions towards a wider conceptualization of mathematics. Naturally, researcher's intentions cannot be simplified as "bad" or "negative." Most of the times, ethnomathematicians express their sincere respect of the community and act consistently with that.

However, within our characterization time emerged as a relevant dimension. The results suggest that the less time invested in fieldwork, the more researchers appear to be prone to do classic ethnography. Despite researchers claiming their good intentions, not infrequently the idea of dialogue in ethnography is in fact a dialogue between ideas interplaying inside the researcher's mind. Doctoral research can hardly open up the opportunity to report implications of the researcher's own work with the community, or at least to discuss the published thesis, because research is often driven by its intention of addressing a theoretical question coming from academic discussions[23]. Conversely, long-

[22]This labeling as short is made in comparison with the experiences in the corpus, because it is obvious that a clear cut about what is a "short" amount of time in research cannot be provided. It is important to realize that institutional frameworks prompt research by imposing measurements of time to assess the quality and the impact of a research study and, accordingly, to provide financial support. This economical rationality of completion rates promotes certain specific styles of research.

[23]Even when some theses proposed educational models to be implemented into the community, they remain at a level of design.

term research deals with several challenges, like the teamwork continuity, the funds for the research, and the community's perception of the presence of the researcher. Interactions and dialogues are developed between people defending their interests and immersed in power relationships.

Questions about the social relevance of the research can appear after some years of continuous work: members of the community ask for the benefits that they can or cannot receive for their participation. Our classification of research according to the time factor does not intend to deny the existence of reciprocity or collaboration in brief ethnographic experiences, but it certainly confirms a basic idea: dialogue requires time, persistence, and patience to be developed.

To conclude, it is not easy to imagine how a dialogue on mathematics could be established between researcher and community. However, there are some insights that can be considered for future projects. For instance, ethnomathematicians could include practitioners' way of thinking, invite them to embrace their own categories, and give them more control in the research process, in order to lose prominence in the report, accepting other voices and other interests. This conveys a change not only of the tematization of out-of-school mathematics but also of the very ways of research.

To rephrase the overall idea developed throughout this paper, I want to appeal to a classic example of ethnomathematics: basket weavings. In the early days, ethnomathematicians wanted to show which part of western mathematics can be identified in the cultural practice (I look for my knowledge in your practice of weaving) and, therefore, mathematical modeling appears as a pertinent tool, as D'Ambrosio and Sebastiani recommended at that time. A turn for reflexivity has drawn the attention of researchers, and nowadays ethnomathematicians are more aware of non-western approaches to mathematical knowledge (I look for your knowledge in your practice, so tell me in your words how you make baskets. But remember, I am the one who analyses). Consequently, methods coming from anthropology arose as useful, as the ones collected by Clifford and Marcus (1986), or the famous "thick description" proposed by Geertz (1973).

This paper makes a call for a symmetrical turn, attempting to do something different: to awake some sensitivity to the interest and expectations of the others for the research experience (I am interested in baskets, do you? If not, in which things are you interested? Maybe we can help each other), and to do so, the methodological discussion moves to the realm of Participatory Action Research (Fals-Borda, 2001; Fals-Borda and Rahman, 1991), Indigenous Research (Cusicanqui, 2012) and other ways to decolonize the research experience (Smith, 2013; Zavala, 2013). This type of theoretical approaches directly addresses the political character of knowledge, by questioning the normalization of spaces, times, agents, and procedures in which knowledge is legitimated. That kind of move supports the core of what I understand as ethnomathematics: an expansion of the history and epistemology of mathematical knowledge.

One of the contributions is to provide a new analytical tool to study ethnomathematics research, and it is left for further research to use this notion to analyse other type of published research. For instance, it can be studied which is the standpoint about symmetry of a specific author (*v.gr.* Gerdes, or Knijnik) and how such standpoint had evolved through time. Another potentiality of this notion is for future research, helping the researchers to structure their projects with communities, by considering some of the tensions and correlations explained here, increasing the sensitivity and awareness of the interests of the communities, and establishing the expectations for the impact of the research experience in a more realistic way, when compared with the type of resources available, specially research time. Finally, I leave for further research if the notion of symmetry can be useful for people that do not work with primary sources but in classroom settings and under the guidance of a pre-established curricula.

In the same way, the reflexivity problem generated a reflexive turn in anthropology; I hope that this symmetry problem can generate a symmetrical turn in ethnomathematics. There is a bunch of promissory examples, as Knijnik (2007), Meaney et al. (2011), Mesquita (2016), Dawson (2013) and Borden (2013). In all those cases, researchers had the necessity to articulate their theoretical interests within a non-academic but socio-political process that surrounds the research and demands some sort of result from the researchers. The relationship transcends the academic instances, gaining a personal dimension that involves ethical, social, and political dilemmas. This does not imply that researchers were employees or militants of those socio-political movements but that they must interact with them, and that a part of the projects' theoretical concerns was related to those non-academic processes.

The symmetry question wants to problematize the means that have been used to reach ethnomathematical goals like healing, respect, valorization, reconciliation, even the dialogue. It is important to examine the discourse that is being naturalized towards the uses of ethnography in ethnomathematics and to ask at what extent classic ethnographical methods have helped or hindered the achievement of such aims. What type of research practices are being considered as participative, respectful, or dialogical in the field? Due to which kind of reasoning? How do researchers understand and perform their notions and beliefs towards respect, dialogue, healing, reconciliation, dignification, valorization, revitalization, and related items? This work tried to describe how these notions are unfolded within the research experience.

The question of symmetry cannot be reduced to a condition to fulfill, as if a research study were or were not symmetrical, in a sort of binary (on/off or 1/0) situation. Nor can symmetry be assumed as a matter of degree or intensity, asking how much symmetry the research had (a kind of percentage of symmetry). With those understandings, symmetry would be reduced to a prescriptive notion, indicating a correct answer and fixing what is desired.

Instead, we can imagine the question of symmetry as the localization in a multidimensional space, demanding a standpoint towards a series of concerns and tensions of representation, impact, voicing, retribution, and decentering. In that regard, symmetry questions the researcher on the ways in which she/he avoids a pure Eurocenteric and self-referential analysis that reinforces the official history of mathematics. As reported cases have shown, ways of escape are multiple and there is not only one correct answer; however, as the cases also show, we can establish what is not desired, imagining a kind of reference point in the multidimensional space, a point of repulsion, which would represent a classic ethnographical account, in which westernized researchers attempt to present westernized audiences the wildness of exotic communities. Following this metaphor of the multidimensional space, symmetry can be established in terms of how far from the zero point the research experience has gone, without prescribing any specific direction. In vectorial terms, symmetry is interested in the magnitude, not in the direction.

Acknowledgements

This research is funded by Departamento Administrativo de Ciencias, Tecnología e Innovación (COLCIENCIAS) and Aalborg University in Denmark. This research also makes part of the NordForsk Center of Excellence "JustEd". I want to thank Rafael Hurtado from Physics department of the Universidad Nacional de Colombia for his comments about networks analysis, and the theses' authors for their willingness and engagement with this study.

References

Adam, A., Alangui, W., and Barton, B. (2010). Bright lights and questions: using mutual interrogation. *For the Learning of Mathematics*, 30(3):10–16.

Alangui, W. (2010). *Stone walls and water flows: Interrogating cultural practice and mathematics*. Phd thesis, University of Auckland, New Zealand.

Alangui, W. and Shirley, L. (2017). *Some Conclusions About Ethnomathematics: Looking Ahead*, pages 357–362. Springer International Publishing, Cham.

Blondel, V., Guillaume, J., Lambiotte, R., and Lefebvre, E. (2008). Fast unfolding of communities in large networks. *Journal of statistical mechanics: theory and experiment*, 2008(10):P10008.

Borden, L. (2013). What's the word for...? is there a word for...? how understanding mi'kmaw language can help support mi'kmaw learners in mathematics. *Mathematics Education Research Journal*, 25(1):5–22.

Chi, R. (2014). *From a typological to network understanding of acculturation.* Thesis, University of Hawai'i at Manoa.

Clifford, J. and Marcus, G. (1986). *Writing culture: the poetics and politics of ethnography / edited by James Clifford and George E. Marcus.* University of California Press, Berkeley, 13 edition.

Cusicanqui, S. (2012). Ch'ixinakax utxiwa: A reflection on the practices and discourses of decolonization. *South Atlantic Quarterly*, 111(1):95–109.

da Silva, A. (2013). *Os artefatos e mentefatos nos ritos e cerimônias do Danhono.* Phd thesis.

Dawson, S. (2013). Mathematics and culture in micronesia: the structure and function of a capacity building project. *Mathematics Education Research Journal*, 25(1):43–56.

Denzin, N. and Lincoln, Y. (2011). *The sage handbook of qualitative research.* Sage, Thousand Oaks, 4 edition.

D'Ambrosio, U. (2006). *Ethnomathematics. Link between traditions and modernity.* Rotterdam: Sense Publishers.

Fals-Borda, O. (2001). *Participatory (Action) Research in Social Theory: Origins and Challenges*, pages 27–37.

Fals-Borda, O. and Rahman, M. (1991). *Action and knowledge: breaking the monopoly with participatory action-research.[Excerpts].* Apex Press, New York.

Fortunato, S. (2010). Community detection in graphs. *Physics reports*, 486(3):75–174.

Geertz, C. (1973). *The Interpretation of Cultures.* Basic Books, New York.

Gerdes, P. (2001). *Ethnomathematics as a new research field, illustrated by studies of mathematical ideas in African history*, volume 5, pages 10–34. Cuadernos de Quipu, Mexico City.

Knijnik, G. (1996). *Exclusão e resistência: educação matemática e legitimidade cultural.* Artes Médicas, Porto Alegre.

Knijnik, G. (2007). Mathematics education and the brazilian landsless movement: Three different mathematics in the context of the struggle for social justice. *Philosophy of Mathematics Education Journal*, 21:1–18.

Krippendorff, K. (2004). *Content analysis: an introduction to its methodology.* Sage, Thousand Oaks, 2 edition.

Latour, B. (1988). *The politics of explanation: an alternative*, pages 155–176. Sage, London.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge University Press, Cambridge.

Meaney, T., Trinick, T., and Fairhall, U. (2011). *Collaborating to meet language challenges in indigenous mathematics classrooms.* Mathematics Education Library. Springer, New York.

Mesquita, M. (2016). Urban boundaries and love: The rebirth of political thought within the disturbing education. *International Journal of Žižek*

Studies, 9(2).

Millroy, W. (1992). An ethnographic study of the mathematical ideas of a group of carpenters. *Journal for Research in Mathematics Education*, 5:i–210.

Newman, M. (2010). *Networks: an introduction*. Oxford University Press, Oxford.

Pais, A. (2013). Ethnomathematics and the limits of culture. *For the learning of mathematics*, 33(3):2–6.

Paranyushkin, D. (2011). Identifying the pathways for meaning circulation using text network analysis. *Nodus Labs*, (December):1–26.

Parra, A. (2015). Intellectual property in ethnomathematics. *Revista Latinoamericana de Etnomatemática*, 8(2):398–414.

Parra-Sanchez, A. (2017). *Ethnomathematical Barters*, pages 89–105. Springer International Publishing, Cham.

Peña-Rincón, P., Tamayo-Osorio, C., and Parra, A. (2015). Una visión latinoamericana de la etnomatemática: tensiones y desafíos. *Revista latinoamericana de investigación en matemática educativa*, 18(2):137–150.

Powell, A. and Frankenstein, M. (1997). *Ethnomathematics: Challenging Eurocentrism in Mathematics Education*. State University of New York Press, Albany.

Powell, A. and Frankenstein, M. (2006). *Respecting intellectual diversity: An ethnomathematical perspective*, pages 161–190. Author House, Bloomington.

Reichardt, J. and Bornholdt, S. (2006). Statistical mechanics of community detection. *Physical Review E*, 74(1):016110.

Roberts, C. (1997). *Text analysis for the social sciences: Methods for drawing statistical inferences from texts and transcripts*. Lawrence Erlbaum Associates, New Jersey.

Rohrer, U. A. B. V. (2010). *Ethnomathematics: New approaches to its theory and application*. Phd thesis, Universität Bielefeld, Germany.

Rohrer, U. A. B. V. and Schubring, G. (2011). Ethnomathematics in the 1930s: the contribution of ewald fettweis to the history of ethnomathematics. *For the Learning of Mathematics*, 31(2):35–39.

Rohrer, U. A. B. V. and Schubring, G. (2013). The interdisciplinarity of ethnomathematics: challenges of ethnomathematics to mathematics and its education. *Revista Latinoamericana de Etnomatemática: Perspectivas Socioculturales de la Educaci?n Matem?tica*, 6(3):78–87.

Salzman, P. (2002). On reflexivity. *American Anthropologist*, 104(3):805–811.

Sebastiani-Ferreira, E. (1994). A importância do conhecimento etnomatemático indígena na escola dos não-índios. *Em Aberto*, 14(62):89–95.

Smith, L. (2013). *Decolonizing methodologies: Research and indigenous peoples*. Zed Books Ltd.

van Atteveldt, W. (2008). *Semantic network analysis: Techniques for extracting, representing, and querying media content*. BookSurge Publishers,

Charleston.

Vergani, T. (2007). *Educação Etnomatemática: o que é?* Cole??o Metamorfose. Flecha Do Tempo, Natal.

Wasserman, S. and Faust, K. (1994). *Social network analysis: Methods and applications*, volume 8. Cambridge University Press, Cambridge.

Woolgar, S. (1988). *Knowledge and reflexivity : new frontiers in the sociology of knowledge*. Sage, London.

Zavala, M. (2013). What do we mean by decolonizing research strategies? lessons from decolonizing, indigenous research projects in new zealand and latin america. *Decolonization: Indigeneity, Education & Society*, 2(1).

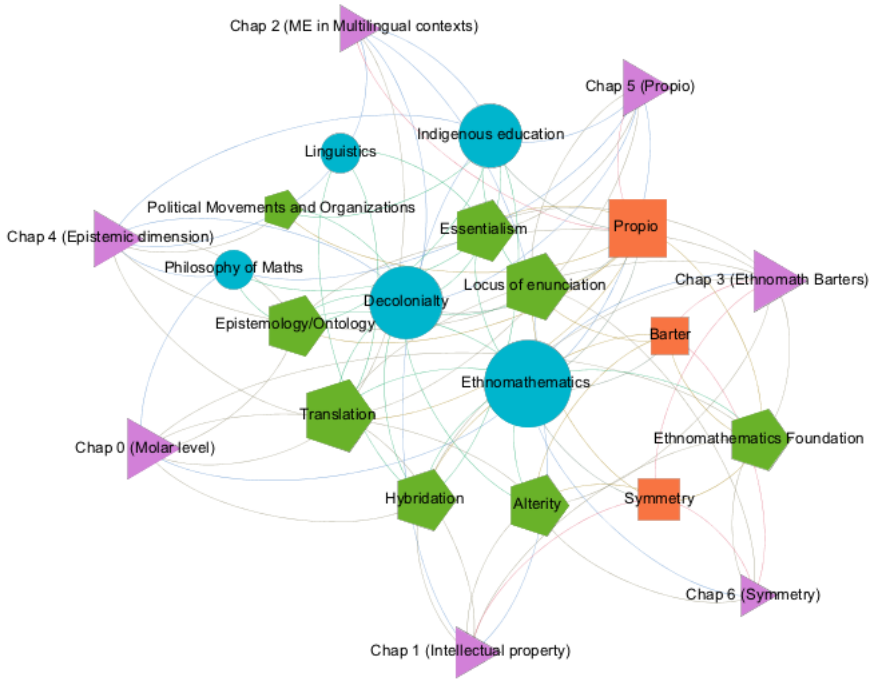


Fig. 6.15: Conceptual entanglement 6. Triangles represent chapters, circles represent areas of study, pentagons represent topics of interest, and squares represent new conceptual tools

Part III

...walking like the Curupira

In this chapter, I want to reflect on the journey and what I have learned through it. Subsequently I will address the two research questions. My starting point was a concern for a new theory of ethnomathematics, but soon I deviated from it to work independently on the different tensions and questions that concerned me for long time. Then, the research experience led me to terrains of intellectual property and intangible cultural heritage, multilingualism and language planning, decoloniality and epistemology, and at the end I combined the readings on discourse analysis with my personal roots as mathematician, when explored Social Network Analysis. Therefore, to propose and provide a foundation for a new perspective for ethnomathematics, I needed to inquire into different realms, entering and leaving ethnomathematics several for times. As time passed by, the importance of decoloniality emerged as a framework to embrace the different concerns that I was facing. Although I realised the impossibility to achieve a new theorisation for ethnomathematics without providing a glossary of binding notions and a toolbox that can make the theoris ation operative, it was difficult to clearly show the relation of those notions among themselves and to the overall idea. Just at the end, when reading a text on connections of Foucault and decoloniality (Castro-Gómez, 2007b), a case of serendipity gave me the deleuzian-guattarian distinction of molar/molecular that allowed me to explain the double effort that I was making. The journey was long and hard; it made me bite the dust many times, but I think it gave me some rewards that I did not expect. Now it is time to share with you, dear reader, the two answers that I could forge during this journey.

III.1 A morphology for a *rara avis*

I get back to the first research question (what set of tensions and concerns for ethnomathematics are identifiable within the Nasa research experience on mathematics and similar ones?) to show how the different chapters delineated a response. I confirmed that the Nasa experience is certainly different from the main branches of ethnomathematics (it is a *rara avis*!), but also, I realised that there is not a sole experience, there are several others operating on the same tensions, sharing many concerns and developing them in its own way. I found that there is certainly a common thread in the experiences being systematically referenced along the chapters.

The common thread of tensions and concerns is about the articulation of ethnomathematics research projects with broader socio-political processes, the conception of culture as battlefield, the active role of language, and the refusal of a "contemplative admiration."

A main characteristic of the Nasa community that shaped the entire experience is the *location of the research within a broader socio-political process.* My research did not start with an academic non-indigenous interest, and to be precise, it was not "my" research either. It was part of the mandate provided by an indigenous assembly made in 2006 about the educational component of the Nasa project of cultural resistance. As explained in Chapter 5, the indigenous organisation that represent Nasa interests (CRIC) started in 1971, and in 1978 launched its educational branch (PEBI). The project in which I participated was just a component of a bigger plan, and the group of researchers that I have interacted with was created before my invitation. This is clearly different from the usual choice in ethnomathematics, where the academic researcher starts the project, defines the research question; gathers, choose, and select the individuals who will participate; edits their interventions and decides in which spaces the research results will be released. A depiction of this type of usual choices was provided in Chapter 6 about symmetry.

The innovative character of the experience is due to the expertise and sensibility of the Nasa people towards participatory and communitarian approaches, accumulated by almost 40 years of continuous work, and sustained by a political agenda that Nasa people develop and renew constantly in the CRIC and PEBI. This agenda constitutes a background that imposes different accountability, methodology, and expectations on the research process, in comparison with the academic ones. A certain autonomy regarding scholar styles of research was allowed, and at the same time a strong demand for social relevance and pertinence was included (this was explored in Chapters 1 and 5). Validation processes also changed accordingly (*v.gr.* the notion of *propio*).

These changes are present in several ethnomathematical experiences around the world, that also share a socio-political project that sustain them, like

the Gelsa Knijnik's experience with Brazilian peasant communities (Knijnik, 2004) organised around the land-less movement [In Portuguese, Movemento Sem Terra, from now on MST]. Similarly, the experience of Tamsin Meaney, Tony Trinick, and Uenuku Fairhall in Aotearoa/New Zealand with the Māori language commission (Barton et al., 1998; Meaney et al., 2011; Trinick, 1994). These research experiences are conducted with organised communities that have non-academic interests and agendas, with their own ways of organisation and validation, that creates a field of tensions with the academic counterpart of the research. These tensions are not necessarily oppositional, but a productive force that adds concerns on accountability.

Moving to another feature that differentiates the Nasa experience, it is important to focus on the role of culture in the research. While Alangui (2010) warned that ethnomathematics conceptualisations about culture were outdated and problematic, the Nasa experience allows me not only to advocate a more robust concept of culture, that can be responsive to anthropological advances; it also intends to conceive *culture as a battlefield*, as a place where struggles for meaning are happening, and in which researchers, voluntarily or not, are immersed and playing a role. The most frequent struggles have an epistemological nature, when discussing the legitimation of what is valid knowledge and who is knowledgeable, as explored in Chapters 4 and 5 with the Nasa and Māori cases. Similar struggles were reported by Knijnik (2004), Mesquita (2016) and more recently by Wagner and Borden (2012).

To acquire awareness of the immersion in a space of struggles implies for ethnomathematicians a commitment to their own agency in the negotiation for meaning. Communities like the MST peasants, Nasa, and Māori indigenous people have built such awareness for more than 30 years, and it is visible in the way that they develop their educational projects to impact *in*, *for*, and *through* culture.

Consubstantial to the conception of culture as battlefield is the *active role of language* in the creation of meaning. As explained in Chapters 2 and 4, language is one of the main scenarios in which culture, worldview, and rationality are expressed and safeguarded. In Chapter 4, it was argued that for Nasa and Māori people, cultural identity is treasured in language and, because of that, any intervention on language entails political and epistemological standpoints. There are several experiences in ethnomathematics reporting the creative force of language, ranging the insightful experiences in Colombia (Cauty, 1998; Cauty and Ramos, 1990), Aotearoa/New Zealand (Barton, 2008; Barton et al., 1998; Meaney et al., 2011), and Canada (Borden, 2013; Borden and Wagner, 2013).

Another break with the previous tradition of ethnomathematics that I perceived due to the research experience with the Nasa people is about the *abandoning of a "contemplative admiration"*. A large part of the ethnomathematics production deals with the recognition of unofficial/non-academic mathematical knowledge. Such recognition is accomplished whether through studies reveal-

ing knowledge and practices from the past (resembling a necropsy or archaeology report) or through the non-participant observation of practices that are currently occurring (resembling a journalistic report or chronicle). The important point is that such knowledge and practices are preexisting to the specific research. This is to say, when a disruption of the official epistemology of mathematics occurs within a community, the ethnomathematical researcher arrives (later) at the community to identify and register the disruption for the academic world, but not to create and generate it with the community. The acceptance of this passive role is what I have called a contemplative admiration.

Certainly, it can be argued that the ethnomathematical activity of producing visibility and respect for some cultural practices within the academic sphere is a kind of active rupture. However, such activity is still a divulgation of practices created beforehand and without the agency of the ethnomathematician. In brief, a contemplative admiration identifies diversity, rather than creating and maintaining it.

As explained in Chapters 1, 3, 4, and 5, the Nasa research experience escaped purposefully from that contemplative admiration. The creation of a book on mathematics to be used outside the school implied the adaptation of different means to communicate and share knowledge in the Nasa communities. The cartographic and toponymical work around the village of Lomitas demanded new technological skills and conceptualisations for teachers, students, and parents. We can mention other cases that also escape from this recurrent understanding of the ethnomathematics activity: the aforementioned experiences of André Cauty in Colombia and Wilfredo Alangui in Philippines, who made their displacement deliberately and attempted to theorise ethnomathematics. We can also include the long-term experience with Māori indigenous people in Aotearoa/New Zealand developed by Tony Trinick, Uenuku Fairhall, and Tamsin Meaney (Barton et al., 1998; Meaney et al., 2011), and the experience of Lisa Lunney Borden (Borden, 2013) with Mi'kmaw communities in Canada.

To summarise this section, this study gathered previous and current initiatives conducted worldwide that correspond to a new trend in ethnomathematics research and that have been not connected under a common frame. Such a frame traced continuities and lines of flight regarding the existing theorisation of ethnomathematics. This trend deserves theorisation, that will be performed in the next section.

III.2 A decolonial answer

With respect to the second research question, which attempts to find some theorisation for current developments in ethnomathematics, this thesis has already answered the question in a tacit way, by making a proposal at two levels: one level looking for an overall account of ethnomathematics as research program

(a molar structure), and other level coining and testing notions that can be useful for working ethnomathematicians (a molecular kernel) in their practice.

Now it is time to formulate an explicit answer: I contend *that a decolonial and performative theorisation for ethnomathematics can address the tensions and concerns raised by the Nasa experience and some other experiences worldwide*. This final section is devoted to explaining the terms of my answer and how they combine the molar and the molecular levels of reflection presented in this thesis, highlighting the principles that underlie in them. I will start commenting on the philosophical-political general framework that inspired the current theorisation.

During my doctoral studies, I made some explorations with Foucauldian post-structuralism and Wittgensteinian post-metaphysical philosophical approaches and with Latourian science and technology studies, in the quest for a framework for ethnomathematics. However, I have found that decolonial theory can collect and articulate the contributions coming from those approaches. This theory also provides a framework in which the tensions and concerns identified in the current wave of ethnomathematics can be addressed.

As decolonial theory was gradually introduced and described along the chapters of the previous section, I will not make another description, but a series of comments showing at what extent the results and tensions explored in the thesis are consistent with central notions coming from decolonial theory. I will also stress on the possibilities that a decolonial framework provide for ethnomathematics to jointly attend the political and philosophical challenges, this is to say, those possibilities depict a way to face Scylla and Charybdis.

To start the comments, I want to consider the decolonial proposal of Boaventura de Sousa-Santos to develop epistemologies of the South as a:

> [...] retrieval of new processes of production and valorisation of valid knowledges, whether scientific or non-scientific, and of new relations among different types of knowledge on the basis of the practices of the classes and social groups that have suffered, in a systematic way, the oppression and discrimination caused by capitalism and colonialism (de Sousa Santos, 2012, p. 51).
>
> The construction of epistemologies of the South must be built by four steps: sociology of absences, sociology of emergences, ecology of knowledges, intercultural translation (de Sousa Santos, 2012, p. 52)[1].

There is an unambiguous connection of epistemologies of the South with ethnomathematical aims, evident in issues like a respectful interrelation of diverse types of knowledge (*v.gr.* the mathematical one). In addition, the four steps that built the epistemologies of the South has been interpreted and unfolded in this thesis through problematisation of a series of tensions like: Who

[1] Definitions of those steps will be introduced in the following subsections.

speaks of knowledge? Where does she/he speak? To whom? In which language is knowledge spoken, and with what purposes? I will follow those questions to show the resonances and communicating vessels of decolonial studies with my theorisation of ethnomathematics.

Who speaks and who does not?

Decolonial studies have introduced notions like *locus of enunciation* (Mignolo, 1999), *sociology of absences* (de Sousa Santos, 2012), or *hubris of the Zero-point* (Castro-Gómez, 2007a) to express a concern on who has (and who has not) the entitlement to speak of knowledge and to analyse the type of practices that produce and sustain such privilege. In this subsection, I argue that ethnomathematics shares with decolonial studies similar concerns on knowledge, but developing them within the specificity of mathematical knowledge, and I summarise how this doctoral study explored and unfolded those decolonial notions and created similar ones.

As explained in Chapters 6 and 1, ethnomathematics has raised recurrently its interest in *dialogue with or respect to* the other. However, there is no consensus about what such respect can be or in what ways the desired dialogues may or may not occur. The ethnomathematical intention to value the voice of the practitioners of cultural practices turned into a sort of desiderata or empty signifier: the level of indetermination and lack of reflection on this matter allowed even notions coming from purely extractive traditions to gain currency in the field (*v.gr.* the emic/etic distinction[2] (Albanese et al., 2017a; Rosa and Orey, 2012)).

Evidence of how timidly the concern for otherness has been treated in ethnomathematics theory can be found in a very recent study conducted by Veronica Albanese, et al. (2017b). Such study proposes two views in the ethnomathematics theoretical framework: (1) the recognition of mathematics in cultural practices, and (2) the discovery of different ways of thinking. The authors explain:

[2]It is enough to note how the Christian linguist Kenneth Lee Pike was interested in achieving faster forms of evangelisation for the Summer Institute in Linguistics (SIL), and because of that he introduced the emic/etic distinction as part of his techniques to assist linguists to extract coherent descriptions of indigenous languages, that can inform robust bible translations. Pike did not have any interest in respect or enter in a dialogue with the metaphysical comprehension of indigenous peoples in South America, neither to respect vernacular deities or cosmogonies. He wanted to introduce the Christian God in an efficient manner. This means, the etic/emic distinction in its origins is not about respect or compassion, it is about efficiency in the extraction of information. To my view, such intention is very far from the intended goals of ethnomathematics. Although promoters of this approach within ethnomathematics are not interested in promoting religious views, their proposals are still permeated of such unilateral view, in which the locus of enunciation is not called into question.

> The first view implies recognizing mathematics in the practices of cultural groups, and this research is *performed by the categories and schemes of thinking of the researcher's culture*. The second view implies discovering different ways of knowing quantity, space and relation aspects of human experience (QRS systems) considering a broader concept of (ethno)mathematics where the categories and schemes of thinking of the studied cultural group are taken into account (Albanese et al., 2017b, p. 324, my italics).

Such categorisation reveals how centred on the academic researcher is the conceptualisation of ethnomathematics[3]. There is no intellectual activity coming from community members or practitioners that deserves to be included in some theorisation. Naturally, these non-academic stakeholders can be informants, and they are invariably indicated as beneficiaries of the research experience results, but they are not conceived as researchers. Their reflections are made invisible and subjected to the realm of "Not-being." As de Sousa Santos states: "Non-existence is produced whenever a certain entity is discredited and considered invisible, non-intelligible or discardable" (de Sousa Santos, 2012, p. 52).

This thesis made several incursions to contest such type of theorisations, by highlighting the issues of voicing and representation in ethnomathematics: the first chapter was devoted to intellectual property. The discussion there raised questions like: How can ethnomathematicians develop their practice in a way that intangible cultural heritage of communities can be recognized? At what extent can intellectual property be respected in ethnomathematical production? Those questions comprise a debate on representation that this thesis stressed, since the beginning acknowledging the complexity, urgency, and relevance of the debate. This concern echoes Alangui's plea: "Any ethnomathematical text produced in the research process must similarly reflect the difficulty of categorisation, the multiplicity, and contingency of knowledge, and confront the issues of representation and legitimation" (Alangui, 2010, p. 73).

The Chapters 2 and 4 also address the issues of representation. In those chapters devoted to multilingualism, it was important to note that current discussions about education for indigenous students are usually based on two assumptions: a) what differentiate students coming from diverse linguistic and cultural contexts is language and not rationality, b) the main aim of the work on multilingualism is to reduce the achievement gap of indigenous students within the non-indigenous school. Such assumptions share a division that locates the ones who can think and decide according a euro-centred perspective on one side,

[3]Of course, any theorisation must declare some intended role for the academic researcher, but I want to stress here that ethnomathematics attempts to be something else than merely an academic area (that is why the most used label is *research program*, instead of discipline). Therefore, a theorisation for ethnomathematics should include some other actions, arenas, and stakeholders.

and the indigenous others on the other side in a subordinate role, waiting for decisions on themselves. With these two chapters problematising such account, the practices, and the monocultural rationality that sustain them, this thesis attempted to make sociology of absences (de Sousa Santos, 2012), understood here as the type of research that unveils the ways in which denial and non-existence is actively produced.

Assuming that sociology of the absences "amplifies the present by adding to the existing reality what was subtracted from it"(de Sousa Santos, 2012, p. 56), to obtain an alternative approach that can be sensitive towards the agency of communities within ethnomathematics research became central to my theorisation, and therefore the notion of locus of enunciation emerged and proved its relevance in Chapters 4, 5, and 6. I have explicitly used the notion of locus of enunciation as "the geo-political and body-political location of the subject that (sic) speaks" (Grosfoguel, 2011, p. 5), to inquire about who is speaking of non-hegemonic knowledge of the theory and practice of ethnomathematics.

Thereafter, two theoretical notions were presented to express this concern with the locus of enunciation in the research practice: *propio* in Chapter 5 and *symmetry* in Chapter 6. *Propio* is a clear example of how research[4] can be decolonised at several levels, by the ones who perform the research; the ways in which information is collected, analysed, published and assessed; the ways in which academy and community can be related. Whereas *propio* focused on the agency of the historically invisible researched subject, the notion of *symmetry* constitutes a concrete theoretical contribution in the direction of providing tools for analysis and action for ethnomathematicians, tools that are sensitive towards the political conditions inherent in research.

Symmetry demarcates an inherent condition of research, demanding a standpoint or positioning from the researcher. Symmetry also turns ethnomathematics closer to working notions of contemporary anthropology like reciprocity, collaboration, and partnership, identified by Robben and Sluka (2012) in their study on the compassionate turn and the emergence of "engaged fieldwork."

Symmetry and *propio* address the political challenge of ethnomathematics as far as they are aligned with concerns coming from anticolonial studies, as those raised by Dei and Kempf (2006):

> What is the final purpose of the process? Who is it supposed to help, and how? Who identifies and defines the social problem? What do they think about the research process? What will be the impact on the participants and communities involved? Who writes the research, and who will take advantage of it? These questions are answered, ultimately, somehow accessorily, through what is called the ethical protocol. Ethical concerns are

[4]Although *propio* is related to educational initiatives, I can extrapolate the notion for the research practice because for Nasa people education is a way to research the own reality. Conversely, to research collectively is a form of communitarian education. Education and research are inextricably intertwined for Nasa people through the *propio* notion.

> often later incorporated to improve the methodological process, but not at the beginning, to inform the research made by an ethical researcher. In this way, scientific knowledge is produced by methodologists seeking an interesting research subject rather than by agents of social transformation adopting appropriate methodologies. (Dei and Kempf, 2006, p. 27)

There is an important issue that I want to mention before closing this debate on the locus of enunciation. Swanson (2015) pointed out that "Western thought is invested in a pre-eminent belief in an individual self, resonant with what Mannheim (2000) might have referred to as Enlightenment 'individualistic liberalism'"Swanson (2015, p. 106). We can see how post-structuralist thinkers like Foucault, Deleuze, and Derrida adhere in various degrees to this image, according to which the individual subject became the centre of the struggles of power, and that battle for the fabrication of the self is what draws attention of these authors. Instead, decolonial theory is interested in rooted approaches pursuing collective efforts (*v.gr. Ubuntu* (Swanson, 2015) or *Minga* (Bolaños et al., 2016)) to build knowledge. Such insight into collective learning and communitarian research is explored in this thesis in Chapters 1, 4, and 5 (intellectual property, epistemic dimensions of multilingualism, and the concept of *propio*), opening an epistemological discussion in a manner that significantly differs from those of eurocentric thinkers. This study has documented the unfolding of possibilities in which knowledge is achieved by the community as a whole, and not as the collection of individual achievements. If must be acquired collectively, what knowledge is that? Certainly, it is not the enlighted one[5]. This important difference makes me prefer decoloniality as a framework for ethnomathematics, over post-structural approaches that also problematise epistemology and power.

To summarise this subsection and introduce the next one, I observe that ethnomathematics has been interested in respect for and dialogue with non-hegemonic communities and cultures, promoting multiplicity and safeguarding diversity of knowledge. This thesis adheres to these interests as well. However, I present a perspective that differs in the way in which diversity and multiplicity are addressed. Most of ethnomathematics production is basically concerned with how the academy can generate conceptions about mathematics that include the cultural and linguistic diversity that society possesses; therefore its target audience is the academy itself. Through concepts such as *propio* and *symmetry*, this thesis asks academically how social groups (not necessarily academic) are generating conceptions about mathematics, based on the cultural and linguistic diversity that these groups have. Part of this study addresses the

[5]I do not find it accidental the fact that among the vast number of explored philosophical foun-dations for ethnomathematics, one of the most referred authors is Wittgenstein, since his philo-sophical insights are least committed to this individualistic liberalism, when focusing on the collective character of a language game.

ways in which the academy can be articulated (nurtured and being nourished by) to social, cultural, and ultimately political movements that find a space of development for their struggles of self-determination within mathematics. It implies that the practice of ethnomathematics should enter the coming in and going out of the academy, involving stakeholders of different nature: academic, popular, vernacular, indigenous, and so on (Cusicanqui, 2012), and strategically performing in the borders of academy with social movements, in the decolonial style proposed by Grosfoguel (2009).

Where is knowledge being spoken?

From a retrospective look, we can imagine the ethnomathematics research program with its effort to expand the epistemology of mathematics as having different stages. One, successfully launched and conducted by Ubiratan D'Ambrosio, consisting in generating awareness in the academic community and the school systems, about the existence, relevance, and importance of non-western and non-academicized forms of mathematical thinking. That stage reaches diverse academic communities, mathematicians, anthropologists, mathematics educators, even policy makers, and curriculum designers towards more culturally sensitive forms of work. The success of that stage can be easily stated, if we consider the socio-cultural turn in mathematics education, the number of papers, journals, and academic events developing sympathetic agendas, even the awards received by some representative scholars (Bishop, D'Ambrosio, Radford) reveals the good health of socio-cultural approaches to mathematics and its education. Without claiming that such efforts do not need to be continued, I can say that the mission of that stage has been accomplished: Nowadays, even the more cognitive scholars and orthodox philosophers make room to include the influence of socio-cultural and political factors in the learning and teaching of mathematics.

Ethnomathematics is posed as contrary to internalist positions on the philosophy of mathematics that are devoted to studying the development of the mathematical objects, avoiding the social and historical conditions that made such development possible. Ethnomathematicians also reject the modernist account that recognises only one scientific rationality. However, followers and critics of ethnomathematics locate their debates within the academic field, characteristic of a modernist canon of knowledge that promulgates an internalist vision. If it is assumed that knowledge is validated only in the arenas that modernity has considered legitimate (*v.gr.* universities, journals, and academic events), how can the modernist account of mathematical knowledge and its internalism be criticised? What is assumed as valid spaces of debate for ethnomathematics corresponds precisely to the same spaces in which internalist views are generated. It is then necessary to break the narrow circles of the academy to allow a real transgressive act for ethnomathematics.

Unfolding that insight, certain questions can be posed: "does ethnomathematics have something to offer to cultural pluralism beyond that of the museum, beyond that of merely exhibiting cultural artefacts and practices through books, paper journals, and at international academic events? Can ethnomathematics be a part of a genuine exchange between cultures based on mutual respect rather than hegemonic academic instincts?"[6]

In those questions underlies the Foucauldian conception of *epistème*. As the French author highlighted, there is no knowledge without a time and space that provides conditions of possibility for such knowledge; instead there are discursive practices that reify regimes of truth and constitute epistemes (Foucault, 1971). In other terms, what is knowledge other than an interconnected system of people, beliefs, values, institutions, and instances that constitute it at certain time and space? In that sense, the attempt at expanding the frontiers of what is accepted as mathematical is an attempt at intervening in such system, calling into question the exclusivity of some instances (*v.gr.* the academic ones) to be legitimate.

As this thesis has shown, there is a new stage emerging for ethnomathematics, interested in promoting a broader vision of mathematical knowledge within other social contexts. The main goal of this thesis was to contribute to the consolidation and theorisation of such emergence. That explains why it was important to raise concerns about intellectual property (Chapter 1), the practice of bartering (Chapter 3), the presence of an epistemological dimension within multilingualism (Chapter 4), and mainly the notion of *propio* (Chapter 5). Those chapters documented and analysed ways in which knowledge is being created, communicated, assessed, and transformed by non-academic communities, within instances that possess culturally rooted dynamics of interaction. I contend that as long as ethnomathematics does not conceive itself as accountable to scenarios other than academy, it will not be able to make a strong academic reading of the epistemological/political dimensions of mathematics.

It is important to stress that this new stage for ethnomathematics is not imagined as concluding or substituting the previous one. Instead, it is conceived as the opening of a new work front in the research program, adding different concerns, agents, and methods to ethnomathematics. This claim for an expansion is not dichotomous and resonates with decolonial contributions about the relation to academic realms:

> Under the ecology of knowledges, granting credibility to non-scientific knowledge does not imply discrediting scientific knowledge. What it does imply is using it in a counter-hegemonic way. This consists, on the one hand, in exploring alternative scientific practices made visible through plural epistemological scientific practices and, on the other, in

[6]These questions are a paraphrases of others that Watson (2002, p. 110) formulated to performing arts interested in intercultural encounters.

> promoting interdependence between scientific and non-scientific knowledge (de Sousa Santos, 2012, p. 57).
>
> What social science should be doing is revolutionizing the episteme. It should be created a playing field between the European heritage and our own inheritance, in which we can autonomously recreate a thought and a gesture capable of overcoming the double bind or colonial schizophrenia of which Tamayo spoke. And this must be done by any means, not only in social science but also in mathematics, in agronomy, in engineering and in the multiplicity of disciplines that are necessary for the here and now of the humanity and the planet, not only of the science. (...) What I assume as the main challenge is to be authentically modern and to connect at the same time with the most ancient, so that, from that contradiction or anachronism, we can create —inside and outside the university— an inclusive, democratic and intercultural public sphere (to put it in conventional terms). For me it is central to recognize that the theory is not enough, social science is not enough, the university and the academy are not enough to understand the world that we have had to live today. And I believe that, throughout Abya Yala, this process of "entering and leaving the academy" is allowing a renewed thinking, a better articulation of such thinking with community, popular and collective practices. On the border between the university world and its outside, there are proliferating initiatives such as the one I have just described, and I have seen it in several countries of our continent (Rivera et al., 2016, p. 4, my translation).

Thus, decoloniality and my theorisation of ethnomathematics share the same interest of interacting within an ecology of knowledges (de Sousa Santos, 2012) and taking advantage of the boundaries. This is not a naive plea for a blurring or a melting in a soft multicultural arcadia, forgetting the political and epistemic consequences of those boundaries. Instead, the approach is conceived as generator of difference, an endeavour for the dignity, and self-affirmation of communities and groups. Therefore, the target of the critique is not the lack of unification, but the isolation and lack of communication among diverse communities and traditions. As Watson (2002) argued, when considering the dynamics of bartering: “Barters are not engaged inthe anthropological notion of identifyingand cataloguing otherness, they are about performing difference in order for a meeting of cultural equals to take place” (Watson, 2002, p. 109).

While I have argued for the unfolding that the academy can have on social and cultural movements, it is important to also defend the contribution that the actions of these movements and their forms of work can have on academic conceptualisations. As Rappaport (2008) points out with respect to collaborative research as a co-theorisation process:

> I purposefully emphasize this process as one of theory building and not simply coanalysis in order to highlight the fact that such an operation

> involves the creation of abstract forms of thought similar in nature and intent to the theories created by anthropologists, although they partially originate in other traditions and in nonacademic contexts. Understood in this sense, collaboration converts the space of fieldwork from one of data collection to one of co-conceptualisation. (Rappaport, 2008, p. 5)

Rappaport's insight reverberates with postulates about a decolonisation of the research methods (Smith, 2013). Within ethnomathematics, Alangui (2010) suggested indigenising the interview method echoing this need to decolonise the research itself. We can find in this thesis another deliberated effort to do such decolonisation of methods, when considering notions as *barter* and *propio*, extracted and elaborated from indigenous Nasa thinking.

The last two subsections have shown how a decolonial framework can explain the analytical exercise made in this thesis to address political concerns of ethnomathematics. The decolonial framing of questions such as who validates and where knowledge is validated are useful for ethnomathematics, because it allows the research program to formulate responses to the concerns of inclusion, dignity, respect, and empowerment of communities, avoiding the risk of ghettoising such communities or freezing them in time. In the next subsection, I will argue that decoloniality can also be a framework for the epistemological concerns that this thesis has considered.

What is being spoken?

In Chapter 0, I presented the question on the ontology of mathematical objects as unavoidable for ethnomathematics theory. In that chapter I reviewed how the scholars in the research program have opted for social and externalist accounts about the philosophy of mathematics, being the contributions of Ludwig Wittgenstein the ones that gained more currency for ethnomathematicians in the last decades. In the subsequent chapters I have subtly delineated my own answer. In this subsection, I will collect the traces and explicitly formulate explicitly my position on this matter.

One of the few common things within the huge diversity of understandings and unfolding of ethnomathematics is the refusal of Platonist explanations on the nature of mathematical objects and mathematical work (Barton, 1999; D'Ambrosio, 1985; Vilela, 2010). The essentialism in the internalist philosophical account driven by Platonist and metaphysical conceptions is rejected by ethnomathematicians worldwide. However, several critics of ethnomathematics have pointed out that an anti-essentialist stance produces a problematic relativism that cannot address features of mathematics, like the complex relationship between mathematic results, time, and space, or the independent finding of similar results for different cultures (Horsthemke and Schafer, 2007),(Rowlands and Carson, 2002, 2004). My position on this matter is different. I look for an approach that avoids both the relativism driven by a Wittgensteinian

post-metaphysics ("mathematical entities are only human conventions"), as the quasi-Platonic universalism that ethnomathematics has criticised so much ("mathematical entities have a non-human essence"). To do so, I appeal again to decolonial standpoints, this time enhanced with discussions coming from Science and Technology Studies (STS).

As a response to the "science wars" that occurred in the 1990's and confronted experimental scientist with human scientists, scholars like Latour (1993, 2004), Woolgar (1988), and Pickering (1992) developed social accounts of the scientific practices, in which facts from nature and values from society become in a mutually constitutive dialectic. In that sense, natural sciences are not merely human or non-human. They are both, simultaneously, human and non-human. Scientific knowledge is the result of the entanglement of human agency and material agency. Although research on STS is often developed on experimental sciences and sustained with the undeniable materiality coming from non-human nature, there is one author, Andrew Pickering, who developed similar argumentation for the specificity of mathematics.[7] I consider his ideas useful for ethnomathematics.

Aiming to analyse the conceptual practice that has a hallmark of of mathematics, Pickering replaced the material agency with the idea of *disciplinary agency*, assumed as the "agency of a discipline that leads us through a series of manipulations within an established conceptual system" (Pickering, 2010, p. 115). Discipline is then a performative agent, without a will or agenda, but capable of establishing procedures that force humans to operate in certain ways. Thus, conceptual practice has "the familiar form of a dance of agency, in which the partners are alternately the classic human agent and disciplinary agency" (Pickering, 2010, p. 116). Within that image, human agency provides *free moves* when extrapolating and bridging old ideas into new realms; conversely, disciplinary agency provides *forced moves* when adapting and transcribing stablished structures. Therefore, the dance is determined neither by the human nor by the discipline, producing "trajectories that cannot be foreseen in advance, that have to be found out in practice" (Pickering, 2010, p. 139).

Although I have no space here to discuss in more extent how the idea of disciplinary agency is not confined to the realm of the social (what would return it to the realm of the human), this Pickering's dance of agency is useful enough for my intended theorisation of ethnomathematics because it goes beyond essentialist/conventionalist accounts, when stressing the complex dialectic

[7] In some sense, this extrapolation of STS standpoints for mathematics is foreseeable. According to Latour, there is no way to claim the existence of the bacillus of tuberculosis before its study by Koch in 1882 (Latour, 2000). That is a polemic and controversial claim. However, the similar contention by Pickering that there was no quaternions before its study by William Hamilton in 1843 (Pickering, 2010) lacks the explosiveness of the Latourian statement and enters within the plausible.

of accommodation and resistance between human and non-human entities that happen in mathematics.[8] However, instead of the image of a dance, I prefer the image of a plastic artist, who sculpts what he/she wants, but is always aware of the kind of raw material being worked. Wood, metal, or stone impose certain specific and non-transferable conditions that the artists need to address and respect. That is not to say that materials have a soul, will, or desire but an agency that is undeniably non-human.

The fact that some mathematical results had been known by different groups throughout history is not explained because certain essence or structure exists (waiting for someone smart enough to discover it), but because those groups were dealing with the same raw material, a metaphysical one, that imposes conditions. As with the plastics artists, it cannot be said that cultural groups are doing the same thing, not at all; groups are not doing the same mathematics, they are just using similar techniques to deal with the material. To conclude the metaphorical game, mathematics are the oeuvres and not the raw material. This approach to the philosophy of mathematics dissolves the dichotomy of essentialism/conventionalism and centres the interest of the ethnomathematician in the dynamic of accommodation and resistance between different agencies, and not merely in the agencies themselves. That dynamic is what Pickering called the "mangle of practice". (Pickering, 2010)

To accept the study of such dynamic means to adhere to the cultural historicity that Radford (2014) claims for reason and for mathematical knowledge. It also means to elaborate on the post-human alternative vein that some ethnomathematicians have observed in Wittgensteinian philosophy:

> This does not mean that mathematics is arbitrary, and thereby open the way for mathematical anarchy. We are free to construct the grammatical rules of mathematics, but not "blindly or capriciously" (Shanker, 1987, p. 319). The arbitrariness Wittgenstein refers to is its autonomy. [...] Cultural mathematics' are not arbitrary in the sense that they could be anyhow. They are arbitrary in that any culture is free to make its own sense of the world. Mathematics is the way it chooses to express that sense (Barton, 1996, p. 182).

> In particular, mathematics or Euclidean geometry, as a set of grammatical rules, are applied because these rules must have an empirical origin and became rules, or forms of intelligibility (Vilela, 2010, p. 352).

What kind of thing is limiting the "mathematical anarchy?" How to explain the "empirical origin?" To my view, Wittgensteinian approaches in ethnomathematics have not achieved its pretended post-metaphysical turn.[9] As noted by

[8] Pickering's dance of agency was already considered in mathematics education by Wagner (2007) and Boaler (2003) to study interactions within mathematics classrooms.

[9] The existence of several *language games* does not explain the universal human capacity to play such games.

Albanese et al. (2017b), many ethnomathematicians[10] inspired by Wittgenstein have embraced cultural relativism for the research program. The recognition of the agency of non-human entities in the conceptual practice of mathematics and its mangle in practice with human agency is one insight of STS that I consider useful for ethnomathematics to overcome cultural relativism without a revival of quasi-platonic universalism. This alternative perspective is an important contribution, as far as it liberates ethnomathematics from being condemned to find a definitive ontology of mathematics and changes the terms of its philosophical discussions, by focusing on a political realm of practice, that makes it possible to develop different and unexpected ways to perform and theorise mathematics.

This philosophical perspective on mathematics that I propose for my theorisation of ethnomathematics is based on STS, but it can be articulated within a decolonial stance due to the multiple communication vessel that these field of studies have. First, STS and decolonial studies share a common interest in developing alternatives to the modernistic project that declares western thinking as exceptional, successful, and the norm of rationality. Harding (2008) combined both trends and formulated a post-colonial Science and Technology Studies (PCSTS) that allow us to realise the previous, current, and future existence of multiple modernities (sic), due to the co-evalness of cultures and communities and the power that they have to create rooted modernities. This thesis has prowled such co-evalness with the notions of symmetry, barter, and *propio*, which recognises the capacity of local communities to formulate rooted explanations and expansions to current developments of western scientific thinking.

Second, both fields of studies refuse isolations between traditions and claim for constant and problematic interactions and translations that enhance the multiplicity of worldviews, looking for the best of traditions involved, accepting conjunctions and transversal cuts whenever they are needed and accepting the impossibilities and singularities whenever they exist. Such multiplicity has been endorsed and fostered by ethnomathematics since its launching in Adelaide and nowadays as well. This interacting multiplicity was also named as *ecology of knowledges* by de Sousa Santos (2012), who claims:

> [...] granting credibility to non-scientific knowledge does not imply discrediting scientific knowledge. What it does imply is using it in a counter-hegemonic way. This consists, on the one hand, in exploring alternative scientific practices made visible through plural epistemological scientific practices and, on the other, in promoting interdependence between scientific and non-scientific knowledge (de Sousa Santos, 2012, p. 57).

This thesis operated in the counter-hegemonic way that an ecology of knowl-

[10] Interestingly, Alangui agrees with Wittgensteinian approaches, but did not adhere to cultural relativism. He proposed the use of mutual interrogation to articulate his position on this matter.

edges suggests, when introduced the notions of *propio* and barter, and when raised the discussions on matters like intellectual property and the epistemological dimension of multilingualism. These efforts explored the hybridity, fluidity, and multiplicity of mathematical knowledge and exemplified how ethnomathematics can go beyond dichotomies of antimodernist fundamentalism versus Eurocentric colonial imposition.

A third and last element that I want to formulate as useful for ethnomathematics is the understanding of reality as continuously transforming and becoming. In another effort to combine STS and decolonial studies, Savransky (2012) contends that worlds and their entities are not there to be represented but "are shaped, sustained and transformed by the social, technical and material practices that take place –and make place– in them" (Savransky, 2012, p. 360).This element has two implications, one is a refinement of the decolonial emphasis on epistemology and representation, complementing the question of the politics of knowledge with a concern for the ontology, which shows that the controversies between western and non-western knowledge-practices constitute a politics of reality. The second implication is to assume knowledge-practices as inventive practices, which returns to human agency the political responsibility to operate in an uncertain and unstable reality.

This blurring of the division epistemology/ontology that decolonial studies and STS promote resonates with the ideas of Barton about conceiving mathematics as the result of the practice of mathematising and gives ethnomathematics a political and philosophical basis for the past, present, and future existence of several, diverse, interrelating, and non-unifiable mathematics. This blurring also entangles power with knowledge in an unsolvable tension, that is in itself a productive force for the research program of ethnomathematics. The next (and final) subsections discuss how this thesis explored such productive force.

Which languages are being spoken?

Along this thesis, I have stressed the theoretical-methodological possibilities that translations offer to ethnomathematics. I used translation in several senses, from the most immediate and direct of the translation of words between languages to create registers (as proposed by Cauty (1998) and enacted by Barton et al. (1998), Meaney et al. (2011), and myself in Chapter 4), passing through the translation of knowledge and practices, such as outlined by Alangui (2010) and Knijnik (2002), and with the notions of *propio* and barter in Chapters 3 and 5. Another sense given to translation in this study refers even to the translation of forms of research, as explored in Chapters 5 and 6, with the notions of *propio* and symmetry.

The accomplishments of translations vindicate the language as the proper scenario for creation and hybridity, because environments in which several languages collide are fertile soil for the generation of knowledge, the introduction of

concepts and practices, and the re-examination of previously known meanings. A foundation of this interest in the tandem translation/creation/hybridation can be found in the theory of linguistic diversity:

> Instead of the often static notions of language implied by concepts of multilingualism, we need to start to move towards concepts such as Jacquemet's (2005) transidiomatic practices: 'the communicative practices of transnational groups that interact using different languages and communicative codes simultaneously present in a range of communicative channels, both local and distant (Makoni and Pennycook, 2007, p. 30).

In the same sense as the notion of multilingualism was introduced by a segregationist colonialism (Makoni, 2011) interested in promoting classifications (and then, hierarchisations) of communites according to the nation-state division, a sort of a "multimathematics" notion that fosters dichotomies like indigenous/white or out/in-school may work under the same segregationist premises. Emulating the invitation of Makoni and Pennycook (2007) to surpass such multilingual framing of diversity through linguistic transidiomatic practices, this thesis invites us to use notions like barter and *propio* to envision alternatives to the situation of a multimathematical framing, advocating instances of mathematical knowledge that are contingent, hybrid, and, as I will argue in the next subsection, performative. Through bartering practices and in the search for their own knowledge, people and communities expand their repertoires to perform in several contexts.

All that comprises an epistemological revision and expansion of what is being accepted as mathematics. I already mentioned in Chapter 0 that insights into the intertwined condition of language and mathematics was previously pointed in ethnomathematics by Cauty (2009), when he made an account of the big revolutions in the history of mathematics, relating them to periods in which languages and cultures collided.

In this thesis, I take the interplay of representations and the creation of new knowledge as fundamental for mathematics and consider that intercultural translations can be accomplished purposefully, aiming not merely to coin new words, but fostering new understandings in culture, language, and mathematics, as exposed in Chapters 2, 4 and 5.

The creation of new understandings, somewhat heretical from the canon of classical epistemology, has a symbolic and political character that was already pointed out for ethnomathematics by Mendes (2004) and Alangui (2010). Mendes analyses how indigenous teachers decide to label some of their practices as "mathematical," in a clear counteract to the dominant power and as a strategy to reaffirm their indigenous identity. This labelling is a deliberate and intentional association, a form of appropriation, in the sense that Nasa indigenous people have proposed (see Chapter 5).

Concerning the translation of practices, Alangui stressed:

> [...] the concept of mutual interrogation suggests that cultures and cultural knowledge systems can and should engage in critical dialogue with each other, because it is only through interrogation and critique that transformation can happen. If ethnomathematics desires the transformation of mathematics, then it has to facilitate critical dialogue between mathematics and other knowledge systems (Alangui, 2010, p. 187).

I find resonances between the concept of *propio* and this main feature of mutual interrogation, as mutual interrogation rejects the suggestion that knowledge systems could not and should not interrogate each other. In a similar manner, Knijnik (2002) explored the capacity to play several language games, the possibility that forms of life can expand to cope with different grammars.

I present this account to show that ethnomathematics have already developed experiences and raised some reflections on the importance of translations and intercultural dialogue. There is an evident connection of those experiences and reflections with the decolonial notion of *intercultural translation*, coined by de Sousa Santos (2012) as a "procedure that allows for mutual intelligibility among the experiences of the world, both available and possible" (de Sousa Santos, 2012, p. 59). I argue that such notion can strengthen future developments of the theory and practice of ethnomathematics, with insights like:

> The 'translation of knowledges ' assumes the form of a 'diatopical hermeneutics'. This kind of work is what makes the ecology of knowledges possible. 'Diatopical hermeneutics' consists in interpreting two or more cultures, aiming to identify isomorphic concerns among them and the different answers they provide. (de Sousa Santos, 2012, p. 59)

> Diatopical hermeneutics stems from the idea that all cultures are incomplete and may, therefore, be enriched by engaging in dialogue with or confronting other cultures. Recognising the relativity of cultures does not necessarily imply adopting relativism as a philosophical stance. It does imply, however, conceiving of universalism as a Western particularity whose supremacy as an idea does not reside in itself, but rather in the supremacy of the interests that support it. (de Sousa Santos, 2012, p. 60)

> The work of translation falls simultaneously on knowledges and cultures, on the one hand, and on the practices and agents, on the other. Moreover, this work what unites and separates them. The common points represent the possibility of an aggregation from bottom up, which is the only alternative to a top-down aggregation imposed by a general theory or a privileged social actor. (de Sousa Santos, 2012, p. 62).

These quotations allow me to stress the importance of language and intercultural translations for ethnomathematics, because they showed the non-relativistic philosophical stances that this thesis adhered in the previous sub-

section and connect with the performativity that will be explained in the next subsection.

Performativity

In this subsection, I explain why the theorisation of ethnomathematics explored in this thesis deserves to be called performative and how such notion is related to the sociology of emergences (one of the components of an epistemology of the South).

In simple and plain terms, an ethnomathematics research experience becomes performative whenever the "contemplative admiration" is broken and knowledge is created or cultural practices renewed through the research experience. However, the idea of performativity can be refined. To assume a performative perspective in ethnomathematics implies that research results are not stated once and for all, but they are constantly demanding for a rephrasing, reframing, a reassessment, they are to be lived, re-enacted again and again. As the results are ephemeral[11] and vanish, ethnomathematics research becomes a type of performance that is different in each instantiation.

As explained earlier in Chapter 3, "the conceptual movement from identification to creation demands that research projects in ethnomathematics become political processes of negotiation of meaning where differences are exposed and fostered" (Parra-Sanchez, 2017, p. 97). The practice of bartering and the relational approach presented in that chapter comprised a performative condition when saying: "Instead of previous and pre-established elements to be uncovered, we might consider the multiple and unexpected possibilities to be developed" (Parra-Sanchez, 2017, p. 101). Such conceptual movement in the results and the activity of ethnomathematics is a coherent consequence of embracing the culture-dependence and historicity of (mathematical) knowledge and the agency of groups and communities in the constitution of new forms of knowledge.

In similar ways, the problem of symmetry explored in Chapter 6 is also connected with a performative condition, as far it is concerned with the relation researcher-researched communities, and such relation needs to be stablished differently with each community, every time, and evolves along the researcher's process. In its essence, symmetry implies a performative action.

The performative character of ethnomathematics that my theorisation envisions is an enactment of the decolonial notion of sociology of emergences because:

> The sociology of emergences consists in undertaking a symbolic enlargement of knowledges, practices and agents in order to identify therein the

[11] Ephemeral not in the sense of insignificant, but in its impossibility of permanence.

> tendencies of the future (the Not Yet) upon which it is possible to intervene so as to maximise the probability of hope vis-à-vis the probability of frustration. (de Sousa Santos, 2012, p. 56)

My theorisation invites us to look back on previous ethnomathematical research with new eyes, to find traces of this sociology of emergences on that production. According to this view, ethnomathematical research basically traces connections between cultural practices and mathematical objects, to show how culturally embedded is knowledge production. Any modelling or mathematical description of cultural practices is a connection. Cultural contextualisations of mathematical practices are also connections. All of them are expanding the frontiers of what is accepted as mathematical knowledge and culture. No matter if they are defrosting mathematics (Gerdes, 2003), finding a family resemblance among practices (Knijnik, 2012), or describing the QRS-systems of a group (Barton, 2008); all of them are arguing for the plausibility of a connection for the future and they are shaking the limits of what is being accepted as mathematics. In that sense, my theorisation is not only interested in the interplay of collective/individual (as explored by Swanson (2015) with the notion of *Ubuntu*), but in the interplay of past/present/future. Thus, the theorisation intends to be a sociology of emergences, that is assumed as "the inquiry into the alternatives that are contained in the horizon of concrete possibilities"(de Sousa Santos, 2012, p. 56).

A glimpse of ending

In the last subsections of this chapter, I argued that the notions that integrate the epistemologies of the South (sociology of absences, sociology of emergences, ecology of knowledges, and intercultural translation) and the critique of the mainstream philosophy of sciences have a clear convergence with founding principles of ethnomathematics. For instance, proposals for an ecology of knowledge(s), in which diverse types of knowledge exist through networks of practices, meets Wittgensteinian conceptions on mathematics as a social practice, which is a central issue for ethnomathematics.

I contend that an unfolding of decoloniality provides robust foundations for the theoretical contributions introduced in this thesis for ethnomathematics. With such foundation, ethnomathematics can build answers to the philosophical and political challenges that this thesis highlighted. Decolonial studies are a response to the epistemicide, defined by de Sousa Santos (2010) as the colonial process of systematic denial and destruction of the knowledge of oppressed people and communities. Ethnomathematics has been clear since its upbringings in its rejection to such epistemicide, and by following the intended theorisation, the research program can find concrete theoretical and methodological tools to counteract the epistemicide.

It is important to observe that the link between ethnomathematics and decolonial theory is not new. It has been raised before by Khan (2011), Melo et al. (2011) and, more recently, by Tamayo-Osorio (2017). In particular Iseke-Barnes (2000) have related language, decoloniality and ethnomathematics. However, this thesis differentiates from those previous decolonial approaches in its emphases on the locus of enunciation and on the concomitant ways to decolonise research. This is what comes to be my specific contribution to ethnomathematics theorisation.

III.3 A last metaphor –with Amazonian Mythology–

The Curupira (a.k.a. Kurupira) is a famous creature in the Amazonian mythology. It is known as the master and guardian of the jungle, who takes care of and reigns over the jungle and its creatures. It is a shape-shifter that can mislead the bad-intended hunters or travellers making them get lost in the jungle.It is described as a boy with ginger hair and feet pointing backwards, which he uses to walk stamping footprints that lead to the starting point. In this way, he confuses those trying to find him.

Music on the Curupira's path

In 2000, a group of academically trained musicians from Bogota (Colombia) decided to form an ensemble to explore the universe of sonorities of Colombian music. They created a band called Curupira and its first album was "pa'lante, pa'trá" (back and forth). Those young multi-instrumentalists, travellers, and music researchers managed to connect different traditions, being conscious of their contemporary urban roots, while, simultaneously, remaining in contact with musical traditions of Colombia. To identify their own musical style, they use the mythical image of the Amazonian Curupira. They refuse classifications like world-music, folk, jazz, or fusion for their sound. They are only making new Colombian music, combining, mixing, and expanding new sonorities.

The band Curupira has released six albums, playing and recording new songs with traditional Colombian musicians and with Ethiopian, Israeli, and U.S. jazz musicians. Curupira tours around the world and inside Colombia. The band plays Colombian instruments like *marimba*, *guasá*, *tambora*, *alegre*, *gaita.* and *llamador*, together with Indian instruments like sitar and tambours, and with international instruments like electric guitars, bass, drums. They perform traditional Colombian rhythms like *gaita*, *chalupa*, *fandango*, *puya*, *champeta*, *currulao* and *joropo.* These are integrated with contemporary urban sounds like jazz, rap, rock, and funk.

Ethnomathematics on the Curupira's path

This thesis envisioned ethnomathematics with the two images of Curupira. If this theory could sound, it would be a Curupira's song. Connecting ancestral and present mathematical traditions to build the future, going back and forth. Embracing local and global traditions, valorising and fostering differences. Without dichotomies and without eclecticisms. To move as the Curupira is to move with a strategy of constant becoming, that will allow us to face the philosophical-political threat of Scylla and Charybdis.

References

Alangui, W. (2010). *Stone walls and water flows: Interrogating cultural practice and mathematics*. Phd thesis, University of Auckland, New Zealand.

Albanese, V., Adamuz-Povedano, N., and Bracho-López, R. (2017a). Development and contextualization of tasks from an ethnomathematical perspective. In Chronaki, A., editor, *9th International Conference of Mathematics Education and Society*, volume 1, pages 205–211. University of Thessaly.

Albanese, V., Adamuz-Povedano, N., and Bracho-López, R. (2017b). *The Evolution of Ethnomathematics: Two Theoretical Views and Two Approaches to Education*, pages 307–328. Springer International Publishing, Cham.

Barton, B. (1996). *Ethnomathematics: Exploring cultural diversity in mathematics*. Thesis, University of Auckland, New Zealand.

Barton, B. (1999). Ethnomathematics and philosophy. *ZDM*, 31(2):54–58.

Barton, B. (2008). *The Language of Mathematics: Telling Mathematical Tales*. Mathematics Education Library. Springer, New York.

Barton, B., Fairhall, U., and Trinick, T. (1998). Tikanga reo tātai: Issues in the development of a māori mathematics register. *For the learning of mathematics*, 18(1):3–9.

Boaler, J. (2003). Studying and capturing the complexity of practice–the case of the. *International Group for the Psychology of Mathematics Education*, 1:3–16.

Bolaños, G., Bonilla, V., Caballero, J., Espinoza, M., García, V., Hernández, J., Peñaranda, D., Tattay, P., Tattay, L., and D., P. (2016). *Nuestra vida ha sido nuestra lucha: resistencia y memoria en el Cauca indígena*. Organización Internacional para las Migraciones (OIM-Misión Colombia).

Borden, L. (2013). What's the word for…? is there a word for…? how understanding mi'kmaw language can help support mi'kmaw learners in mathematics. *Mathematics Education Research Journal*, 25(1):5–22.

Borden, L. and Wagner, D. (2013). *Naming Method:"This is it, maybe, but you should talk to…"*, pages 105–122. Springer.

Castro-Gómez, S. (2007a). Decolonizar la universidad. la hybris del punto cero

y el diálogo de saberes. *El giro decolonial. Reflexiones para una diversidad epistémica más allá del capitalismo global*, pages 79–91.

Castro-Gómez, S. (2007b). Michel foucault and the coloniality of power. *Tabula Rasa*, (6):153–172.

Cauty, A. (1998). Etnomatemáticas: El laboratorio kwibi urraga de la universidad de la guajira. In *Congreso de Antropología – Simposio de Etnoeducación, 7*, volume 7, pages 267–365. Fondo de Publicaciones de la Universidad del Atlántico.

Cauty, A. (2009). *Como nascem e se desenvolvem as tradições escritas matemáticas. Exemplos mesoamericanos*, pages 29–52. Editora da Universidade Federal Fluminense, Niteroi.

Cauty, A. and Ramos, A. (1990). Vigilancia etnocultural: el caso de la numeración tradicional nasayuwe. *Boletín de Lingüística Aborigen*, 2:3–15.

Cusicanqui, S. (2012). Ch'ixinakax utxiwa: A reflection on the practices and discourses of decolonization. *South Atlantic Quarterly*, 111(1):95–109.

de Sousa Santos, B. (2010). *Descolonizar el saber, reinventar el poder*. Ediciones Trilce, Montevideo.

de Sousa Santos, B. (2012). Public sphere and epistemologies of the south. *Africa Development*, 37(1):43–67.

Dei, G. and Kempf, A. (2006). *Anti-colonialism and education*, volume 7. Sense Publishers, Rotterdam.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics*, 5(1):44–48.

Foucault, M. (1971). *The order of things. An archaeology of the human sciences*. Vintage Books, New York, april 1994 edition.

Gerdes, P. (2003). *Awakening of geometrical thought in early culture*. MEP Publications, Minneapolis.

Grosfoguel, R. (2009). A decolonial approach to political-economy: Transmodernity, border thinking and global coloniality. *Kult*, 6:10–38.

Grosfoguel, R. (2011). Decolonizing post-colonial studies and paradigms of political-economy: Transmodernity, decolonial thinking, and global coloniality. *Transmodernity: Journal of Peripheral Cultural Production of the Luso-Hispanic world*, 1(1).

Harding, S. (2008). *Sciences from below: Feminisms, postcolonialities, and modernities*. Duke University Press, Durham.

Horsthemke, K. and Schafer, M. (2007). Does' african mathematics' facilitate access to mathematics? towards an ongoing critical analysis of ethnomathematics in a south african context. *Pythagoras*, 2007(65):2–9.

Iseke-Barnes, J. (2000). Ethnomathematics and language in decolonizing mathematics. *Race, Gender & Class*, pages 133–149.

Khan, S. (2011). Ethnomathematics as mythopoetic curriculum. *For the Learning of Mathematics*, 31(3):14–18.

Knijnik, G. (2002). Curriculum, culture and ethnomathematics: The practices

of 'cubagem of wood' in the brazilian landless movement. *Journal of Intercultural Studies*, 23(2):149–165.

Knijnik, G. (2004). *Lessons from research with a social movement*, pages 125–141. Kluwer Academic Publishers, New York.

Knijnik, G. (2012). Differentially positioned language games: ethnomathematics from a philosophical perspective. *Educational Studies in Mathematics*, 80(1-2):87–100.

Latour, B. (1993). *We have never been modern, trans. C. Porter*. Cambridge, MA: Harvard University Press.

Latour, B. (2000). *On the Partial Existence of Existing and Nonexisting Objects*, page 247–269. University of Chicago Press., Chicago, IL.

Latour, B. (2004). *Politics of nature*. Harvard University Press, Cambridge.

Makoni, S. (2011). Sociolinguistics, colonial and postcolonial: an integrationist perspective. *Language Sciences*, 33(4):680–688.

Makoni, S. B. and Pennycook, A. (2007). *Disinventing and Reconstituting Languages*. Multilingual Matters, Clevedon.

Meaney, T., Trinick, T., and Fairhall, U. (2011). *Collaborating to meet language challenges in indigenous mathematics classrooms*. Mathematics Education Library. Springer, New York.

Melo, T., Fantinato, M., Thees, A., Silveira, A., and Soares, G. (2011). O programa etnomatemática como humanizador do ensino de matemática (co). In *XIII Conferência Interamericana De Educação Matemática*.

Mendes, J. (2004). *Aspectos políticos e simbólicos na apropriação do discurso da etnomatemática: o caso dos professores Kaiabi do Parque Indígena do Xingu*, volume 1, page 364–376. EDUNISC, Santa Cruz do Sul.

Mesquita, M. (2016). Urban boundaries and love: The rebirth of political thought within the disturbing education. *International Journal of Žižek Studies*, 9(2).

Mignolo, W. (1999). I am where i think: Epistemology and the colonial difference. *Journal of Latin American Cultural Studies*, 8(2):235–245.

Parra-Sanchez, A. (2017). *Ethnomathematical Barters*, pages 89–105. Springer International Publishing, Cham.

Pickering, A. (1992). *Science as practice and culture*. University of Chicago Press, Chicago.

Pickering, A. (2010). *The mangle of practice: Time, agency, and science*. University of Chicago Press, Chicago.

Radford, L. (2014). *Cultura e historia: dos conceptos difíciles y controversiales en aproximaciones contemporáneas en la educación matemática*, pages 49–68. Livraria da Física, São Paulo.

Rappaport, J. (2008). Beyond participant observation: Collaborative ethnography as theoretical innovation. *Collaborative Anthropologies*, 1(1):1–31.

Rivera, C., Domingues, J., Escobar, A., and Leff, E. (2016). Debate sobre el colonialismo intelectual y los dilemas de la teoría social latinoamericana.

Cuestiones de sociología, (14):e009.

Robben, A. and Sluka, J. (2012). *Ethnographic fieldwork: an anthropological reader*. John Wiley & Sons, Pondicherry.

Rosa, M. and Orey, D. (2012). The field of research in ethnomodeling: emic, ethical and dialectical approaches. *Educação e Pesquisa*, 38(4):865 – 879.

Rowlands, S. and Carson, R. (2002). Where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? a critical review of ethnomathematics. *Educational Studies in Mathematics*, 50(1):79–102.

Rowlands, S. and Carson, R. (2004). Our response to adam, alangui and barton's "a comment on rowlands & carson 'where would formal, academic mathematics stand in a curriculum informed by ethnomathematics? a critical review"'. *Educational Studies in Mathematics*, 56(2-3):329–342.

Savransky, M. (2012). Worlds in the making: Social sciences and the ontopolitics of knowledge. *Postcolonial Studies*, 15(3):351–368.

Smith, L. (2013). *Decolonizing methodologies: Research and indigenous peoples*. Zed Books Ltd.

Swanson, D. (2015). Ubuntu, radical hope, and an onto-epistemology of conscience. *Journal of Critical Southern Studies*, 3:96–118.

Tamayo-Osorio, C. (2017). A colonialidade do saber: Um olhar desde a educação matemática. *Revista Latinoamericana de Etnomatemática*, 10(3):39–58.

Trinick, T. (1994). *Mathematical Māori discourse and the language of quantification*. Thesis, The University of Auckland, New Zealand.

Vilela, D. (2010). Discussing a philosophical background for the ethnomathematical program. *Educational Studies in Mathematics*, 75(3):345–358.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. *Mathematical Thinking and Learning*, 9(1):31–50.

Wagner, D. and Borden, L. (2012). *Aiming for equity in ethnomathematics research*, pages 69–87. Springer, Dordrecht.

Watson, I. (2002). The dynamics of barter. *Negotiating cultures: Eugenio Barba and the intercultural debate*, pages 94–111.

Woolgar, S. (1988). *Knowledge and reflexivity : new frontiers in the sociology of knowledge*. Sage, London.

SUMMARY

This thesis adopts a decolonial standpoint to introduce a new theoretical framework for ethnomathematics. There is an emerging trend in ethnomathematics, that considers challenges posed by socio-political movements worldwide. That trend deserves a solid foundation and this thesis is aiming to provide it. The proposed framework calls into question the current theorization (and critique) of ethnomathematics, by claiming a performative, non-essentialist, and interactional approach that assumes the inseparability of socio-political concerns and philosophical questions within the field.

ISSN (online): 2446-1636
ISBN (online): 978-87-7210-217-7

AALBORG UNIVERSITY PRESS