

Mathematics education and the political

An ideology critique of an educational research field

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Publication date:
2011

Document Version
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Pais, A. (2011). *Mathematics education and the political: An ideology critique of an educational research field*. Aalborg Universitet. Institut for Læring og Filosofi.

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Mathematics Education and the Political: An Ideology Critique of an Educational Research Field

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PhD Thesis

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September 2011

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Summary

This thesis seeks to posit mathematics and its education within the political and economical spectrum of our times. It takes advantage of the contemporary combination of Hegel's dialectics, Lacanian psychoanalysis and Marx's critique of political economy, carried out by the Slovenian philosopher Slavoj Žižek, to develop an ideology critique of mathematics education as a field of research. It proceeds by acknowledging the groundbreaking critique carried out by cultural, social and political approaches on a field historically closed around the disciplines of mathematics and psychology. These approaches brought to the research scene issues of social justice, equity, culture, power and identity that are becoming widely recognized as influential when learning mathematics. Nonetheless the problem of failure and inequity that characterizes school mathematics persists worldwide. Although acknowledging the political dimension of the problem, the vast majority of the research within the sociopolitical turn performs as if such problem could be solved within mathematics education. It will be argued that such disavowing of the Political makes the community hostage to an ideology that holds back a comprehension of the problem in its totality; thus restraining the possibility of change. The problem of failure in school mathematics, and education in general, is an integrative part of current school education. And as such, it cannot be conceptualized without an understanding of the relation between scholarized education and capitalism as the dominant mode of social formation. It is the wager of this thesis that positing schools as crucial modern institutions for the reproduction of capitalist economy and ideology can bring us closer to the complex situations teachers and students experience in schools.

The analysis focuses on recent research revolving around five recent key research areas in mathematics education: the role of theory, the achievement of equity in mathematics education, the importance of the use-value of mathematics, ethnomathematics, and critical mathematics education. These are the areas addressed in the eight peer-reviewed articles that compound the core of the thesis. The method used to analyse research can be called *ideology critique*, and is based on Žižek's recent revitalization of this Marxist concept. It consists in showing the incongruities between the discourse emanating from research and its actualization within a life world context—in this case, schools. What normally runs well within the research discourse, when actualized in a specific practice, often encounters a series of obstacles which end up perverting the official intention. Usually research proceeds by eliminating such obstacles, so that the official aims can be fully actualized: equitable mathematics education, valorisation of different cultures, useful and critical

mathematics. However, an ideology critique is instead interested in analysing these obstacles since they stand for the symptomatic points which allow one to grasp the political and economical relevance of the school system.

One important implication of this study is to call attention on the importance of developing not only research reporting successful experiences, but also of investigating failed attempts to promote a meaningful mathematics instruction. ‘Failure’ has the potentiality to point towards the antagonistic character of schooling, thus ‘estrangle’ the mathematics education community to the self-evidence of its established identity. Therein resides the importance of taking social antagonism—once upon a time called *class-struggle*—as a constitutive element of the Political.

The thesis is organized in two parts. The first part starts by giving a general overview, addressing the issues that led me to conduct this analysis. Those reasons are profoundly related with my nine years experience as a mathematics teacher. The major bulk of this chapter is occupied with a discussion addressing theoretical and methodological aspects of my work. The purpose is to give the reader a philosophical account of the axis Hegel-Marx-Lacan-Žižek and how it is used in the different articles. A section is also reserved to explain the organization of the thesis and to introduce the articles. This chapter ends with a small section in the guise of a conclusion. The second part consists of the eight peer-reviewed articles that make the main core of the thesis.

Resumé

Afhandlingen *Matematikkens didaktik og det Politiske – En ideologikritik af et uddannelsesforskningsfelt* har til formål at placere matematik og didaktisk forskning i et nutidigt politisk og økonomisk perspektiv. Den baserer sig på den nuværende kombination af Hegels dialektik, Lacans psykoanalyse og Marx' kritik af politisk økonomi, udviklet af den slovenske filosof Slavoj Žižek. Der udvikles en ideologikritik rettet mod matematikkens didaktik som internationalt forskningsfelt. Afhandlingen anerkender det banebrydende bidrag fra den ”kulturelle, sociale og politiske” forskning i et felt, der historisk set har været lukket omkring disciplinerne matematik og psykologi. Denne tilgang har tilført forskningsområdet opmærksomhed omkring problemstillinger som social retfærdighed, lighed, kultur, magt og identitet i relation til matematikundervisning og -læring. Men på trods af forskningsresultater findes der fortsat problemer med nederlag og ulighed, hvilket er karakteristisk for matematik som skolefag overalt i verden. Selvom den politiske dimension af problemet er anerkendt, agerer langt størstedelen af forskningen inden for det socio-kulturel-politiske område, som om dette problem kunne løses inden for matematikundervisningens grænser. Der argumenteres for, at en sådan benægtelse af det Politiske tager samfundet som gidsel for en ideologi, som tilbageholder en forståelse af problemet i hele sin udstrækning; hvorved muligheden for forandring begrænses. Problemet med nederlag i skolens matematikundervisning —og i undervisning i det hele taget— er en integreret del af den moderne skoleundervisning. Og derfor kan det ikke konceptualiseres, uden at man også forstår relationen mellem skoleundervisning og kapitalisme som en dominerende faktor i den sociale dannelsesproces. Det er denne afhandlings påstand, at man ved at anskue skolerne som vitale moderne institutioner, som reproducerer den kapitalistiske økonomi og ideologi, kan nærme sig de komplekse situationer, som lærere og elever oplever i skolerne.

Analysen fokuserer på den seneste forskning, som beskæftiger sig med fem nyere nøgleområder inden for forskning i matematikkens didaktik: teoriens rolle, opnåelsen af lighed i matematikundervisning, vigtigheden af matematikkens brugsværdi, etnomatematik, og kritisk matematikundervisning. Disse områder behandles i otte peer-reviewede artikler, som udgør kernen i afhandlingen. Den metode, som er anvendt i analysen af forskningsresultaterne, kan kaldes *ideologikritik*. Den baserer sig på Žižeks nylige revitalisering af dette marxistiske begreb. Det består i at påvise uoverensstemmelserne mellem den diskurs, der udspringer af forskningen, og realiseringen af denne i en levende sammenhæng i verden—i dette tilfælde i skolerne. Det,

der normalt flyder let i en forskningsdiskurs kan, når det realiseres i en specifik praksis, ofte støde på forhindringer, som kan ende med at pervertere den officielle hensigt. Sædvanligvis kommer forskningen videre ved at fjerne sådanne forhindringer, således at de officielle hensigter kan realiseres fuldt ud: lighed i matematikundervisningen, valorisering af forskellige kulturer, nyttig og kritisk matematik. Men en ideologikritik er i stedet interesseret i at analysere disse forhindringer, fordi de repræsenterer de symptomatiske pointer, som sætter os i stand til at begribe skolesystemets politiske og økonomiske relevans.

En vigtig konsekvens af afhandlingens undersøgelse er at gøre opmærksom på vigtigheden af at udvikle ikke alene forskning, som beretter om succesfulde eksperimenter og oplevelser, men også at undersøge fejlslagne forsøg på at fremme en meningsfyldt matematikundervisning. 'Nederlag' har i sig et potentiale til at pege hen imod undervisningens antagonistiske karakter. På den måde er der mulighed for at 'fremmedgøre' matematikdidaktikere over for det, der er blevet selvindlysende i feltets etablerede identitet. Heri ligger vigtigheden i at betragte social antagonisme —tidligere betegnet som "klassekamp"— som et grundlæggende element i det Politiske.

Afhandlingen organiseres i to dele. Den første del begynder med at give et samlet overblik, idet den beskriver de problemstillinger, der foranledigede mig til at foretage denne analyse. Her er der en tæt forbindelse til mine ni års erfaring som matematiklærer. Hovedparten af dette kapitel udgøres af en diskussion, som beskriver teoretiske og metodologiske aspekter af mit arbejde. Hensigten er at give læseren en filosofisk redegørelse for Hegel-Marx-Lacan-Žižek-aksen og anvendelsen heraf i de forskellige artikler. Der er også et afsnit, som forklarer, hvordan afhandlingen er organiseret, og som desuden præsenterer de otte artikler. Dette kapitel slutter med et lille afsnit, som har karakter af en konklusion. Den anden del indeholder de otte peer-reviewede artikler, som udgør kernen i afhandlingen.

Acknowledgements

In a time when university discourse is becoming increasingly colonized by an economic idiom, where expressions such as ‘growth’ and ‘profit’ dictate the overriding goals of a whole swathe of social, cultural and intellectual activities which had previously been understood and valued in other terms, it is remarkable that a governmental institution responsible for funding scientific research opens its online site with the following quotation by Albert Einstein: “Science is a wonderful thing if one does not have to earn one’s living at it”. Such an institution exists. It is called Fundação para a Ciência e a Tecnologia, and has been since 1997 giving money to people to do science. During the last four years I have been one of these people. Without the support of this Portuguese institution this thesis would not exist. Thus, my first and foremost important acknowledgement goes to all the people who have made it possible for an entire generation of researchers, from all around the world and from all areas of knowledge, to work on science without having to sell themselves for a mere song.

Because there are things that money cannot buy, I also express my deepest gratitude to the team of accountants, bank managers, husbandman, housemaids, hosts, motorists, cooks, and creditors who, with all their heart, supported me during these last four years. They are my parents, João and Aurora. They made it possible for me to go abroad without leaving behind what matters most: the sense of belonging. In Portuguese we call it *saudade*. Alone, they will always be there for me, unreservedly.

I like to think of life as floating along. Nonetheless, from time to time, it appears as if everything was already meant to be. This is what I feel regarding my relation with Paola. A series of inexplicable episodes—from the day of her PhD defense in Copenhagen to a wonderful afternoon in Albufeira or a conversation in Monterrey—crystallized three years ago, when I came to Denmark to work under her supervision. Her enthusiasm for investigation and the formidable knowledge of mathematics education as a field of research has made this supervision a true pleasure. She has always nurtured great interest for my work and shown endless willingness in opening the doors of the field to me. Her generosity is breathtaking. Looking back, it is difficult to point out all the things I learned from her. We share a lot of ideas, and diverge in others. But deep down, what unites us is a desire to push things further, to complicate what appears to be clear, to disobey. This is Paola’s methodology, and I felt like a fish in the water in it.

[G]reat psychologists, and even the psychoanalysts are full of the idea of “total personality.” At any rate, it is always the unifying unity which is in the foreground. I have never understood this, for if I am a psychoanalyst I am also a man, and as a man my experience has shown me that the principal characteristic of my own human life and, I am sure, that of the people who are here —and if anybody is not of this opinion I hope that he will raise his hand—is that life is something which goes, as we say in French, à la dérive. Life goes down the river, from time to time touching a bank; staying for a while here and there, without understanding anything—and it is the principle of analysis that nobody understands anything of what happens. The idea of the unifying unity of the human condition has always had on me the effect of a scandalous lie.

J. Lacan,

Of Structure as the Inmixing of an Otherness Prerequisite to Any Subject Whatever

(Talk at John Hopkins University, Baltimore, 1966)

Part 1

How to Do a Totality with Failures: The Road to the Articles

Getting Here: The Motives and the Spirit of the Thesis

Eleven years ago, and after four years studying mathematics, didactics and all the sort of pedagogical theories, I started my work as a teacher in a public Portuguese school. Until this day I had never questioned my formation as a mathematics teacher, which I praised as one of high value. With three years of mathematics and one more of pedagogy and didactics I was about to start my mission: being a ‘teacher’. And what a mission: teaching to young people this wonderful piece of knowledge called mathematics. I spent a whole month preparing the lessons for the first trimester, imagining a classroom where all the nice things I had learned in the previous years could be applied. Although I was aware of what can get wrong in a classroom—after all, I had been a student almost my entire life—my feeling was one of completion: I knew mathematics, I had prepared well my lessons, and I was enthusiastic about getting started. This feeling, however, did not last long. Indeed, it ended right after I enter the classroom, put my briefcase on the table, and look to what I had in front of me: twenty eight teenagers, gazing at me, waiting for me to say something. There is no university course that can prepare you for the real of a classroom. My whole experience of schooling which had filled the last two thirds of my life was suddenly put under a completely new perspective. Performing the role of a teacher places you in an entirely new relation with schooling: looking at those adolescents I felt as being interpellated by something bigger than myself. I was carrying the insignia of schooling and of mathematics. This turns education into something more than the teaching of a piece of knowledge. I remember mentioning to one of my teacher colleagues: “you feel like doing someone else’s job”. You feel that you are participating in something which goes beyond the aseptic and ‘clean’ image of school conveyed by my former education. I realised that being a teacher was not the end, the corollary of an intense education, but the beginning of something completely new.

In the following years, I taught in six Portuguese schools, and completed a master degree at the University of Lisbon. Eight years ago I got a permanent position in a suburban school known for receiving students from a lower social, cultural and economical

background. I worked for four years in this school, until I decided it was time to stop and wonder. I needed time to reflect on my role as a teacher, and all the problems I felt during my work at the schools. A PhD project appeared as the true excuse to do all the reading, thinking, and writing that being a full time teacher strategically inhibits. I wanted to find a language to articulate the contradictions I felt as a teacher.

However, I soon realised that, as a field of research, mathematics education, with its emphasis on mathematics and psychology, was not the most prolific field to find such a language. Being dominated by a discourse that orbits around the development of better stratagems to teach and learn mathematics, the field lacks a comprehension of an entire set of problems that cannot be addressed within the theoretical straitjacket of ‘learning’ that drives mathematics education research. These problems are in a way more mundane than the increasingly specialized research on topics as diverse as cognitive strategies for teaching geometry proof, or teaching methods for modelling students’ self-efficacy expectations. They are related with contradictions in my role as a teacher, and the role of school in society. Why in a class of thirty students will there always be some (or many) students who fail? Why do some students develop strategies to pass without learning, many times with the complicity of the teacher? Why are teachers asked to give a grade to each student by the end of the year? Why do students tend to forget what was apparently well learned? These were some of the questions I could not find an answer to in mathematics education research. Indeed, the impression I started to have of research is that it applies to ‘types’ rather than to the diversity of childhood and the complexity of schooling. I realised that research in mathematics education, by creating an imaginary student and an idyllic school that only very seldom can be found in reality, did not offer me tools to analyse the problems I felt as a teacher. There is plenty of research available if you want to know more about problem solving, the use of technology or the learning of quadratic functions. However, only recently has the field opened its doors to research which seeks to theorize mathematics education as something broader than the process of teaching to someone a piece of knowledge. There were, by the end of the 1990s, a significant number of discontent researchers with the way research was being carried. They wanted to emphasize cultural, social and political aspects of school mathematics that were until then absent from research. This led some people to talk about a *social turn* in mathematics education research (e.g., Lerman, 2000). It was within this uprising arena of research that I found the kind of readings that allowed me to engage critically with school mathematics. The work of Ole Skovsmose, Roberto Baldino, Tânia Cabral, Thomas Popkewitz, and Paola Valero are among the ones I most greedily read. They were an important part of my Master thesis, which I developed under the supervision of João Filipe Matos. Soon after having finished the Master, I started with the PhD, still having João Filipe as a supervisor. However, after a brief passage by São Paulo where I worked

with Maria do Carmo Domite, the opportunity came of working with Paola. Before the end of the first year I was in Aalborg, doing research with someone in tune with my growing criticisms on the field of mathematics education.

My interest in Paola's work has to do with her approach to research. Coming from a background in political sciences, Paola has been calling researchers' attention to the importance of, on the one hand, investigating not just the didactical aspects of mathematics and its education, but also the role this science plays within the broader social and political spheres that frame our activity, and, on the other hand, the necessity of considering the common objects of research—teacher, student and mathematics—as embedded in a sociopolitical context that goes beyond the constricted view of a “cognitive learner” (Valero, 2004) and a piece of knowledge ready to be learned. In this way, what Paola calls the “network of sociopolitical practices of mathematics education” (Valero, 2010) allowed me a space where I could analyse many of the problems I felt as a teacher, which I had identified not as being didactical, but as social and political problems.

The last decade has been prolific in research concerned with issues of social justice and equity, and some authors have found in postmodern theories a language to address problems, which have been historically unconnected from mathematics (e.g., Walshaw, 2004; Valero & Zevenbergen, 2004; Atweh, Graven, Secada & Valero, 2010; Gutiérrez, 2010). These theories brought to the research gaze cultural, social and political aspects of mathematics education that are now widely recognized as influential when learning mathematics. Fields such as *critical mathematics education*, *ethnomathematics*, and *sociopolitical perspectives* have been deploying theoretical frames, views of society and conceptualizations of the learning process that resemble many of the postmodern assumptions, even if the authors do not explicitly situate their work within the postmodern. For instance, critical mathematics education as developed by Ole Skovsmose in the early 1990s (e.g., Skovsmose, 1994), owes more to critical theory and the late development carried out by Jürgen Habermas¹, than to the postmodern. However, crucial notions in the recent work of Skovsmose (e.g., Skovsmose, 2005) such as uncertainty, doubt and *aporia*, and the use of authors such as Michel Foucault, touches some of the postmodern tenets. Also the field of ethnomathematics, especially the epistemological critique made to the enduring belief in the universality and neutrality of mathematical knowledge, has taken advantage of postmodern theories to show how epistemology is less a discussion on whether knowledge is itself universal or ‘local’, but, what intentions, what politics, allow us to claim that some knowledge (like academic mathematics) is universal (e.g., Powell & Frankenstein, 1997; Knijnik, 1998, 2007). Finally, the sociopolitical perspective is arguably the one of the three that more openly shows the flag of the

¹ Who, as we know, has been a critic of the postmodern idea.

postmodern. Authors such as Paul Ernest, Paola Valero, Margaret Walshaw or Rochelle Gutiérrez have been writing or editing books that disturb the taken-for-granted assumptions, ‘truths’ and certainties of mathematics education research, whether they concern the modern rationality fuelling mathematics and its education, or the mainstream didactical and individually centred approach to learning (e.g., Ernest, 1994; Walshaw, 2004; Valero & Zevenbergen, 2004; Gutiérrez, 2010).

The spirit animating research within these three trends is one that stimulates a positioning of mathematics within the social and political spectrum of our times. Emancipation is also a flag of research in the social turn. There is at the core of this research a desire for betterment. No more the naïve modern idea of utopian progress, in which human beings through the deployment of their rationality will achieve the ideal high-tech society with plenty of resources available to anyone, but a more localized concern with empowerment and liberation from a plurality of social constraints and injustices associated with race, sexuality, ecology, language, cultural minorities, colonialism, religion, rurality, and class. In line with the postmodern paradigm, change is not conceived as a change in the totality—a change in the global mode of production, for instance—but as being based in local struggles which take into account the complexities of particular contexts (D’Ambrosio, 2003; Gutstein, 2003; Valero & Stenoft, 2010; Skovsmose & Borba, 2004; Ernest, 2004; Knijnik, 2007; Gutiérrez, 2010).

After finishing the Master thesis, I spent the next three years reading mathematics education literature within the social turn, especially the one explicitly adopting postmodern theories, such as the philosophy of Michel Foucault. Reading his work had the effect of an eye-opening regarding many of the problems I felt as a teacher. I started to posit school not as a necessary institution of our society, but as the result of a historical process tamed by power relations and deeply involved in the fabrication of the modern Subject. It may be that in school people learn specific contents (mathematics, history, geography, etc.), but they also learn the symbolic norms that structure their acting in the world. In this sense, mathematics, being the dorsal spine of the modern world as Ubiratan D’Ambrosio (2002) has referred to it, when inserted in a school frame, will function as a disciplinary device of subjectification (Foucault, 1979; Rose, 1999). Mathematics education research, by being primarily focused on pragmatic approaches to the improvement of classroom practice, lacks a theoretical comprehension of how its own activity is at the same time entangled and generating broader systems of reason that format what is possible to think about practice (Valero, 2004).

Within what Gutiérrez (2010) recently called the *sociopolitical turn in mathematics education*, Foucault is a popular author. An increasing number of researchers have been using his ideas to analyse different aspects of the practice of teaching and learning mathematics (e.g., Walkerdine, 1988; Walshaw, 2001; Cotton & Hardy, 2004); in

particular to analyse mathematics education as a field of research (Valero, 2004; Popkewitz, 2004). Valero and Stentoft (2010) argue that we should move beyond a Marxist idea of social justice which was based in the inequalities generated by the system of Capital, to an idea of social justice revolving around the way modern institutions and discourse formations fabricate the modern subject. Mathematics education, both as a school practice and an academic field of research, is deeply involved in the fabrication of what Popkewitz (2004) calls the modern Subject. It conveys in its discourse an idea of normality about what it means to be a (mathematical) subject in our contemporary society. Doing so, it creates boundaries that exclude all of those who do not fit this stereotype. This is particularly the case with school mathematics, which stands as no other school subject for the power of rationality and universality that characterizes the discourse of modernity.

Notwithstanding the importance Foucault's work has had in sharpening my critique of mathematics education, I was progressively feeling that there were important dimensions of schooling that a Foucauldian approach could not reach. Such uneasiness found an echo in the work of Roberto Baldino and Tânia Cabral. While mathematics education research was roughly divided between a 'didactical' approach, with no social or political concerns whatsoever, and a 'postmodern' approach, with its emphasis in discourse and power relations, Baldino and Cabral were using old Marxist categories to analyse their work as mathematics teachers at the university (e.g., Baldino, 1998a, 1998b; Baldino & Cabral, 2006). They have been suggesting that considering school as a place of economic production may shed light on students' learning difficulties. Their entire research is an attempt to show the inconsistencies of capitalist schooling. They bring forward the usually disavowed role of mathematics within what Vinner (1997) called the *school credit system*, by developing a succinct account of the role of schools in guaranteeing the necessary *capitalist* and *worker* ideology that is to be reproduced in society. Their work allowed me to express in a new language many of the problems I felt as a teacher. And it introduced me to what would be the main theoretical support of this thesis: the philosophy of the Slovenian philosopher Slavoj Žižek.

I spent the first year of my PhD time reading Žižek without a clear understanding of what he was saying. Nonetheless, I felt that there was something in his writings that resonated in my growing concerns regarding the role of school mathematics within a world characterized by its capitalist economics and neo-liberal ideology. By the end of the first year, I had to make a choice: either I continued to read Žižek as leisure, that is, as a challenging reading with all his inspiring examples from popular culture, or I really got into it and used it as the main theoretical support for my thesis. I chose the latter, and, after one month dedicated to the reading of *For they know not what they do* (Žižek, 2008b), I started to make connections between his analysis and the current state of

mathematics education. I got acquainted with notions such as the *symptom*, the *Real* and the *Symbolic*, the *quilting-point*, *ideology*, the *big Other*, *fantasy*, *dialectics*, among others that I would later use as central notions in the articles that compound this thesis. Perhaps the main difficulty in understanding the work of Žižek had to do with my ignorance regarding his three main influences: Karl Marx's critique of political economy, Lacanian psychoanalysis, and Georg W. F. Hegel's philosophy. Reading Žižek made me go through the work of these thinkers. With the exception of Hegel, which remains on the shelf, I have been in the last two years reading Marx, and especially Jacques Lacan. These readings allowed me a stronger understanding of the concepts deployed by Žižek, and crucial insights for criticising mathematics education as a field of educational research started to emerge.

In particular, Žižek's theorizations revealed to have a great potentiality to analyse mathematics education research emanating from the sociopolitical turn. At this point, I had already decided that I would not collect any kind of 'empirical data'. If I followed the common research approach in mathematics education, I was expected to collect empirical data resulting from the observations and speech of teachers and students interacting in a math class. This would make me going back to school, now as a researcher, to investigate the same reality I had experienced during the last nine years. Somehow, I felt that I had already enough of school and I was instead interested in elaborating a theoretical argumentation which could situate my experience in the social and political arena. Since I was discontent with the way research portrays the role of school mathematics, I decided to develop a critique on mathematics education as a field of academic research, thus using research itself as data. Instead of questioning teachers and students, I decided to question research.

Thus, the analysis I develop in the eight articles that compound this thesis revolves around certain topics that I considered to be symptomatic of the way the research perceives its role in the everlasting battle of assuring 'mathematics for all'.² Philosophy was the field where I found the most fruitful theorizations to engage in this critical analysis. The work of Michel Foucault stands on the background of some of the articles, but it is above all the philosophy steaming from Marxian theory and psychoanalysis that substantiates this work. Since the earlier times of the Frankfurt School of Critical Theory that Marxian theory and Sigmund Freud's psychoanalysis have been combined to develop a critique of society. More recently, Lacan took Freud's work into an unprecedented level of deepness (some would say obscurity!) and pointed out its implications for Marxism. These were taken by Louis Althusser, which brought together Marx, Freud and Lacan to widen the Marxist concept of ideology. Within contemporary theory, the philosophy of Žižek, Fredric

² A slogan propagated both by national institutions (e.g., NCTM, 2000) and researchers (e.g., Presmeg, 2010).

Jameson's characterization of postmodernism as the cultural logic of late capitalism, and the analysis of hegemonic systems developed by Ernesto Laclau and Chantal Mouffe, all base their analysis on Marxism and Lacanian psychoanalysis. The work of these scholars was the object of my reading, and I used them throughout the eight articles of this thesis.

My critique is targeted at mathematics education as a field of research. First and foremost towards a didactical approach where a focus on 'learning' and 'mathematics' permeates the way researchers apply and develop different theorizations. This critique has already been initiated by Paola Valero (2004, 2010), and substantiates the three articles that we co-authored.

However, my critique moves beyond this and points in a different direction. I previously mentioned that, when reading literature within the social turn, I was feeling more and more unease with the way crucial problems of schooling were being conceptualized. This has to do with what I felt to be a *disavowal of the economic dimension of schooling*. As I later realised, despite the diversity of studies animating both the social and sociopolitical turn, what binds them is the rejection of a central organizing principle which could take into account the totality that capitalism is today. This is also one of the defining traces of the postmodern. As posed by Elizabeth Atkinson, a postmodern approach to education is interested in "systems of control beyond the economy and the labour market, focusing in particular on control, and 'self-control', within educational systems" (Atkinson, 2001, p. 88, quoted in Cole, 2003 p. 497).

Doing so, postmodernism emphasizes institutions and issues of subjectivity instead of production and the economic infrastructure. Mathematics educational research for social justice within a postmodern trend refuses to be tied down to a simple model, but offers many domains of action (Valero & Stentoft, 2010; Cotton & Hardy, 2004; Skovsmose & Borba, 2004; Ernest, 2004; Gutiérrez, 2010). As defended by Ernest (2004), the metanarratives that dominated scientific discourse in the 20th century are now being replaced by a multiplicity of accounts, which are related to social groups and their political agendas. According to Gutiérrez (2010), a sociopolitical perspective tends to move from Marxist views of society, where structures are conceived as determining individual agency, to critical and poststructuralist approaches where the focus is on *identity* and *power* issues. Therefore, a shift has occurred "from examining school structures and institutions to examining discourses and social interactions" (p. 3), and "educators who take a socio-political perspective stance recognize that mathematics education is identity work" (p. 17), engaged in transforming mathematics education in ways that privilege more socially just practices towards marginalised students. Against the old-fashioned categories of labour, class and political economy, the postmodern educational research privileges issues related to sexuality, gender, race, dynamics of

discipline, normalization, subjectification, administrative control, and consumerism (Seidman, 1994a).

As a consequence, the central problem of failure in school mathematics is not understood as an all-encompassing reality permeating the whole of schooling, but as having to do with the particularities of specific groups of students. Such a conceptualization has established itself as a preponderant research industry (Vighi & Feldner, 2007) enthusiastically struggling for social justice and equity.³ Indeed, as a teacher, I took advantage of this research to change some features of my lessons. I started to emphasize the history of a mathematical content through the exploration of examples which shown the importance of culture in the constitution of mathematics, I tried to start my teaching from the local knowledge of students, I made use of technology and explored real life situations in order to show the importance of mathematics for people's daily life, and I changed my discourse around the importance of mathematics. I also became aware of how the discourse, the kind of tasks and the kind of work I proposed to students comprised particular views of mathematics, and what it means to be a 'mathematical learner'. Lessons structured around the *exercise paradigm* (Skovsmose, 2001) and fomenting individual work contribute to the fabrication of what Foucault (1979) calls a *docile body*, needed for the warfare of the modern industrial age. However, when trying to change the lessons I bumped into external demands that pressure me to maintain 'traditional' classes. The increasing emphasis in national and international examination which occurred in Portugal and worldwide in the last two decades led teachers with no other options than to tailor their instructional practices to the format of the test out of the concern that if they design their teaching differently, their students fail. And a test has nothing to do with solving problems or working in groups (not to mention the use of technology, or the exploration of mathematics in real life situations). Notwithstanding my awareness of the role of schools and the mechanisms of subjectification involved in it, the crude reality of failure did not changed: I still had to put a grade in each student by the end of the year, and, ultimately, that was all it matters.

At some point I started to realize that all my particular struggles to promote success among my students were in vain. Trying to change particular aspects of my practice was not really it. There was something 'wrong' and 'frustrating' in this very effort to promote success. I was starting to conjecture that there was something inherent to schooling itself, which makes failure inevitable. The problem of achieving equity beats mathematics education as a field of research. The problem of failure in school mathematics is properly speaking a political and economical one, having to do with the way schools are structured as credit systems, where year after year teachers are asked to mark students with a grade—

³ As Cotton and Hardy (2004) symptomatically say, "social justice is a way of working" (p. 90).

preferably expressed in numbers—that will determine (sometimes in quite severe ways) their future possibilities. Apparently, there is no way of getting out of such an accreditation system, and mathematics education research ends up taking it for granted.

What is lost in postmodern approaches is precisely the capacity of finding a common denominator which colours all the particular struggles which animate the postmodern research. I agree with Valero (2004) and Valero and Stentoft (2010) that postmodern research is an opportunity to make visible what has been ignored, excluded and silenced from research—what they call the ‘noises’. However, we cannot dismiss as a remainder of the past the continuous, omnipresent noise of capitalism. Indeed, an economy based on the accumulation of capital is so omnipresent, so familiar to us, that we tend to understand it as part of the human nature. Since we are born, capitalism is the system that regulates our social relations to a point where we no longer realize its noise. Indeed, it seems that in postmodern thinking capitalism is no longer a noise but the sound of life. Therefore, a central component of my critique concerns the disavowal of the *economy* of schooling within studies which can be covered under the umbrella of social, cultural and political perspectives.⁴ Such research studies are most of the ‘data’ for my analysis in the different articles.

The fields of ethnomathematics, critical mathematics education and sociopolitical perspectives—around which most of the articles revolve—have been developing a tireless effort in disassembling traditional philosophies of mathematics, in linking mathematics with the cultural, social and political context, in criticizing the ideology of certainty that accompanies mathematics and its formatting power regarding the constitution of reality and the modern subject. These fields have evidenced the role of school mathematics in the exclusion of particular groups of people considered to be in disadvantage. All these particular efforts are not ‘wrong’, yet it is my contention that they become more durable forces and movements by developing in the direction of what Marx called *class consciousness* (e.g., Marx, 1852). Thus, my plea is not for the detriment of the issues raised by postmodern approaches in mathematics education research (namely within critical mathematics education, ethnomathematics and sociopolitical perspectives). Rather I am insisting that such approaches run the risk of being harmless if school mathematics as part of capitalist economics remains unaddressed. As I explore in some of the articles in

⁴ Although I am emphasizing studies within the postmodern trend, I am aware that there are studies adopting a cultural, social or political perspective that nonetheless do not use postmodern theory. For instance, the work of Eric Gutstein is without a doubt within the sociopolitical turn; however, his approach is instead substantiated by so-called post-Marxist theory. In one of the articles of this thesis—*Methodological issues in critical mathematics education*—I specifically address studies which own more to critical theory than to postmodernism. However, my point regarding the disavowing of the economical dimension of schools is still valid. As I explore in this article, even studies within a post-Marxist tradition (for instance, the ones steaming from Paulo Freire’s work, as it is the case with Gutstein’s) end up disavowing the *economic*. It is what I call a *domestication* of Marx (and critical theory as such).

this thesis, it is the fact that schools need to produce failure what colours many of the students' difficulties usually identified by the research community as 'cognitive' or 'sociocultural' impediments. In short, I am pleading for what authors such as Žižek (2008a), Fredric Jameson (1991) or Terry Eagleton (2001), in contemporary philosophy, and Baldino and Cabral (2006), in mathematics education, call the *return of the primacy of the economy*, not to disregard all the important insights of research around the postmodern concerns, but precisely in order to create the conditions for the more effective realization of them.

Therefore, while highlighting the insights that postmodern approaches brought to mathematics education research, I also point to its limitations. Especially, I stress the importance of considering the totality of the system if the purpose is to change it. The dissemination of an irreducible plurality of struggles seems to produce no change in the global dynamics of capitalist schooling. It is the wager of this thesis that the functioning of schools as credit systems cannot be challenged on a local level. The plurality of different struggles has to be condensed in a global 'target'. Thus this thesis is an attempt to posit mathematics and its education within the political and economical spectrum of our times. It is my contention that the postmodern spirit animating the recent research on the cultural, social, and political dimensions of mathematics education disavows a broader comprehension of schools as places of economical production. It involves, as I explore in the articles in the thesis, a tacit acceptance of capitalist economics as the unquestioned framework structuring our social life.

Perhaps the field of mathematics education research is dismissing too quickly theories which allow us to grasp what Butler, Laclau and Žižek (2000) call a *sense of universality*, of what is this all encompassing system that, despite all the efforts towards change, continues to posit school mathematics as a privileged piece of school's credit system. The position I take is that the crucial conundrum colouring all the particular problems teachers and students feel around the teaching and learning of mathematics is precisely the way schools are articulated within capitalist economics. And mathematics education research, by repressing analysis of the school system as a whole, plays an important role in the perpetuation of capitalist schooling.

Organizing the Thesis and Introducing the Articles

The thesis consists of a collection of eight articles produced during the three years of the PhD project. Five of them are already published in proceedings of congresses, books and journals. Three of them were already submitted to international journals, and are now in the final process of second revision after peer review. They should be published by the

end of the year. In the following table I present a scheme with the order in which the articles will going appear, their reference, the ‘code’ I will use to refer to them throughout the next section, and their status.

Reference	Code	Status
Pais, A. (2010). Portrait of an influence. In H. Alrø, O. Ravn and P. Valero (Eds.), <i>Critical mathematics education: Past, present and future</i> (pp. 133-144). Rotterdam: Sense.	<i>Portrait of an influence</i>	Published
Pais, A., Stentoft, D., & Valero, P. (In press). Broadening the role of theory in mathematics education research. In C. Bergsten & E. Jablonka (Eds.), <i>Skrifter från Svensk förening för matematikdidaktisk forskning MADIF 7</i> . Stockholm: Stockholm University.	<i>Broadening the role of theory</i>	In press
Pais, A. & Valero, P. (In press). The specificity of mathematics learning and the disavowal of the political in research. <i>Educational Studies in Mathematics (special issue on contemporary theory)</i>	<i>The specificity of mathematics</i>	Under second review
Pais, A., & Valero, P. (2011). Beyond disavowing the politics of equity and quality in mathematics education. In B. Atweh, M. Graven, W. Secada & P. Valero (Eds.), <i>Mapping equity and quality in mathematics education</i> (pp. 35-48). New York: Springer.	<i>Beyond disavowing</i>	Published
Pais, A. (2011). A critical approach to equity in mathematics education. In B. Greer & O. Skovsmose (Eds.), <i>Critique and politics of mathematics education</i> . Rotterdam: Sense.	<i>Equity</i>	Published
Pais, A. (2011). Criticisms and contradictions of ethnomathematics. <i>Educational Studies in Mathematics</i> 76(2), 209-230.	<i>Ethnomathematics</i>	Published
Pais, A., Fernandes, E., Matos, J. & Alves, A. (forthcoming). Methodological issues in critical mathematics education. <i>For the Learning of Mathematics</i> .	<i>Critical mathematics education</i>	Under second review
Pais, A. (forthcoming). A critique of ideology in the issue of transfer. <i>Educational Studies in Mathematics</i> .	<i>Transfer</i>	Under second review

Besides the articles, which are gathered in the second part, I develop in the next section a discussion addressing the theoretical and methodological aspects of this work. My intention is to provide a philosophical background regarding crucial topics and concepts I use in the articles. The purpose of this discussion is threefold. Firstly, it allows me to theoretically substantiate the critique I do on cultural, social and political approaches in mathematics education research, by means of exploring the philosophy of Žižek, together with other scholars who have been criticizing what has been called the postmodern turn in social sciences and humanities. Secondly, I explain the reason why I decided for *ideology* as a concept of analysis, instead of *discourse*. Ideology enables me to articulate the particular problems addressed in research—the *threes*, to use a common metaphor—within the entire political and economical spectrum —the *forest*. I shall argue that

discourse analysis, by discarding evaluations of the system as a whole, falls short to account for the relation between these two dimensions. Thirdly, I organize the section in a way that a constant reference to the eight articles which compound the thesis can be possible. The purpose is to articulate the different analysis made in the articles with the broader theoretical and methodological discussion. This will give the reader a ‘background image’ to situate the different articles in the context of the critique of ideology I sought to develop. It also allows me to connect the analysis of the different topics developed in the articles. As the articles already address many of theoretical and methodological issues, I refer to the articles instead of repeating what is in them. When referring to the articles I use the ‘code’ name listed in the table.

With the exception of the first article, that supplements the previous section, all the others use research texts as the material for analysis. Mathematics education is in nowadays an immense field of research, with studies covering every imaginable particularities of the process of teaching and learning mathematics. It was impossible to cover all the research, and indeed this was not my intention. My intention was to capture the ideological injunctions at work in the way research constitutes its problems. Although research in mathematics education deals with a huge variety of problems, as can be easily noticed by reading the proceedings of a congress such as The International Congress on Mathematics Education (ICME)⁵, there is a common shared assumption that transverses the field: Research should contribute to the improvement of the teaching and learning of (most of the times, school) mathematics. As Paola and I explore in the articles *Beyond disavowing* and *The specificity of mathematics* this concern can be traced back to the beginning of the 20th century, and has arguably reached its peak today. There seems to be an assumption that a meaningful mathematics education equals societal development (e.g., Niss, 1996; English, 2008; Gutiérrez, 2010) and, as such, measures has been taken both by governments (e.g., NCTM, 2000) and by scientific research to achieve what has been known as “mathematics for all” (e.g., Lubiensky, 2000; Davis, 2001; Niss, 2007; Presmeg, 2010). Failure in school mathematics is thus perceived as *the* problem justifying the immense research industry developed around particular predicaments (from the learning of number systems and arithmetic to mathematical modelling or new technologies).

This is specially the case within the social and sociopolitical turn, which encompasses issues of equity, social justice, and fields such as critical mathematics education and ethnomathematics. These approaches, together with the studies addressing the use-value of mathematics, explicitly assume a concern both with connecting mathematics with the social reality and to allow mathematical access to all. This justifies my choice for these

⁵ For instance, in the last ICME which took place in México, there was thirty eight study groups and twenty eight discussion groups (not to mention the survey teams, sharing experience groups and affiliated study groups) all addressing different research predicaments.

areas of research as my material for analysis. Although I feel as if the kind of ideology critique I develop could be applied to other approaches in mathematics education (experimental psychology or constructivism, for instance), research on the sociopolitical turn gave me more ‘food for analysis’, since it is explicitly addressing the role of mathematics in society. Finally, the issue of theory transverses the entire field, and presents itself as an excellent ‘ideological barometer’ of the assumptions behind how the problems are conceptualized and how solving strategies are conceived.

I am aware how problematic it could be to cover under the same umbrella research as diverse as the one emanating from these fields and approaches. In the analysis I did on the issues of equity, theory, critical mathematics education, ethnomathematics, and transfer, I was not so much interested in identifying the differences between the research developed within this fields, and they are immense, but in what remains the same despite all the differences. In the majority of the articles, I sought to identify the common shared assumptions about crucial aspects of mathematics education, regarding the role of theory, the importance of achieving equity, the importance around the use-value of mathematics, and the importance of bringing local cultures into schools. Despite the diversity which characterizes the research around these topics, there are prepositions that permeate the way researchers address them. ‘Catching’ these prepositions is a central strategy in ideology critique, as I shall develop in the next section. In what follows I shall give a brief overview of the articles. This is important since I will recurrently refer to them in the next section.

In the articles *Broadening the role of theory* and *The specificity of mathematics* I took advantage of the significant literature produced in the last decade concerning the role of theory in mathematics education research. Most of these studies can be found in handbooks, special issues of the most highly valued international journals of the field, and proceedings of conferences designed especially to address a particular topic. This was also the case with the articles about equity—*Beyond disavowing* and *Equity*—currently a central topic within the sociopolitical turn. Reading this research allows one to conceptualize how the field constitutes what are the best theoretical approaches to mathematics education and the worldwide problem of failure in school mathematics. In the article *Ethnomathematics* I analysed research pointing towards the contradictions in the ethnomathematical program. Having Portuguese as my native language was an advantage, since many of the research being done currently in ethnomathematics comes from Brazil and is written in Portuguese. In the articles *Critical mathematics education* and *Transfer*, although still structured as a critique of research I used empirical data already collected by other researchers involving students and teachers working in a math class. These were written as part of the project LEARN, which is one of the activities of the Technology, Mathematics and Society Learning Research Group of the Centre for

Research in Education at the University of Lisbon, coordinated by João Filipe Matos. I have been member of this group and participated in its activities. One of the purposes of this project was to analyse, from a different theoretical perspective, data already collected in previous research work done by the participants in the project. It was our contention that instead of collecting more data we should analyse in a new light what was already available in research. This economy or data recycling gave me more time to explore theory. In each of the different articles I used different approaches, and I will not repeat their description in the following section. As I said, my intention here will be to give a philosophical explanation of the theories and methods I deployed, and how they allowed me to address the inconsistencies and contradictions of the field.

In terms of the content, the articles are quite diverse and address different topics of research. In *Broadening the role of theory* and *The specificity of mathematics* I addressed the way mathematics education research, by being primarily driven by a ‘didactical’ approach, disavows a broader comprehension of the problems involved in school mathematics. I argue that many of the problems related with school mathematics cannot be fully conceptualized without some kind of ‘political mapping’ that situates them, not at the level of ‘learning’, but at a political and economical one. This implies for research to be open to the use of other theoretical frameworks than the ones revolving around ‘learning’ and ‘mathematics’. I suggest that positing schools as crucial modern institutions for the reproduction of capitalist economy and ideology can bring us closer to the complex situations teachers and students experience in schools. In the articles *Beyond disavowing* and *Equity*, I address recent research dealing with the problem of equity (in connection with quality, in the case of the later). My intention was to show that the vast majority of the literature addressing the issue of equity, although acknowledging the political dimension of the problem, performs as if it could be solved within mathematics education. This way, by disavowing the political and economical dimension of equity, the community becomes hostage to an ideology that holds back a comprehension of the problem in its totality; thus restraining the possibility of change. In the case of *Equity*, I developed a dialectical materialist theorization, based on the recent revitalization of Hegel and Marx carried out by Žižek (2008a, 2008b). This article is arguably the most substantial of them all, and functions as a “metaphoric universalisation” (Žižek, 2008d, p. 243) of all the others. It was in it that I most clearly develop the argument that exclusion and inequity within mathematics education, and education in general, are integrative parts of current school education. As such, they cannot be conceptualized without an understanding of the relation between scholarized education and capitalism as the dominant mode of social formation. In the articles *Ethnomathematics* and *Critical mathematics education*, I picked two of the most important trends within the sociopolitical turn and developed an ideology critique of the assumptions informing its research. I

pointed out the contradictions arising when one tries to implement powerful insights coming from these fields into schools. In the case of ethnomathematics, I showed that research runs the risk of conveying an idea of culture where the Other is squeezed from its otherness, thus ending up having a result opposite to their aims. In the case of critical mathematics education, I argued that a severe disavowing of its Marxist heritage could make critical mathematics education become one of the dozens of ‘school mathematics contents’ completely inserted into the maintenance of school as an institution at the service of capital. In the article *Transfer* I address the crucial importance of mathematics as use-value. While the majority of the literature in this issue commends the possibility of transfer, thus assuming both the desirability of transfer and the importance of school mathematics for the professional and mundane lives of individuals, I was interested in developing an ideology critique on the belief sustaining the research investigating this issue. I showed that the use-value attributed to school mathematics disavows its value as part of a political and economical structure, which requires school mathematics to perform other roles than the one related with utility, namely the role associated with its exchange-value. It is in this article that I better deploy Žižek’s ideology critique.

As I previously mentioned, I shall in the next section develop an argumentation which justifies the importance of having chosen these topics of research—besides the reasons already given in the previous section, regarding my path as a teacher and researcher—and how I proceeded in the analysis of research. Thus, reference to the articles—by using its ‘code’ name—is a recurrent feature of the next section.

Theoretical and Methodological Considerations: The Axis Hegel-Marx-Lacan-Žižek

A critique of the postmodern spirit

I am aware how murky the use of labels such as ‘postmodern’ and ‘critical’ can be. Scholars such as Michel Foucault, Jacques Derrida, Gilles Deleuze or Jean Baudrillard, although individually they may resist common affiliation, are routinely associated with postmodernism in philosophy, critical theory and cultural studies. Indeed, none of them uses the term *postmodern* to describe their work and I suspect they would resist association with ‘-isms’ of any sort. Regarding the term *critical*, things are perhaps even more unclear. While usually associated with the revitalization carried out by the Frankfurt school of Marxian and Freudian theories, by authors such as Herbert Marcuse, Theodor Adorno, Max Horkheimer, Walter Benjamin, and Jürgen Habermas, it is also used in a broader sense to encompass feminist theory, race critical theory and postcolonial studies. There is also a clear overlapping between postmodernism and critical theory, with some

authors (such as Foucault) usually catalogued as being both postmodern and critical.⁶ Notwithstanding my personal discomfort with the use of such categories to address the diversity which characterizes the work of these scholars, I shall in this section engage in a discussion using these terms since they have been used to characterize contemporary research in social sciences, particularly in mathematics education. The emphasis, however, will be given to postmodernism. This is because in the article *Critical mathematics education* I already develop a criticism of the way critical theory (especially the one steaming from Frankfurt School and Paulo Freire's work) has been *sanitized* by contemporary approaches that disavow its Marxist roots. At least in one point, which is also the most important, my critique of postmodernism coincides with my critique of the way contemporary critical theory has been interpreting Marx: both repress the antagonistic character of social reality, as I shall explore throughout this section.

My first and foremost important uneasiness regarding a postmodern approach to research is its disavowal of theories that seek to grasp society in its totality, most notably, Hegel's dialectics of spirit, Martin Heidegger's hermeneutics, and Marx's historical materialism. According to Lyotard (1984) this incredulity towards metanarratives, together with the *hybridization* of traditionally fixed and compartmentalized areas of knowledge, the recognition of the strict interlinkage between science and politics, and a denaturalization of the notion of truth are the main features of the postmodern condition. Conversely, the question posed by Lyotard—do we still live in modernity?—created a discussion that remains actual, and different suggestions have been given to characterize our current society. Conceptualizations such as Ulrich Beck's *risk society*, Anthony Giddens' *late modernity*, Zygmunt Bauman's *liquid society*, or Manuel Castells' *informational society* try to reinforce or oppose the idea that we live in a postmodern époque. Whether we have surpassed the modern, and have already inaugurated a 'new era', or we are just living a radicalization of modernity, what seems to be common to all these conceptualizations is the shared assumption that there are core features of today's society that cannot be fully explained with modern theories, specially the 'meta-theories' that conceive the world as a totality. Uncertainty, complexity, multiplicity, and pluralism are central concepts when

⁶ If we dare to obscure even more this discussion, we could add the term *poststructuralist*—an American invention to cover under the same umbrella the diversity of the French philosophy of the second half of the 20th century—and *post-Marxism*. However, for the sake of the clarity of the discussion, I will restrict the labelling to postmodern and critical which, by themselves, already show a propensity for confusion. I challenge the reader to survey the incredible amount of books and journal articles that have been attempting to catalogue different scholars and research, and make sense of it. I particularly like Philip Goldstein's clear cut definition of post-Marxism as a theoretical viewpoint that, unlike traditional Marxism, which emphasizes the priority of class struggle, reveals the sexual, racial, class, and ethnic divisions of modern Western society (Goldstein, 2005). This is, as we shall see, also the flag of the postmodern. No wonder that the author equals post-Marxism with what he calls 'poststructuralist Marxism'. Things could not get clearer about the obscurity that labelling creates.

thinking about today's world, and they are part of what is commonly called the postmodern condition.

Postmodern theorists turn up their noses at concepts such as 'universality' or 'totality'. Instead they emphasize the existence of multiple realities, each one with their own universality. Steven Seidman, arguably one of the most important catalysts of the postmodern turn, poses this shift in the following terms:

The shift from metanarratives to local narratives and from general theories to pragmatic strategies suggests that in place of assuming a universal mind or a rational knowing subject, we imagine multiple minds, subjects, and knowledges reflecting different social locations and histories. (Seidman, 1994a, p. 5)

No one doubts that different realities exist. Indeed, from a psychological point of view, each individual stands for a specific reality, a unique way of making sense of life. The postmodern claim that modern theories generally had been discarding the historical discontinuities, local struggles and forms of resistance, in favour of a 'universal history'—which, as we know, is the history of the Western White Men—is thus completely justified. As Foucault so minutely explored in his work (e.g., Foucault, 1970), history does not stand for 'what really happen', rather, it is a meticulous process of *storytelling*, where the emergence of new knowledge is always mediated by the correlative power relations involved in the proclamation of some statement as 'true'. Therefore, what appears to the individual as a historical necessity—for instance, the existence of schools—is the result of an endless set of contingent occurrences that become crystallized and seen as necessary—as if schools were always here. With Foucault we realise the radical contingency of historical processes, thus offering tools for postmodern theory to deconstruct the modern claims of universality and truth and engaging is the new *politics of difference* which asserts "the value of individuality, difference, heterogeneity, locality, and pluralism" (Seidman, 1994a, p. 7)

However, within Žižek's dialectics of necessity, this is just half of the story. As I said, for Foucault, 'truth' emerges from considering what is no more than a historical contingency as a necessity. Again the example of schools which, although in our days are perceived as a necessity, are in fact the result of a set of historical contingencies (Foucault, 1979; Rose, 1999). But, with a dialectical twist, the opposite also holds: it is equally possible to perceive as contingent what always has been a necessity. For instance, as I develop in the article *Equity*, school failure is dealt with as a contingent occurrence of a necessary system, when it is a necessity to maintain failure in school. Whereas Foucault offers us tools to unmask reality as a historically contingent, symbolic fiction—the radical contingency of historical processes—Žižek's dialectics confronts us with the task of

recognizing what he calls, drawing on Lacan, the *Real* in what appears to be mere symbolic fiction (Vighi & Feldner, 2007). As I shall explore later, the Real is, in this case, the radical necessity of social antagonism: the necessity of some ‘static’ core that remains the same in all different worlds.

Although we live in a world of multiple social, cultural and political realities, we must ask what, in all these different sets, remains unchangeable. That is: what is necessary for the existence of this multiplicity as such? In cultural and social terms, there is no doubt that the world is diverse. European culture and sociability, despite all the similitude, is different from North American culture, as we can easily notice in literature, cinema or philosophy. Not to mention the perhaps even deeper differences between these and Asian and African cultures. Also in political terms, we have around the world all sorts of political organizations: neo-liberal American ideologies, European social-democracy, China’s (capitalized) communism, Arabic religious states, etc. However, when talking about economy, we get stuck. Can we say that we have a plurality of different economical systems around the world? No, *the* global economical system present in all these multiplicity of cultural and social formations is capitalism.⁷ Even communism, whether we are talking about state communism as in China or the communist parties that subsist within capitalist democracies, follows the rules of capital. Capitalism dresses diverse clothes in order to keep reproducing, no matter how many different the ‘philosophies’ of political organization could there be around the world (monarchy, socialism, religious fundamentalism, dictatorship, neo-liberalism, etc.). What is common in all them is that, despite the different apparent ‘clothes’, human relations are based on capital. Therein resides the Real of today’s society: capitalist economics is the unchangeable core that remains the same in all the multiplicity of social, cultural and political spectrums. A proper postmodern theory, with its disavowal of *necessity* and the *universal*, lacks the tools to address this constancy that transverses all the pluralisms so dearer to the postmodern. I am not rejecting the multiplicity, heterogeneity and pluralism of social reality. But we need to add to this pulp of particularities its supplement, that is, to express it in Hegelian terms, the concrete universality which colours the multitude of all particularities.

Sociopolitical approaches in mathematics education adopt a pluralistic approach to reality (D’Ambrosio, 2003; Gutstein, 2003; Valero & Stentoft, 2010; Skovsmose & Borba, 2004;

⁷ Not all societies are at the same level of capitalist development, and, evidently, some are still indeed in pre-modern systems of production. However, with the advent of global, multinational capitalism, the way to get into the ‘market’ is through capitalism. For instance, we can say that indigenous tribes in Amazonia do not live in a capitalist economy. However, when exchanges with the ‘outside world’ are needed (and they are increasingly needed since the neo-colonization taken care of by capitalist expansion obliges these people to search for products that cannot be produced within the community) they are automatically inside the capitalist mode of production. So the choice is between accepting capitalism or to perish.

Ernest, 2004; Knijnik, 2007; Gutiérrez, 2010). As posed by Cotton and Hardy (2004) “we do not believe that there is such a thing as reality of experience within schools—rather a multiplicity of ‘reality’” (p.100). There is no doubt that schools are different, and that teachers and students experience a multiplicity of problems. Each school, each teacher and each student stand for a unique constellation of life experiences. However, we have to ask: what is unchangeable in all these different schooling realities? Albeit schools being different (are they that different?), what does remain the same in all these complex universes? What is the Real of school? As I develop in most of the articles, especially in *The specificity of mathematics*, with Paola, and the one on *Equity*, what remains unchangeable is the way schools function as credit systems (Vinner, 1997), by continuously redistributing students submitted to its examination. As we know from the work of Bourdieu and Passeron (1977), access to privilege is justified by success in a meritocratic system. At the same time exclusion is justified by failure. This is the Real of schools, the worldwide accreditation system that runs amok indifferent to the particular and local changes that in vain try to overcome, in our case, the endemic problem of mathematics failure.

Therein resides the importance of taking into consideration the notion of *totality*: capitalism today *is* a totality. That is, capitalism is the all-inclusive economical system that ends up subsuming the cultural, social and political spaces of our lives. It has become common sense to argue for the increasing capitalization of the academy, technology, education, media, leisure and, of course, politics, where the logic of the market dictates the, many times invisible, rules for acting and being acted in these social spaces. Making use of Hegel’s dialectics, Žižek (2004) poses this *overdetermination* carried out by capitalism as follows:

More than ever, capital is the ‘concrete universal’ of our historical epoch. What this means is that while it remains a particular formation, it overdetermines all alternative formations, as well as all noneconomic strata of social life. (p. 185)

A postmodern approach rejects the idea that there is such a central organizing principle. It emphasizes instead a multitude of organizing principles, what Wendy Brown (1995) refers to as the postmodern mantra of “race, class, gender, sexuality” (p. 61). This debate, opposing Marx’s notion of *class struggle* and what is usually called *identity politics*⁸ (e.g., Brown, 1995), is becoming a central one in contemporary theory.⁹ While authors such as Laclau or Butler (Butler, Laclau & Žižek, 2000) argue that class struggle is just one type of identity politics, and one which is becoming less and less important in the world today,

⁸ But also *politics of recognition* (Butler, Laclau & Žižek, 2000), or *politics of difference* (Seidman, 1994).

⁹ For an account of the terms in which this discussion is carried out see, for example, Butler, Laclau and Žižek (2000), Eagleton (2001) and Vighi and Feldner (2007); and within education, Cole (2003).

Žižek (2008a) or Jameson (1991), in accordance with Marx, defend the opposite: class *qua* structuring principle of the social totality colours all the other particular struggles. According to Žižek (Butler, Laclau & Žižek, 2000), it is not a matter of neglecting the achievements produced by the politics of recognition, but to supplement them with a critique of capitalism that, in current postmodern politics, is clearly disavowed:

Postmodern politics definitely has the great merit that it ‘repoliticizes’ a series of domains previously considered ‘apolitical’ or ‘private’; the fact remains, however, that it does *not* in fact repoliticize capitalism, because *the very notion and form of the ‘political’ within which it operates is grounded in the ‘depoliticization’ of economy.* (p. 98)

Such a depoliticization of economy has been responsible for what Kurtz (2003) calls an *ontologization* of capitalism: we accept capitalism as the all encompassing economical system to which there seems to be no alternative. Notwithstanding all the emancipatory breakthroughs made possible by the feminist, postcolonial, queer, multicultural and ecological studies and movements, I take the standpoint that the crucial emancipatory step is still to come and it has to do with the way society is organized around capitalist economics. In this sense, the fundamental *modern* problem is still to be solved. Therefore, it is the wager of this thesis that before proceeding claiming that we entered in postmodernity, we should attempt a return to Marx, and seriously address this still very modern problem called capitalism. And, in this task, to paraphrase Jameson (1991, p. 330), the epistemological razor of postmodernism nominalism which shears away apparent abstractions as the economic system and the social totality itself do not seem to provide us the best theoretical framework to do it.

Some authors have been arguing that not only postmodernism does not provide an effective theoretical apparatus to address capitalist society, it can even endorse its spirit (Brown, 1995; Cole, 2003; Jameson, 1991; Žižek, 2008a). Their arguments is that the postmodern disavowal of a central organizing principle of society suits very well Capital as the all encompassing economical system that remains unchangeable despite all the political and intellectual differences. Postmodern theories, by rejecting analysis of the system as a whole, leave the system of Capital free from critical analysis. The Capital starts to function as the non-questioned set of coordinates setting the scene for all the local postmodern struggles. This way, as posed by Cole (2003), “poststructuralism/postmodern acts as an ideological support for national and global capitalism” (p. 490).

Against this background, we can speculate how postmodernism is not a ‘new beginning’, a new logic of approaching research, but an important aspect of capitalist reproduction. This is the thesis developed by Fredric Jameson in his notable book *Postmodernism, or the Cultural Logic of Late Capitalism*. Jameson (1991) does not deny that we entered a

new époque, and he explores—in fields such as theory, arts, architecture, technology and economics—some of the changes that characterize the break with the modern. There is no doubt that fundamental changes in science, technology, economics, media, politics, and even in capital—now global, multinational and *affective*, to use a Deleuzian concept¹⁰—have turned modern conceptualizations on these fields outmoded. However, and this is the crucial insight of the book, Jameson does not see any emancipatory feature in this break. Instead, he entangles this shift towards the postmodern within the logic of what he calls, taking the expression first deployed by the economist Ernest Mandel, *late capitalism*. Postmodernism is less a new cultural form than it is a reflex and the concomitant of yet another systemic modification of capitalism itself. Thus, contrary to the assumption defended by many economists (most notably Daniel Bell's *post-industrial society*) that we have arrived to a new type of society, where the new social formation in question no longer obeys the laws of classical capitalism, late capitalism signals instead that this 'new society' is a *purser* stage of capitalism than any of the moments that preceded it. In this sense, every position on postmodern in culture "is also at one and the same time, and *necessarily*, an implicitly or explicitly political stance on the nature of multinational capitalism today" (Jameson, 1991, p. 3). This is another way of saying that Capital, being the "concrete universal" (Žižek, 2004, p. 3) of our times, ends up co-opting the multitude of local struggles that animate the postmodern trend.

In this sense, I risk saying that, concerning emancipation, the postmodern plea is a failed appeal from the beginning—it is ignited by the renunciation of changing not just some variables of the 'game', but the entire system of coordinates that maps the way we play the game. As phrased by Žižek (1997), the very domain of the multitude of particular struggles with their continuously shifting and displacements and condensations, is sustained by the repression of the key role of economic struggle¹¹:

¹⁰ Affects in Deleuze are those pre-conscious 'processes' that are beyond signification or coding, but which, nonetheless, structure our sense of reality (Deleuze, 1990). They belong to the field of the Virtual, the "‘real but abstract’ incorporeality of the body" (Massumi, 2002a, p. 21). Brian Massumi and Patricia Clough have been developing Deleuze's theory to analyse political economy in terms of what they call the "capitalization of affects", which can be seen as one of the crucial extensions of late capitalism.

¹¹ It has become a common place to quarrel for sexual equality, gender equality, cultural equality and race equality. However, nobody seems to dispute anymore for *economical* equality. What would it mean to argue for an equal distribution of the material goods needed (or not) for life? There is no doubt that we are far from a world where racism and sexism are remainders of the past. Nevertheless these issues are *posited*, they are 'thinkable', they are the wager of governments and they fuel an entire research industry. On the contrary, to posit equality in terms of *material* equality is not even part of the discourse. Why is it unthinkable for people to defend an equal distribution of money, for instance? As Wendy Brown (1995) defends, the issue of economical equality is no longer 'an issue'. And she asks: "to what extent do identity politics, require a standard internal to existing society against which to pitch their claims, a standard by that not only preserves capitalism from critique, but sustains the invisibility and inarticulateness of class—not incidentally, but *endemically*?" (p. 96, my emphasis). This is the reason why 'class' is invariable named but not theorized in the mantra "race, class, gender, sexuality".

The Leftist politics of the ‘chains of equivalences’ among the plurality of struggles is strictly correlative to the abandonment of the analysis of capitalism as a global economic system—that is, to the tacit acceptance of capitalist economics relations and liberal-democratic politics as the unquestioned framework of our social life. (p. 162)

The giving up of a common struggle against Capital—Francis Fukuyama’s idea that capitalism is the end of history—in favour of a multitude of particular struggles is the result of an accommodation to or a *naturalization* of capitalism: we are no longer able to change as a whole, and the least we can do is to content ourselves with our private struggles. In this sense, as Žižek (2004) argues, the image of a truly fundamental political and social immobility is the evolutionary image of a postmodern society as a complex network of endless transformation, developments, and innovations, in which the more we change, the more things remain the same.

There are no reasons to assume that the postmodern trend is in itself an emancipatory force. Instead, the postmodern can very often be in tone with the reproduction of social forms: “the simple prolongation of more of the same under different sheep’s clothing” (Jameson, 1991, p. xiii). In this sense, postmodernism, far from being inconsistent with Marx’s 19th century analysis, constitutes, on the contrary, the purest form of capital yet to have emerged, an incomparable extension of capital into hitherto uncommodified areas (Jameson, 1991). Thus it entangles a very fruitful paradox to be explored: at the same time that the postmodern presents itself as breaking with the past, it offers a ‘cultural window’ to posit this break within the current latest stage of capitalism. Jameson (1994) poses as follows:

I occasionally get just as tired of the slogan ‘postmodern’ as anyone else, but when I am tempted to regret my complicity with it, to deplore its misuses and its notoriety, and to conclude with some reluctance that it raises more problems than it solves, I find myself pausing to wonder whether any other concept can dramatize the issues in quite so effective and economical a fashion. (p. 418).

Jameson is arguing that it is through the analysis of postmodern culture that one can develop a critique of capitalism today. Postmodern claims emerged because modern forms have ceased to be functional in reproducing capitalism. In this sense, the postmodern wave is at the same time a condition for the capitalistic reproduction and a consequence of it. This was one of the reasons why I found the research within the sociopolitical turn a fertile arena for ideology critique. I challenge the reader to find, within this turn, an articulation between the cultural, social and political dimensions of mathematics and the economical system. Except for the work of Roberto Baldino and Tânia Cabral (Baldino,

1998a, 1998b; Baldino & Cabral, 1998, 1999, 2006), and, more recently, Julian Williams (2011), there has been no attempt to theorize and develop the economical dimension of school mathematics. These authors are usually not considered to be part of the sociopolitical perspective (e.g., Valero, 2004; Gutiérrez, 2010). Thus, despite the diversity of studies within the sociopolitical turn, what is common in them is a disavowal of the *economic*.¹² And, according to Žižek (2008a, 2004) and Jameson (1991, 1994), this is the defining sign of the postmodern.

Regarding the postmodern disavowal of the notion of totality, the crucial question to be made is not how postmodern theories could offer us better tools to grasp reality, but, in a Foucauldian fashion, “why it is that ‘concepts of totality’ have seemed necessary and unavoidable at certain historical moments and, on the contrary, noxious and unthinkable at others” (Jameson, 1991, p. 402)? Why at a time when Capital seems to affect all dimensions of life, research in social sciences has been repressing evaluations of the system as a whole? Postmodern theories could very well be a function of the contemporary expansion of capital:

It makes sense to suggest that the waning of our sense of history, and more particularly our resistance to globalizing or totalizing concepts like that of the mode of production itself are a function of precisely that universalization of capitalism. (p. 405)

This universalization of capitalism coincides with postmodernity as the third stage of classic capitalism, where many of the hitherto surviving enclaves of economic difference have been eradicated. Late capitalism, in order to full realize itself as a totality, must efface the theoretical instruments that allow us to address it as a totality: “the need to avoid evaluations of the system as a whole is now an integral part of its own internal organization as well as its various ideologies” (p. 350).

Another philosopher who provides us tools to understand the postmodern as the cultural logic of late capitalism is, not without surprise, Gilles Deleuze. Deleuze is the philosopher of the *virtual*, of the *bodies without organs*, of *percepts* and *affects*, of *flux* and *intensities*. In the words of Žižek (2004), Deleuze is the philosopher of late capitalism: with his lexicon and logic he analysed the crucial changes allowing capitalism to enter in its third

¹² This is obviously also the case with the vast majority of mathematics education research (which disavows not only the economical but also shows a historical tendency to disavow the social and cultural dimensions, by being centred in a psychological approach). As Paola and I explore in the article *The specificity of mathematics*, even socioculturalism and its use of Marxist psychological theories such as the ones of Lev Vygotsky and Alexei Leontiev, end up focusing on the cultural and historical dimension of learning, thus completely obliterating its economical dimension. Nonetheless my criticism concerns also research that, although seeking to go beyond a ‘didactical’, ‘psychological’ and ‘sociocultural’ perspective of school mathematics, by means of emphasizing ‘political’ issues, restrains from analysing the relation between school mathematics and the Capitalist system.

phase. Contemporary capitalist ideology no longer functions towards centralization, consolidation, homogenization, and against diversity. Contrary, the latest trend in corporate management is to diversify, devolve power, and try to mobilize local creativity and self-organization, instigating decentralization as the flag of the ‘new’ digitalized capitalism. On the other hand, the old Foucauldian notion of *normalization*—that power/knowledge relations mould the subject towards the Norm—so dear to industrial capitalism, seems today no longer holding its power. Instead of the logic of ‘totalizing normality’ today’s capitalism adopted the logic of the ‘erratic excess’.¹³

the more varied, and even erratic, the better. Normalcy starts to lose its hold. The regularities start to loosen. This loosening of normalcy is part of capitalism’s dynamic. It’s not a simple liberation. It’s capitalism’s own form of power. It’s no longer disciplinary power that defines everything, it’s capitalism’s power to produce variety—because markets get saturated. Produce variety and you have a niche market. The oddest of affective tendencies are okay—as long as they pay. Capitalism starts intensifying or diversifying affect, but only in order to extract surplus-value. It hijacks affect in order to intensify profit potential. It literally valorises affect. The capitalist logic of surplus-value production starts to take over the relational field that is also the domain of political ecology, the ethical field of resistance to identity and predictable paths. *It’s very troubling and confusing, because it seems to me that there’s been a certain kind of convergence between the dynamic of capitalist power and the dynamic of resistance.* (Massumi, 2002b, p. 224, quoted in Žižek, 2004, p. 184, 185, my emphasis)

That is, *affect* has been engineered to produce a politics, which reinforces capitalist dynamics. And, as Massumi suggests, the power of Capital—a strong machine of deterritorialization that generates new modes of reterritorialization, as Deleuze and Guattari (2004) put it—to produce variety is coupled with its power to co-opt what in principle are resistant forces against it. The capitalist *machinic* (to use a Deleuzian term) integrates in itself the different and fragmented local forms of resistance.

In four of the articles that compound this thesis—*Equity*, *Critical mathematics education*, *Ethnomathematics*, and *Transfer*—I sought to show that, what in principle are forces of resistance and change in mathematics education, end up being co-opted by the same logic they openly criticize. The system satisfies the societal demand of bringing into school other cultures (ethnomathematics), issues of social justice and empowerment (critical

¹³ Perhaps Foucault was the (disavowed) philosopher of 20th century capitalism, while Deleuze provides us with the lexicon to map today’s capitalism.

mathematics education and equity), and ‘reality’ (transfer), while assuring that such insertion will not actually change any of the core features of the school system.

At stake here is also the difference between *choice* and *selection*. Within today’s society, choice seems to be reduced to select between a set of pre-given and non-crucial features. The endless myriad of choices available today conceals the fact that the ‘true’ choices are not available. The impossibility of choosing the core features that map our lives must be disguised by the availability of a multitude of choices that do not change anything. As Žižek (2006) argues, “[t]his appearance of choice, however, should not deceive us: it is the mode of appearance of its very opposite: of the *absence* of any real choice with regard to the fundamental structure of society” (p. 348).¹⁴

In two of the articles of this thesis I explore how this dialectic of choice is operationalized within *Critical mathematics education* and *Ethnomathematics*. Regarding the latter, there is already a considerable array of didactical proposals aimed to nurture a multicultural education. A teacher can choose materials to work with students topics of school mathematics from ethnomathematical areas as disparate as the construction of houses in an Amazonia tribe, to the work of carpenters in the south of Italy. For instance, in South Africa a whole knowledge industry is developing around the idea of Indigenous Knowledge Systems of which ethnomathematics is one component.¹⁵ The choices of materials to work with students multicultural topics are endless. However, a radical transformation of the school system as advocated by some ethnomathematicians¹⁶—to redefine what mathematics is, for instance—is not available. The same happens with critical mathematics education. Today, many curricula around the world already contemplate the possibility for teachers to work with students topics of critical mathematics education (as I explore in the article *Critical mathematics education*), where issues related with the formatting power of mathematics (Skovsmose, 1994) or the way mathematics is involved in process of social exclusion (e.g., Gutstein, 2003) can be addressed. In the case of Ana, the teacher who co-authored the article and to whom experience the case analysed refers, she even started from students’ interests who were free to choose the topic to be explored. However, when confronted with the rigidity of the school evaluation system based on exams, she decided not to jeopardize this same system, developing instead her critical mathematics with a particular group of students outside the official mathematics class. Again, the core choice—changing the evaluation system—was

¹⁴ This was already thematized by Adorno and Horkheimer (1979) in their critique of modern capitalist society: “All are free to dance and enjoy themselves, just as they have been free, since the historical neutralisation of religion, to join any of the innumerable sects. But freedom to choose an ideology—since ideology always reflects economic coercion—everywhere proves to be freedom to choose what is always the same” (p. 130).

¹⁵ I thank one of the anonymous reviewers of the article *Ethnomathematics* for this information.

¹⁶ Most notably D’Ambrosio (2007).

not available. The more variety we have to choose, the more we forget that the core features of the system cannot be chosen. The multiplicity of choices available—from technology, didactical materials and textbooks to topics of school mathematics—mesmerize so many teachers that they cannot realize how captive they actually are.

Discourse analysis or ideology critique?

It is clear, even if one admits that Marx will disappear for now, that he will reappear one day. (Foucault, quoted in Cole, 2003, p. 489)

In this section I shall explain my choice for *ideology* as a central category in my analysis. Since, as mentioned by Jameson (1994), discourse analysis is what has allowed scholars conveniently, in a postmodern age, to practise ideological analysis without calling it that, I shall develop the discussion in the guise of an encounter between discourse and ideology.

Foucault wrote the above quoted sentence in a period where ideology critique was starting to give way to discourse analysis. Despite the efforts of Louis Althusser, fellow countryman and contemporary of Foucault, to revitalize the Marxist notion of ideology as a central one to theorize politics, his work has hitherto become too much associated with Marxism, thus petrified. Contrary, the studies developed by Foucault flourished, and have become in the last three decades one of the most powerful theoretical frameworks being used in humanities and social sciences. Concepts such as ideology, class, production, economics, infrastructure, reproduction, labour, value, commodity and capital, have become residual in social sciences' literature since the nineties. Conversely, terms such as discourse, group, power, gender, consumerism, population, normalization, discipline, body, control and citizenship have been used intensively, given rise to new ways of mapping the Political generally associated with the postmodern trend. As argued by Eagleton (2001), postmodern theorists tend to see ideology as teleological, 'totalitarian' and metaphysically grounded and, as such, not useful to analyse the contingent and plural world in which we live today. Foucault himself implicitly criticized Althusser's Marxism and structuralist anthropology for employing concepts of ideology and culture that were universalist, synchronistic and ahistorical (Vighi & Feldner, 2007). Grossly travestied in this way, the concept of ideology has been writing itself off from research in social sciences (Eagleton, 2001; Jameson, 1991; Vighi & Feldner, 2007; Žižek, 2008a). In contrast, Foucault's brand of discourse analysis offered theoretical frameworks for conceptualizing epistemic practices and technologies of power as historically situated and contingent (Vighi & Feldner, 2007).

However, against this abandonment in the course of the 1980s of the notion of ideology in favour of discourse, some scholars have been recently criticizing the later, showing what is lost when we reduce ideology critique to discourse analysis (Eagleton, 2001; Jameson, 1991; Vighi & Feldner, 2007; Žižek, 2008a). Eagleton (2001), for instance, shows surprise

for the fact that in a time when we are witnessing a remarkable resurgence of ideological movements throughout the world (among others, the coming back of nationalism in Europe, Islamic fundamentalism as a political force, or the noxious brand of Christian Evangelicalism in the United States), the concept of ideology almost disappeared from academia as something worth being studied:

How are we to account for this absurdity? Why is it that in a world racked by ideological conflict, the very notion of ideology has evaporated without trace from the writings of postmodernism and poststructuralism? (p. 79)

Whereas the concept of ideology is inextricably connected with the Marxist concept of class struggle, discourse appears to address also and more adequately the conflicts of gender and race, which became paramount topics for the fast growing feminist, post-colonial and cultural studies (Vighi & Feldner, 2007). This emphasis in discourse and the disavowal of class struggle contributed to the effacement from academic research of the economic dimension of society, as I previously described.

I completely endorse the critique that discourse analysis makes of the Enlightenment notion of ideology (up to Jürgen Habermas). As Žižek (1994) puts forward, in the Enlightenment tradition, ideology describes some state of illusion, a biased comprehension of reality. In this way, ideology is usually understood as a poor form of knowledge that does not aim to understand reality (as the scientific knowledge does), but to look at reality from a position that we wish to defend. It is as if scientific knowledge was the true one, and all the misconceptions of it are just ideological attempts to make some political position stand for through obscurantism. For discourse analysis, the very notion of an access to reality unbiased by any discursive devices or conjunctions with power is ideological. Instead of ideology falsifying reality for the sake of pathological interests (power strategies), both discourse analysis and Žižek's conceptualization of ideology start from the assumption that there is no way to access and conceptualize reality which is not already stained by discourse. In this sense, ideology is not a distorted representation of a true reality. Rather, ideology is the reality, we are 'naturally' in ideology, our natural, immediate sight is ideological (Žižek, 1994).

There is no space outside a given discursive regime of power-knowledge, in the same way that ideology has no 'outside'. This makes ideology not an occultation or dissimulation of reality, but a *real representation of the individual imaginary* (Althusser, 1994). Likewise, discourse analysis also rejects the idea that there is some undisturbed and true reality occulted by discursive regimes, as if we could lift the veil of discourse and see the world as it really is. If this was true, it will be possible for subjects to assume a position somewhere beyond the microphysics of power and, from a non-polluted position, evaluate

the ideological distortion. Both Althusser and Foucault denounce as ideological the very attempt to draw a clear line of demarcation between ideology/discourse and actual reality.

Where then does Žižek's idiosyncratic conceptualization of ideology diverge from Foucault's discourse analysis? First, in the same way that already Althusser (1994) diverged from Foucault regarding the conceptualization of power. For Foucault (2004), power concerns the disciplinary apparatus that operate at a 'micro-level', through a plurality of discursive mechanisms that constitute themselves from below, never from some unique summit, as it was the case with Althusser's *ideological state apparatuses*. Within a Foucauldian framework, such a summit (the Monarch or some other form of sovereignty) emerges as an effect of the multiplicity of micro-practices, of the complex network of their interrelations. However, as pointed out by Žižek (1994), when Foucault, in his last works, is compelled to display the concrete mechanisms of this emergence, he

resorts to the extremely suspect rhetoric of complexity, evoking the intricate network of lateral links, left and right, up and down... a clear case of patching up, since one can never arrive at Power this way—the abyss that separates micro-procedures from the spectre of Power remains unbridgeable.

(p. 13)

This is partly the reason why Žižek refers to Foucault as the “anti-dialectician *par excellence*” (2008d, p. 300). Foucault conceives the macro and the micro as two separate identities, where the former results from some kind of ‘bundling’ of the later. A dialectical approach is instead concerned with the evaporation of such dichotomy, to conceptualize micro and macro not as separate identities, but as ‘phases’ or mechanisms of Power as such.

However, the crucial distinction between ideology and discourse concerns an aspect that was leaved unaddressed by both Foucault and Althusser. It has to do with the uncanny Lacanian notion of *jouissance*, or, in its anglicised version, enjoyment¹⁷. As Lacan (1999, 2007) explored in the domain of psychoanalysis, in order for a person to overcome a symptom disrupting her life, it is not enough for her to acknowledge the reasons of her discontentment. In some eerie way, a person *enjoys* her symptom, and refuses to abandon it, even if, at a rational level, she is well aware of the reasons that cause her malaise. Žižek (2008a, 2008d) adds this logic of enjoyment to the Althusserian mechanism of ideological interpellation (Althusser, 1994). He suggests that the subject actually *enjoys being* ideologically interpellated. As such, breaking the ideological spell is not just a matter for a subject of knowing the ideological mechanisms that constrains her, but, at a more fundamental level—that is, at an *unconscious* level—, *to traverse the fantasy* (Lacan, 2004) mapping the relation of the subject with the kernel of her *jouissance*. Something has

¹⁷ This is a central concept in Lacan's late work. See, for instance, Lacan (1999, 2007).

to change not just at a rational level—knowing the reasons that lead me to act in particular ways—but at the level of enjoyment. In this sense, ideology is for Žižek (2008a) precisely what regulates the relation between the Symbolic—the smokescreen of ideological/symbolic illusions—and the Real of enjoyment.

At stake here is the radical split between knowledge and action, as Žižek (2008c) explores in endless examples ranging from politics to pop culture.¹⁸ The Žižekian formula of *fetishistic disavowal*—I know very well, but nevertheless...¹⁹—calls our attention for the fact that we may very well know that we are dealing with a fetish, but nevertheless we continue to endorse it²⁰. For instance, in Nazi Germany, people knew that the official ideological line—“Jews are responsible for all evils”—was false (since the problems of Germans at that time were instead of a socioeconomic nature), but nevertheless people stuck to it as a matter of belief. Another example explored by Žižek (2008a) is the case of the Communist Party in Stalinism. People knew very well that the propagated motto of the Party as a representative of the people was false (it represented only the interests of the highest picks in Party’s hierarchy), yet people endorsed their habits and acted accordingly.²¹

In the articles *Equity*, *Critical mathematics education* and especially in *Transfer*, I explore this ideological mechanism in mathematics education research. I can summarise it in the following way. Researchers engaged on social justice know very well that achieving equity is a problem that surpasses mathematics education as a field of research, nevertheless they continue to act as if it could be solved within the field. Ana, the teacher in the article *Critical mathematics education*, knows very well that implementing a critical mathematics in her classes will fail due to schooling constraints. Yet she decides to implement it. The blacksmith apprentices in the article *Transfer* know very well that school mathematics was of any use for their work; nevertheless they stick to teacher’s discourse and pretend that it was indeed important. In the same article, researchers know that people do not transfer the mathematics learned in schools to out-of-school practices. Still, they continue to argue for the importance of school mathematics for mundane activities.

¹⁸ Baldino (1998a), and Baldino and Cabral (2005) explore this split concerning the pedagogical relation between teacher and student in mathematics education.

¹⁹ Originally developed by French psychoanalyst Octave Mannoni: “Je sais bien, mais quand-même...”

²⁰ For examples of this formula being operationalized in pop culture see, for example, Žižek (2008a, 2008b).

²¹ There are many examples of fetishist disavowal. A paradigmatic one in our current academia concerns the means by which research is being evaluated: “I know that the current state of evaluating the quality of research by the number of articles produced is a farce, nevertheless I struggle to publish as many articles as I can”.

To identify such incongruities between knowledge and action, between what people know and what people do, was a crucial methodological strategy of this study. It justifies my preference for ideology as an analytical concept. It is not that people are not aware of the discourses moulding their subjectification: researchers (*Equity, Transfer*), teachers (*Critical mathematics education*) and students (*Transfer*) know the ‘falsity’ involved in their actions. But they do not renounce it. A renunciation will imply tearing apart the entire fantasy sustaining their professional or even personal life. In some cases, as I explore in the article of *Transfer*, doing differently would jeopardize the central role mathematics has in education, with obvious implications for those of us who rely on the social importance given to mathematics. It seems that, although we may know the falsity involved in the way the community approaches crucial topics as equity or the use-value of mathematics, we continue to play the game and to obey it in order not to disturb the usual run of things. What ultimately legitimates the system is not what we know about it, the truth of it, but simply that it works: “ideology can lay its cards on the table, reveal the secret of its functioning, *and still continue to function*” (Žižek, 2007a, p. 200). This attachment to something we know is ‘wrong’ can only be explained in terms of enjoyment: after the ideology has been exposed we still do not change our behaviour because *we enjoy it*.

Knowledge is not enough. And this is what Foucault fails to explain.²² After having depicted with an unprecedented minutia how different disciplines and technologies mould the modern Self, showing how the subject is discursively constructed by modern institutions such as the asylum, prisons or schools, and by the knowledge emanating from modern sciences such as psychiatry, how does Foucault conceive resistance towards these modes of subjectification? That is, after being aware of the processes behind one’s constitution as a subject, how could one put in motion some kind of resistance against them? Foucault addresses this question in his last works, where he develops his *history of sexuality* (Foucault, 1990, 1992, 1998). Foucault conceptualizes emancipation from oppressing and normalizing practices as a matter of the subject building his own mode of self-mastery, what he calls, drawing on pre-Christian Antiquity, “the care for the Self” (Foucault, 1990). The subject must, so to speak, produce himself as a subject, find his own idiosyncratic way of living beyond the disciplinary/confessional mode of power from early Christianity to psychoanalysis. What Foucault does not explain is why people may very well know what constraints them, but still continue to act accordingly to these constraints. At stake here, as explored by Judith Butler (1997), is how the very submission of oneself to discipline also produces libidinal activity. That is, the subject actually *enjoys*

²² Here I will be drawing on Žižek (2008a, p. 296-301), where he explores Foucault’s work.

being submitted to disciplinary mechanisms.²³ As phrased by Butler (1997), “the repressive law is not external to the libido that it represses, but the repressive law represses to the extent that repression becomes a libidinal activity” (p. 49, quoted in Žižek, 2008d, p. 300). This explains why people continue attached to discursive or ideological forms of submission, even after being completely aware of them. What binds us to explicit ideologies or discursive practices of subjectification is not a rational decision to do so, but an unconscious mode of enjoyment.

Foucault misses to accomplish this dimension of the Real. For Foucault the subject is the result of its own process of subjectification, by means of submitting herself to a given discourse regime of power-knowledge. There is no ‘subjective’ position outside discourse: the subject is always already a discursive formation. However, despite this all-encompassing discourse, Foucault was well aware of the non-All character of discourse. When he points out that “there, in the midst [of discourse] is an essential void: the necessary disappearance of that which is its foundation” (Foucault, 1970, p. 16, quoted in Vighi & Feldner, 2007, p. 152) or, “a void, a moment of silence, a question without answer ... a breach without reconciliation where the world is forced to question itself” (Foucault, 1965, p. 288 quoted in Vighi & Feldner, 2007, p. 152), he is calling our attention to something that exceeds discourse. Nonetheless, as pointed out by Vighi and Feldner (2007), Foucault did not theorized this notion of a “void in the middle of discourse”, because that would had made him consider the possibility of a *cause* that is never presented in his discourse analysis:

In his endeavour to sideline historical metanarratives, Foucault eventually abandoned any explanation based on principles that would transcend the discursive regime under scrutiny, and plumbed instead for the principle of absolute *immanence*, i. e., the notion of a positive cause which is immanent within the field of its effects. (p. 153)

What Foucault overlooked, in his denaturalization and de-reification of entrenched notions of the social, was precisely the ‘voids’ where the universe of discourse cracks. These black holes are taken into account by Lacan. As I previously anticipated, they stand for the Real: that uncanny dimension of subjectivity that remains unregistered in the symbolic field of discourse. In this sense, ideology, in Žižek’s Lacanian coordinate system, is not only an effect of discourse—the enumeration of a network of reasons, the explicit text, the ‘spoken’—but also what fills the gap between the Foucauldian discourse and the Real qua non-discursive kernel of jouissance (Vighi & Feldner, 2007). Ideology is a defence against

²³ An extreme case is the one of the ascetic who flagellates himself in order to resist temptation and ends up finding sexual pleasure in this very act of inflicting wounds on himself (see Žižek, 2008a, p. 300).

the Real understood as the repressed, resisting and disturbing surplus of discourse. Real is what remains unspoken, what escapes the order of discourse.²⁴

If we are talking about the subject, ideology is a defence against the libidinal content of our unconscious desires. A defence against what *causes* our desire.²⁵ Unfortunately for us, men and women seeking for happiness, the language of desire is not an open book that can be analysed through discursive techniques. Or perhaps this is precisely what makes life worth being lived. The choice seems to be between a conceptualization where reality is reduced to a hermetic universe of self-enclosed discourses, or, to recover Lacan's quotation at the beginning of this thesis, reality as something which goes *à la derive*.

As I anticipated, in the Lacan-Žižek axis, the Real is not some transcendent beyond that forever eludes symbolization—as it was the case with the traditional concept of ideology, as previously mentioned—and it is not to be confused with reality. Reality stands for the Symbolic, the entire discursive/symbolic framework that needs to be presupposed if the subject is to engage in any communicative activity or social exchange. What for Foucault is the discursive (power-knowledge) order. However, whereas it follows from Foucauldian theory that all we are dealing with is an all-encompassing 'physics' of symbolic fictions, a plurality of discourse universes, for Žižek this negation of externality *tout court* is defective. As explained by Žižek (1994), symbolization ultimately always fails. It never succeeds in fully 'covering' the real. Ideology, in this sense, is precisely what allows us to conceive reality as consistent and meaningful. Ideology is an attempt to 'cover' the real. Take, for instance, the slogan 'mathematics for all'. It is no more than a neurotic attempt to cover the fact that mathematics is not for all. It is an ideological injunction to mask the crude reality of all of those for whom mathematics is no more than a stumbling block for an academic and professional future. These people stand for what Žižek (2008b) calls *spectral apparitions*: "[t]his real (the part of reality that remains non-symbolized) returns in the guise of spectral apparitions" (p. xvii). In the midst of a field living a heavenly fantasy around the importance of mathematics for all sorts of things, these 'ghosts' appear once one puts a foot on a school. This real of school mathematics—where students fail and teachers get burnout, to say the least—is precisely what research tries to conceal by means of propagating "successful experiences" (Gutiérrez, 2010, p. 14).

²⁴ When Lacan (Seminar XXI, unpublished, quoted in Fink, 1995, p. 142) says that "[t]he real is what does not depend on my idea of it", he is pointing to the dimension of human subjectivity that is independent of our knowledge of it. As I shall explore in the next two sections, such conceptualization is what allows Žižek to transpose the Real *qua* psychic dimension to social analysis. His argument is that we may very well know that our economical system is unfair, nonetheless its functioning is *real*, i.e., it does not depend on our knowledge of it.

²⁵ The Lacanian name for the cause of desire is the (in)famous *objet petit a*. See, for instance, Lacan (1999, 2004).

All attempts towards a full symbolization—the liberal-democratic utopia, the all-encompassing power-knowledge society, or the ‘mathematics for all’ meta-aim of mathematics education—are deceptive. Although they can be presented as a ‘whole’ at the level of ideology—the way politics talks about an all inclusive society, or mathematics educators about equity—when actualized in a concrete practice, problems arise that postpone what at the level of the discourse is complete. When it comes to the level of the subject, ideology reveals its truth: it is no more than a defence against the impossibility of closure. I cannot be fully symbolized because the process of symbolization is already an attempt to fill up the void that I am. Ideology in this sense is always not-All, it always presupposes a non symbolized kernel around which is constituted as an attempt to mask its incompleteness.²⁶ If I recall the clash between the university discourse and my first entrance in a school classroom, what at the level of the discourse runs smoothly—teachers may be well aware of the aim of equity in mathematics education, on how to plan a meaningful lesson, and so on—when confronted with the unfathomable density of a classroom context, this discourse reveals to be incomplete, always lacking a full comprehension of the sometimes ‘unreal’ situations which constitute the Real of a classroom. They seem unreal (or surreal, in a postmodern mode) not because they do not happen—strange things do happen in classrooms—but because they were not predicted in advance neither by ideology neither by the all sort of discourses emanating from research. As argued by Brown (2008), research in mathematics education often fails to address the Real of schools.

Ideology, as the entire universe of discursive-symbolic fictions, functions precisely as a shield against this Real. It enables us to put order in a situation that is inherently chaotic. Although a person—whether a student, a teacher or a researcher—may be fully aware of the ‘right thing to do’—the right way of solving a quadratic equation, allowing all students the same opportunities to learn, or to know about the falsity of the use-value of mathematics—, she could easily end up doing exactly the opposite: continue to solve quadratic equations in the wrong way, privileging some students instead of others, commending the importance of working with students real life problems. As I previously said, what attaches people to a practice cannot be fully accounted by what people know about this practice, but by the way people enjoy doing it. This non-discursive excess of ideology is what lacks in discourse analysis, and it is one of the reasons why I found the revitalization of the concept of ideology carried by Žižek more fascinating to analyse mathematics education as a field of research.

²⁶ What Godel’s Incompleteness Theorems proved was precisely this non-All character of formal structure. The process of mathematical formalization engenders its own impossible spots. These spots, although not ‘beyond’ the formal structure, are impossible to reach (that is, to demonstrate). This is why mathematics, according to Lacan (1999), was the first discourse to perceive that the symbolic order itself contains elements of the Real.

Marx, a postmodern *avant la lettre*

Within the Žižekian field, to conceptualize social reality as non-All is to assume the irreducibility of *social antagonism*. This was the groundbreaking step taken by Ernesto Laclau and Chantal Mouffe in their book *Hegemony and Socialist Strategy*. Against a liberal-corporative view of society based in the idea of *consensus*, they assert the constitutive status of social antagonism. According to them, the appeal for a consensus always implies a hegemonic act, which *hegemonizes* by means of excluding the dissident voices, i.e., the ones who have to be excluded so that consensus could be reached (Laclau & Mouffe, 2001). The attempt to eliminate confrontation and conflict stands in opposition to what they call *radical democracy*. As explored by Mouffe (2005), in liberal-democracy the troubled ‘other’ is heard insofar as it is the voice tending to the consensus, the voice expected to be heard. According to her, the decisive achievement for democracy today is to recover the radical meaning of the *Political*—not as a utopian space of distributive justice (Rawls, 1999) or unpolluted communication (Habermas, 1984)—but a place where the voices of those who disagree are not silenced by authority or ideology.

It is in this sense that Žižek (2004) says that today we live in a *post-political* world. The field of the Political no longer stands for the place to discuss alternative ways of living together, but the field of expertise administration, where major ideological fights have been declared death. We have already established the Goal—a harmonious society of equal and free people, who happily and in a multicultural fashion communicate with each other—and all the work to be done consists in an asymptotic approach to the Habermasian regulative idea of a free unconstrained communication, where the obstacles are perceived as being of an empirical nature (Mouffe, 2005).

According to Mouffe (2005), the political challenge today, where categories such as ‘left’ and ‘right’ have fall into a centralized acceptance of capitalism as the ontologized *modus operandi*, is to affirm that capitalist liberal-democracy, with its emphasis in consensus, equality and inclusion, conceals a severe exclusion. Beyond left and right, we are thrown into a world where politics is reduced to a technical or, as defended by Giddens (1994), dialogical enterprise. As placed by Mouffe (2005):

Nowadays politics operates supposedly on a neutral terrain and solutions are available that could satisfy everybody. Relations of power and their constitutive role in society are obliterated and the conflicts that they entail reduced to a simple competition of interests that can be harmonized through dialogue. (p. 111).

Indeed, this new all-encompassing category of people—the Citizen—stands precisely for this imaginary space of (neo)liberal-democracy where there is no antagonism: equal

people discussing towards a common consensus. Against this view, Mouffe (2005) argues for the ruthless *antagonistic* constitution of the Political.

Žižek (1994) identifies the antagonism that perpetrates the social edifice with the Real that has to be ‘repressed’ so that the discourses of ‘social cohesion’ (from traditional ethics of Good up to Rawls and Habermas) can be established. As phrased by Žižek (1994), “what the spectre [of ideology] conceals is not reality but its ‘primordially repressed’, the irrepressible X on whose ‘repression’ reality itself is founded” (p. 21). What follows from Žižek’s theorization—and from the previous discussion opposing the postmodern identity politics to the Marxist notion of class—is that this ‘repressed X’ is the *economic*—class struggle—on whose repression society itself is founded. Class struggle is *real* in the strict Lacanian sense: something that impedes society from closing itself into one unified corporation. Vighi and Feldner (2010) place this way:

From Žižek’s Marxist perspective, the key manifestation of the Real upon whose disavowal society is erected is, of course, class struggle. Although its disavowal makes social coexistence possible, it simultaneously makes it impossible as it perpetually haunts our symbolic space, reminding us of its inconsistency and demanding every-new symbolisations. (p. 37)

In this sense, class struggle is not (necessarily, at least) a name for two positive entities—workers and capitalists—struggling against each other, but designates the very antagonism that prevents the objective (social) reality from constituting itself as a self-enclosed whole (Žižek, 1994). As expressed by Žižek (1994):

The question of the suitability of the term ‘class struggle’ to designate today’s dominant form of antagonism is secondary here, it concerns concrete social analysis; what matters is that the very constitution of social reality involves the ‘primordial repression’ of an antagonism (p. 25)

Žižek’s point is that, once upon a time, this primordial antagonism was called class struggle. There is no doubt that the old communist measures to deal with the impasses of the present are no longer effective. However, the antagonism inherent to the form of Capital disclosed by Marx has not disappeared. In our current post-ideological world they came back as the *return of the repressed* to use Freud’s famous expression (Freud, 1915). My plea is thus not for some kind of ‘back to the past’, but, to put it in Deleuzian terms, to *repeat* the past in its becoming (Deleuze, 2004). ‘Becoming’ for Deleuze is strictly correlative with the concept of repetition. As stated by Žižek (2004), in his analyses of Deleuze’s philosophy,

far from being opposed to the emergence of the New, the proper Deleuzian paradox is that something truly New can only emerge through repetition.

What repetition repeats is not the way the past ‘effectively was’, but the virtuality inherent to the past and betrayed by its past actualization. (p. 6)

Repeating the New is to repeat what in the Old remains new, that is, its potentiality for emancipation, for becoming. If we take Marxism, we can say that to ‘repeat’ Marx is not to repeat the statements which provide his solutions—Soviet Union’s communism, for instance—but to rediscover the radical novelty of his breakthrough, of his predicaments around the categories of value, capital and commodity. Today everyone is willing to mumble that no society can function efficiently without the market. Against this background, how can one return to the friction, to the resistance of the earlier moments where capitalism was not yet ‘in human nature’?

For Marx, the production and distribution of the material goods necessary to human existence is the primary force determining social life, what is usually called the *primacy of the economy* (Marx, 1859). This way, contrary to the idea that Marxism proposes an utopian formulation for the world, where communism stands for some kind of idyllic mode of living together, I wish to assert the ruthless *actuality* of Marx’s analysis. Marx’s work consists in a scrupulous, groundbreaking analysis of capitalism as an economical system. Both Žižek and Jameson interpret Marxism not as a political philosophy (although it entangles a practice of politics), but as being primarily concerned with the economic organisation of society and how people cooperate to organize production. Marxism is primarily concerned, not with the end of a certain political thinking, but with the end of an economical system.²⁷ In this sense, as pointed out by Jameson (1991), Marx is truly *modern*, insofar as for him to really enter in modernity implies not just a *defetishization* of God—a society ruled by clericalism—or the King—an economical system based in feudalism—, but above all a defetishization of the market. For Marx, ‘modern’ men have only replaced the fetish of God with the fetish of commodity, thus remaining essentially pre-modern:

Market ideology assures us that human beings make a mess of it when they try to control their destinies (‘socialism is impossible’) and that we are fortunate in possessing an interpersonal mechanism—the market—which can substitute for human hubris and planning and replace human decision altogether. We only need to keep it clean and well oiled, and it now—like the monarch so many centuries ago—will see to us and keep us in line. (Jameson, 1991, p. 273)

²⁷ In this sense, as argued by Jameson (1991, p. 265), Marxists have something in common with neo-liberals: for the latter ‘politics’ means simply the care and feeding of the economic apparatus (in this case the market rather than the collectively owned and organized means of production).

This belief in a transcendental dimension regulating our lives is the sign of the pre-modern. Although being a purely fictional category strictly belonging to the field of human relations, the market is seen, perhaps today more than ever, as an ‘extra-human’ entity with its self-regulated functioning that escapes human control: we live according to the rules of the market, and no alternatives to a market economy seem possible any longer²⁸. This is fetishism at its purest:

The point of Marx is that the commodity universe provides the necessary fetischistic supplement to the ‘official’ spirituality: it may well be that the ‘official’ ideology of our society is Christian spirituality, but its actual foundation is none the less the idolatry of the Golden Calf, money. (Žižek, 1994, p. 20)

Thus, at a time where Humankind was getting free of all fetishism—the Enlightenment—Marx pointed out how society was only substituting one fetish for another. This resistance of people to take their future in their own hands is the sign of the pre-modern. And it is in this sense that Marx argues that we still live, to some extent, in pre-modernity, since we still anchor our being in the world in some ‘invisible hand’ regulating our lives. For Marx, there is no emancipation without economical emancipation and, in this sense, to paraphrase Jameson (1991, p. 343), the conquest by humans beings of the otherwise seemingly blind and natural ‘laws’ of socioeconomic fatality remains the irreplaceable will of Marx’s heritage.

The emphasis on economics when thinking social change is, to some extent, what makes Marxism a dangerous theory. If we consider the work of other crucial figures of modernity such as Immanuel Kant, Max Weber and Émile Durkheim, or even contemporaneous such as Foucault and Habermas (but not Althusser, obviously), the use of their theories in the academia do not seem to rise more than confined academic conflicts between different schools. However, using Marx immediately transfers us to an imaginary of ideology, totalitarianism, and radical social solutions. It does not seem suitable for scientific research to be befouled with such ‘political’ theories. Marxism is usually discarded as being non-scientific, as carrying with it ideologies from the past that we have to get rid of.²⁹ However, in a strictly postmodern manner, we should ask: what in science is not ideological?

²⁸ Take today’s Europe, for example. People are been asked to change their lives dramatically, to give away social rights acquired during modernity, in the name of the ‘market stability’. Paradigmatic is the case of the so-called rating agencies. How does a group of people, seated in front of a computer in the other corner of the world alter the entire future of a country just by changing a ‘rate’? This was what happened in Portugal last March.

²⁹ Recall, for instance, Karl Popper’s famous criticism of Marxism as a *pseudo-science*, based on dogmatism and, as such, immune to refutability (Popper, 2002).

On the other hand, does the fact that Marxism had been associated with totalitarian regimes entirely justify its disavowal? Perhaps, but then we should ask: why was Marxism the only theory that has until now offered both a detailed analysis of capitalism and an alternative to it? And we should also add: the only one which originated a process of *revolutionary becoming* materialized in the October Revolution? Because there is something in Marxism that is pure ‘evil’: it strikes right at the core of the hegemony of Capital, thus making revolution—as opposed to reform—a necessity. Regarding research, you cannot be simultaneously faithful to Marx’s work and use it as an ‘exotic pet’ in order to praise some particular point of your study. Getting into Marx’s studies is dangerous because it changes your positioning in research: you realise that all the particular problems that you are trying to address (in the case of mathematics education, the endless obstacles standing in the way of a meaningful learning) do not change if the problem as a whole remains unaddressed. That is why Marx hurts. Taking Marx’s insights seriously into consideration implies the lost of hope for emancipation within capitalism—we realise the wilderness of the task ahead.

The disavowing of Marxism by the postmodern can thus be understood as the way critical research (in general) found to be able to continuing its activity, without really changing anything. Mouffe (2005) argues that the postmodern transmutation of antagonism into a multiplicity of equally preponderant ‘struggles’ is one of the elementary ideological operations: it disavows the antagonism that inheres to the field of the Political: “[t]he kind of pluralism they [postmodern approaches] celebrate implies the possibility of a plurality without antagonism, of a friend without an enemy, an agonism without antagonism” (p. 134). This disavowal of antagonism is what leads Jameson (1991) to assert that poststructuralism/postmodernism acts as an ideological support for national and global capital. Postmodernism misses to acknowledge social antagonism—class struggle—not as a part of objective social reality (part of a discursive order to use a Foucauldian language), but Real in the precise Lacanian sense: “that which remains the same in all possible universes” (Žižek, 1994, p. 25).

Throughout the last six articles in this thesis I address the importance of having in consideration the economy of schooling when doing research in mathematics education. It is not enough to change the discourse, the “frame of mind” as Gutiérrez (2010, p. 4, 9, 12) calls it, wherewith we research in mathematics education. We have to address the role schools play within capitalist economics, by means of being, at the base³⁰, an accreditation system. Studies in ethnomathematics, critical mathematics education, and sociopolitical perspectives are often embedded in a humanist rhetoric that posits emancipation as a matter of assuring that human beings “are equally concerned with the most universal

³⁰ In the article *Transfer* I address Marx’s articulation between base—the economic structure of society—and *superstructure*—the legal, political and intellectual life.

problem facing mankind, that is, survival with dignity” (D’Ambrosio, 2007, p. 25), of “help[ing] students becom[ing] productive and active citizens” (Malloy, 2008, p. 23), of “creating solidarity” (Gutiérrez, 2010, p. 13). As Paola and I explore in the article *The specificity of mathematics*, even post-Marxist approaches in mathematics education—in this case, the work of Luis Radford—disavow the fundamental dimension of economy. Consider, for instance, how Radford (in press) conceives emancipation:

In contemporary cultural-historical approaches, emancipation is a political project that can only put in motion through the encouragement of forms of solidarity, trust, sharing, and a commitment to improving the quality of human life. (p. 9)

What is in fact a structural problem, endemic to a mode of production, is transformed into an abstract problem of solidarity, trust, sharing and commitment. Painted this way, Marxism uncannily resemblances a catechism, with charity as the main safe guard of Humankind. Economical exploitation, the fundament of capitalism, is reduced to political domination, which can be solved through the goodwill of engaged people. As posed by Souza, Linardi and Baldino (2002), presented in this way “Marxism becomes completely palatable to the Bourgeoisie, transvestite of humanist ideology, castrated from its fundamental concept, the “surplus-value” (p. 28). That is, exploitation is not structural of capitalism, it is not the exploitation of labour by capital, but the abstract exploitation of ‘man by man’, which can be solved through the efforts of well intended human beings engaged in the amelioration of their praxis. In other words, “no dishes need to be broken” (p. 28).

Two faces of the same coin

The Foucauldian notion of *bio-politics* (as Paola and I explore in the article *The specificity of mathematics*) has been important to understand how in our contemporary society fundamental social problems are being addressed as if they were the object of expert management and administration (Foucault, 1991, 1997). The field of the Political as elaborated by Lefort (1986) or Laclau (1990)—the moment of openness, of undecidability, when the very structuring principal of society is called into question—is given place to a more anthropomorphic, corporative notion of politics as a positively determined subsystem of social relations in interaction with other subsystems (culture, institutional, religious, ethnic, ecological, gay, etc.). The time of the big ideological questions seems to have passed, and a consensus was established regarding the core structures of society—Capitalism and Liberal Democracy. The task ahead is reduced to one of polishing up the particular obstacles standing in its way, what has been known as ‘capitalism with a human face’.

Notwithstanding the comfort that such position may bring, it is the wager of this thesis that one should reject it without mercy. The idea that we *still* have problems standing in the way of a harmonious sociopolitical system is wrong not because these obstacles cannot be overcome, but because they are generated by the same system who sees them as mere dysfunctionalities. In the articles *Equity* and *Critical mathematics education* I extensively develop this dialectic that, at a philosophical level, opposes Kant to Hegel. What I call, based on Lacan (2008), the *evolutionistic* thesis, which substantiates liberal (e.g. Rawls, 1999) and deliberative (e.g., Habermas, 1984) views of democracy, implies the belief in a Supreme Good, in a final goal of evolution which guides its course from the very beginning. Such a perspective assumes that although the modern project based on capitalist economics and liberal ideologies has been not without its problems, these are contingent, conceived as ‘deviations’ from an otherwise good system (Mouffe, 2005). The message conveyed is that modernity gave us the ideals for which we should strive (equality, freedom, etc.), and our task consists in eliminating all the impediments standing in the way of their full actualization. Marxism offers us the *negative* of such a worldview. Through the deployment of Hegel dialectics, Marx (Marx & Engels quoted in Jameson, 1991, pp. 261-262) showed us that, not only the Ideal cannot be achieved amidst capitalism, the ideal discourse itself functions as the proper staged discourse—ideology—that makes capitalist reality sustainable and acceptable. We need to know that the goal for which we all strive is equality and freedom (that the presupposition of the system is a ‘good’ one), so that we can accept the unequal reality in which we live.

As I mentioned, there are two different world views launched by the philosophies of Kant and Hegel at stake here. Kant is the philosopher of the Enlightenment, of the Bourgeois revolution, social-democracy, and citizenship. Kantian philosophy is the philosophy of progress and modernity, the philosophy of the Universal Human Rights and the faith in the free autonomous subject that, under the law of duty, works for the common good. Hegel colours Kant’s picture of bourgeois society with its negative, through the deployment of a method vulgarly called *dialectics*. Hegel makes visible how Kant’s philosophy of progress entails its opposite—darkness, poverty, death and misery, ultimately, as posed by Adorno and Horkheimer (1979), the Holocaust—as the fellow correlative of modern progress. We cannot have one without the other; they are the two faces of the same coin. The universality sustaining Kantian society is accompanied by an excess produced by the idea of universality itself. Hegel showed us that with Liberal progress comes, as its correlative, exploitation and poverty. We cannot eliminate the later so that the former can be fully actualized, since they are two phases of the same development. Capitalist development requires some kind of ‘throwback’ in order to progress:

the amassing of wealth ... is intensified on the one hand, while the subdivision and restriction of particular jobs, and thus the dependence and distress of the class tied to work of that sort, increases on the other hand. (Hegel, 1952, para. 243, quoted in Marcuse, 2008, p. 53)

Despite the excess of wealth, capitalist society is not rich enough to “check excessive poverty and the creation of a penurious ramble” (idem). Therein resides one of the contradictions of late capitalism: on the one hand, there is a saturation of wealth where we have today more goods and more abundance than ever before in the history of Humankind. On the other hand, there is a growing number of the excluded, of those who cannot satisfy the basic necessities of life. As Žižek (2007a) poses, excess and lack, suffocation and deprivation, are structurally co-dependent, therefore:

It is inappropriate to claim that since some people live in abundance while others live in deprivation, we must strive for universal abundance: the ‘universalization’ of the form of abundance which characterizes late capitalism is impossible for structural reasons, since, as Hegel pointed out in his *Philosophy of Right*, in capitalism, abundance itself produces deprivation. (p. 231)

This brings us to another Žižekian concept that I use in some of the articles: the *symptom*. Whereas from a point of view of liberal-democracy the problems standing in the way of the common Good are conceived of as *empirical* (Mouffe, 2005), as vicissitudes of a good system, from a Marxist point of view, they are the truth of the system. Žižek calls them the symptoms (Žižek, 2007b, 2008a, 2008c), and phrases in this way:

for the standard capitalist view, crises are ‘temporary, correctable glitches’ in the functioning of the system, while from the Marxist point, they are its moment of truth, the ‘exception’ which only allows us to grasp the functioning of the system. (Žižek, 2007b, p. 6)

In the article *Critical mathematics education* I tried to conceive the everyday problems that a teacher lives in her work when trying to implement a CME in a regular school not as temporary, correctable glitches, but as core points where is possible to see the entire purpose of the school system: sorting people by means of (school) credit accumulation. My intention was to theorize the symptoms at which the Supreme Goal of a critical mathematics education fails, not as something that can be ‘healed’—through better teaching and learning practices, for instance—but as a window into the entire contradiction of schooling. Critical and sociopolitical research in mathematics have been playing an important role in identifying many of the contradictions standing in the way of a meaningful and equitable mathematics education—the symptoms are recognized. However, as I explored in this article, and also in *Equity*, mathematics education research

often falls into an evolutionistic understanding of them. Problems exist, but they are conceived as external to the ideal image of an equitable and inclusive school. We have to get rid of these problems, so that the ideal can be fully established. Consider for instance how Ernest (2010) conceives educational improvement:

In education we turn promise into reality, and we have more time and opportunity to do so than ever before in the history of humankind. Despite these increased opportunities, or perhaps through them, school education still serves as a powerful fractional distillation device that separates off different sectors of the population for different rewards (p. 82)

The idea is that we *still* have inequalities in school mathematics. Nevertheless we are on the right path, which just has to be polished. In *Equity* I analyse this conflict between the *real*—school exclusion—and the *ideal*—school equity—using Žižek’s exploration of the Hegelian concept of *negative particularity* (Žižek, 2008b). I sought to show that exclusion is not some marginal problem of schooling, but its constitutive element. Exclusion functions not as an obstacle to full schooling, but as a ‘filler’, as something school needs in order to affirm its identity. The traditional way of positing the problem of inequity conceals the fact that achieving equity is not about dealing with particular forms of exclusion, but a matter of addressing school as a capitalist state apparatus.

This way, the symptom stands for universality as such. By showing the counterpart of the universal discourse, it shows it all: the rule and its constitutive exception. This was an important methodological strategy I followed in my analysis, and allowed me to tackle a common postmodern criticism to the concept of ideology. According to postmodernism (e.g., Lyotard, 1984; Seidman, 1994b), theories who deal with the notion of universality end up flying over the forest, leaving aside the particular constellations of power relations which constitute the interstices of human sociability. However appropriate this critique might have been for the deconstruction of so-called metanarratives, one should reject it as a false dichotomy. The idea that if we look at the global we miss the local is based in an epistemology where these two dimensions are perceived as different realities that, although linked, remained separated. This implies that we have to choose: either we research the local, and miss the ‘big picture’, or we research the global, and end up in a metanarrative which no longer accounts for the local. What Žižek allowed me to do, with his revitalization of Hegel’s dialectics, was precisely to overcome this dichotomy, to realise how it is in the concrete situations that the universal manifests itself. It is not a matter of having one in detriment of the other, but to acknowledge how the universal operates within the particular which, in return, colours its very universality and accounts for its efficiency (Žižek, 2008b).

This was precisely my purpose with the articles *Critical mathematics education*, *Ethnomathematics* and *Equity*: to realize how the ideals steaming from ethnomathematics, critical education and sociopolitical approaches are ultimately determined by the concrete practices in which they are going to be operationalized, in this case, schools. When universal ideas such as Multiculturalism, Social Justice and Equity are inserted into schools, they turn out to be monoculturalism, social injustice and inequity. It is in the realm of the particular that the universal become fully actualized. This is the fundamental dialectical law of the determination of a form by its content (Jameson, 1991). The universal form is always determined by the concrete practice which renders it ‘livable’. The proper way to carry a critique of ideology is precisely to confront what is usually called ‘ideology’—the global, universal proclamations—with its actualization within a *life world context*:

In order to pass from abstract propositions to people’s ‘real lives’, one has to add to the abstract propositions the unfathomable density of a life world context—and ideology are not the abstract propositions in themselves, ideology is this very world density which ‘schematizes’ them, renders them ‘livable’. (Žižek, 2010, p. 6)

The practice of the blacksmith apprentices, as I explore in the article *Transfer*, and Ana’s classes in the article of *Critical mathematics education*, are cases where I sought to actualize within a concrete practice the sonorous claims steaming from research assuming the importance of mathematics for the mundane life of people. Such claims, when actualized in a concrete practice, reveal to be a farce. At stake here is what Žižek (1994) calls, drawing on the Hegelian notion of *concrete universality*, “the concrete existence of an ideological universality” (p. 24). The inequalities of schooling or the irrelevance of the use-value of mathematics are not violations of the principle of equality and of the principle of the importance of mathematics, but are absolutely inherent to its logic. They are “the paradoxical results of its logical realization” (Žižek, 2006, p. 325). It is the concrete social circumstances of schooling—exchange-value, accreditation—that provide the ‘true’ meaning of pathetic slogans such as ‘mathematics for all’.

Hence the method I deployed in my analysis was not intended to compare the universality of a discourse or ideology—the macro—with the reality which is suppose to correspond—the micro—, but to compare the research discourse *to itself*, to its concrete content. This is what Žižek has in mind when he argues for a displacement of the difference between universal and particular into the particular itself (Žižek, 2006). The Hegelian concrete universal is precisely the unity of the abstract universal with its constitutive exception. In this sense, as I explored above regarding the discourse/ideology discussion, the Universal is in itself non-All. It ‘needs’ the “unfathomable density of a life world context” (Žižek, 2010, p. 6).

This brings us to a Lacanian distinction which proved to be crucial for my analysis: the distinction between the *subject of the enunciation* and the *subject of the enunciated* (Lacan, 2001, 2004), already present in Hegel (cf. Žižek, 2008b). This distinction consists in a comparison between what the subject enouncing a thesis *wanted to say*—enunciated—and what he *actually said*—enunciation.³¹ Consequently,

[o]ne subverts a universal thesis when one demonstrates to the subject who enounced it how, *by means of its very enunciation*, he said something entirely different from what he intended to say; as Hegel points out again and again, the most difficult thing in the world is to say exactly what one ‘wants to say’. (Žižek, 2008b, p. 160-161)

As I explored in the article *Ethnomathematics*, researchers may very well, at the level of the enunciated content, argue for a valorisation of different cultures, while in their practice—the position of enunciation—they end up *desubstantializing* the Other, depriving them of all the social, cultural, political or religious dimensions that characterized their culture. Also in the article on *Critical mathematics education*, what at the level of the enunciated is a genuine will to promote a critical mathematics education, ends up reasserting, at the level of the enunciation, the primacy of traditional mathematics ruled by the exam. In these articles I sought to show that research may very well contradict its declared intentions. As phrased by Žižek (1997),

what, on the level of the enunciated content, is the critical rejection of an ideological hegemony can well involve the full endorsement of this same hegemony on the level of the position of enunciation. (p. 56)

This was a central strategy of my analysis: to disclose how a considerable part of mathematics education research concerned with social justice, emancipation, in short, in achieving the universality of a ‘mathematics for all’, misses to acknowledge, in the corrupted reality in which they lament, the ultimate consequence of their won act.

This happens because what gives ‘meaning’, what colours what a subject says is not determined by him, but by what Lacan (2000, 2001) calls the logic of the signifier: the entire intersubjective symbolic order that a subject inhabits. And, as I said previously, if it is the logic of Capital which stands today for the concrete universal, colouring our ‘acting’ in the world, the conclusion to be drawn is that in the end, at the level of the enunciation, what (over)determines our action is Capital itself. This becomes clear when I confront, in the different articles, the discourse of research with the reality of schooling. No matter

³¹ It was Freud who first extensively explored this discrepancy which is repeated between what the individual wants to say and what he actually says. According to Miller (1999) it is in this discrepancy that Freud situated what he called the *unconscious*—as if for this wanting-to-say of mine, which is my ‘intention of signification’, another wanting-to-say was substituted, which would be that of the signifier itself and which Lacan designated as “the desire of the other” (online text).

how prosaic and emancipatory the discourse may sound, when actualized in schools, it ends up being adjusted to the logic of school accreditation. As noticed by Žižek (1997), this was already acknowledged by Walter Benjamin:

One is tempted to invoke here Walter Benjamin's crucial distinction between the attitude that a cultural product assumes *towards* the dominant relations of production, and the position of this same product *within* these relations of production: a product whose explicit attitude is very critical towards the dominant relations of production often fits the frame of these relations perfectly. (p. 56)

In the article of *Transfer* is where I explore more clearly this dialectics in relation with the discourse of the blacksmith appendices. Whereas at the level of the enunciated content, the discourse of students, teacher and master urges for the importance of school mathematics in their work, in their practice as blacksmiths—their position of enunciation—they do not make any use of the mathematics learned in school. What, at the level of the official discourse, runs smoothly—the argument of the utility of mathematics in becoming a worker or a citizen is well established—when actualized in a specific practice often encounters a series of obstacles which ends up perverting the official intention. The usual way of research to proceed is to eliminate such obstacles, so that the official aims can be fully actualized: better teacher formation, a stronger connection between school and workplace, more realistic problems, etc. However, I took another approach. I argued that such mismatch—between what we say and what we do—is the proper ideological mechanism that makes effective the presence of mathematics in school. While presenting school mathematics, at the level of the enunciated content, as an important subject in terms of knowledge and competence, the other, surreptitious, functions of mathematics—its exchange-value—can, *actually*, become operative.

Closing Up

Paraphrasing Žižek (2008b, p. 160), and to recapitulate, the central strategy of my analysis was based on the Hegelian assumption that every totality, posited as 'thesis', necessarily contains within its particular elements 'at least one' which *negates* the universal feature that defines it. This is, as I previously explored, what Žižek calls the symptom: "the element which—within the field of universality—holds the place of its constitutive Outside, of what has to be 'repressed' for the universality to constitute itself" (2008b, p. 160). Therein resides the logic of the dialectical exercise I sought to develop: to 'catch', within mathematics education research, what the universal secretly excludes. Ultimately, what it excludes is the value of school mathematics within capitalist economics.

With the exception of the first article, all the others are attempts to ‘catch’ this disavowed dimension of school mathematics. Starting by the way mathematics education conceives its own role as a field of academic research. In the articles *Broadening the role of theory* and *The specificity of mathematics* I sought to show how the concepts of ‘learning’ and ‘mathematics’ are the ones that allow the field to avoid the confrontation with the Real of schooling, preventing it from taking into consideration its economical dimension. In the articles *Equity*, *Beyond disavowing*, *Critical mathematics education*, *Ethnomathematics* and *Transfer*, I choose specific topics of research—equity, transfer, ethnomathematics, critical mathematics education—and address it as ideologies designed precisely to conceal the Real of school antagonism. When sociopolitical perspectives claim that the problem of failure in school mathematics is due to the exclusion of particular groups of people whose subjectivity differs from the one conveyed by schooling, when critical mathematics education suggests that school mathematics will empower students by allowing them to disclose ‘oppressive’ uses of mathematics in society, or when studies on ethnomathematics argues that it is through the valorization of ‘other’ cultures that school mathematics can become meaningful, they all are performing an ideological move. They are positing both ‘particular subjectivities’, ‘mathematics’ and ‘culture’ as what Žižek (2008a) calls *sublime objects of ideology*. A sublime object is an object that seems to be missing where something is in discord.³² All the research texts I addressed in the articles of this thesis somehow deal with objects such as: more equitable mathematics, more real life problems, more critical mathematics, more local culture, and so on. They all identify what Sverker Lundin (in press) calls the *missing link*—what is standing in the way of a meaningful and successful mathematics for all—and design strategies to overcome it. That is, they all present ideological attempts to close the irreducible gap between the ideal aim of a ‘mathematics for all’ and the reality of a school which forever postpones such aim. This is the basic definition of Lacan-Žižek’s notion of ideology: a totality set on effacing the traces of its own impossibility, by means of displacing the internal and all-pervasive contradiction of schooling—where failure is a necessity—onto an external and contingent series of events. Instead of conceiving the gap properly, research ends up creating ideological injunctions whose purpose is to mask the real of capitalist schooling. The question underlying research is not why there is a gap, but how to make the gap disappear. It follows from what I have been theorizing that the gap is constitutive, thus it cannot

³² An important example of this object comes from mathematics. During the sixties, when Soviet Union launched the Sputnik and become the first country to put a satellite in the Space, the United States reacted by positing mathematics and its education as crucial features for surpassing such a crisis. Mathematics was perceived as what was missing in United States society to equal and surpass Soviet Union not just in the conquest of Space, but also in scientific, technological and economical terms.

‘disappear’. But it can be masked, forgotten, only to appear later in the guise of what Žižek (2008b) calls “spectral apparitions” (p. xvii).³³

Hence, my analysis was not concerned with ‘fixing’ the problems of teachers or researchers, but it sought to *complicate* the usual way in which the community formulates and addresses some of its most preeminent problems (equity, culture, use-value, critical mathematics, the political, theory).³⁴ One implication emerges from this work of mine. It is the importance for mathematics education research to investigate *failure*. That is, more research efforts need to be made to study failed attempts to promote a meaningful mathematics education for all. Contrary to Gutiérrez’s assumption that “it is important to highlight the features of practice that coincide with certain kinds of students engaging/succeeding in school mathematics (and this form is much more productive than focusing on failure and/or disengagement)” (2010, p. 15), I suggest that the community should take the difficulties and impediments in achieving a meaningful mathematics education not as particular obstacles to get rid of, but as central issues for educational research. It appears that mathematics education is confronted with a similar challenge to the one of psychoanalysis.³⁵ The latter is ultimately the theory of why its clinical practice is doomed to fail (Lacan, 2008). And, in the same line, perhaps mathematics education should be the field of research of why people are doomed to fail in school mathematics; and not so much, as it is today, the field of research on how people could achieve success.

Taking failure seriously into research implies considering antagonism as the irreducible and constitutive kernel both of the Subject and of the Political. This was what the axis Hegel-Marx-Lacan-Žižek allowed me to do. It led me to reject, on the one hand, the idea that the individual relates with the social by ‘interaction’—where an autonomous subject *objectifies* some external phenomena, as argued by socioculturalism. On the other hand, at the level of the Political, I also rejected the idea that antagonisms can be surpassed through an amelioration of the system. In the article *The specificity of mathematics*, the analysis Paola and I make of socioculturalism provides the basis to articulate the antagonistic constitution of the Subject and of the Political. We take socioculturalism as a paradigmatic example of the disavowal of the antagonistic character of social reality. By assuming a complementarity between individual and culture where the former objectifies the exterior knowledge, socioculturalism reflects a corporative view of society, where there is no class struggle but different groups (directors, managers, technicians, factory workers, etc.) working each for some common Good, from their freely chosen social

³³ In the *Equity* article I analyse a series of these spectral apparitions, from Nolan’s dream to the chagrin of teachers in the final evaluation meetings of the school year.

³⁴ Many others could be mentioned, and it is part of my future plans to develop similar analysis around teacher education, the use of technology and identity.

³⁵ Indeed, Freud considered psychoanalysis, education and politics the three impossible societal tasks.

position. This disavowing of the constitutive *negativity* of the Subject—who refuses to learn—and of Society—where people dramatically continue to fail in mathematics—allows socioculturalism to avoid the Real of schools—the blockages, the violence, the negation of students in complying with the symbolic mandate conferred upon them. Socioculturalism ends up creating a ‘research heaven’ where what fails in schools can be fixed and harmonized in research. Lacanian theory, by emphasizing ‘what fails’, that is, the symptoms, gives researchers a space where the ‘negativities’ felt by teachers and students in their daily school lives can be addressed in two ways. On the one hand, the negativities are not marginalities, or contingencies of an otherwise good system, but fundamental blockages resulting from the splitting nature of the Subject itself. On the other hand, the worldwide problem of failure in school mathematics can be taken as being co-substantial to a certain ‘failure’ that is endemic to civilization as such. In this sense, Lacanian theory is a theory of tragedy. Thus tuned with the tragic reality of many children who year after year, in schools around the world, fail to ‘unite’ with the culture of school mathematics.

Therefore, what the axis Hegel-Marx-Lacan-Žižek can offer to education is not another ‘formula’ to conceptualize the process of learning, but an understanding of why education and learning are ultimately destined to fail. It is the wager of this thesis that, to paraphrase Žižek (1994, p. 7), such ‘failures’ have the potentiality to point towards the system’s antagonistic character, and thus ‘estrangle’ us to the self-evidence of its established identity. This ‘estrangement’ takes place when we no longer take as for granted the liberal-democratic capitalist view of school as a neutral environment purged of ideology. Instead, we realize the purely manufactured, contingent nature of such discourse. Ultimately, this discourse functions as an ideological mechanism aimed at obliterating the traces of its traumatic origins: the fact that school is a place of social promotion. There is no Supreme Good waiting for us at the end of the tunnel. Failure is a necessary condition for the consistency of the same system that strives to abolish it. Therefore, it should be investigated as such.

This implies that mathematics education researchers have to get dirty hands. It implies coming down from the heaven where they have been living, and address the Real of schooling. This same Real I encountered for the first time eleven years ago, where I realised that being a mathematics teacher has nothing to do with mathematics or didactics. It has to do with people, with their unknown desires deeply structured by ideology. While the community continues to implicitly assume that people will engage in mathematics for the satisfaction of exploring mathematics, the Capital will remain the disavowed structuring principle dictating what happens in the mathematics classroom.

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Part 2

The Articles

ALEXANDRE PAIS

10. PORTRAIT OF AN INFLUENCE

Does mathematics keep our imagination on a short leash?

Ole Skovsmose

RENDEZ-VOUS

One of the first images I have of Ole Skovsmose was generated in a very German city, located in Brazil. At that time the idea I had of a researcher, especially one with the name of Ole Skovsmose, was that of a serious and distant figure, formally dressed and with haughty manners. Six years of university education leaves its trace. At the day of his presentation I was obviously anxious and for several reasons. To start with, it was also the day of my presentation, which was based on a critical view of the relation between Critical Mathematics Education (CME) and Ethnomathematics (E), therefore a controversial topic. But also because I would see for the first time in action the name that I had read so many times in papers. So there I was seated and expectant. When I saw this person dressed in a training suit, wearing tennis shoes and with ruffled beard climbing the small stairs to the pedestal, I remember wondering if he was giving an address or an aerobics class. In some way, that vision mirrored the first impression I had of the city of Blumenau. The same anachronism I felt being in a German city inside Brazil, I felt with the way Ole presented as an academic inside a training suit. I felt comfortable, with the city and with Ole.

But that was not the first time I meet Ole. Few months before, I went to Copenhagen to attend Paola Valero's PhD defence. At that time I was taking my Master Degree at University of Lisbon, and I was also a mathematics teacher in a public school in Portugal. I was beginning to think about mathematics education research beyond the didactical approach, focusing on a broader perspective of what it means in our contemporary world to teach and to learn mathematics. But it was still not clear to me how this new approach could be related with my work as a teacher, with my students in the confines of a mathematics classroom. After the defence, Ole approached me (it is not common for someone to travel many kilometres to see a PhD defence) and we engaged in a brief chat. I told him I was a teacher, and he mentioned something that immediately resonated with some doubts I was starting to raise about my work as a teacher. He said that the majority of teachers were like missionaries, their mission being to teach the mathematics of the curriculum. According to this mission, most of the research on mathematics education was dedicated to researching better ways to teach and learn mathematics,

H. Alrø, O. Ravn, and P. Valero (eds.), Critical Mathematics Education: Past, Present and Future, 133–144.
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in a didactical perspective. In fact most of the teachers do not raise questions as *why do I teach mathematics?* Or *why is the mathematics of the curriculum the most important for students' lives?* We rarely question the necessity of teaching the things we teach. We simply accept the appropriation of content, incorporate the curriculum discourse, and try to do the best we can. How could it be otherwise? In metaphoric terms, school is closer to a factory than to a laboratory. In school life an environment of criticism and questioning is absent. The teacher has a mission consisting of transmitting a particular body of knowledge or, in current language, a set of competences. Her main concern is to develop that mission, to be a missionary. Questioning, criticising or mobilising for change are not verbs of her mission. Ole's main point was that a missionary teacher does not assume a critical perspective about what she teaches. Should not the teacher question the social and political reasons of her mission, instead of following them in a blind way? These were not comfortable ideas for a young teacher, engaged in his career, to hear. They raised a sense of discomfort, but also a willingness to know more, to reflect on the radical questions posed by Ole.

APPROACHING OLE'S IDEAS

At that time, I had read just a couple of Ole's articles. The first one I read is relatively well known (at least in Portugal). I am referring to "Escenarios de investigación" (Landscapes of investigation, Skovsmose, 2000). However this reading did not awaken me to the radicalism involved in a critical approach to mathematics education. Rather I read it as a framework to classify the several tasks a teacher could propose in his mathematics teaching. It helped me understand the difference between an exercise and an investigation, as the difference between tasks involving pure mathematics, semi-reality and reality. I started to categorise the tasks I proposed to the students according to this scheme, and constructed learning scenarios where mathematics interplays with reality in a reflective way.

The second article, "The end of innocence" (Skovsmose & Vithal, 1997), written with Renuka Vithal, was a precious article, not just because it is a very sharp critique of ethnomathematics, but also because it provides a good description of the field and how ethnomathematics could be positioned in relation to critical mathematics education. By that time, I started also to be interested in ethnomathematics, under the influence of Mônica Mesquita, who was at the time developing an ethnomathematical study with children living in the streets of São Paulo. Both critical mathematics education and ethnomathematics brought new insights to the mainstream research I was doing —insights that tore at the sameness that characterise the field. However, every time I re-read "The end of innocence", I got the impression that Ole and Renuka were trying to delineate one field (CME) by neglecting another (E), when, in my opinion, they should 'dance' together. Moved by this nuisance, and together with Valéria Lima and Helena Gerardo, we wrote an article to be presented at XICIEAM in Blumenau (Pais, Gerardo & Lima, 2003), where we tried to defend the thesis that a critical mathematics education that does not pay due attention to the cultural dimension becomes poorer, and could lose its emancipatory purpose.

The presentation in Blumenau went well, and I had the opportunity to listen to Ole making some comments on our ideas. At night all the people from the group went to dinner at a typical German brewery. There was Ole. I remember looking at him and sensing a willingness on his part to talk with me. But I was gutless. It could have been the fact that at that time my English was no better than Ole's Portuguese; the fear of not having arguments to engage in a discussion; or simply because Ole remained an idealistic reference to my ideas in mathematics education. Like a guru that you want to read and listen but, when the time of confronting comes, you just feel that you are not up to take the challenge. Or probably it was the same feeling you have when you fall in love with someone but do not have the courage to take the first step. You just keep your admiration at a platonic level.

My stay in Blumenau was a turning point in my life for several reasons. One of them was the opportunity to listen for the first time to Ole giving a presentation where I realised that his work was bringing to Mathematics Education an insight that went beyond the classroom. The title of his presentation was "Ghettorising and globalisation: A challenge for mathematics education", and it ranged over ideas such as globalisation, questioning enlightenment and belief in progress, the 'wonders and horrors' that mathematical knowledge provides (as Ubiratan D'Ambrosio so eloquently put it), aporetic situations, social-political functions of mathematics education as something critical, social exclusion and others. This whetted my appetite for exploring more deeply Ole's work. So I started to read *Towards a Philosophy of Critical Mathematics Education* (Skovsmose, 1994).

Briefly, Skovsmose (1994) understands critical education as education that addresses the conflicts and crisis in society: "critical education must disclose inequalities and oppression of whatever kind" (p. 22). In such a task, there is a desire for emancipation, where a critical education must not simply contribute to prolonging existing social relations. Skovsmose substantiates his idea of emancipation from the work of Jürgen Habermas, who connected emancipation with a critique to the positivist way of researching in social sciences, and the need for social sciences to be founded on an interest in emancipation:

By so doing [falling into the trap of logical positivism], social sciences will be colonised by the technical-manipulative research paradigm, according to Habermas. It is not possible to find any platform of neutrality. Social sciences must be 'committed'. A pretended neutral registration of facts will result in an acceptance of the social status quo. (Skovsmose, 1994, p. 12)

But for me the strongest idea presented in the book was on the first pages —the idea that mathematics can be considered as a language. The relation established by Ole between the 'Newspeak' of Orwell's *Nineteen Eighty-four* (Orwell, 1950) and mathematics, provided me with some insightful ideas to develop my Master's thesis around the concepts of language and ideology. In respect to the way the control was exercised by the 'Thought Police' in Orwell's novel, Ole suggested that the control in our contemporary societies might appear to be more sophisticated and 'humane':

It can be carried out by ideological structures generating a grammar for what is possible to think and what is not. Then people preserve their freedom to

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think whatever they want but within limitations which are difficult to identify and therefore to recognize as being restrictive (Skovsmose, 1994, p. 16).

Could mathematics be one of those grammars? Could it be that mathematics keeps our imagination on a short leash? These were disturbing questions. Especially when you are a mathematics teacher actively engaged in the teaching of that language. Disturbed by these and others discomforts I started to inspect my relationship with mathematics, which is intrinsically connected with my relationship with school.

PRACTICE AS THEORY

Since the age of six, when I first went to school, schooling has been part of my life. I completed twelve years of non-university education and then went to the university for four years to study mathematics. With only one month in between, I started teaching in a public school, very similar to the one I first entered, seventeen years before. During the last nine years I have worked as a mathematics teacher in several public schools in Portugal.

My ideas about school, education and all the processes of learning and teaching have not always been the same. The first years as a student were not very happy ones. I felt for the first time the power of castration that schools carry. Being all day closed in a room, listening to an adult speaking, being obliged to talk and interact with children of my own age, feeling the pressure to succeed, starting to realise the competition involved in the school life, had been difficult for me. But it was the natural course of life: going to school because everyone does the same. The first years passed and, despite all the problems I faced, I managed to keep up with the school routine. I made friends, fell in love (several times a day), admired some teachers, learned some things. School progressively started to be less a prison and more a place where everybody of my age was. The discontentment felt in the first years gave way to a sense of inevitability, in the sense that other alternatives to school life became nonexistent. School was the right (and only) path to walk. And I walked.

By the age of fifteen I had the chance to enter a class of ambitious students. It was considered the first class in school, implicitly designated to ensure students entered good universities. From that time, I started to enjoy playing the school game. I was in school no longer just because everyone was there, but because that way I could be someone. School became meaningful to me, not as institution of 'knowledge', but as an institution of 'possibility'. School was no longer an endurance but an opportunity.

At that time mathematics was a game for me. It was not the subject I most liked, but it was the subject where I had best understood the strategy to pass (the result of three years of private teaching): doing lots of exercises, together with memorising and training procedures. It was easy for me to study mathematics: I had learnt the rules of the game. Besides that, I think I liked the aura of eccentricity that surrounds anyone who studies mathematics. Of course, at that time in Portugal it was very easy for a mathematics teacher to find a job, and earn lots of money for private lessons (the last part remains true).

While in university, I continued to play the game, although at a different level: things had become much more difficult, which required many more hours of isolated study: classes, thinking, training, examining. No research had been undertaken. After three years I ended my mathematical ‘training’, which consisted of studying hard and passing exams. I ended up being one of the best in my year, and when I started to feel what it could mean to do mathematical research, I began the fourth year adding the educational sciences. Can I say that I never experienced what it is to do research on mathematics? I think so. What I did was a training process. Trained for what? To teach others how to play the same game.

As I said before, during the years as a teacher, mathematics started to appear to me even more as a language. Not just a language that *formats* reality, as Skovsmose (1994) put it, but also as a language that formats people. Mathematics in school alters people’s conduct and lays down who is and who is not capable. Capable of what? Not of learning mathematics¹, but of continuing studies, going to a good university and finally finding a well paid job. I started to realise, helped by some marginal readings on Mathematics Education (e.g., Popkewitz, 2004) that the mathematical curriculum functions as a disciplinary device that through the transmission of rules, behaviours, and norms, shapes the kind of people our society needs. This was my Foucaultian *époque*. By reading Foucault, and his archaeological and genealogical analyses of formations like the human sciences and sexuality, and institutions like the prison, the asylum, and the hospital, I started to look at the school not as a necessary, or natural institution of humankind, but rather as a specific and contingent institution, with specific purposes that have less to do with the transmission of knowledge, and more with the governance of people.

I started to realise how education has become *disciplinarised*, *schoolarised*, *academicised*—schoolarisation has become the only legitimate modality of thinking in education. In school, education is a disciplinary device (Foucault, 2003), which fabricates the individual. Following Foucault’s thought, school has become one of the bigger modern disciplinary centres of the body. Obligatory in modern societies, in school we are introduced to a disciplinary society, via academically recognised knowledge, in the way we submit our bodies and minds to training devices (there exists in schools a huge load of corporal discipline, whether in the organisation of spaces, or in rules and norms about what is considered to be good and bad behaviours). It is in school that the human being, no longer a person but a student, starts to understand the hierarchy of behaviours and knowledge, by means of the creation of classificatory systems that limits, integrates and excludes them. School plays the role of an apparatus to govern the population by concocting the kinds of subjects that hegemonic society stipulates as normal, through the dissemination of norms that function as calibration devices. This fabrication of subjectivity, by means of mechanisms, techniques, norms, rules, discourses, positions us in order to become knowledgeable and possible of being administered. Universal and compulsory schooling catches up the lives of all young children into a pedagogic machine that operates not only to inculcate knowledge but also to instruct in conduct and to supervise, evaluate, and rectify pathologies.

These concerns were far from being a theoretical caprice of mine. I was gradually feeling contradictions about my role as a teacher, and the school's role in society. Although the Portuguese curriculum explicitly mentions the importance of working with student topics of mathematics and society (which opened me the opportunity to develop with students topics of Critical Mathematics Education), it is content orientated and the high-stakes tests are always present, putting pressure on teachers and students to be bound to specific mathematical content. I felt that more audacious enterprises (like working with students topics of mathematics and society in a critical and reflexive way) were suppressed by a school structure that has other purposes for education, visible in mechanisms like the high-stakes tests and selection role taken by school; the culture of individualism that students felt, and the appeal to be competitive; the central mathematical curriculum. The contradictions arose when I started to criticise, in an academic field, the same practices I developed as a teacher: I continued to propose lots of routine exercises, preparing kids for exam, reproving children, conveying the idea that mathematics is only for some, taking an authoritarian role as a teacher, and controlling students. I was feeling more and more like someone being used to perform somebody else's job.

THE SURPLUS-VALUE OF MATHEMATICS EDUCATION

During this period of questioning my missionary profession, I had the chance to participate in a seminar invited by João Filipe Matos to comment on a presentation of Ole, at Faculty of Sciences of University of Lisbon. I felt proud both for the invitation, and to have the chance to comment on Ole's work that had so much influenced my critique of school mathematics. At that time I had already read most of Ole's papers, and I felt comfortable engaging in some provocative questions. Basically the main doubt I entertained in Ole's work was also one of his (many) doubts. It has to do with the role of school and with the yelled aims for school mathematics. On one side we have one of the most important goals proclaimed in our day for mathematics education: the importance of educated people to be critical participants in a more and more *mathematised* world. This means to reflect on the presence of mathematics in society, by developing the competence of "mathemacy" (Skovsmose, 1998) as the capacity to deal with mathematical notions, to apply them in particular contexts and to reflect among those applications in order to allow the desired transformation of social oppressive structures in which mathematics is implicated. On the other side, we face the fact that school, by definition, has the implicit goal of maintaining dominant social structures in society. As mentioned by Ole "the fundamental principles that structure the curriculum derive or are in accordance with the dominant power relations in society" (Skovsmose, 2001, p. 5, my translation). How to solve this paradox that I, as a teacher, felt on a daily basis?

Of course I was not expecting Ole to have an answer, but I couldn't resist raising it. These questions led me to think broadly about the implications of a critical mathematics education. Not just as a way of developing mathemacy, but also as a way to specifically address the fact that mathematics has other social and political functions that are not restricted to the application of mathematical knowledge on reality.

The discourse implicit in most mathematics education research is that mathematics is one of the biggest achievements of mankind, considered to be the main pillar of our technological society, and an indispensable tool to become an active participant in a more and more *mathematised* world. Therefore learning mathematics is posited as a critical requisite to be a citizen. According to NCTM (2000), it gives students power to participate in the world. But how does mathematics gives power? Is it the power of knowing or the power resulting from succeeding in school mathematics? Is it through school mathematics that we learn mathematics or do we learn a set of processes for how to achieve success in school mathematics? Raising these questions led me to understand school in an Althusserian way (Althusser, 1994), that is, as a crucial ideological state apparatus in the reproduction of capitalism. This current mode of living tends to 'capitalise' all social and political dimensions of our daily life. It promotes the all-embracing view of society as a big market place, where everything (including education and research) can be produced and consumed in a capitalist economical logic. It is in school that the education for capitalism begins, school being above all a credit system.²

Ole is well aware of these vicissitudes that mould school and mathematics education. To change them is not an easy task, and if we want to work as researchers we must, in some way, close our eyes to them. But not without a sense of contradiction. And one of the contradictions I started to be interested in is the one related with the notion of empowerment through mathematics, or why it is important to learn mathematics. Using the theoretical framework developed by Skovsmose and Valero (2008) to deal with the issue of power in mathematics education, we can say that mathematics education can empower people through the intrinsic characteristics of mathematics itself (logical thinking, abstraction); by providing students with psychological meaningful experiences (solving problems, metacognition); by enhancing the relation between cultural background and foreground therefore allowing students to learn 'in context' (connection between every day practices and school mathematics; providing opportunities to envision a desirable range of future possibilities); and finally students can get empowered through school mathematics by exploring situations of 'mathematics in action' which makes visible the way mathematics formats reality (exploring real mathematical models in a critical way). However what is missing in these four perspectives of conceiving the way mathematics empowers people is the fundamental one — mathematics empower people not so much because it provides some kind of knowledge or competence to them, but because it gives people a value. It allows students to accumulate credit in the school system that will allow them to continue further studies and later to achieve a place in the sun. Mathematics empowers people because it is posited as a socially valuable resource. Therefore, I agree with Skovsmose and Valero (2008) that people need mathematics to become democratic citizens, to become included in the social mode of living. However I suggest that the reasons why they need it are not related with mathematical knowledge or competences, but with the school valorisation that mathematics gives to people. People need school mathematics not because they will use it directly in the democratic participation (as knowledge or competence), but to continue having

success in school, to take a university course and to find a stable job, so that they become workers and citizens —the two goals for mathematics education (Bishop & Forgasz, 2007). I argue that the importance of mathematics must be discussed not just in the field of knowledge but also in the field of value. This is just one of the contradictions I feel today, concerning mathematics education research, and which I am trying to develop in a research environment. Yes, because being a teacher is a full time profession, which leaves you no time to put questions of *Why* —the emphasis in on the *How*.³ And I needed time to think.

So I had the brilliant idea of doing research, in the form of a PhD. This way I killed two birds with one stone: I got out of school where I was becoming worn-out, and went to university to study the socio-political problems I experienced as a teacher. I escaped from the core of the problem, so no big change was to be expected, but at least I was having time to think in a relatively free environment —a PhD thesis.⁴

There is obviously a fissure between teacher practice and researcher practice. In some way that can be explained because of the fact that a teacher is, in fact, in school, all day, dealing with contradictions, mechanisms and discourses that compel him to exercise power, to change people's lives (mostly students' lives). He will, sooner or later, in a superficial or profound way have to deal with those contradictions. From my experience, most teachers find refuge in educational doctrines to avoid dealing with these contradictions, and so continue their work. But when you start to erase all the dogmas and certainties about education, you get lost. And that is an important step in becoming critical.

BACK TO CRITIQUE

Ole helped me criticise the field of mathematics education research as a field stem from psychology, where the main focus of research has been the classroom, with all the knowledge on how to better teach, learn and evaluate teachers and students in their school tasks. I realise that most of the mathematics education research takes school and mathematics for granted. The focus is not in the 'why' but in the 'how'. And huge amounts of research have been produced based on that unquestionable certainty. But above all Ole showed me that mathematics is not this innocent and inoffensive endeavour, as Hardy (1940) eloquently characterised it. Mathematics needs a critique, not just in its applications but also, Althusser would say, as an ideological state apparatus. And that is far from happening, even in critical mathematics education.

We know since Marx that our capitalist society in order to reproduce itself demands perpetual reforms by means of integrating what could be new and potentially emancipatory acts into well-established social structures. The word 'critique' has become a common place among educational research and curricular documents, being used as a signifier implicitly convey different ideologies about what it means to be critical. Today we can find notions of "critique" in a variety of contexts such as school curricula ("educate people to become critical citizens"⁵), in teacher education ("Tips for teaching critical thinking skills"⁶), professional education ("Education and Knowledge in Safety-Critical Software"⁷), online education

(“Role of critical thinking in online education”⁸), and so on. One consequence of this extensive use is the loss of meaning. That is, words begin to function as empty signifiers, representing no more than a way of symbolising some assumed shared meaning. Very often, the use of these words lacks a deeper concern for understanding what could be the ideologies filling the empty space conveyed by these words.

We can argue that the notion of critique has suffered from a kind of “domestication” in the field of education and in particular mathematics education. In Pais (2008) and Pais, Fernandes, Matos and Alves (2010) we developed a critique on how the notion of critique can be lost when inserted into social frames (like schools) that aim not for emancipation but reproduction. We suggest that more research should be done that explicitly addresses not so much the ‘benefits’ of bringing topics of critical mathematics education to school, but the problems and contradictions felt by the different actors in such implementation and to avoid engaging in salvation discourses which, by blindly misunderstanding the true problems facing mathematics education, only perpetuate existing realities.

Having in mind the characterisation of school as an institution for the reproduction of capitalism, I risk saying that this ‘domestication’ of the notion of critique is due to the fact that all emancipatory actions are thought and put into action within capitalism. If we recover the critique of political economy developed in the first years of the Frankfurt School, which informs a great part of Ole’s work (specially the late development made by Jürgen Habermas), we can say that emancipation from capitalism failed completely.⁹ No radical alternative was made. But this fact contrasts with the proliferation of the idea of critique, especially in education. We can read in the curricula all around the world the word critique, how it is important to allow students a critical education, to become critical citizens. It is in that sense that I argue that the word critique has become ‘domesticated’, it has lost its most radical meaning. It is a case of what Žižek (2005) calls “progressive amnesia” (p. 9): We recover critical theory but it is deprived from its true transformative core. It is fine to take a critical stance as long as you do not raise questions that imply a societal change that goes beyond mathematics education — we are allowed to be critical as long as we do not criticise the capitalist system itself.

CERTAINTY OF DOUBT

During my PhD, I spent a period of time at Aalborg University, under the supervision of Paola Valero. Here I met Ole again, who showed the same sympathy and willingness to talk, share ideas and above all to know what I was doing. We talked for a few hours on topics as disparate as the enjoyment he has of being with students in doctoral courses, to the displeasure of the already unbearable administrative work. He mentioned the sameness that characterises the Mathematics Education research, and told me something politically incorrect: he has been losing interest in reading literature in mathematics education. The reason was simple: papers become predictable. From the first lines we can already predict the development of the paper and its conclusions. Something like a Hollywood movie: it repeats the same story all

over again, although with different scenarios and characters. I couldn't agree more. Ubiratan D'Ambrosio, awarded with the ICMI Felix Klein Medal, in a regular lecture last ICME11 made the same claim: research on Mathematics Education lacks alternative ways of understanding the problems posed by mathematics and its education. Is there some kind of a 'sameness virus' that infects the majority of mathematics education research?

During those days in Aalborg we talked a few more times, and shared some common concerns. Ole shared with me what could be a possible explanation for the fact that, although mathematics education as a scientific field has grown immensely in recent years, students are not getting better at mathematics.¹⁰ Making some rough calculations, and based on his experience, he estimates that around 90% of the research on mathematics education is focussing on a 'prototype classroom' that represents only (if it represents any at all!) 10% of the world classrooms. Something similar to the distribution of wealth worldwide: 90% of the 'economic goods' belongs to 10% of the world population. Why should it be different in Mathematics Education? That makes me wonder if we really desire our desire for change.¹¹

As Skovsmose would say, doubts and uncertainties inhabit our world. Doubts and uncertainties of a guru who far from assuming a posture of a well established knower, takes uncertainty as his main certain. That was, to me, the main lesson: the more we know the more we doubt, and being critical is precisely not the resolution of these doubts and uncertainties, but the deconstruction of the referential, the habits, the practices and discourses (like the modern myth that mathematics is) that make things appear to us as certain and secure. To be in doubt is the first step to allow alternative ways of understanding and doing mathematics education. And doing this is vital if we do not want mathematics education to keep our imagination on a short leash.

ACKNOWLEDGEMENT

This paper is part of my PhD study, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007.

NOTES

¹ It is always useful to remember the research made by Baldino & Cabral (1998), where they show how students in school are primarily worried to pass (and not necessarily to learn).

² See for instance the work of Shlomo Vinner (2007), Dowling (1998), or Baldino & Cabral (1998).

³ Even in a social and political approach to mathematics education, many of the questions posed are on the how: How to teach in multicultural classrooms? How to teach for social justice? How to educate teachers for social justice? How to integrate immigrant students in the learning of mathematics? How the socio-cultural contexts of students influence the learning of the concepts of chance and probability? All these questions were retrieved from papers presented at the last *Mathematics Education and Society Conference*, which took place at Albufeira, Portugal, 2008.

⁴ Of course it all depends on your supervisor. In my case I will be forever in debt with Maria do Carmo Domite, Paola Valero and especially João Filipe Matos for the space they always gave me to explore my ideas.

⁵ Portuguese, Colombian, South African curricula.

⁶ <http://www.modernghana.com/news/203119/1/tips-for-teaching-critical-thinking-skills.html>

- ⁷ <http://ercim-news.ercim.org/content/view/446/699/>
- ⁸ http://www.masternewmedia.org/education/critical_thinking/educational_role_of_critical_thinking.htm
- ⁹ We should remember that the beginning of Critical Theory was based on a political critique (and only after the appearance of the critical on instrumental reason) having as focus the capitalist system. Despite major differences between members of the Frankfurt School in their assessment of the development of capitalism, it may be noted from the outset that their respective analysis were informed by Marxian tenets (Held, 1980). According to Benhabib (1994) the core feature of critical theory, as it emerged in the works of Horkheimer, Adorno, Marcuse, Lowenthal, Pollock and Benjamin, was the realisation that a revolutionary transformation of capitalism from within capitalism itself was doomed to fail. Critical theory was confronted with the enterprise of thinking a “radical alternative”.
- ¹⁰ In last ICME11, in México, during a plenary debate session with Paul Cobb, Mariolina Bussi, Teresa Rojano and Shiqi Li, entitled *What do we need to know? Does research in mathematics education address the concerns of practitioners and policy makers?*, Shiqi Li asked a very insightful (and provocative) question: Why does the students of the research always increase their capabilities (to solve problems, to learn with meaning, of communication, to use technology, etc.) but in reality, in the daily routine of worldwide classrooms, failure in mathematics persist, and students are far from reaching those desirable capabilities achieved in research settings?
- ¹¹ As argued by Baldino and Cabral (2006).

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Broadening the Role of Theory in Mathematics Education Research

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We intend to engage in a theoretical discussion on the limitations that the concept of learning imposes to mathematics education. After a brief analysis based on recent key publications dealing with the issue of theory in mathematics education research, we show how the concept of learning configures the theoretical framework of the field. This leads us to develop a critique that aims at highlighting the dimensions that are lost when we think of education exclusively as a process of learning. As a result of this critique, we argue for a broader theoretical approach to mathematics education that takes into account the social and political dimensions involved in that process. [1]

Introduction

The issue of theory in mathematics education research is currently on the agenda. At the 11th *International Congress on Mathematics Education* (ICME11) one of the survey teams was responsible for developing a study on the notion and role of theory in mathematics education research. This survey team had the task of identifying, surveying, and analysing different notions and roles of ‘theory’, as well as providing an account of the origin, nature, uses, and implications of specific theories pertaining to different types of research in mathematics education. The *Second handbook on mathematics teaching and learning* (Lester, 2007) contains two articles dealing with the issue of theory: “Putting philosophy to work: Coping with multiple theoretical perspectives” (Cobb, 2007), and “Theory in mathematics education scholarship” (Silver & Herbst, 2007). In the Congress of the European Research in Mathematics Education (CERME) there has been a working group dealing with the problem of linking, contrasting and comparing the wide variety of theoretical approaches found in the field to tackle the teaching and learning of mathematics. The second number of the fortieth volume of the international journal *ZDM* was published in 2008, collecting some of the results of the work of this group. Finally this year the theme of the 33rd *Conference of the International Group for the Psychology of Mathematics Education* (PME) is “In search for theories in Mathematics Education”. There seems to be a widespread desire for understanding the role of theory in mathematics education research:

The moment seems propitious for a serious examination of the role that theory plays and could play in the formulations of problems, in the design and methods employed, and in the interpretation of findings in education research. (Silver & Herbst, 2007, p. 41)

We wish to make a modest contribution to this discussion by engaging in a critical discussion on the way theory is used in mathematics education research. The educational sciences are generally construed around concerns of providing research that informs practices of learning and teaching in educational institutions. This research shows a propensity to emphasize the “technical” aspects of education, being primarily concerned with providing solutions to practical problems (Biesta, 2005). We will argue that mathematics education as a research field is not an exception. Thus we wish to understand in more detail how research perspectives in general and theoretical perspectives in particular construct and/or ignore particular discourses and, in this, our possibilities for addressing these basic yet powerful questions. In what follows we briefly present the groundings of our analysis. We will then address the texts mentioned above and show how they construct issues pertaining to theory in mathematics education research. Then we will refer to the problems that this type of construction leaves unattended. Drawing inspiration from the work of the philosopher of education Gert Biesta and on current socio-political research in mathematics education, we argue for an opening in the understandings of theory in the field.

Our theoretical and methodological standpoint

As the “linguistic turn” in the social sciences has touched mathematics education research (Lerman, 2000), it appears increasingly important to pay attention to the discourses that mathematics education research constructs about itself and the contributions and limitations of these constructions. By discourses here we understand the ways of naming and phrasing the ideas, values and norms that emerge from the constant and complex interactions among human beings while engaged in social practices. Academic fields construct particular discourses about themselves and their objects of study. Such discourses constitute systems of reason that regulate what is possible to think and do in a given field (Popkewitz, 2004). Discourses thus both generate a space for possibilities as well as limitations of what we can imagine and construct as alternatives to existing orders.

Mathematics education as a field of research is not an exception. As researchers engage in studying the field, they not only define what is characterizing legitimate practices of mathematics education. They also define the ways in which it is valid and legitimate to research those practices. We have elsewhere engaged in examinations of the discourses generated in and by the field of mathematics education research, such as the idea of mathematics

education being “powerful” (Christensen, Stentoft, & Valero, 2008), the conceptions of students as mathematics learners (Valero, 2004), the concept of learners’ identity in mathematics (Stentoft & Valero, in press b) and the concept and view of ethnomathematics (Domite & Pais, 2009). In this paper we examine the construction of “theory” and its implications for mathematics education research. In our investigation we point to some of the blind spots of theoretical construction and try to broaden the theoretical horizon as to embrace aspects of mathematics education which we are constantly witnessing as fundamental parts of the everyday practices of teachers, parents, students and school leaders in real schools. In addressing these blind spots we intend to enlarge the research gaze of mathematics education to embracing the “noises” that are often ignored in mathematics education research, in a search for new possibilities for our field of study and for the educational practices in mathematics (Stentoft & Valero, in press a). This we see as the role of a critical, socio-political approach to research in mathematics education.

We are aware that the “social turn” (Lerman, 2000) in mathematics education brought to the field new concerns and new theories that progressively de-emphasise cognitive psychology as the only interpretative framework and instead favour socio-cultural theories. In this we have witnessed a move from an understanding of children’s learning focused on the individual subject and his/her cognition to an understanding that perceives learning as a product of social activity, where not only the cognition of the subject is at stake but also his/her relations with other individuals and their shared discourses. Some of the research bearing social, cultural and political connotations, has opened up the field of mathematics education by conceiving theory as more than “theory for learning”, and posing questions that do not imply a “technical” response or solution but rather an intellectual and philosophical reflection.

However, despite this invigorating openness, we argue that a significant part of research in mathematics education labelled socio-cultural-political research shows a tendency to understand mathematics education in a didactical sense and to aim primarily to address questions of how: How to teach in multicultural classrooms? How to teach for social justice? How to educate teachers for social justice? How to integrate immigrant students in the learning of mathematics? How the socio-cultural contexts of students influence the learning of the concepts of chance and probability? These questions were found in the proceedings of the Mathematics Education and Society, MES conference in Albufeira, Portugal in 2008 (Matos, Valero, & Yasukawa, 2008), and shows how even in a research environment where the emphasis is on the political, the research persists on the question addressing the technicalities of the field. In Pais, Stentoft and Valero (2010) we developed an analysis that shows how even in a research environment

characterized by a social, cultural and political approach, a concern with the technicalities of education, namely with “learning”, persists.

In this paper we will rather focus on the mainstream research in mathematics education. Therefore we selected for our analysis some recent and mainstream publications in mathematics education research addressing the issue of theory, the collection of contributions emerging from the ICME11 survey team and the *Second handbook on mathematics teaching and learning*. We will show that the central idea in these texts is that the understanding of theory in mathematics education is mainly reduced to that of learning theory. This trend is not exclusive to the field of mathematics education research, but has over the last two decades also proliferated in broader discourses of education. The language of education has largely been replaced by a technical language of learning (Biesta, 2005). The contradictions on the role of school and the goals of education that fuelled part of the educational debate during the last century seem to have been surpassed. We seem to have reached a consensus on the benefits of schooling, we need to make it more effective and, therefore, we live an apparent consensus in what concerns education. The problems with schooling and school subjects are not anymore to be political or ideological, but have become primarily technical or didactical. In most cases, solutions to educational problems are being reduced to better methods and techniques to teach and learn, to improve the use of technology, to assess students’ performance, etc. Education has progressively been reduced to be a controllable, designable, engineerable and operational framework for the individual’s cognitive change. Although the prevalence of theory as “learning theory” has allowed us to gain deeper knowledge on the processes of teaching and learning mathematics, we suggest that it has left important problems faced by the educational communities in their everyday practices unaddressed. We will argue that in order to bring these problems seriously into the gaze of research, we need a broader theoretical frame which allows us to understand theory not just as “theory of learning”, but also as “theory of education”.

Mathematics education as a learning science

There seems to be a consensus that the main concern of mathematics education research is to improve students’ performance in mathematics. Niss (2007) is very clear when answering the question of why do we do research in mathematics education:

We do research on the teaching and learning of mathematics because there are far too many students of mathematics, from kindergarten to university, who gets much less out of their mathematical education than would be desirable for them and for society. (Niss, 2007, p. 1293)

If this is the main concern of mathematics education research, it is not surprising that the field has been designed as a space for researching “the

problems of practice” (Silver & Herbst, p. 45), defined as problems relating to teaching and learning, in a systematic, scientific way. According to Boero (in press) “this is a rather obvious widely shared position” (p. 1). In this framework, the work of mathematics educators is:

To identify important teaching and learning problems, considerer different existing theories and try to understand the potential and limitations of the tools provided by these theories. (Boero, in press, p. 1)

Cobb (2007) addresses the issue of philosophy in mathematics education. The author makes it clear that although the invitation to write the article was to focus on the philosophical issues of mathematics education, he decided to engage on a personal perspective rather than developing an exhaustive overview of currently philosophical ideas in the field. However, the fact that it is the article dealing with philosophy in one of the most significant publications in mathematics education is, in our opinion, symptomatic on how the majority of researchers understand the role of philosophy in relation to mathematics education. Cobb suggests that mathematics education should be understood as a “design science” (2007, p. 7), and provides as an example the NCTM standards. By design science Cobb understands “the collective mission which involves developing, testing, and revising conjectured designs for supporting envisioned learning process” (p. 7). The ultimate goal of a science designed this way will be to “support the improvement of students’ mathematical learning” (p. 8). Under the pragmatic realism philosophy adopted by Cobb a substantial part of his article focuses on the comparison between four significant theoretical perspectives used in mathematics education research (experimental psychology, cognitive psychology, socio-cultural theory and distributed cognition). The discussion revolves around how these theoretical perspectives could help improving students’ learning of mathematics. We can research at the level of the national educational system, school or classroom, but the ultimate goal remains the same. Theory is understood as a tool to give insight and understanding into learning processes.

An alluring analogy made by Silver and Herbst (2007) between mathematics education and medicine helps us to better understand the meaning of theory as “theory for learning”. The authors place mathematics education as a science of treatment, and by understanding the symptoms that characterizes the difficulties of students’ learning of mathematics we can propose the proper treatment:

The evolving understanding of the *logic of errors* has helped support the design of better instructional treatments, in much the same way that the evolving understanding of the *logic of diseases* has helped the design of better medical treatments. (Silver & Herbst, 2007, p. 63)

In this perspective, students are seen as patients in need of treatment, and the role of mathematics education is to understand students' problems and elaborate designs that threat those learning diseases.

Problems left behind

After describing the decadent state and lack of minimal conditions (broken windows, holes in the roof, full exposure to climate change) in many South African classrooms serving mainly black African students, Skovsmose (2004) asks a very innocent question: "What seems to be the most obvious learning obstacle to the children in this school: their colour of skin, their dominant father, or the hole in the roof?" (p. 35). According to Skovsmose, the hole in the roof, absent from the majority of mathematics education research, calls any deficiency theory of the child into question. Which kind of treatment "for learning" should mathematics education and mathematics education research deploy in this case?

Schmitz (2006) analyzed the way six mathematics teachers developed and interpreted their practice in a typical Brazilian school. They represent what we can consider traditional, unqualified teachers. They use the textbook as the main resource in their classes; confine the teaching to the specific content of the official curriculum; do not incorporate students' culture in the learning process (at best they mention anecdotal examples with the purpose of illustrating the true knowledge of the curriculum); and perform the traditional way of teaching mathematics. Students are on their seats listening, and the teacher at the black-board speaking. They go through lots of exercises. The teachers do not invest in their classes. In an interview one of the teachers drops the veil: she would like to use the textbook less, and build her own materials; but unfortunately she has to work in two different schools in order to have two salaries to pay all her needs. Working more than twelve hours a day to earn less than three thousands *reais* [2] per month does not leave time or energy to prepare her classes. How can the research in mathematics education address the problems of these teachers? [3]

These are only two examples of the problems that are predominately unaddressed by research in mathematics education. A reason for this lack of attention to these problems could be found in the language theories offer to the research field, namely a language concentrating on issues of learning and lacking possibilities to properly formulate questions to addressing socio-political aspects of mathematics education. They are what we call, inspired by the work of Valero (2004), socio-political problems emerging by the way in which the social practices of mathematics education are part of the larger social, political, economic, cultural and historical frameworks for education, schools and classrooms. To understand these problems we need to start questioning not just how students learn mathematics, but also why do students learn mathematics.

And we need to construct a language through theory that allows us to move beyond research into processes of teaching and learning mathematics.

So, why do students learn mathematics? Vinner (2007) argues that students have very good reasons to study mathematics. However, these reasons are not related to the common aims associated with mathematics education (utility, professional, or to educate critical citizens in an increasingly *mathematized* world). The good reasons students have to learn mathematics are rooted in “the selection role that mathematics has in all stages of our educational system” (p. 3). Thus, the predominant reason for students to learn mathematics is to become winners in a world where mathematics is a gatekeeper in accessing further studies and well paid jobs. Indeed, they need mathematics to participate in this world, not directly with the knowledge of school mathematics, but in the form of a diploma, as a schooling valorisation, first, and then professional. They need to learn mathematics to become economically relevant and revealed. This seems to be the case when students of different socio-economic backgrounds are asked about their reasons to engage in the learning of mathematics (i.e., Skovsmose et al., 2008a, 2008b).

The past twenty years have seen a remarkable rise of the issue of measurement and testing in education. The mass scale comparative studies as the *Trends in International Mathematics and Science Study* (TIMSS) and the OECD’s *Programme for International Student Assessment* (PISA) represent the most prominent manifestation of this phenomenon. These international, comparative, measurement studies are to an increasing extent brought into the political sphere placing pressure on national governments to regulate their educational systems according to the standards stipulated by those tests (Biesta, 2009; Wilson, 2007). This is what has been happening in the last eight years in very many developed countries where education tends to be transformed, by the pressure of politicians’ demands for accountability, into an evidence-based profession. Consequently, political measures contribute to formatting teaching and learning of mathematics in a clear and crude way. Teachers tend to tailor their instructional practices to the format of the test out of concern that if they design their teaching differently, their students will fail. Although they might know all the didactical novels tricks and methods to promote learning in a way meaningful to the students, if what counts is to pass the test, that is how they will ‘educate’ their students (Wilson, 2007; Lerman, 1998).

The idea that we should take seriously the political and social context in which teachers work and students learn has been growing in the last years. Covalieskie (1993) argued that the institutional arrangements, in ways no one quite seems able to pin down, make even the most able and intellectually capable of teachers to tone down their teaching to the level of the approved curriculum materials. De Freitas (2004), drawing on the work of Michael Apple, Basil

Bernstein and Thomas Popkewitz, made it clear that any attempt to reform teacher education programs will dash against the “insidious structural elements operating normatively in maintaining ‘common sense’ practices within schools” (p. 259). The question remains whether it pertains to mathematics education research to address these issues, and how to construct a theoretical language that goes beyond theories of learning, which offers a scope for this kind of research.

Broadening the horizons of theory: From learning back to education

As we argued at the beginning of this article, the ultimate goal for mathematics education appears to be improving students’ mathematical learning. The idea of mathematics education as a therapy or a design science described previously constructs education as a technological endeavour, where mathematics education is understood as a technical engineering of students’ mathematical thinking and learning. We acknowledge the contributions that this learnification has brought to our understanding of what happens in a mathematical classroom at a micro-scale. Nevertheless, we argue that reducing the possible meaning of “mathematics education” to “mathematical learning” can narrow our perspectives. And thus it becomes impossible to think and act in ways that could open spaces of possibilities inside and outside mathematics education research. Cobb (2007) is well aware of this. When referring to the theory that informs the researcher he mentions that “the constraints on what is thinkable and possible are typically invisible” (2007, p. 7). This awareness also emerges strongly in much research and it is clear that addressing mathematics education from a narrow perspective pointed out here, reconfirms the fact that “if we look strictly at events as they occur in the classroom, without consideration of the complex forces that helped to shape those learning conditions, our understanding is only partial [and] the solutions to the problem [are] ineffectual” (Rousseau & Tate, 2008). Very few researchers, however, have addressed these limitations in serious ways. In the remaining lines we wish to sketch a modest contribution to understand theory differently in mathematics education, by focusing on the field of teacher education.

Part of the problems felt by teachers in their profession are not only problems related to finding better ways to improve students’ mathematical learning from a micro-didactical perspective. They are, as mentioned above, sociopolitical problems that constitute part of the embeddedness of teachers’ practices in a socio-political context. These problems are not present in the “cognitive subject” (Valero, 2004) or in the “prototypical classrooms” (Skovsmose, 2005) that dominate discourses of mathematics education research. These problems are obliterated, and made invisible. They are excluded through the application of ‘orderly’ research methods and theories that need “sanitized” environments (Vithal & Valero, 2003). But those are problems that real teachers

face in their real everyday activity. The problems can sometimes make their teaching skills seem obsolete. We therefore appeal for a teacher education that makes these problems visible to the teacher so he can act and react appropriately, critically and responsibly. As Vinner (2007) pointed out, we as researchers and educators responsible for teacher education have to tell our teacher-students the truth about the system in which they operate. This implies confronting the future teachers with more than desirable and visionary goals of mathematics thinking and learning, by making the real importance and the role of mathematics in school explicit. School cannot be taken for granted. It needs to be critically questioned as an institution capable of promoting new opportunities as well as stratifying children in social hierarchies by stipulating who is capable, who will have access to the best university courses, and who will enjoy well paid jobs. This implies going beyond the idea that mathematics teacher education consists only on giving teachers the didactical tools necessary to enhance students' mathematical learning. Teachers need to know about the social and political context in which this enhancement occurs, to understand which their contribution in the school is and to their students and which are the limits of their ability.

In order to address these problems, teachers and researchers need alternative discourses. If the main discourse in mathematics education is a didactical one, it is not possible to formulate these problems adequately. The discourse of mathematics education from a critical and socio-political perspective can offer an alternative. Any discourse will of course have blind spots. So our point is not to simply replace a discourse of learning for a discourse of education, but rather to make them both available as competing and complementing discourses available to researchers and practitioners to engage and explore the otherwise limited possibilities that the dominance of only one may offer.

Endnotes

1. This paper is part of Alexandre Pais's PhD study, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007.
2. Brazilian currency.
3. The research developed by Schmitz, contrary to what it may seem, doesn't take these aspects into consideration. They are treated as marginal political vicissitudes of teachers' life.

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THE SPECIFICITY OF MATHEMATICS LEARNING AND THE DISAVOWAL OF THE POLITICAL IN RESEARCH

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Abstract

Mathematics education as a field of academic research has been historically concerned with the improvement of the process of teaching and learning mathematics in schools. Despite the diversity of theories used and developed within the field (from cognitivism and constructivism to more recent sociocultural approaches), a focus on ‘learning’ and ‘mathematics’ permeates the way researchers apply and develop different theorizations in mathematics education. In recent years, however, a discontentment with the way research disavows the social and political dimension of ‘learning’ has led the community to search for theories that provide a different conceptualization for the problems of the field. In this article we take advantage of contemporary philosophy to develop an analysis of mathematics education as a field of research. We shall argue that many of the problems related with school mathematics cannot be fully conceptualized without some kind of ‘political mapping’ that situates them not at the level of ‘learning’ but at the political and economical one. Our suggestion is that positing schools as crucial modern institutions for the reproduction of capitalist economy and ideology can bring us closer to the complex situations teachers and students experience in schools.

Key words: Political economy of mathematics education, specificity of learning mathematics, mathematics education research, capitalism.

1. Introduction

The will to provide a quality mathematics education to all people plays a central role in the formation of mathematics education as a scientific field of research. At the beginning of the 1900s, when the *Comission International de l’Enseignement Mathematique* (CIEM or ICMI) was established, it was clear how the importance of teaching mathematics was conceived of as a social problem. In his review of the extensive work published to commemorate the one hundred years of *L’Enseignement Mathématique* (Coray, Furinghetti, Gispert, Hodgson, Schubring, 2003), Radford (2004) points that mathematics could no longer be seen as a luxury for elites who managed to climb the highest peaks of education. In pace with the emerging concept of Humanity—men as a coherent rational subject, his own source of meaning, knowledge and action, where civilization was equated

to scientific and technological process—mathematics became a social need, in which the success of the civilized world depended on the inclusion of informed and participative masses. School mathematics in modern, massive educational systems started providing an important element in constituting the rational, cosmopolitan minds of 20th century citizens (Popkewitz, 2009).

In order to satisfy such societal demand, fields of expertise such as mathematics education are called to investigate new ways of assuring mathematical success to all students. In the last three decades, mathematics education research has been growing exponentially, with an array of different highly specialized research topics covering mathematical learning processes, initiatives for change, detailed and in depth studies, comprehensive and interactive studies and data, new teaching strategies, broad studies on the use of technology, and, recently, growing numbers of studies addressing the social and cultural aspects of school mathematics. In spite of all these research efforts, achieving the desirable equitable and quality mathematics education is far from becoming a reality (e.g., Atweh, Graven, Secada & Valero, 2011). Baldino and Cabral (2006) raise the question: “Why do so many people insist in asking for more that which cannot be said to have produced results for change so far?” (p. 21).

We observe that the great majority of mathematics education research, by being primarily focused on pragmatic approaches to the improvement of classroom practice, lacks a theoretical comprehension of how the problems of the field are related with broader social and political structures. Without such strong social and political theorisation, researchers risk moving blindly because we do not “take certain distance and develop consistent research theoretical frameworks to appreciate our practices” (Baldino & Cabral, 2006, p. 31). This concern resonates with calls for developing a more reflexive research: “[i]t appears to be one of the weaknesses of our profession that many of us, myself included, tend to write and speak too much and read and contemplate too little” (Niss, 2007, p. 1311). Despite these appeals, mathematics education researchers seem not to be appreciative of research that is not immediately concerned with action in the sense of providing solutions or strategies for improving the teaching and learning of mathematics. We argue that without a theoretical conceptualization of the role that mathematics education plays within the Political, we run the risk of leaving unaddressed crucial problems standing in the way of an equitable and quality mathematics education. For such openness to happen, discourses defending the specificity of ‘mathematics’ and ‘learning’ as defining characteristics of the field need to be questioned.

In this paper we start by making evident how mathematics education research has been primarily focused on developing optimal learning scenarios for school mathematics, being the focus on learning and the specificity of mathematics invoked as the characteristics which distinguishes it from other fields of research. Such a way of structuring the field will be analysed referring to what the philosopher of education Gert Biesta calls *learnification*, and through Foucault's concept of *biopolitics*. Afterwards, we take advantage of the recent revival of Marxist theory carried out in contemporary theory by authors such as Slavoj Žižek and Frederic Jameson, in order to read schools as institutions responsible for the reproduction of capitalist ideology and economics. We then explore, from this theoretical perspective, a recent socio-cultural theory for the learning of mathematics. We conclude with a call for opening up the view of which types of theories and of which views on the mathematical specificity can be fruitful for mathematics education research if we would like to grasp the constitution of mathematics education practices in the Political.

2. The specificity of mathematics and learning as disavowal mechanisms

The political dimension of mathematics education has been a growing concern of teachers as well as of researchers. Different trends such as critical mathematics education (e.g., Skovsmose, 1994), ethnomathematics (e. g. Powell & Frankenstein, 1997), discursive approach (e.g., Morgan, 2009), multicultural education (e.g., Planas & Civil, 2002), and post-structuralist studies (e.g., de Freitas, 2004) have presented both inspiration for teaching practices, and an analysis of the way power is implicated in the constitution of the mathematics curriculum and all its associated practices.

In our own work we have built on the literature available in these different trends in order to study the discourses of mathematics education research concerning 'power' and the 'political' (e.g., Valero, 2008; Pais & Valero, 2011). Our fascination with research has to do with the fact that we conceive of research as an activity that, by examining mathematics education practices in classrooms and schools, generates particular systems of reason (Foucault, 2004) that format what is possible to think about practice. In this sense, mathematics education research is not an innocent activity producing a diagnosis of the state of mathematics education practices or proposing evidence-based solutions to the problems of practice; rather, it is an active participant in shaping, discursively, the possibilities of seeing and inventing practice. Research produces a language, which words and structures format what we as researchers end up saying about the very same world of mathematics education. As Brown (2008b, p. 249) argues, the theoretical and analytic lenses we deploy in our research "comprise particular choices in terms of the analytic filters that we apply,

governed by underlying ideological motivations and trends of which we are not always aware”. For us, a study of the political dimensions of mathematics education, or said in better words, the study of the political constitution of mathematics education has to necessarily cover research and its discourses, and the way in which such discourses contribute to the formation of particular subjectivities in and through mathematics education.

Despite the awareness that school mathematics performs a strong political role, there is a lack of studies that seek to understand school mathematics as part of dominant political and economic systems such as capitalism: “little has been written in mathematics education that addresses how mathematics might play a role in broader politics” (Gutierrez, 2007, p. 38). Neglecting the political dimension, we argue, is connected with two main features of the way mathematics education research as a field of study has been constituted during the 20th century. One has to do with the interpretation of the word ‘education’ in terms of ‘learning’. The other has to do with the specificity of mathematics as defining the borders of what counts as legitimate research in the field.

2.1. *Theory as ‘learning’*

What is theory and what are its roles in mathematics education research is currently on the agenda of the research community. In recent years a number of groups have worked on this issue and have produced a considerable number of papers¹. An examination of this production has led us to argue elsewhere (Pais, Stentoft & Valero, 2010, forthcoming) that ‘theory’ is taken to refer to ‘learning theory’. Whether adopting sociological or anthropological perspectives, socio-constructivist or radical constructivist perspectives, cognitive science perspectives, all these theories are deployed to study people’s thinking and engagement with mathematical topics and ways of reasoning. The theories help constituting the field as a ‘domain of learning’ (Valero, 2010). The role of theory is understood as providing ‘tools for action’, where action is normally the practice of school mathematics. This approach to theory lacks a deeper theoretical understanding of the concepts involved, by reducing research to a matter of providing the solutions for the problems of practice.

This tendency towards the *learnification* of education (Biesta, 2005), that is, the reduction of the study of educational phenomena to the study of administrable, engineerable learning processes

¹ See, for example, Cobb (2007) and Silver and Herbst (2007) in the *Second handbook on mathematics teaching and learning* (Lester, 2007); a survey team at ICME 11; the working group “Different theoretical perspectives and approaches in research in mathematics education” at CERME and their special number in *ZDM* in 2008; and even the theme of the 33rd PME.

contribute to erase political considerations from educational research. This is part of a larger societal trend where fundamental social problems are addressed as if they were the object of expert management and administration (Foucault, 1991, 1997; Agamben, 1998). Foucault (1991, 1997) shows us that the government of life is achieved through two fundamental *technologies* that act upon the individual and the population. On the one hand, the *technologies of the self* refer to the processes of subjectification that force individuals to bind themselves to their own identity, defined by the degree of adherence to social norms. On the other hand, the *political techniques* or *bio-power* refer to the way the State assumes and integrates the care of natural life of individuals into its very centre.

As an example of the first, we can think about the mathematical curriculum as a technology of the self. Popkewitz (2004) evidences the mechanisms through which the alchemy of school mathematics constructs a set of learning standards that are closely related to the administration of children rather than with an agenda of mathematical knowledge. This alchemy is carried out by pedagogy. Based on psychology and social psychology, pedagogy generates knowledge about children and how they can effectively appropriate the mathematical content to acquire competences, behaviours and attitudes (e.g., being participative, competent, having self-esteem). In this perspective, school mathematics serves as an alibi to the appropriation of behaviours and modes of thinking and acting that make each child governable. Mathematics education research provides the precise labels and techniques to effectuate the *governmentalization* of children through school mathematics. The concern of researchers for improving mathematical learning is the fuel for the effective instalment of technologies of the self.

As an example of bio-power, we can mention all the recent emphasis on measuring and evidence based research that reduce fully human beings to numbers representing mathematical performances. The mass scale comparative studies as the Trends in International Mathematics and Science Study (TIMSS) and the OECD Program for International Student Assessment (PISA) represent the most prominent manifestation of this phenomenon. These international, comparative, measurement studies are to an increasing extent brought into the political sphere placing pressure on national governments to regulate their educational systems according to the standards stipulated by those tests (Biesta, 2009; Wilson, 2007). This is what has been happening in the last eight years in very many developed countries where education tends to be transformed, by the pressure of politicians' demands for accountability, into an evidence-based profession. Consequently, political measures

contribute to formatting teaching and learning of mathematics in a clear and crude way. Teachers tend to tailor their instructional practices to the format of the test, out of the concern that if they design their teaching differently their students will fail. Although they might know all the didactical novelties and methods to promote learning in a meaningful way to the students, they will ‘educate’ their students in the ways the system considers to be legitimate (Wilson, 2007; Lerman, 1998). Research supporting the construction of these systems and their implementation by teachers in classrooms are highly implicated in setting these mechanisms in operation.

The interplay between the two mechanisms of subjectification—techniques of subjective individualization and procedures of objective totalization—creates a twofold political strategy which Foucault (1997) calls *bio-politics*: the growing inclusion of humans’ natural life (as opposed to their political life) within the mechanisms and calculations of power. In this way politics is made operational. Its purpose is no longer to be a place where alternative emancipatory ways of living together can be thinkable, but to engage in the global regulation for the sake of the species’ biological reproduction. For Agamben (1998), who amplified the work of Foucault, the only real question to be decided is which form of organization would be the most suitable for the task of securing the care, control and use of *bare life*: human life stripped from its entire political dimension, and reduced to its biological entity. Human bare life is that type of existence that can be measured, calculated and predicted. In other words, it is the object and result of technical expertise. Recognizing this condition, Žižek (2006) argues that today we live in a *post-political* society: politics have surrendered to specialized social administration, targeting the bare life of the individual by controlling its fluctuations according to global standards of normality.

Just as politics is being replaced by administration, education has given up its place in favour of learning and specialized pedagogy and didactics. In the case of mathematics education research, the discursive construction of students as cognitive subjects and “schizomathematicslearners” (Valero, 2004) is a good example of the way mathematics education research reduces full political and historical human beings to ‘bare learners’, whose cognition can be scrutinized with the interest of devising appropriate and effective techniques for learning mathematics. All the complexity of the social and political life of the student is wiped out from the research focus. The student is reduced to a biological entity, likely to be investigated in a clinical way. With this move, research effectuates bio-power. For example, some researchers find it useful to draw an analogy between mathematics education research and medicine. Mathematics education research is formulated as a

science of treatment that, by understanding the symptoms that characterize students' learning difficulties in mathematics, aims at designing and applying proper treatments, with the hope of curing what is a defect in students' learning: "The evolving understanding of the logic of errors has helped support the design of better instructional treatments, in much the same way that the evolving understanding of the logic of diseases has helped the design of better medical treatments" (Silver & Herbst, 2007, p. 63).

To some extent, philosophy offers an alternative to this tendency to 'technicize' educational problems. By taking a step back, by resisting the temptation to engage immediately in some form of action, we suggest that sometimes the best way to act is to stop acting, in the sense of doing more of the same research which proved not to have the solutions for the core problems the community faces.

2.2. *The mathematical specificity*

The adoption of learning theories is also connected with the emphasis on the mathematical specificity of the field. Mathematics education research is defined as the discipline studying "the practice of mathematics teaching and learning at all levels in (and outside) the educational system in which it is embedded" (Sierpiska & Kilpatrick, 1998, p. 29). In this field, "[...] mathematics and its specificities are inherent in the research questions from the outset. One is looking at mathematics learning and one cannot ask these questions outside of mathematics." (p. 26). Mathematics is usually posited as the "thing" that differentiates mathematics education research from other academic fields. Questions, problems, theories and methods not allowing for mathematical specificity tend to be considered irrelevant, and out of the scope of mathematics education research.

There are two important aspects of this issue. Firstly, mathematics education, as an educational practice is not and can never be specific because it is immersed in the political arena of schooling. Although it seems clear that learning mathematics is different from, for instance, learning music, there are important common educational problems that outweigh the specific problems of any school subject. If the community recognizes that, when dealing with the process of teaching and learning mathematics in schools, there are social and political aspects that influence it, how to address such dimensions? How to develop research which seriously takes them into consideration (instead of a brief mention usually at the beginning or end of articles)? It is difficult to sustain that we can analyse them with the theoretical tools that have been used in mathematics education

research to address the teaching and learning of mathematics. When problems appear that cannot be explained within the theoretical straitjacket of learning—such as, for instance, the institutional framing of schooling which hampers many of the insights for action coming from research, or the exclusionary role of school mathematics within the school credit system—they are, as noted by Gates & Zevenbergen (2009, p. 162), discarded since it is not the responsibility of mathematics education to address such “political” issues. For example, Abreu, Bishop and Presmeg (2002, p. 4), state that changing school mathematics practices “much depends of course on changing the formal educational structures that determine and shape the particular mathematics education practice experienced by the students in their schools”. However, they promptly add that such a task “is beyond the scope of this book”. So the situation is this: although we acknowledge that the problem has an economical and political nature, going way beyond the classroom, the specificity of mathematics is not compromised: we end up investigating as if it could be solved through better mathematics education.

Secondly, the mathematical specificity of the field does not reside in the particular mathematical content, notions or competencies being directly addressed in a research. Rather, its specificity is associated to the privileged role that school mathematics has played in the constitution of particular subjectivities during the 20th century, in a dominant cultural, social and political order. Without an understanding of order, classification, universalism, and objectivism incarnated in the organization of the mathematical rationality in the school mathematics curriculum, the cosmopolitan ‘homeless mind’ necessary for the advancement of the Modern project would not have been possible (Popkewitz, 2009). That we study mathematics-related practices and their relation to the meaning of mathematics education has therefore a social and political significance, even if there is no apparent mathematical content involved (Valero, 2010).

In this way, we suggest that the ‘specificity of mathematics’ has been functioning as the “fantasy-scenario” (Žižek, 2006) that allows keeping a safe distance from confronting the socio-economical roots of the worldwide problem of failure in mathematics. That is, that which allows us to disavow the role school mathematics plays in the Political. It is our contention that the internalism of the field around the learning and the specificity of mathematics make the research produced virtually innocuous if the purpose is to change the current state of affairs of school mathematics.

3. Positing mathematics education within the Political

Important contemporary philosophers, of which Slavoj Žižek is the most boisterous example, have in recent years been developing theory which articulates insights from Lacanian psychoanalysis with a Marxist critique of political economy.² In this article we take the standpoint that capitalism, both as ideology and as economy, has become the “concrete universal” (Žižek, 2004) of our historical époque, that is, “while it remains a particular formation, it *overdetermines* all alternative formations, as well as all noneconomic strata of social life” (p. 3). Thus, every position in education is also at the same time an implicitly or explicitly political stance on the nature of what Jameson (1991) calls “late capitalism”. However, the strength of capitalism, as Žižek (2006) has been exploring through a revitalization of the notion of *ideology*, resides precisely in the way it presents itself as a non-ideological economical system:

Although it [capitalism] is global, encompassing all worlds, it sustains *strict sensu* “worldless” ideological constellation, depriving the great majority of people of any meaningful “cognitive mapping”. The universality of capitalism resides in the fact that capitalism is not a name for a “civilization”, for a specific cultural-symbolic world, but the name for a neutral economic-symbolic machine which operates with Asian values as well as with others. (p. 318)

In today’s society capitalism has become the disavowed all-inclusive economical system that ends up subsuming the cultural, social and political spaces of our lives. It has become common sense to argue against the increasing capitalization of the academy, technology, education, media, leisure and, of course, politics, where the logic of the market dictates the, many times invisible, rules to act and be acted in these social spaces. Although we live in a world of multiple social, cultural and political realities, we must ask what, in all these different sets, remains unchangeable? In cultural and social terms, there is no doubt that the world is diverse. European culture and sociability, despite all the similitude, is different from North American culture, as we can easily notice in literature, cinema or philosophy. Not to mention the perhaps even deeper differences between these and Asian and African cultures. Also in political terms, we have around the world all sorts of political organizations: neo-liberal American ideologies, European social-democracy, China’s (capitalized) communism, Arabic religious states, etc. However, when talking about economy, we get stuck. Can we say that we have a plurality of different economical systems around the world? No, *the* global economical system present in all this multiplicity of cultural and social formations is capitalism. Even communism, whether we are talking about state communism as in China or the

² Frederic Jameson, Chantal Mouffe, Ernesto Laclau and Alan Badiou are also, each one in his or her own way, good examples.

communist parties that subsist within capitalist democracies, follows the rules of capital. Capitalism dresses in diverse clothes in order to keep reproducing, and no matter how different the “philosophies” of political organization could be around the world (monarchy, socialism, religious fundamentalism, dictatorship, neo-liberalism, etc.) what is common in all of them is that, despite the apparently different “clothes”, the human relations are based on capital. By explicitly mentioning capitalism we want to point to the very core of the problem—this (so often) unaddressed reality that permeates all social relations and for which we seem to have no alternatives.

It is our contention that, if capitalism is the ‘concrete universal’ of our times, colouring our ‘acting in the world’, mathematics education cannot continue without questioning itself, as a field of research, what role it is playing in the reproduction of capitalist economy and ideology. How then shall we address such capitalist overdetermination within mathematics education?

The usual way to understand the relation between schools and capitalism is to conceive education as an increasingly commoditized social space. It is a common place in critical educational studies to assert how education has become merchandise and schools some kind of corporation. For instance, D’Ambrosio (2003) explicitly compares schools with factories, where people are components of big machinery that aims uniformity. In this view, education is conceived as something originally pure that has been progressively contaminated by the capitalistic structure of society. Educational industries, from publishing houses producing text books to computer firms developing technology, see schools as a profitable market; administrators and politicians use the metaphor of schools as companies to envision ways of managing education; governments attribute primordial importance to results in high stake tests as a means to do school evaluation and make grades and scores a matter of profit; demands of the labour market and industries for the production of highly qualified people needed; all these are few examples of such views. In this perspective, education has become capitalized, and the ‘solution’ would be a *de-capitalization* of education, to return to its original purity, based on humanistic ideas of schools being the place to learn the cultural heritage of humanity, to educating the free man, to help learners’ advancement through formative assessment instead of summative assessment, etc. The purpose is to keep the capitalist logic of production/consumption outside the educational enterprise.

Despite being true, this characterization does not exhaust the relationship between capitalism and education, nor is it the crucial aspect. The problem is that school itself, more than just being contaminated by some capitalistic ideas has in its kernel the capitalist logic. It is against this

background that we should conceive education not as being contaminated by capitalism, or a part of capitalism, but as sustaining the capitalist system itself, by assuring its reproduction. Education is not just a product (education as a piece of profitable market) but a means of (ideological and material) reproduction (Althusser, 1994).

However, this role of school as an ideological apparatus is concealed by means of a ‘naturalization’ of schooling. Capitalist ideology represents school as a neutral environment purged of ideology (Althusser, 1994). Žižek (1994) calls this the ideological function of “self-disguising”: in order to become efficient, ideology must conceal its own ideological assumptions, so that we can be able to act as if our actions were deprived of all ideological content: “the very logic of legitimizing the relation of domination must remain concealed if it is to be effective” (p. 8). We must not perceive ourselves as being interpellated by some big Other³ but as individual subjects who freely choose to believe and act according to utilitarian and/or hedonistic motivations. For instance, when the NCTM (2000) argues for the importance to educating students to become active participants in society, they disregard any pathetic ideological phrases in sustaining their argument. The argument is either a pragmatic one—we need competent people in mathematics to become the future workers of our high-tech society—or a hedonistic one—people get empowered through mathematics. What we cannot miss here is that this attitude remains an ideological one: it involves a series of ideological presuppositions (e. g. about what it means to be an active citizen in a more and more commoditized society) that are necessary for the reproduction of existing social relations.

One of the few exceptions within mathematics education research to acknowledge the importance of mathematics not as knowledge or competence but as *value* is the work of Roberto Baldino and Tânia Cabral. They have analysed schools and mathematics education as part of capitalist economy (Baldino, 1998a, 1998b; Baldino & Cabral, 1998, 1999, 2006). Their suggestion is that we should look at school not so much as a place of knowledge but as a place of production, and students’ failure as a necessity. It is because some of us fail, that others can achieve higher social hierarchies. The value of the ones who flunk is appropriated by the ones who pass as surplus value. At school the student learns, above all, to participate in and accept the conditions of production and seizure of surplus value. Failure is posited as a *necessary* condition for schooling: “in order to perpetuate the process of production/seizure of surplus value, a certain amount of failure is necessary” (Baldino,

³ In this context, the Lacanian notion of big Other stands for all the State, Justice and Law that give symbolic meaning to our social life.

1998a, p. 5); therefore, “failure of students means success of the institution” (Baldino & Cabral, 2006, p. 34).

However, in order for school to be the most important ideological apparatus, to function as a credit system, as Vinner (1997) noticed more than ten years ago, it is not productive to be presented as an exclusionary institution. This will cause criticism from the whole of society, and will be unbearable from an educational or political point of view. In order to perform well in the role of credit systems, schools need to be presented as inclusionary and emancipatory places. Places where phenomena such as exclusion and failure are seen not as necessary parts of the same system which tries to abolish them, but as contingent problems, malfunctions of an otherwise good system.

Thus, by positing schools in the Political, we are not suggesting a capitalistic metaphor either to better understand education or to help school functioning. We are suggesting that teachers are teaching a basic capitalist practice in their classrooms even if they refuse to recognize it; and students are having the opportunity to learn, besides the specific knowledge the student is supposed to learn (in our case mathematics), the necessary capabilities of a *capitalist* and a *worker* (Baldino and Cabral in this issue). Indeed, this should not be surprising since it is precisely what people as Foucault and Bourdieu had been showing in the past: Knowledge in school functions as an alibi to the appropriation of symbolic norms for structuring our lives. And if capitalism is today the neutral matrix organizing our social relations, the conclusion to be drawn is that students in school learn, above all, to participate in and accept the conditions of production and seizure of surplus value.

4. Moving the field towards the Political

In this section we would like to put in operation our analytical tools in doing a political reading of recent socio-cultural theorizations of mathematical learning. The *cultural theory of objectification* (Radford, 2006a, 2006b, 2008a, 2008b) is arguably one of the most solid and well-documented theorizations about teaching and learning mathematics within a socio-cultural framework. Taking advantage of Vygotsky’s and Leontev’s theories, and also Husserl’s and Pierce’s phenomenological epistemologies, Radford presents learning as the reaching of a culturally-objective piece of knowledge that students acquire through a social process of *objectification* mediated by signs, language, artefacts, the body, and social interaction as the students engage in cultural forms of reflecting and acting. He deals with the dichotomy of the individual and the object of knowledge by introducing the notion of *learning as being*, as a dialectical process where learning is both a process of objectification (knowing) and subjectification (being or becoming). Learning is seen as more

than constructing logico-mathematical, mental structures or picking up ready-made knowledge. It is also viewed as an ethical and political activity where the subject is constantly renewed and constructed in the meeting with culture. Learning thus is not just about knowledge but also entangles an ethical and political dimension: “The meanings circulating in the classroom cannot be confined to the interactive dimension that takes place in the class itself; rather they have to be conceptualized according to the context of the historical-cultural dimension” (Radford, 2006b, p. 21, 22).

By doing so, the theory opens the space to address the historical and cultural context, within which the meanings of being a student and a teacher (that is, the social identifications they are subjected to) are constituted. However, when reading analyses of empirical material from classrooms using the theory, the details in the micro-situations under the research gaze seem to leave aside their ‘historical-cultural context’. The historical-cultural dimension seems to get concentrated in the (mathematical) history of the mathematical object being objectified, and in the teacher’s awareness of such constitution to guide the student’s explorations. From the analyses we get the impression that the subject (and his/her historical, cultural, political and social constitution) is thrown out of the equation: what remains is a mathematical learner with the desire to learn mathematics. The usefulness of the theory for presenting a strong interpretation of the ‘learning of mathematics’ seems to force the researcher to ignore all the ‘non-mathematical’ complexities of classrooms which so often stand in the way of a meaningful learning. The result is that learning is portrayed as an encounter between a piece of historic content and a subject who desires to learn it.

But is this the only possible analysis that can be performed with this theory? Our suggestion is that what Radford calls the “meaning circulating in the classroom” should be conceptualized not just as the meaning related with the specific mathematical object, but as involving also an *ideological* dimension for which neither students nor teachers (and apparently not even researchers) are always aware. As mentioned by Brown (2008b), the point is “not to get at the intention of what the teachers were saying, but rather to understand how the policy was operating through the teachers” (p. 250). That is to say, how ideologies are present and conveyed through students and teachers’ discourse. This is exactly what Baldino and Cabral do in the article they submitted to this special issue. By positing schools in the Political, they open the possibility of conceiving schools not so much as places of learning but places of production, completely entangled by capitalist economics. Doing so, they offer us a different interpretation for many of the learning difficulties they encounter in their

daily work (they both teach mathematics at university), by positing them not at the cognitive or socio-cultural level—that is, at a level where it can be analysed through “theories for learning”—but at the Political. Their suggestion, as we already noticed, is that considering school as a place of economic production may shed light on students’ learning difficulties.

The theory and the analyses called our attention because the theory takes support in Marxism (Presmeg & Radford, 2008), as clearly expressed in Radford and Presmeg’s response to Brown (2008b) in a vivid discussion carried out recently in this journal around the issue of subjectivity. In our view, the construction of a Marxist inspired theory of learning without the full recognition of the political *economy* that Marx put forward ends up amputated. This disavowing of the *economic*, as mentioned by Ozselçuk and Madra (2010, p. 333), is in pace with “tendencies within political economy that read Marx in ways that contain and even annul the constitutive negativity of class antagonism”. The cultural theory of objectification, therefore, can be taken to be a historical and cultural framework of mathematical objectification in mathematical learning. But it could be interesting to explore whether it offers possibilities for a broader analysis of the dialectics of objectification and subjectification in the full Political constitution of schooling and the school mathematics curriculum.

In the way we read Marx, there is no emancipation without *economic* emancipation and, in this sense, as Jameson (1991) argues, the conquest by human beings of the otherwise seemingly blind and natural ‘laws’ of socioeconomic fatality remains the irreplaceable will of Marx’s heritage. Both Žižek and Jameson interpret Marxism not as a political philosophy (although it entangles a practice of politics), but as being primarily concerned with the economic organisation of society and how people cooperate to organize production. Marxism is primarily concerned not with the end of a certain political thought, but with the end of an economical system. This way, by disavowing the fundamental economic dimension of Marxism, a theory such as that of the one discussed above produces a case of what Žižek (1995, p. 9) calls “progressive amnesia”: Marxism is recovered but deprived of its most fundamental core. It is, so to speak, a sanitized Marx. By disavowing the centrality of political economy in Marx’s work—and emphasizing instead the historical and the cultural—, a theory of mathematical learning falls short to bring an understanding of schools as credit systems, and of learning being more than just the acquisition by the student of a piece of cultural-historical mathematical content.

5. Final remarks

If during the venture of opening mathematics education research to theories not primarily concerned with learning for the purpose of designing optimal instructional situations, the specificity of mathematics gets redefined into more blurry issues of mathematical rationality in society, culture, politics and economy—and the same argument could be made for any other school subject—, then for sure we are dealing with a problem that is consubstantial with the constitution of school mathematics in the arena of schooling itself. The teaching and learning of mathematics in classrooms around the world does not occur in an ideal heaven of equal opportunities, where students desire to learn mathematics and teachers to teach it. It happens in schools with all the political charge such institutions have in our current societies. To detach mathematics from the political terrain where it is taught and learned is to produce innocuous research if the researcher's intention is to envision new educational possibilities.

Such is, in our view, the potential of the research carried out in this article. This kind of research should be read not as research on the teaching and learning of mathematics, but “research on research”. That is, research that, by teasing out other theories—that is, theories which are not primarily concerned with the teaching and learning of mathematics— allow us to ‘estrangle’ us from the self-evidence of mathematics education as an established field of research. It is our contention that such an approach, although not directly aimed at providing some kind of insight for action, can help us in redefining the coordinates we use to map and make sense of the problems of the field.

We want to make clear that we are in no ways disavowing the importance of the insights gained by the research in mathematics learning; rather we are insisting that all the research done on better ways to teach and learn mathematics runs the risk of being harmless if schools as credit systems remain unaddressed. It is the fact that schools need to produce failure that colours many of the students' difficulties usually identified by the community as cognitive or sociocultural impediments. In short, we want to emphasize that a reading of mathematics education practices within the Political economy has an important contribution to make to the field of research.

6. Acknowledgements

We would like to thank our colleagues from the Science and Mathematics Education Research Group (SMERG) for their comments and discussion to previous versions of this manuscript. This article is part of Alexandre's PhD project, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007.

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Chapter 3

Beyond Disavowing the Politics of Equity and Quality in Mathematics Education

Alexandre Pais and Paola Valero

Introduction

Who would publicly deny that mathematics education should be concerned with “equity” and “quality”? Definitely, a concern for “quality” has been behind the constitution of mathematics education as a field of scholarly inquiry from its very beginning. In several accounts of the history of mathematics education and its related institutions, considerations about the “quality” of teachers’ instruction and of students’ learning have been at the core of the justifications for the development of the field. The constant call for the improvement of the “quality” of mathematics education characterizes a field of research that more often than not documents the shortcomings of “quality” in the mathematics being taught, in the teaching practices of teachers, and in the learning of students.

Nowadays the terms “equity” and “quality” appear side by side, and it seems “natural” to have them together. It also seems “natural” to know what is understood by the terms. In fact, notice that we have already used them without a clear signaling of the meaning given to them. In this paper we react to the “naturalization” of the meaning of these terms and their relationship. Thus, it is our purpose to provide a critique of how equity, and its relation with quality, is addressed in current mathematics education research. For doing so, we adopt a socio-political perspective that, on the one hand, views mathematics education practices as a large network of social, economic, political and historical practices and discourses where mathematical rationalities are constituted (Valero 2010). On the other hand, we view research in mathematics education as a series of practices that contribute to the construction of naturalized discourses about what constitutes mathematical rationalities in the social world. Such a perspective allows us to engage in an examination of the types of discourses that mathematics education research has produced when tack-

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ling “equity” and “quality,” and the relationship between both. We see such critique as an important activity of mathematics education research. Without it, it becomes impossible to imagine different alternatives to the language that we use and to the world that such language creates.

In particular, we emphasize the fact that discussions of equity and quality are necessarily political since they allow us to address the values and ideologies that make part of educational practices, as well as of the whole set of practices and social organizations that extend beyond mathematics classrooms. However, it is our observation that the great majority of mathematics education research, by being primarily focused on pragmatic approaches to the improvement of classroom practice, lacks a theoretical comprehension of how “equity” and “quality” are related with broader social and political structures. Without such theoretical comprehension, Baldino and Cabral (2006) argue, we, as researchers, risk moving blindly because we do not “take certain distance and develop consistent research theoretical frameworks to appreciate our practices” (p. 31). In this chapter, we contribute to this comprehension by analyzing how social discourse and ideology permeate the way mathematics education research phrases and tackles “equity” and “quality”. In our analysis we draw on the work of Gert Biesta, and Slavoj Žižek. Biesta, a philosopher of education, offers us tools to enter into a critique of mathematics education research in relation to the politics of education as a human activity. Žižek offers us tools that allow us to understand how ideology permeates today the field of mathematics education research.

The Framing of the Research Field

In our recent work, we have been paying attention to the historical constitution of mathematics education research as a field of study. Inspired in the work of Michel Foucault, we are interested in the effect of the fields of academic inquiry in the construction of discourses about the social world that they intend to study. Researchers through their practice formulate languages and forms of doing and acting that constitute the world that they themselves are studying. When we adopt a critical strategy to examine the discursive constructions of mathematics education research, we do not intend to dismiss the achievements of the field in relation to how to diagnose and improve the practices of teaching and learning. We are interested in denaturalizing what seems to be taken for granted with the aim of opening a space for other possibilities of phrasing mathematics education. Such a strategy allows us to transgress established ways of seeing and understanding practices in a search for the impossibilities, disturbances and hidden potentials within the established order (see e.g., Biesta 2005; Stentoft and Valero 2010).

Recently, we have been developing a critique on the theories used in mathematics education research (Pais et al. 2010). We argued that there is a strong tendency to reduce the selection of theories to mainly theories of learning. This is due to the way in which the overarching aim of the research field is formulated. There seems

to be a consensus on the proposition that the main concern of mathematics education research is to improve students' performance in mathematics (Boero 2008). Niss (2007, p. 1293) is very clear in stating: "We do research on the teaching and learning of mathematics because there are far too many students of mathematics, from kindergarten to university, who get much less out of their mathematical education than would be desirable for them and for society." If this is the main concern of mathematics education research, it is not surprising that the field has been constructed as a space for researching in a systematic, scientific way "the problems of practice" (Silver and Herbst 2007, p. 45), defined as the predicaments of the teaching and learning of mathematics. The work of mathematics education researchers is "to identify important teaching and learning problems, considerer different existing theories and try to understand the potential and limitations of the tools provided by these theories". Cobb (2007) also suggests that mathematics education research should be understood as a "design science," that is, "the collective mission which involves developing, testing, and revising conjectured designs for supporting envisioned learning process" (p. 7). The ultimate goal of this science is to "support the improvement of students' mathematical learning" (p. 8).

The trend to focus on issues of learning—and thereby of teaching—is not exclusive to the field of mathematics education research, but has over the last two decades also proliferated in educational research in general. The language of education has largely been replaced by a technical language of learning. The contradictions on the role of schooling in society and the goals of education that fueled part of the educational debate during the last century seem to have been surpassed. We seem to have reached a consensus on the benefits of schooling. Therefore, a central concern now is to make it more effective. The problems of schooling and school subjects are not anymore political or ideological, but have become primarily technical or didactical. In most cases, solutions to educational problems are reduced to better methods and techniques to teach and learn, to improve the use of technology, to assess students' performance, etc. Education has progressively been reduced to be a controllable, designable, engineerable and operational framework for the individual's cognitive change. Such tendency is what Biesta calls the *learnification* of education (Biesta 2005). Although the dominance of learning theories for researching mathematics education has allowed us to gain deeper understandings on the processes of teaching and learning mathematics, we suggest that it has also left unattended important difficulties and dilemmas faced by the educational communities in their everyday practices. This reduction of education to learning disavows the political magnitude of education. Learning is conceived as a nominal activity, isolated from what Valero (2010) calls the network of socio-political practices that constitute mathematics education—that is, the entire social, political, economical and historical configuration where mathematics education practices are given meaning. We argue that in order to bring the many difficulties and dilemmas of educational communities seriously into the gaze of research, we need broader theoretical frameworks which allow us to understand mathematics *education*, and not only mathematical *learning*.

Education has given up its place in favor of specialized pedagogy and didactics. In the case of mathematics education research, the discursive construction of

students as cognitive subjects and “schizomathematics learners” (Valero 2004a) is a good example of the way mathematics education research reduces full political and historical human beings to “bare learners,” whose cognition can be scrutinized with the interest of devising appropriate and effective techniques for learning mathematics. All the complexity of the social and political life of the student is wiped out from the research focus. The student is reduced to a biological entity, likely to be investigated in a clinical way. For example, some researchers find useful to draw an analogy between mathematics education research and medicine. Mathematics education research is formulated as a science of treatment that, by understanding the symptoms that characterize students’ learning difficulties in mathematics, aims at designing and applying proper treatments, with the hope of curing what is a defect in students’ learning: “The evolving understanding of the logic of errors has helped support the design of better instructional treatments, in much the same way that the evolving understanding of the logic of diseases has helped the design of better medical treatments” (Silver and Herbst 2007, p. 63).

The Framing of “Equity”

In recent literature, the concern for “equity” is addressed in different forms by different authors. It is actually interesting to notice that few authors clarify their understanding of the term. It is written in between the lines that the problem of equity has to do with the differential achievement in mathematics. The systematic underachievement and its consequences for certain groups of students is not acceptable, particularly at a time when the agenda of “mathematics for all” seems to have permeated policy documents all around the world. The understanding about what it means to address and achieve equity also diverge, and some authors prefer to use terms such as social justice (e.g., Gutstein 2003), democratic access (e.g., Skovsmose and Valero 2008), inclusion/exclusion (e.g., Knijnik 1993). It is also common to find the declaration that research on equity requires social and political approaches that situate the problem in a broader context than the classroom or schools (Valero 2004b, 2007). For instance, Nasir and Cobb (2007) state that all the contributors to their book “view equity as situated and relational and as being informed both by local schooling practices and by practices and ideologies that transcend school” (p. 5). However, when reading the contributions in the book, we find that all the research reported is centered on improving the process of teaching and learning mathematics. Although politics is acknowledged as determinant in equity, and some authors explore the connections between mathematics education and politics (e.g., Gutiérrez 2007), the contributions lack a theoretical analysis on how these “ideologies than transcend school” influence what happens in schools, and its contribution—or not—to equity. As mentioned by Gutiérrez (2007), “little has been written in mathematics education that addresses how mathematics might play a role in broader politics” (p. 38).

One of the most extensive reviews on the issue of equity in mathematics education is the article by Bishop and Forgasz (2007). The authors provide an overview of the different research approaches on the issues of access and equity in mathematics education. Right from the beginning they call our attention to the artificiality present in the construction of groups of people as being in disadvantage—girls, ethnic minorities, indigenous minorities, western “ex-colonial” groups, non-Judeo-Christian religious groups, rural learners, learners with physical and mental impairments, and children from lower class—and how such constructions can in themselves convey discriminatory actions. This problem has been recently labeled by Gutiérrez (2008) as the *gap-gazing fetish in mathematics education*. Gutiérrez’ provocative formulation generated a debate with Sarah Lubienski. Roughly speaking, they discuss whether research focusing on the achievement-gap benefits or not the purpose of achieving equity. The position of Gutiérrez (2008) is that there are dangers in concentrating on the “achievement gap” because such research helps “offering little more than a static picture of inequities, supporting deficit thinking and negative narratives about students of color and working-class students, perpetuating the myth that the problem (and therefore solution) is a technical one, and promoting a narrow definition of learning and society” (p. 358). She argues that such research, which usually leans on quantitative methods, does no more than providing a description of the problem without presenting understandings that allow a change. She argues that less research focusing on the “gap” should be made, and more research should analyze qualitatively successful experiences among groups of people considered to be in disadvantage. On the other hand, Lubienski (2008) argues that more skilled and nuanced gap analyses are necessary: “analyses of gaps also inform mathematics education research and practice, illuminating which groups and curricular areas are most in need of intervention and additional study” (p. 351). Lubienski is concerned with the question of whether there is a gap, to what follows studies analyzing when the gaps manifest, under what conditions they grow or shrink, and what consequences underserved students ultimately suffer because of the gap. In contrast, Gutiérrez is concerned with the question of how to diminish the gap, to what follow studies oriented toward effective teaching and learning, making research more accessible to practitioners and more intervention by the researcher.

Some authors have been trying to list which practices can be carried out in order to achieve equity in mathematics education. For Schoenfeld (2002, quoted in Langrall et al. 2008, p. 127), achieving equity requires four systematic conditions to be met: a high-quality curriculum, a stable, knowledgeable and professional teaching community, a high-quality assessment aligned with curricular goals, and stability and mechanisms for the evolution of curricula, assessment and professional development. Alternatively, Lubienski (2002) claims that it is necessary to learn more about the complexities of successful implementation of meaningful instructional methods with students who differ in terms of social class, ethnicity and gender. For Goldin (2008), the most important is “to create teaching methods capable of developing mathematical power in the majority of students” (p. 178). Finally, Gates and Zevenbergen (2009) identify a common basis for how to deal with equity, summarizing existing research: “What might we all agree on then as fundamentals of a

socially just mathematics education? Perhaps we can list: access to the curriculum; access to resources and good teachers; conditions to learn; and feeling valued.” (p. 165).

Although we can discuss the better ways to do research addressing equity and what needs to be done, there is a fundamental question that seems to be unaddressed: Why is there inequity? Why is there a gap at all? That is, why does school (mathematics) systematically exclude/include people in/from the network of social positionings? Why do schools offer low-quality curricula and do not have a stable group of teachers in schools serving underprivileged population? Why does school perform the selective role that inevitably creates inequity? As Bishop and Forgasz (2007) put it, “in every country in the world mathematics now holds a special position, and those who excel at it or its applications also hold a significant positions in their societies” (p. 1149). Why does society need to have an institution that guarantees an accumulation of credit? These questions are rarely posed by research in mathematics education addressing equity. Research only recognizes it as a fact. Posing the questions above dangerously opens the field of mathematics education to politics, and it seems few researchers are ready to take a risk.

In the previous discussions, it is evident that the problem of equity is recognized as an economical and political problem. However, research in mathematics education transforms it to be a problem pertaining to mathematics teaching and learning. This type of displacement reduces the aim of researching the problem of equity to a matter of developing the best “instructional methods” to allow mathematical success to all students. The absence of a political conceptualization of the problem of equity is evident in much of the existing literature. Disavowing politics as part of the conceptualization of equity is one of the best ways of turning research innocuous for social change.

The Framing of “Quality”

A quality mathematics education research is constructed to be the one that allows students to improve their mathematical learning. Why is this important? The literature on mathematics education research is full of statements that justify the necessity of mathematical learning (e.g., Niss 1996). In most of such statements, mathematics and its education are viewed as powerful knowledge and competence for people to become full citizens and competitive workers. Are these formulations enough to justify mathematics education and to define “quality”?

We look for support in the philosophy of education. Biesta (2009) analyzes the functions that education fulfills in society nowadays. The function of qualification has to do with the role of education to providing people with the knowledge, skills and understanding necessary to fulfill a productive function. The function of socialization has to do with the role of education in enculturating people to become members of a particular society, by the insertion of the “newcomers” into existing social and cultural orders. An analysis of the justifications for mathematics teaching

and learning leads us to see that mathematics education builds fundamentally on the fulfillment of qualification and socialization functions. Consider the following assertion as an example:

Mathematics education in schools is thus seen to have a dual function: to prepare students to be mathematically functional as citizens of their society—arguably provided equitably for all—and to prepare some students to be the future professionals in careers in which mathematics is fundamental, with no one precluded from or denied access to participation along this path. (Bishop and Forgasz 2007, p. 1152)

On the one hand, mathematics teaching and learning is important because it allows the nurturing of the next generation of mathematicians and of those who will use mathematics in their work, therefore assuring the development of a working force equipped to compete successfully in the global economy of our high-tech society. On the other hand, mathematics teaching and learning secures the insertion of people in a society where mathematics is seen as an indispensable tool to become a citizen. The goal of citizenship concerns a wide range of competences: providing mathematical skills for dealing with situations of everyday life, intellectual enrichment, acknowledging mathematics as equally a part of humankind's cultural and aesthetic heritage, or making accessible powerful tools to analyze critique and act upon the way mathematics is used in society. The way quality is understood both in mathematics education research and in school mathematics seems to be in resonance. Since school mathematics is posited as indispensable to become both a productive and competent worker and an active and participative citizen, the purpose of mathematics education research should be to improve students' mathematical learning.

What is the problem with this view, then? We would like to argue that this way of conceiving quality conceals the ideology informing what it means to be a worker and a citizen in a capitalist society. At first glance, the aims for school mathematics mentioned above are worthy aims for any compulsory schooling system. Becoming a successful worker and an informed and participative citizen seems to fulfill the desire of students, parents, politicians, teachers and others participants in the educational process. So why do we feel uneasy about these aims? On the one hand, the listed aims for school mathematics are formulated on the assumption that subjects are conscious of themselves (Althusser 2000). The assumption disavows the political substrate that informs what it means to be a worker and a citizen in current societies. On the other hand, these aims conflict with the politics of accountability where quality is often defined as having the best ranking positions both in national and international examinations.

Allow us to explore these two aspects in more detail. Althusser (2000) argues that the ideology of capitalism is based on the idea that individuals are self-conscious subjects, responsible for their own acts. They can be persuaded, consciously, to obey rules that otherwise would be imposed by force. It is only under this condition that human beings become *homo economicus*. Marx (1989) showed that, contrary to the assumption that the subject is a coherent rational being, subjects are not conscious about the "nature" of the place they occupy in the structure ruled by the laws of capitalist society. Marx allows us to understand that behind the

ideology that asserts the equality of individuals in the free market lays a profound inequality.

One of the ideological modes dissected by Žižek (1994) conceives ideology as “a doctrine, a composite of ideas, beliefs, concepts, and so on, destined to convince us of its ‘truth,’ yet actually serving some unavowed particular power interests” (p. 10). The strategy to criticize this mode of ideology is to carry a *symptomal reading* (Althusser 1994) that exhibits the discrepancies between the public discourse and the actual intention of it. The Standards of the National Council of Teachers of Mathematics of the United States (NCTM 2000) is a prolific document to engage in such a reading. The basic discrepancy of discourses in this document has to do with how behind the public discourse of forming students to become active and participative citizens in society, there is a concern in maintaining the economic and scientific dominance of the United States. The NCTM Standards can also be read as a case of what Žižek (2006) called *staged democracy*. The document expresses an official discourse with all the virtues and democratic goals that society stands for, but when put in practice the actions deriving and resulting from the formulations will almost secure that the democratic goals continue to fail. In this case, the ideological critique will be concerned not with the understanding of why in practice those desirable aims continue to fail—as if it were a problem of “implementation”—but to understand how the discrepancy is already being created at the level of the official discourse by completely obliterating the real reasons why inequality and lack of democracy continue to exist. In the case of the NCTM standards, the interesting point is not to focus on why their implementation fails in the hands of “incompetent” authorities, administrators and teachers, and of “deficient” children. An ideological critique would see how the document bears in itself the impossibility of achieving its stated goals.

However, in order to become efficient, ideology must go under a process of “self-disguising,” so that we can be able to act as if our actions were deprived of all ideological content: “the very logic of legitimizing the relation of domination must remain concealed if it is to be effective” (Žižek 1994, p. 8). We must not perceive ourselves as being questioned by some big Other¹ but as individual subjects who freely choose to believe and act according to utilitarian and/or hedonistic motivations. When the NCTM standards argue for the importance of educating students mathematically to become active participants in society, the document disregards any pathetic ideological phrases in supporting their argument. The argument is a pragmatic one—competent people in mathematics are needed as the future workers of our high-tech society—or a hedonistic one—people gain power through mathematics. However, we cannot miss here that the choices in the document are highly ideological: The formulations involve a series of assumptions about what it means to be an active citizen in an increasingly commoditized society, and how such type of people are necessary for the reproduction of existing social relations. A staged

¹ In this context, the Lacanian notion of big Other stands for all the State, Justice and Law that give symbolic meaning to our social life.

discourse is needed so that school mathematics continues to perform other roles than those present in the official discourse.

Furthermore, how does one assess the quality of students' mathematical learning? It seems to be an unachievable task to assess if students, at the end, become or not desirable workers and citizens. However, society cannot live in this state of uncertainty regarding the mathematical performance of students. Society craves for results, for evidence that shows if people are becoming desirable subjects. Therefore, rigorous instruments should be created so that it is possible to objectively know if students are performing well in school mathematics. Indeed, such instruments exist under the form of national tests and international comparative studies. In our days where accountability reigns, what counts as quality is the performance of students in high-stakes examinations and in international tests. Ultimately, are not the results from these examinations what define quality in mathematics education? We are confronted with the inconsistency of a system that, on the one hand, defines quality as a matter of achieving the desirable "mathematical subject"—the mathematically competent, informed worker, and critical citizen. But on the other hand, what ultimately decides the quality of mathematics education is the results of the exam. Again, the question to be posed is why the type of society we live in needs a staged discourse concealing what everybody recognizes.

The Disavowing of Politics in Mathematics Education

We explore now the disavowing of politics involved when addressing equity and quality in mathematics education. We argue that the political disavowal keeps research at a "technical" level, which contributes to reaching just the opposite of the stated aims. Furthermore, we argue that this is precisely one of the strongest limitations for bringing equity and quality together. A quality mathematics education is not one that attends mainly to the intrinsic characteristics of mathematics as the foundations for educational practices, neither one that proposes pragmatic and hedonistic justifications for why to teach mathematics to all students, but rather one type of education that recognizes the possibilities of the meeting between human beings and the school subject of mathematics within the social, political and historical frame in which such meeting is being constituted. This means that definitions of equity and quality in mathematics education that do not attend to how both notions as well as the practices of mathematics education are shaped in power relationships are partial definitions that can only place hysterical demands to practitioners.

We showed previously how researchers address the societal demand of mathematics for all and make it a research concern. They engage on this demand by assuming that through their studies on the teaching and learning of mathematics, on better curricular and instructional design, on better connections between researchers and practitioners, they are contributing to achieving equity in school mathematics. However, if equity, or rather, inequity, is an economic and political problem that surpasses school, then the demand of mathematics for all is impossible to satisfy

(cf. Žižek 1991). Why then do we keep doing research to address equity issues in mathematics education as if the problem could be solved within the realm of mathematics teaching and learning? Could it be that keeping us occupied doing innocuous research inhibits us from looking at other issues?

As we know, dominant social systems demand for perpetual reforms as a means of integrating what could be new and potential emancipatory acts into well-established social structures. In other words, dominant systems such as capitalism today are constantly changing something so that nothing really changes (Žižek 1991). The novelties research produces on how to promote equity by improving the teaching and learning of mathematics are part of these superficial transformations. According to Gutiérrez (2007), “[e]quity is threatened by the underlying belief that not all students can learn” (p. 3). Although in a first reading we agree with Gutiérrez, we see that other beliefs are at stake, namely the not underlined but publicly assumed belief that all students can learn. The interplay between these two discourses makes visible how ideology works today. The view that all students can learn—the official view, present in curricula, political documents and research, attesting that mathematics is for all—conceals the commonly shared but not assumed belief that there will always be some who will fail. Following Žižek (2010), when we read an abstract “ideological” proclamation such as “mathematics for all,” we should be aware that people’s experiences are different—for teachers and students know and experience that in any mathematics class there will always be some—or many—who fail. The official discourse functions not as some kind of utopian state to be achieved, a desired good to strive for, but rather as a pure mechanism to conceal the fact that mathematics is not for all. The obliteration of the “background noise”—the voices of those who will always fail—is the very core of utopia. The “background noise” conveys “the obscenity of barbarian violence which sustains the public law and order” (Žižek 2010, p. 10). In the case of mathematics education, the obscenity of the barbarian violence that school exercises year after year when it throws to the garbage bin of society thousands of people, under the official discourse of an inclusionary and democratic school (mathematics education).

As far as society remains organized under capitalist tenets, there will always be exclusion because exclusion is not a malfunction of capitalism, but the very same condition that keeps it alive (Žižek 1989). In such organization, having (certain) students failing in mathematics is not an abnormality of mathematics education, it is the necessary condition for its very same existence. So, why do we need an official discourse affirming that mathematics is for all? It is because such discourse masks a crude fact: The capitalist lie that presupposes equity in schools as an extension of the equality in the market. In other words, the official discourse conceals the inconsistency of a system that, on the one hand, demands mathematics for all while, on the other hand, uses school mathematics as a privileged mechanism of selection and credit.

The denial in confronting the core of the problems of equity is the result of an ideological injunction that systematically leads us to repeat the same “abstract” mottos of discourse: School is a place for emancipation; mathematics is powerful knowledge and competence; mathematics is for all; etc. In order to critically

analyze such discourses, we should replace the abstract form of the problem with the concrete scenes of its actualization within a life-form: “In order to pass from abstract propositions to people’s ‘real lives,’ one has to add to the abstract propositions the unfathomable density of a life world context—and ideology are not the abstract propositions in themselves, ideology is this very world density which ‘schematizes’ them, renders them ‘livable’” (Žižek 2010, p. 6). In other words, in order to understand the real aims of school mathematics, or the real motives that students have to be in school, we must not repeat ideologically loaded discourses conveyed by the curriculum, the political statements, and the research. Rather, we need to look at schools selecting the future workers of the labor market by means of credit accumulation. That is, what Gutiérrez calls the “underlying belief that not all students can learn” must be posited not as a threat to equity, but as the truth of a system in which equity is forever postponed. Following Žižek’s (1989), this implies asking research to pass from the notion of crisis—in this case, the fact that people fail is school mathematics creating exclusion—as an occasional contingent malfunctioning of the system, to the notion of crisis as the symptomatic point at which the truth of the system becomes visible. Some will say that such an awareness of the problem of equity takes us to a deadlock. Indeed, by realizing that exclusion is something inherent to the school system in a capitalist society, we realize that ending exclusion implies finishing schooling as we know it. In the current myriad of world social structure, this does not seem possible. However, what dooms us to constant failure is precisely experiencing the change as impossible. We acknowledge that the problem of equity requires a fundamental societal change, which may be impossible. The question is whether it is impossible or it is ideologically posited as impossible.

Threshold

The key feature here is that to see the true nature of things, we need the glasses [glasses as a metaphor for critical ideological analysis]: it is not that we should put ideological glasses off to see directly reality as it is: we are ‘naturally’ in ideology, our natural, immediate, sight is ideological. (Žižek 2010, p. 6)

We would like to argue that mathematics education research needs such glasses. The “natural” way in which we relate to reality is ideological—in our practice we convey discourses that conceal more than what we know. Ultimately, the purpose of this chapter was to attempt an ideological critique in the way we address issues of equity and quality in mathematics education research. Apparently, there is no doubt that definitions of quality and the discourses for equity live side by side and are equally political. However, it is almost nonexistent in mathematics education research studies that aim to understand in depth such problems that are identified as political in their nature. If mathematics education research desires to address them, it must open its gates to research that locates the complexity of mathematics education within the network of social and political practices that permeate all educational act. Without that we run the risk of falling in the trap of what we criticize. On the issue

of equity, our premise is that exclusion and inequity within mathematics education and education in general is an integrative part of current school education, and cannot be conceptualized without understanding the relation between school education and the social mode of living that characterizes our current world. On the issue of quality, a serious challenge is also to politicize our understanding of what is taken to be the significance of valued forms mathematical thinking within capitalism.

Therefore, our intention with this text is not to give a solution to the problems of equity and quality, neither is it to propose an alternative way of doing research on these topics. Our purpose is much more modest. We wanted to raise the awareness on the fact that there are broader issues involved when discussing equity and quality in mathematics education, than doing research on better ways to teach and learn mathematics, to improve students' mathematical performance so that they could become better workers and citizens. As mathematics education researchers actively engaged in the field, we find the need for developing a deeper understanding of our practices and the discourses we convey. The way we found to do this was to explicitly look at the inconsistency of discourses that make the apology of equity and quality without considering the meaning that these terms have in a society where capitalism has become the ontologized substrate for all social relations.

Acknowledgment This chapter is part of Alexandre Pais' PhD study, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007.

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ALEXANDRE PAIS

2. A CRITICAL APPROACH TO EQUITY IN MATHEMATICS EDUCATION

INTRODUCTION

Equity has been on the agenda of mathematics education research for at least two decades. The (first) *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992) contains two chapters dealing with issues of equity and access: one focusing on gender (Leder, 1992), and the other on race, ethnicity, social class, and language (Secada, 1992). In 1995, a collection of pioneering contributions concerning the research on equity within mathematics education was published (edited by Secada, Fennema, and Adajian). In the same year, Rogers and Kaiser (1995) edited a book compiling research on the relation between equity and gender. This interest in equity was followed by a proliferation of theories in mathematics education research that progressively de-emphasized cognitive psychology as an interpretative framework for the learning of mathematics in favour of more socio-culturally oriented frameworks – what Lerman (2000) called the “social turn” in mathematics education research.

Acknowledging this trend, Valero (2004) coined the term “socio-political perspectives on mathematics education” to describe the branch of research emerging when the practice of mathematics education is investigated as part of the larger social, political, economical, cultural, and historical frameworks for education, schools, and classrooms. The scope and diversity of the research around these issues is vast. Authors such as Ole Skovsmose, Leone Burton, Marilyn Frankenstein, Paul Ernest, Ubiratan D’Ambrosio, Alan Bishop, Paola Valero, Robyn Zevenbergen, Mogens Niss, Paul Cobb, Gelsa Knijnik, among many others, have been developing research concerning issues that in some way handle social and political aspects of mathematics education. These subjects include equity, democratic access, social justice, critical mathematics education, social exclusion, and ethnomathematics.

Regarding the issues of equity and social justice, the last decade was prolific in publications. In 2002, a special edition of the journal *Mathematical Thinking and*

Learning, edited by Na'ilah Nasir and Paul Cobb, was dedicated to the issue of *Diversity, Equity, and Mathematical Learning*. Five years later, the same authors edited a book (Nasir & Cobb, 2007) collecting contributions from several well-known mathematics education researchers on the issue of diversity and equity in the classroom. The theme of the *28th International Conference of the International Group for the Psychology of Mathematics Education* (proceedings edited by Johnsen Haines and Anne Fuglestad) was *Inclusion and Diversity*. The *Handbook of International Research in Mathematics Education* (English, 2008a) was particularly concerned with social, cultural, economical, and political influences in mathematics education. It collected several articles dealing with the issue of equity, under the label of "democratic access to powerful mathematical ideas". Also in 2008, three articles, by Sarah Lubienski and Rochelle Gutiérrez, were published in the *Journal for Research in Mathematics Education*. The authors engage in a discussion about the most fruitful way to address issues of equity in mathematics education research, concerning the existence of an achievement gap in the USA between middle-class white (and certain Asian) students and groups of people considered to be disadvantaged. Finally, in 2009, this same journal dedicated two special editions to the issue of equity, considered to be one of the current top priorities in mathematics education by the NCTM Board of Directors. Therefore, there is no risk in saying that the issue of equity is currently on the agenda, and the moment seems propitious to develop a theoretical analysis on the way equity is being addressed in mathematics education research.

Although notions of what it means to achieve equity diverge – and some authors prefer to use other terms such as "social justice" (e. g. Gutstein, 2003), "democratic access" (e.g. Skovsmose & Valero, 2008), or "inclusion/exclusion" (e. g. Knijnik, 1993) – it is usually acknowledged that research on equity requires social and political approaches that situate the problem in a broader context than the classroom or school (Anderson & Tate, 2008; Gates & Zevengergen, 2009; Valero, 2004, 2007). Although studies dealing with equity in mathematics education acknowledge the social and political dimensions of the problem, I shall argue that such studies insist on addressing the problem of inequity as if it could be understood and solved within mathematics education. It is as if we admit that the problem has an economical and political nature, going way beyond the classroom, but, since we are mathematics educators, we must investigate it in the classroom.

This approach – which consists in reducing a political problem to a didactical one, thus possible to be solved through the development and implementation of better stratagems to teach and learn mathematics – cannot be said to have produced the desired results, namely the commonly shared desire of "mathematics for all". Indeed, authors such as Roberto Baldino and Tânia Cabral have called our attention to the discrepancy between the huge amount of research targeted at diminishing the worldwide problem of failure in mathematics and the fact that failure persists and, as they highlight, is getting worse:

So the situation is this: in spite of all research efforts along the lines such as the few ones above mentioned that are more directly concerned with exclusion issues, the social gap continues to increase. Authors ask for more re-

search, better understanding of exclusion processes, more initiatives for change, more detailed and in depth studies, more comprehensive and interactive studies and data, new teaching strategies, and, recently, more studies addressing the social and cultural aspects of school mathematics. In one word, our problem is: *why do so many people insist in asking for more of that which cannot be said to have produced results for change so far?* (Baldino & Cabral, 2006, p. 21, my emphasis)

The question posed by Baldino and Cabral makes us wonder if mathematics education is on the right track in relation to research on equity.¹ The discrepancy between the increasing sophistication of research and the persistence of failure reminds me of an old joke about a man who lost a needle. Someone asks him: "Why are you searching for your needle in the kitchen if you lost it in the toilet?" He responds: "Well, the light is better in the kitchen." I will return to an exploration of this joke latter. For the moment, I suggest that most mathematics education research on equity may be leaving unaddressed crucial dimensions of the problem.

The main purpose of this article is to provide a deeper theoretical understanding of how equity is addressed in mathematics education research. The appeal for stronger theoretical conceptualizations has been endorsed by some authors (e.g. Gutiérrez, 2008; Nasir & Cobb, 2002), and I strongly believe that solution-oriented research is not enough when addressing problems that by their very nature are political and economical. One can suggest that, in the case of equity, perhaps it is a good idea to stop and ponder rather than repeating existing research. Nevertheless, a number of renowned researchers have been calling for the importance of developing a more reflective research: "[i]t appears to be one of the weaknesses of our profession that many of us, myself included, tend to write and speak too much and read and contemplate too little" (Niss, 2007, p. 1311). Despite Niss's appeal, mathematics education research has not been appreciative of research that is not immediately concerned with action, in the sense of providing solutions or strategies for improving the teaching and learning of mathematics (Pais, Stentoft & Valero, 2010). However, I believe that without deep criticism of our own actions we run the risk of acting blindly.

My premise is that exclusion and inequity within mathematics education, and education in general, are integrative parts of schooling and cannot be conceptualized without understanding the relation between scholarized education and capitalism as the dominant mode of social formation. As mentioned, it is common to find research that presupposes the idea that the problem of inequity transcends school and mathematics education. However, little research has been done that explicitly

¹ At the last ICME Conference, ICME11 in México, during a plenary debate session with Paul Cobb, Mariolina Bussi, Teresa Rojano and Shiqi Li, entitled *What do we need to know? Does research in mathematics education address the concerns of practitioners and policy makers?*, Shiqi Li asked a very insightful (and provocative) question: Why do the students participating in the research always increase their capabilities (to solve problems, to learn with meaning, of communication, to use technology, etc.) but in reality, in the daily routine of worldwide classrooms, failure in mathematics persist, and students are far from reaching those desirable capabilities achieved in research settings?

tries to understand exclusion as an integral part of schooling; that is to say, as something consubstantial to schooling itself.

The article is constructed as follows. After presenting the theoretical and methodological background, which leans heavily on Slavoj Žižek's philosophical interpretation of the works of Marx, Lacan, and Hegel, I proceed to an analysis of recent research on equity, trying to identify: a) the problematic issues affecting research on equity within mathematics education; and b) common shared assumptions in the discourse on equity. The analysis leads to two conclusions. On the one hand, the discourse that vindicates the importance of mathematics to becoming a full citizen, and therefore requires mathematics for all, can carry the germ of exclusion. On the other hand, the way mathematics education research addresses the problem of equity is mostly a technical one, leaving aside a social and political comprehension of it. This imbalance creates an inconsistency since it is acknowledged that the problem of equity is a political and economical one, that goes beyond mathematics and its education, yet the strategies for "solving it" presuppose that it can be solved within mathematics education. The last part of the article is an attempt to address the problem of inequity in its materiality. I argue that inequity is not a marginal problem of a "good" system; rather, it is what gives consistency to the system itself. These conclusions will be related with capitalism wherein, from a Hegelian point of view, equity amounts to universal exclusion.

CRITIQUING RESEARCH ON EQUITY IN MATHEMATICS EDUCATION

Positing equity in the Political

As a field of research, mathematics education has been historically concerned with the improvement of the process of teaching and learning mathematics in schools (Cobb, 2007; Sierpiska & Kilpatrick, 1998; Silver & Herbst, 2007). As a consequence, the theories used in the field, even within a sociocultural framework (which sees meaning, thinking, and reasoning as products of social activity), are, in the majority of cases, theories of learning, designed to improve students' performance in mathematics (Pais, Stentoft, & Valero, 2010). In this sense, the work of researchers is "to identify important teaching and learning problems, consider different existing theories, and to try to understand the potential and limitations of the tools provided by these theories" (Silver & Herbst, 2007, p. 45).

As explored by Paola Valero and me (Pais & Valero, 2010), the tendency to focus on issues of learning – and thereby of teaching – is not exclusive to the field of mathematics education research, but has over the last two decades also proliferated in educational research in general. The language of education has largely been replaced by a technical language of learning. The contradictions of the role of schooling in society and the goals of education, that fueled part of the educational

debate during the last century,² seem to have been superseded. We seem to have reached a consensus on the benefits of schooling. Therefore, the central concern now is to make it more effective. The problems of schooling and school subjects are no longer political or ideological, but have become primarily technical or didactical. In most cases, solutions to educational problems are reduced to better methods and techniques to teach and learn; to improve the use of technology, to assess student's performance, and so on. Education has progressively been reduced to be a controllable, designable, engineerable, and operational framework for the individual's socialization.

Such a tendency is what the philosopher of education Gert Biesta calls the *learnification* of education (Biesta, 2005), and can be understood as part of a larger societal propensity to address fundamental social problems (such as the worldwide pressure to succeed in school mathematics) as if they were the object of expert management and administration (Agamben, 1998; Foucault, 1991, 1997).³ On the one hand, if we take a look at two of the recent articles on the role of theory in mathematics education research⁴ (Cobb, 2007; Silver & Herbst, 2007), we can notice how "theory" is perceived as providing "tools for action", where action is normally the practice of school mathematics, thus reducing research to a matter of providing the solutions for the problems of practice. On the other hand, the discursive construction of students as cognitive subjects and "schizomathematicslearners" (Valero, 2004) is a good example of the way mathematics education research reduces full political and historical human beings to "bare learners"⁵, whose cogni-

² For instance the discussions fuelled by the work of John Dewey, Ivan Illich, Louis Althusser, and Paulo Freire.

³ This characterization of modern, technological, societies is in line with what Heidegger called "enframing" and Adorno the "administered world".

⁴ Theory is another "hot topic" in recent publications within the field. At the 11th *International Congress on Mathematics Education* (ICME11) one of the survey teams was responsible for developing a study of the notion and role of theory in mathematics education research. This survey team had the task of identifying, surveying, and analysing different notions and roles of "theory", as well as providing an account of the origin, nature, uses, and implications of specific theories pertaining to different types of research in mathematics education. The *Second Handbook on Mathematics Teaching and Learning* (Lester, 2007) contains two articles dealing with the issue of theory: "Putting Philosophy to Work: Coping With Multiple Theoretical Perspectives" (Cobb, 2007), and "Theory in Mathematics Education Scholarship" (Silver & Herbst, 2007). At the Congress of European Research in Mathematics Education (CERME) there has been a working group dealing with the problem of linking, contrasting, and comparing the wide variety of theoretical approaches found in the field to tackle the teaching and learning of mathematics (see ZDM, 40(2), published in 2008). Finally, in 2009 the theme of the 33rd *Conference of the International Group for the Psychology of Mathematics Education* (PME) was "In search for theories in mathematics education". There seems to be a widespread desire for understanding the role of theory in mathematics education research: "The moment seems propitious for a serious examination of the role that theory plays and could play in the formulations of problems, in the design and methods employed, and in the interpretation of findings in education research" (Silver & Herbst, 2007, p. 41). This chapter is also intended to be a contribution for how theory is understood and used in mathematics education research.

⁵ For Agamben (1998), who amplified the work of Foucault, the only real question to be decided in contemporary society is which form of organization would be the most suitable for the task of secur-

tion can be scrutinized in the interest of devising appropriate and effective techniques for learning mathematics. All the complexity of the social and political life of the student is wiped out of the research focus. The student is reduced to a biological entity, likely to be investigated in a clinical way. For example, some researchers find it useful to draw an analogy between mathematics education research and medicine. Mathematics education research is formulated as a science of treatment that, by understanding the symptoms that characterize students' learning difficulties in mathematics, aims at designing and applying proper treatments, with the hope of curing what is a defect in students' learning: "The evolving understanding of the logic of errors has helped support the design of better instructional treatments, in much the same way that the evolving understanding of the logic of diseases has helped the design of better medical treatments" (Silver & Herbst, 2007, p. 63).

To some extent, philosophy offers us an antidote against this tendency to "technicize" educational problems. By taking a step back, by resisting the temptation to engage immediately in some form of action, one can propose that sometimes the best way to act is to stop acting, in the sense of doing more of the same research which proved not to have the solutions for the core problems the community faces (Baldino & Cabral, 2006). When addressing the problems of the field – most notably the persistent failure in school mathematics worldwide, the senseless mathematics instruction which reduces mathematics education to the apprehension of a set of routine procedures to be reproduced in exams, and the problem of inequity and how school mathematics is associated with processes of social exclusion – I concur with Brown (2008, p. 229) that "greater attention to policy domains, rather than focusing primarily on developing apparatus for working with individuals' minds" is necessary.

I argue that this judgment is especially true when the topic under research is equity, because achieving equity transcends the field of mathematics education. The problem of guaranteeing an equitable and just society, where all people have access to the material and cultural goods they need to become fully integrated citizens has been at the same time the flag of Modern Liberal-democratic politics, and its prevalent disenchantment. In line with Marx's (1857) critique of capitalist societies, modernity carries the spirit of universalizing bourgeois ideology wherein equality, freedom, civil rights, humanitarianism, free speech, and open media stand for the Common Good. Of course, as noted by Jameson (1991), Marx sees nothing

ing the care, control, and use of *bare life*: human life stripped from its entire political dimension, and reduced to its biological entity. Human bare life is that type of existence that can be measured, calculated, and predicted; in other words, the object and result of technical expertise. Recognizing this condition, Žižek (2006) argues that today we live in a *post-political* society; politics has surrendered to specialized social administration, targeting the bare life of the individual by controlling its fluctuations according to global standards of normality.

wrong with these principles in themselves, only the assumption that they can be achieved under capitalism.⁶

In this chapter, and taking advantage of the recent revitalization of capitalism's critique carried out by philosophers such as Slavoj Žižek and Frederic Jameson, I take the Marxian standpoint that equity within capitalism (both as an economy and an ideology), is doomed to fail. Indeed, there should be no need for sophisticated theories to recognize this, since we are living today at the pinnacle of world economical inequality⁷ at a moment where capitalism has become what Žižek (2004) calls the "concrete universal" of our historical époque: "what this means is that while it remains a particular formation, it *overdetermines* all alternative formations, as well as all noneconomic strata of social life" (p. 3). In this way, contrary to the assumption defended by many economists (most notably Daniel Bell's *post-industrial society*) that we have arrived at a new type of society, where the new social formation in question no longer obeys the laws of classical capitalism, "late capitalism" (Jameson, 1991) signals instead that this "new society" is a *purier* stage of capitalism than any of the moments that preceded it. In this sense, every position on postmodernism in culture "is also at one and the same time, and *necessarily*, an implicitly or explicitly political stance on the nature of multinational capitalism today" (Jameson, 1991, p. 3). And this implication also goes for mathematics education research: what is the political stance towards capitalism informing the studies around the issue of equity? As will be argued in this text, the way the community engages in the research on equity is based on the assumption that despite the recognized problems in achieving equity in school mathematics, these can be overcome through an amelioration of the system (better teacher education, valorising students' learning diversity, more equitable school practices, use of technology, etc.). The system itself is not questioned. Change is conceptualized as a change within the system.

This view of change assumes that although the modern project based on capitalist economics and liberal ideologies has been not without its problems, these are contingent, conceived as "deviations" from an otherwise good system. The message conveyed is that modernity gave us the Ideals for which we should strive (equality, freedom, etc.), and our task should be to eliminate all the impediments standing in the way of their full actualization. In *The Ethics of Psychoanalysis*, Jacques Lacan (2008) describes this ideology – *evolutionism* – as implying a belief in a Supreme Good, in a final goal of evolution which guides its course from the very beginning.

⁶ The day I was writing these lines, the US government decided to bar online access to five of the world's most important newspapers to its Air Force personnel. Such censorship, in a country that professes to stand as no other for the modern ideals of equality and freedom, clearly shows the inherent limits of such ideals: they are "universal" insofar as they don't jeopardize the same system which posits them as universals. From the moment they put the system at risk, they are sacrificed. This should make us suspect the values of a system which presents itself as the pinnacle of democracy, freedom, and equality.

⁷ For evidence that this is the case see, for instance, Cole (2003).

This perspective is notably evident in the influential works of John Rawls and Jurgen Habermas. Despite their differences, they share the assumption that a theory of the social should be primarily concerned with the delineation of a set of “universal” principles that should guide our action towards a better society: Habermas’s society of fully communicative beings and Rawls’s society of distributive justice. According to Chantal Mouffe (2005), who developed a groundbreaking critique of both, Rawls and Habermas don’t deny that there will be obstacles to the realization of the ideal discourse, but those obstacles are conceived as *empirical* ones. That is, when dealing with a universal structuring principle, it is assumed that it is possible to apply this principle to all its potential elements, so that the principle’s empirical non-realisation is merely a matter of contingent circumstances. What in this thesis are subsidiary problems of a “good” system are, in Marxian theory, the points at which the “truth”, the immanent antagonistic character of the system, erupts – what Žižek calls, after Lacan, the symptom:

A symptom, however, is an element which – although the non-realization of the universal principle in it appears to hinge on contingent circumstances – *has* to remain an exception, that is, the point of suspension of the universal principle: if the universal principle were to apply also to this point, the universal system itself would disintegrate.⁸ (Žižek, 1997, p. 161)

Capitalism and its ideology posit progress, equality, and freedom as natural ideals shared by all humankind. These are presented as the goals we have to strive for – we know what we want, so the question is how to achieve it. The fact that we are today (still) living in an unequal society, where democracy has been reduced to elections and our freedom to freedom to select among a set of pre-given conditions⁹, is seen by today’s liberal-democracy as degenerations of the normal functioning of society, and as such capable of being abolished through amelioration of the system. By inventing the symptom (as Lacan (2008) put it), Marx called our attention to the fact that such “empirical obstacles” are the necessary conditions for

⁸ We can speculate how, if everyone experienced success in school mathematics, it would disintegrate as part of the school’s credit system. In the pages below, I develop further the characterization of schools as credit systems. For the moment, by “credit system” I am referring to the role schools, particularly through school mathematics, perform as gatekeepers.

⁹ In one of his many pop illustrations of ideology, Žižek explores that uncanny button placed in elevators which enables the user to close the doors, thus apparently speeding up the journey. He argues that this can be seen as a case of a placebo effect, and, as such, a metaphor for freedom in Liberal-democratic capitalist politics. In short, we are led to believe that by our personal, free actions we are making a contribution for the “progress” of society. Philipp Oehmke, in an article in the German magazine *Der Spiegel*, gives this explanation: “One of his famous everyday observations on this subject relates to the buttons used to close the door in elevators. He has discovered that they are placebos. The doors don’t close a second faster when one presses the button, but they don’t have to. It’s sufficient that the person pressing the button has the illusion that he is able to influence something. The political illusion machine that calls itself Western democracy functions in exactly the same way, says Žižek.”

(Available at <http://www.spiegel.de/international/zeitgeist/0,1518,705164-2,00.html>)

the maintenance of the system which generates them, and it is through them that we can perceive the antagonistic structure of society:

... for the standard capitalist view, crises are “temporary, correctable glitches” in the functioning of the system, while from the Marxist point, they are its moment of truth, the “exception” which only allows us to grasp the functioning of the system. (Žižek, 2007, p. 6)

In this light, the problem of inequity appears not as a contingency of a good system, but as a *necessity* of the same system that posits equity as a goal to strive for. Inequality is a necessity of capitalist economics, while equality functions as the necessary ideological supplement concealing the obscenity of what is going on. That is, the values of equity and freedom are generated by the market itself, as the necessary “double” concealing the lack of freedom and inequality of the system. Inequity is not foreign to the same system which struggles to eliminate it. Rather, it is its own motor. As Marx and Engels put it:

Exchange value, or, more precisely, the money system, is indeed the system of freedom and equality, and what disturbs in the more recent development of the system are disturbances immanent to the system, i.e., the very realization of equality and freedom, which turn out to be inequality and unfreedom. It is an aspiration as pious as it is stupid to wish that exchange value would not develop into capital, or that labor which produces exchange value would not develop into wage labor. What distinguishes these gentlemen [the Proudhonists of the time, or the Habermasians of today] from the bourgeois apologists is, on the one hand, their awareness of the contradictions inherent in the system, and, on the other, their utopianism, manifest in their failure to grasp the inevitable difference between the real and the ideal shape of bourgeois society, and the consequent desire to undertake the superfluous task of changing the ideal expression back into reality, whereas it is in fact merely the photographic image of this reality. (Marx & Engels quoted in Jameson, 1991, pp. 261- 262)

Ideals such as freedom and equity are indissolubly linked to capitalist economics which, in practice, turn out to be unfreedom and inequality. In this sense, as stated by Jameson (1991), the only thing real about the ideal is its unrealizability: “everybody wants to want them [freedom and equality]; but they cannot be realized. The only thing that can happen to them is for the system that generates them to disappear, thereby abolishing the ‘ideals’ along with the reality itself” (pp. 262-263). This is a courageous statement to make. It posits change in its *radicality*, that is, not aiming for a multitude of particular changes that, with time and faith, will allow us to correct the errors standing in the way of the Ideal, but to change the Ideal itself, that is, the whole system that coordinates our desire for equality and freedom. At stake here is the importance of conceiving, in a proper dialectical way, the relation between ideal and real: not just that the ideal cannot be actualized (an equitable economic organisation of society), this unrealization is disavowed by the presence of the ideal. That is, the ideal discourse functions as the proper staged

discourse – what we usually call ideology – that makes the real sustainable, accepted, and reproduced. We need to know that the goal for which we all strive is equality and freedom (that the presupposition of the system is a “good” one), so that we can accept the unequal reality in which we live.

Ideology and dialectics of necessity and contingency

The kind of analytical exercise carried out in this chapter conceives critique as something more than the critique of particular features of a system. Rather, it seeks to criticize the system as a whole, in this case, the whole of a system that posits mathematics and its education as crucial factors in becoming a citizen. According to Žižek (1993, p. 2), this is the most radical meaning of critique: an exercise of suspending what exists so that an experience of wondering, of conceiving possible alternatives to the system as a whole, becomes possible. However, the need to avoid evaluations of the system as a whole is now an integral part of its own internal organization as well as its various ideologies (Jameson, 1991, p. 350). Suffice to recall the recent measures deployed by governments to deal with the well-known “economical crisis”. Despite some disruptive voices calling our attention to the need for a complete re-evaluation of the current economical system as a whole, the measures taken to solve the crisis are what Paulo Freire (1998, p. 508) called “superficial transformations”, that is, transformations designed precisely to prevent any real change in the core features of Capital. In the same way that, for capitalism, achieving social equity is a matter of correcting market mechanisms (increasing taxation of private profit, governmental limitations on capital speculation, the criminalization of individual magnates, etc.), similarly the vast majority of research on equity to achieve equity in school mathematics is a matter of developing better teaching and learning strategies. In both cases, a philosophical and political critique of the system as a whole, whether economical or schooling, is foreclosed.

In this chapter, in the form of a critical essay, I seek to re-map the way the community of mathematics education research gives meaning to the problem of equity. This reconceptualization implies not accepting what exists as given, but, rather, raising the question of how what we encounter as actual is also possible. That is, it implies that we should conceive as contingent what is usually addressed as necessary. This interplay between necessity and contingency brings up one of the central concepts of Žižek’s philosophy: Ideology. According to Žižek (1994), ideology can emerge from considering what is no more than a historical contingency as a necessity. For instance, in our times the existence of schools is perceived as a necessity, yet is, in fact, the result of a set of historical contingencies (Foucault, 2003; Rose, 1999). But also the converse: ideology can emerge from considering contingent what was always a necessity. For instance, school exclusion is dealt with as a contingent occurrence of a necessary system, whereas exclusion is what is necessary to maintain school. The task of a critique of ideology is precisely to “discern the hidden necessity in what appears as a mere contingency” (Žižek, 1994, p. 4). This task is also the central purpose of a dialectical approach – to articulate necessity with contingency.

On the one hand, necessity is only understood as such (as “necessary”) retroactively, after a set of contingent episodes crystallizes and gives place to a sense of necessity. In this sense, necessity is a retroactive effect of a contingent process:

“Dialectics” is ultimately a teaching on how necessity emerges out of contingency: on how a contingent *bricolage* produces a result which “transcends” its final conditions into internal necessary moments of its self-reproduction. It is therefore Necessity itself which depends on contingency: the very gesture which changes necessity into contingency is radically contingent. (Žižek, 1991, p. 129)

On the other hand, to complete the dialectics, what appears as a mere contingency of a necessary system is to be posited as a necessary constitutive – and thereafter foreclosed as a mere contingency – of the same system. Take the slogan “mathematics for all” as an example. “Mathematics for all” has to be posited as a necessary goal if researchers, teachers and politicians are to find some meaning in their task of providing an equitable mathematics education. The fact that people continue to fail in school mathematics is seen as an “excess” introduced from the outside; its elimination would enable us to obtain a stable, inclusive school mathematics. All obstacles impeding the full actualization of the Ideal are aliens to the Ideal, thus susceptible of being overcome through a correction of “empirical” intruders. However, although the goal should be easily within our grasp, it appears as if, to paraphrase Žižek (1997, p. 164), the entire universe has somehow been adjusted to produce, again and again, the unfathomable contingency of failure blocking the full actualization of “mathematics for all”. The kind of dialectical twist I am suggesting here is one that posits this “unfathomable contingency” as a *necessity*. Žižek articulates the argument this way:

The other side of this necessity which realizes itself in the guise of a series of contingent intrusions which again and again prevent the universal notion of the project from realizing itself (...), is the necessity, the absolute certainty, that within the field of a universal Lie the “repressed” truth will emerge in the guise of a particular contingent event. (1997, p. 165)

In our case, the “universal Lie” is no more than the slogan “mathematics for all”, the “repressed truth” being the crude reality of those who year after year continue to fail in school mathematics. The systematic failure of people in school mathematics points towards the system’s antagonistic character: the condition of impossibility of realizing the common goal (mathematics for all) is simultaneously its condition of possibility. That is, what makes schooling such an efficient modern practice is precisely its capability of excluding people by means of promotion. Thus, schooling is possible (as an institution *overdetermined* (Althusser, 1994) by capitalism) only to the extent that universal schooling, where everybody will be successful, remains impossible.

The motto “mathematics for all” functions as the necessary ideological double concealing the crude reality that, as any mathematics teacher knows, mathematics is not for all. This “social fantasy”, as Žižek (1997) calls it, keeps us on the “right

track” by avoiding putting in question the system as a whole: who will dare to challenge a system that seeks the Common Good, in this case, mathematics for all?

Ideology simultaneously conceals its “motives” whilst making them actual and effective. It is in this sense that Žižek says that ideology always appears in its sublated form, that is, its injunctions make effective what it “officially” conceals. In the example of “mathematics for all”, this official claim conceals the obscenity of a school system that year after year throws thousands of people into the garbage bin of society under the official discourse of an inclusionary and democratic school. It is in this discrepancy between the official discourse and its (failed) actualization that ideology is made operational. Within the official discourse, what is *necessary* is the abstract motto of “mathematics for all”, all the exceptions to this rule (the ones who fail) being seen as contingencies. However, from the critical/dialectical discourse I am deploying here, what is *necessary* is precisely the existence of those who fail, the abstract proclamation being a purely contingent result of the frenetic activity of individuals (researchers, teachers, politicians) who believe in it. The antagonistic character of social reality – the crude reality that in order for some to succeed others have to fail – is the *necessary* real which needs to be concealed so that the illusion of social cohesion can be kept.

Mapping the “hot topics” in mathematics education research on equity

One of the most extensive reviews on the issue of equity in mathematics education is the article by Bishop and Forgasz (2007) published in the *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, 2007). The authors try to give us an overview of the different research approaches on the issues of access and equity in mathematics education. Right from the beginning, they call our attention to the artificiality present in the construction of groups of people as being in disadvantage (girls, ethnic minorities, indigenous minorities, western “ex-colonial” groups, non-Judeo-Christian religious groups, rural learners, learners with physical and mental impairments, and children from lower classes) and how such constructions can in themselves convey discriminatory actions. This problem has been recently labelled by Gutiérrez (2008) as the “gap-gazing fetish in mathematics education”, which triggered an interesting discussion between her and Sarah Lubienski, published in the *Journal for Research in Mathematics Education*, in which the two authors confronted their ideas and different strategies for approaching the issue of equity. Roughly speaking, the dilemma is how to know if research based on an achievement gap focus can benefit or not the purpose of equity. The position of Gutiérrez (2008) is that there are dangers in conceiving and performing research focused on an “achievement gap”. These dangers include:

[O]ffering little more than a static picture of inequities, supporting deficit thinking and negative narratives about students of color and working-class students, perpetuating the myth that the problem (and therefore solution) is a technical one, and promoting a narrow definition of learning and society. (p. 358)

She argues that such research, which usually leans on quantitative methods of data collection, does no more than provide a description of the problem without presenting understandings that allow us to change it. She argues that less research should be done focusing on the “gap”, and more on qualitative analyses of successful experiences among groups of people considered to be in disadvantage.

On the other hand, Lubienski (2008) assumes the position that research on the achievement gap is necessary. She is against the suggestion of Gutiérrez of lowering the intensity of gap analysis, arguing, rather, for moving towards a more skilled and nuanced analysis, since: “analysis of gaps also inform mathematics education research and practice, illuminating which groups and curricular areas are most in need of intervention and additional study” (Lubienski, 2008, p. 351).

While Lubienski is concerned with the question “Is there a gap?”, which leads to all the studies that analyse when those gaps begin, under what conditions they grow or shrink, and what consequences underserved students ultimately suffer because of the gap, Gutiérrez is concerned with the question “How to diminish the gap?”, leading to studies orientated towards effective teaching and learning, making research more accessible to practitioners, and more intervention by the researcher. Despite the difference regarding the ways to conduct research on equity, both authors share a common assumption: “For both of us, the goal is for students to gain access to dominant and critical ways of viewing the world so that they might become empowered citizens” (Lubienski & Gutiérrez, 2008, p. 367). We already start to notice how this common assumption – that mathematics empowers people by giving them the tools for “viewing” the world – functions as a *quilting-point*¹⁰ in mathematics education research.

Although we can discuss better ways to do research on equity, there is a fundamental question that cannot be left unaddressed: Why is there a gap at all? That is, why does school (mathematics) systematically exclude and include people in the network of social positioning? Why do schools perform this selective role that inevitably creates inequity? As Bishop and Forgasz (2007) note, “in every country in the world mathematics now holds a special position, and those who excel at it or its applications also hold significant positions in their societies” (p. 1149). Why does our society need to have such an institution that guarantees from very early ages an accumulation of credit? This question is rarely posed by the community of researchers in mathematics education when addressing equity. Gates and Zevenbergen (2009) state that “mathematics and social justice has been the focus of much research – however this has largely focussed on such issues as the process of learning, the content of the curriculum and its assessment” (p. 162). They also make a very suggestive point. They argue that it is common in mathematics education research to discard such “political” questions, since it is not the responsibility of mathematics education to address them (p. 165). One can argue that such a position

¹⁰ As explored by Lacan, an entity that enables us to unify in a single large narrative all the antagonisms – something which “quilts” the social edifice. In Pais & Stenoft (unpublished manuscript) we link this notion with another Lacanian one: the Master-Signifier.

removes the possibility of subversiveness in research on equity, rendering it harmless.

In the same article, Gates and Zevenbergen (2009) take advantage of Bourdieu's theory on social reproduction to theorize about social justice in mathematics education in relation to teacher education. They categorize research on social justice into three types. The first type, which they call "moderate forms of social justice" (p. 166), does not relate the failure in mathematics to structural inequalities in society. These perspectives do not challenge the status quo and show a tendency to see the social inequalities as something natural, as a result of people's different capabilities and merit. In this sense, the exclusion provoked by the school system is seen as a natural selective process of our societies. The second, "liberal forms of social justice" (p. 167), recognises structural inequalities in the way mathematics is taught in schools, and proposes as a solution more detailed and accurate research on the process of mathematical learning, teacher education, assessment, curriculum, and so on. Most of the research compiled by Nasir & Cobb (2007) assumes this perspective, the idea that social inequalities will be solved through better classroom practices. Finally, the "radical forms of social justice" (p. 167) also recognize structural inequality, as well as social class and ideology. This perspective avoids engaging in salvation discourses, and assumes that the only way for mathematics education to become more equitable is through a deep change in the class structure of society, to which we can contribute by ways of developing with students a "class consciousness".¹¹

Perhaps we should extend the radicalism of the third type. What still appears problematic is the notion of empowerment through mathematics, or why it is important to learn mathematics. Using the theoretical framework developed by Skovsmose and Valero (2008) to deal with the issue of power in mathematics education, we can say that mathematics education can empower people through the intrinsic characteristics of mathematics itself (logical thinking, abstraction); by providing students with psychologically meaningful experiences (solving problems, metacognition); by enhancing the relation between cultural background and foreground, therefore allowing students to learn "in context" (connection between everyday practices and school mathematics; providing opportunities to envision a desirable range of future possibilities); and finally students can get empowered through school mathematics by exploration of situations of "mathematics in action", which make visible the way mathematics formats reality (exploring real mathematical models in a critical way). What is missing in these four conceptualizations of the way mathematics empowers people is a fundamental element – mathematics empowers people not so much because it provides some kind of knowledge or competence to them, but because it gives people a value. It allows students to accumulate credit in the school system that will allow them to continue studying and later to achieve a place in the sun. Mathematics empowers people because it is posited as a socially valuable resource.

¹¹ See for instance the work of Marilyn Frankenstein.

This powerful dimension of mathematics is often absent in mathematics education research. One feels tempted to conceive such obliteration as the repression of a trauma. The discourses about the way mathematics empowers people through knowledge and competence disguise the traumatic role of school mathematics as a social gatekeeper. So, if one wishes to extend the radicalism of the last approach mentioned above, it must be added that it is not just that the problem of failure is a structural problem, a solution to which mathematics education can contribute to by developing a class awareness, but that mathematics education research itself is part of the problem as long as it continues to neglect the importance of school mathematics outside of knowledge and competence.

Nolan (2009) developed a critique on how the issue of social justice has been researched in mathematics education. She calls our attention to the risk of considering issues of equity and social justice just as a current fashion, and explores what in her view is missing in the current push to marry off mathematics with social justice. She starts by criticizing the usual way in which issues of social justice are translated into practices of teaching and learning, which are basically in terms of contents: “[t]he most common approach to realizing a mathematics education in and through social justice is by integrating the facts and figures of poverty, exploitation, and discrimination into ready-to-use problem-based lesson plans” (p. 207). According to her, this is the easiest approach to realizing a simplified consensus on the nature of the complex union between mathematics and social justice. Unsatisfied with this conceptualization and practice of social justice within mathematics education, she suggests that for a lasting relationship between social justice and mathematics we need to focus our attention on teacher education, by preparing teachers to develop in their classrooms practices of social justice. She then continues the article by exploring her own experience in teacher education for social justice, and all the problems involved in it.

The author makes a good point by saying that, if we wish to take social justice seriously, it is not enough to work with students’ issues where mathematics appears as a tool to explore socially unjust situations. The mathematical content approach “seems to leave the dominant characteristics and personality of mathematics intact, while molding and shaping the concerns of social justice to fit into life-as-usual of mathematics” (Nolan, 2009, p. 207). She suggests, in a similar way as ethnomathematics (e.g. Powell & Frankenstein, 1997), that we must not just deconstruct social reality using mathematics as a tool, but to deconstruct mathematics itself – its image of neutrality, social independence, universalism, and so on.

Despite her good will, producing “teachers for social justice” suffers from repeated failure. Notwithstanding the fact that prospective teachers revealed some “theoretical” interest in issues of social justice during the classes with Nolan, when they go to schools they feel that it is nonsensical to explore such issues with their students. At most they feel compelled to explore some ready-to-apply examples of mathematics as a tool to understand reality. This gap between knowing the research findings (in the case of social justice: the nature of mathematics, issues of critical mathematics education, issues of democracy, etc.) and the practice of teaching in schools is common. Although teachers can be aware of the social and political di-

mensions of mathematics, they tend to reproduce discourses that posit mathematics as an apolitical subject (de Freitas, 2004). There are many possible explanations for the fact that teachers do not simply transfer to their practices the theoretical ideas learned in the academy. Nolan suggests that to reduce this gap is a teacher education problem. In her view and in her work, the way to decrease the gap is to alter teachers' conceptions about mathematics and the role of school mathematics in social justice.

We already have research within mathematics education that shows how teacher education cannot be taken for granted. Although some say that teachers are not prepared to conveniently implement reforms, Klette (2004) takes another approach. She conceptualizes the lack of change in mathematics education reforms as an embedded part of research itself. The author suggests that researchers are an important element in the non-concretization of school reform. This idea goes against the commonsensical one, which posits the problem on the side of the practitioner. The idea is that in research everything goes well, we know the best methods, theories and strategies; the problem lies in its application. Klette criticizes this view, and argues that the denial of change is being constructed from the beginning, in the theoretical, methodological and conceptual ways in which research is done. The author suggests that more is needed than merely investigating how reforms change schools; we should investigate how schools change reforms. This is precisely what is missing in Nolan's work: not so much concern with how teachers will change classroom routines, but how classroom routines change the (weak) theoretical awareness of teachers.

Finally, I wish to argue that although the issue of equity is usually conceived as having to do with specific groups of people (women, indigenous, poor, etc.), other issues are at stake. Valero (2007) analysed a paradigmatic episode of discrimination in a regular Danish mathematics classroom. The case of the "lonely girl" (p. 227) is about Gitte, a Danish teenager who was positioned by the school administration and by her teacher as a student with learning difficulties in mathematics. The girl was isolated in the classroom, performing banal tasks like sharpening the pencils for her colleagues or picking up dropped rulers. She was allowed to be in school, but everybody unassumingly knew she wasn't learning any mathematics. The exploration of this case led Valero to assert that "disadvantage is being built even at the heart of an educational system that has inclusion and democracy as an organizational principle" (pp. 228- 229). The problem of equity is not exclusive to people who are positioned as being in disadvantage due to their association to some category (ethnicity, gender, linguistic, socio-economical, etc.). Indeed, I wish to argue for a displacement of the problem of equity that conceptualizes inequity not so much as a problem affecting particular groups of people, but a generalized problem of the school system, that affects everyone by the way schooling is involved in social stratification. The paradox is that such systematic "social selection" is happening at the core of a school organized around democratic and inclusionary principles.

CRITIQUING THE IMPORTANCE OF MATHEMATICS

Common shared assumptions: the importance of mathematics in becoming a worker and a citizen

I shall argue that beneath all the different understandings on how to deal with the issue of equity in mathematics education research, there are some common shared assumptions that function as neutral camps where everybody should agree – in Lacanese, as quilting-points. These common shared assumptions, it will be argued, convey the germ of exclusion.

The literature on equity and mathematics education is full of statements that posit mathematics as a powerful knowledge and competence, required to become a full citizen and worker. These two educational functions, that Biesta (2009) calls qualification – having to do with the need for people with the knowledge, skills, and understanding that allow them to “do something” on a professional basis – and socialization – having to do with the role of education in allowing people to become members of a particular society, by the insertion of the “newcomers” into existing social and cultural orders – comprise the two main goals of mathematics education:

Mathematics education in schools is thus seen to have a dual function: to prepare students to be mathematically functional as citizens of their society arguably provided equitably for all – and to prepare some students to be the future professionals in careers in which mathematics is fundamental, with no one precluded from or denied access to participation along this path. (Bishop & Forgasz, 2007, p. 1152)

Mathematics is posited as indispensable knowledge and competence to participate in the world – the idea that through mathematics we become empowered citizens. This idea presupposes another one: that mathematics is everywhere. In our times, it is a commonplace to state that mathematics is the dorsal spine of our high-tech world (D'Ambrosio, 1993; Ernest, 1991; Skovsmose, 1994). This paramount presence and influence of mathematics poses a challenge to mathematics education: since most of the mathematics “ruling” our world are “under the veil”, students need to critically deconstruct the way in which mathematics formats reality, so that they can socially participate as informed citizens. Such claims can be read in many articles dealing with the issue of equity. From this point of view, to guarantee equity means to provide to all students learning experiences that allow them to become full participative citizens:

Students are facing a world that is shaped by increasingly complex, dynamic, and powerful systems of information and new ideas. As future members of the work force, students will need to be able to interpret and explain structurally complex systems, to reason in mathematically diverse ways, and to use sophisticated tools and resources. (English, 2008b, p. 11)

Democratic education – accessible to all students – rests on the assumption that all students can learn, given the right circumstance, provides students with an avenue through which they can learn substantial mathematics, and, at the same time, can help students become productive and active citizens. (Malloy, 2008, p. 23)

The marginal performance in mathematics of minority students, language-minority students, poor students, and to some extent, girls have led several American scholars to raise concerns about the opportunities for members of these groups to compete in an increasingly technological world. (Nasir & Cobb, 2002, p. 91)

It is impossible to be a democratic citizen and not be proficient in mathematics. Every decision that a citizen must take requires complicated calculations. (Pearl & Knight, 1999, p. 119 quoted in Malloy, 2008, p. 24)

Indeed, this discourse on the importance of mathematics to become a full citizen is not new. Amit and Fried (2008) locate the emergence of this goal for mathematics education in the middle of the 20th century. They refer to the School Mathematics Study Group (SMSG) founded in 1958 in the United States. In one of the documents of this study group we can read:

Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. (SMSG newsletter, quoted in Amit & Fried, 2007, p. 389)

This statement seems more contemporary than ever, and it can be read, not just in research, but also in many curricular documents around the world. Its basic structure consists of two steps. First, we realise that mathematics is everywhere in our technological society. We live in a society reigned over by mathematical modelling, which influences our life decisions, most of the time without us being aware of it. Second, this “under the veil” mathematics needs to be understood if we wish to fully exercise our democratic rights. So, a mathematics education concerned with citizenship must prepare students to use mathematics in a way that allows them to participate in an informed and critical way. This is the premise behind the aim of equity: to achieve equity in mathematics education means to provide meaningful mathematics education for all students, so that they can become full workers or citizens. The ways in which this can be done could diverge, but this “neutral camp” is common to all of them. This discourse seems to be accepted as benign for the majority of mathematics education research. So, why do I feel uneasy with it?

The problem arises when one forgets that such abstract aims for mathematics education are to be implemented in a concrete reality where not everyone will manage to achieve success. If one seriously looks at the Pearl and Knight quotation above, one realises how, behind a plea for the importance of mathematics – mathematics education as a right – we are indeed constructing a barrier in becom-

ing a citizen – mathematics education becomes a duty. In the desire to provide significant mathematics education to all students, we are constructing a discourse that puts anyone who is not proficient in mathematics (the majority of the population?) outside citizenship and democracy. It is not that people need mathematics to become democratic citizens. This necessity is retroactively constructed by this discourse itself.

The dissemination of discourses like this one – mathematics as a prerequisite to becoming a citizen – ends up creating the ideological injunction that you really need mathematics to attain citizenship. Given that mathematics is socially posited as a prerequisite to be a citizen, if you fail in school mathematics you will find difficulties in school and professional advancement. Here lies the importance of learning mathematics. The proper act of enunciation functions retroactively to perform mathematics as a powerful thing. It is not that school mathematics is powerful because people use it in their daily lives; mathematics is powerful because it gives people school and professional credit. That is, mathematics is constructed as a prerequisite to citizenship because school needs mathematics to perform its role as a credit system.

What I am suggesting is an inversion of the cause-effect relation in the discourse on the importance of mathematics. At a first glance it seems that the relation between cause and effect is as follows: people need mathematics in their daily lives, so they must learn it in school (preferably by exploring “real” examples). From this point of view, learning school mathematics is a consequence of the necessity people have to use it. But what if one reverses the argument: since people need to learn mathematics in school, a discourse attesting its importance must be constructed by characterizing daily practices as being mathematical practices. Therefore the importance of mathematics in daily life is the result of a performative act that *a posteriori* performs this importance for the sake of school mathematics. From this perspective, the importance attributed to mathematics is a consequence of the need for teaching school mathematics.

The discourse on the importance of mathematics

The last statement is polemic, and requires justification. I agree that people need mathematics to become citizens, that is, to become included in the social formation. However, I suggest that the reasons why people need it are not related with mathematical knowledge or competences, but with the school valorisation that mathematics gives to people. People need school mathematics not because they will use it directly in democratic participation (as knowledge or competence), but to continue having success in school, undertake a university course and find a stable job, so that they become “normal” social beings. I argue that the importance of mathematics must be discussed not in the field of knowledge but in the field of value. Hence it will be necessary to support the claim that people do not really use school mathematics in the exercise of citizenship or as workers.

On a personal note, I begin with a story about my grandfather. He was a cultivator during the last sixty years of his life, in a rural village near Lisbon. He never

went to school. Until now, I never met anyone so knowledgeable and with such an affinity with nature. He knew everything about agriculture, from irrigation, to the calendar routines of planting all kinds of plants, not to mention the mechanical aspects of dealing with a tractor and all the heavy machinery used in agriculture. He was not just a good farmer, but also a reasonable citizen. He had an acute critical sense of politics and of the social discrepancies that pervade society. He voted and, although a solitary man, he complied with his communitarian obligations. From my point of view, my grandfather performed well his duties as a worker and a citizen. He never needed school mathematics to do so, and he was not excluded from participation in society. From the moment someone urges that if you do not know mathematics you are not a full citizen, then my grandfather, who never felt excluded, was to be found in that position. Not because he suddenly realised that he did not know any school mathematics necessary to live his life, but because someone asserted that in order to be a full citizen you need to know mathematics.¹² Then my grandfather was placed in the difficult situation of being sixty years old and being obliged to go to school and learn mathematics if he wished to continue working. This happened because new laws came up saying that people should have some kind of schoolarization if they wanted to be employed. My grandfather realised that he needed a minimum scholarship to continue doing what he did all his life. So, my grandfather went to school, not to acquire mathematical skills and knowledge, but to have a diploma attesting that he can now be a citizen.

To be a participative citizen does not have to do with how much you know about the world that surrounds you (my grandfather was a master in cultivating the land, dealing with nature in a way that remains original to me) but how the world that surrounds you – Society – recognizes, valorises or not what you know about the world. The problem of participation is not a problem of skills or knowledge, but a matter of what is valorised in our social symbolic order. And the same goes for equity. What if guaranteeing equity were not a problem of providing meaningful mathematical learning to pupils, but guaranteeing that even if someone fails in school mathematics, they will be no less a citizen and no less able to participate in society than those who succeed?

Fortunately, there has been research in mathematics education that criticizes the idea that we use school mathematics in our daily lives. The most well known example is arguably the eye-opening work of Dowling (1998). He identifies three fundamental myths that pervade the field of mathematics education. For our purpose, we will focus on the myth of participation, which has to do with the way mathematics justifies its existence in the school curriculum. Dowling states that the current trend in mathematics education “is orientated more towards the widespread dissemination of mathematical use-values: not more mathematicians, but a more mathematically competent workforce and citizenry” (p. 3). Thus, he continues, “[m]athematics justifies its existence on the school curriculum by virtue of its utility in optimizing the mundane activities of its students. This is the *myth of partici-*

¹² Although my grandfather dealt with what we can call ethnomathematical knowledge, the point is that he didn't need to realize that what he was doing involved any mathematics.

pation.” (p. 9). Dowling suggests that the way in which mathematics is incorporated in mundane/everyday practices leads to the individual who lacks mathematics being acknowledged and, ultimately, acknowledging himself/herself as being “handicapped” (p. 9). So, in order not to be handicapped, the individual must learn mathematics. But the notion of “handicapped” is already being constructed “inside” the discourse of the importance of mathematics in daily life:

However, Sewell’s assertion that a lack of mathematical skills constitutes a handicap within these contexts is untenable; in fact, mathematical skill is neither necessary nor sufficient for optimum participation within these practices. (p. 10)¹³

The myth of participation “recognizes” the operation of mathematical tools in diverse practices. It constructs a role for mathematics education in providing the toolbox and a pathological lack on the part of the yet-to-be-tutored. (p. 11-12)

What Dowling (1998) calls the myth of participation — the idea that mathematics is a necessary feature of everyday practices — ends up creating a school curriculum where mundane activities are *mythologized* in a way that privileges mathematical rather than everyday principles. That is to say, everyday activities, in order to be introduced in school, need to be amputated of all the complex vicissitudes which makes them what they are. This amputation — the result of casting the mathematical gaze onto public domains — privileges what Dowling (2001, p. 22) calls the “esoteric domain”, while, at the same time, concealing the purely fictional status of the importance attributed to mathematics.

Another useful resource is the study conducted by Riall and Burghes (2000), who gathered together employees from a wide range of industry, commerce, and the public sector. Their intention was to evaluate the extent to which these people use mathematics in their professions. They conclude that “almost the entire population of the study said that they had had to learn at school some maths that they had never then used again” (p. 110). In the voice of one of the workers, who stands for the general opinion: “I think that a lot of maths that is taught is not used in later life. I’ve forgotten most of what I had to learn and I never use it” (p. 104). Also Hudson (2008) found that the people of his study did not transfer what they learnt in school mathematics to their daily work activities. Rather, they developed their mathematical skills in the workplace. Within a philosophical approach, Ernest (2007), when discussing epistemological issues in the internationalization and globalization of mathematics education, generalizes these findings by arguing that the mathematics behind our high-tech society is just a small part of the huge amount of research being done in mathematics. Some kind of applied mathematics

¹³ Dowling is referring to the recommendations for school mathematics that emerged from the Cockcroft Committee, in which Bridgid Sewell defended the importance of percentages to operate in a shopping environment.

that ends being *routinized* and used in a “technical” way. This mathematics is learned by people in practice, outside school:

It is not academic mathematics which underpins the information revolution. It is instead a collection of technical mathematised subjects and practices which are largely institutionalised and taught, or acquired in practice, outside of the academy. (p. 31)

There are two important dimensions here. First, the argument that what people use is just some kind of “applied mathematics” does not in any way diminish the importance of research in “pure” mathematics: we could argue that it is because there is research in “pure” mathematics that we can apply more particular results. But we realise how the mathematics behind the “information revolution” is not directly academic mathematics. The second dimension involves another concern. Ernest argues that the mathematics we use in our daily lives as workers and citizens is acquired not in school but in practice. Indeed, this statement is in line with the research on the situated dimension of learning (Lave, 1988), which has been largely applied in mathematics education research. If we consider the research on the sociocultural aspects of knowledge and learning we can assert that all knowledge is eminently situated in the places where it is used, whether a workplace or an indigenous community. The meaning of some practices and knowledge is deeply involved in the community of practice (Lave & Wenger, 1991) where it is exercised and developed. There is no guarantee that people transfer knowledge from one practice to another without some kind of “misrecognition”. School mathematics, although it can explore “real” situations, will always be school knowledge, learned in a specific place called school where students are not necessarily concerned with learning.¹⁴

Also, the studies carried out in mathematics education under the auspices of Activity Theory reveal that “the activity of situated problem solving in the school context seems to be fundamentally different from decision-making in the real world because of the difference of the activity systems that govern them” (Jurdak, 2005, p. 296). This difference is reflected in the responses of the students¹⁵, who suggest that they operate under different rules when solving a school problem and a real problem, no matter if they are, from a mathematical point of view, the same (e.g., car loans or buying a cell phone): “they define their own problems, operate under different constraints, and mathematics, if used at all, plays a minor role in their decision making” (p. 296). Williams and Wake (2007) also take advantage of Activity Theory to study the relations between workplace mathematics and college mathematics. In a similar way they conclude that:

... while it is true that some mathematics can readily be identified by academics in workplace practices, we find that workplace mathematics has its

¹⁴ See Baldino & Cabral (1998, 1999) for an interpretation of why students in school engage not in learning but in passing.

¹⁵ Thirty-one grade 12 students dealing with real-life mathematical problems.

own distinct genre, inflected by the local practice and its activity system, its instruments and division of labour and power, as well as the productive goal of the whole activity. (p. 336)

From the point of view of Activity Theory, it seems that school and out-of-school practices are fundamentally different due to differences in the activity, the goal, and the operation, or the conditions under which the action is carried out.

EXCHANGE-VALUE AND CREDITATION

Shifting the importance of mathematics from knowledge to value

Thus a question should be raised. If the value of school mathematics for citizenship is deceptive, what *could* be the value of school mathematics? Dowling (1998) calls our attention to the *exchange-value* of school mathematics:

[m]athematics justifies its existence on the school curriculum by virtue of its utility in optimizing the mundane activities of its students. This is the myth of participation. It constructs mathematics, not as a system of *exchange-values*, but as a reservoir of use-values. (p. 9, my emphasis)

Exchange-value is a notion explored by Karl Marx in his book *Das Kapital* to refer to one of the two values a commodity can take. While the use-value of a commodity is strictly related to the concrete use someone makes of a commodity – the mathematical know-how necessary to perform a profession, for instance – the exchange-value posits this commodity in relation to all the others, that is, as part of a structure of equivalences where its value can be gauged. Thus exchange-value has a purely *relational* status: it is not inherent to a commodity. It expresses the way this commodity relates to all the others. When looking at a commodity, say a table, we see its use-value. What we cannot see is its exchange-value, which remains invisible. If we transpose this line of thought to school mathematics, we can speculate how the gesture of positing the value of mathematics in its use hides its exchange-value – that is, the formal place school mathematics occupies within capitalism. The fundamental gap between an object and the structural place it occupies (Žižek, 2006) is at work in school mathematics: while perceiving the importance of school mathematics as use-value, we neglect the importance of this subject in maintaining schools' functioning as credit systems.

To my knowledge, it was Shlomo Vinner (1997) who first called mathematics educators' attention to the fact that issues stemming from the recognition that "the educational system is, above all, a credit system" (p. 68) have not been dealt with in mathematics education. In Vinner (2007), the author makes a crucial distinction between the importance of mathematics in maintaining important aspects of our present life and the importance for everybody to be proficient in mathematics: "No doubt mathematical knowledge is crucial to produce and maintain the most important aspects of our present life. This does not imply that the majority of people

should know mathematics” (p. 2). And, going further, Vinner argues that the real importance for students to learn mathematics is because through a good performance in mathematics they will be able to achieve a higher social position:

I suggest that the students have very good reasons to study mathematics. It is not the necessity of mathematics in their future professional life or their everyday life. It is because of the selection role mathematics has in all stages of our educational system. (p. 3)

Despite Vinner’s plea we lack research which explicitly connects these social phenomena – shamefully associated with school mathematics – with the broader political and economical spectrum. How, then, shall we understand the relation between school and capitalism? The usual way is to conceive education as an increasingly commodified social space. It is a commonplace in critical educational studies to assert how education has become merchandise and schools some kind of corporation.¹⁶ In this view, education is conceived as something originally pure, that has been progressively contaminated by the capitalistic structure of society. Educational industries, from publishing houses producing textbooks to computer firms developing technology, see schools as a profitable market; administrators and politicians use the metaphor of schools as companies to envision ways of managing education; governments attribute primordial importance to results in high-stake tests as a means to do school evaluation and make grades and scores a matter of profit; the labour market and industries demand the production of the highly qualified people needed; all these are few examples of such a view. In this perspective, education has become capitalized, and the “solution” would be a *decapitalization* of education, to return to its original purity, based on humanistic ideas (the place to learn the cultural heritage, educating the free man, formative assessment instead of summative assessment, etc.). The purpose is to keep the capitalist logic of production/consumption outside the educational enterprise.

Despite being true, this characterization does not exhaust the relationship between capitalism and education, nor is it the crucial aspect. The problem is that school itself, more than just being contaminated by some capitalistic ideas, is the crucial ideological state apparatus in the reproduction of capitalism (Althusser, 1994). Education in its *schoolarized* form has in its kernel the capitalist logic.¹⁷ It is against this background that we should conceive education not as being contaminated by capitalism, or a part of capitalism, but as sustaining the capitalist sys-

¹⁶ For instance, D’Ambrosio (2003) explicitly compares school with factories, where people are components of big machinery that aims for uniformity.

¹⁷ From a strictly economical perspective, schools have been performing a crucial role, without which our current mode of living could not be possible. Schools guarantee a place where children could be deposited when their parents go to work. But is not just a matter of “guarding” children. It is also a matter of sorting them, by means of stipulating who is capable of performing specific roles in society. Therefore, at the same time school performs three crucial economical functions in our societies: it guarantees a space where parents can put their children so that they can work, keeps children away from production while sorting them.

tem itself, by assuring its reproduction. Education is not just a product (education as a piece of a profitable market) but a means of (ideological and material) reproduction.

However, this role of school as an ideological apparatus is concealed by means of a “naturalization” of schooling. Capitalist ideology represents school as a neutral environment purged of ideology:

[W]here teachers respectful of the “conscience” and “freedom” of the children who are entrusted to them (in complete confidence) by their “parents” (who are free, too, i.e. the owners of their children) open up for them the path to the freedom, morality and responsibility of adults by their own example, by knowledge, literature and their “liberating” virtues. (Althusser, 1994, p. 20)

This concealment is essential to maintain the role of school as an ideological state apparatus. Seeing school as a place free of ideology disables bringing ideological struggle to school. All enterprises undertaken by teachers to unmask the “invisible” ideology are immediately accused of being ideological acts. In this way, the dominant ideology ensures that no ideology is present in school except, of course, the dominant one. The dominant one is precisely the one which presents itself as ideologically free, by positing the importance of mathematics as knowledge and competence.

One of the few exceptions within mathematics education research to acknowledge the importance of mathematics not as knowledge or competence but as exchange-value is the work of Roberto Baldino and Tânia Cabral. They have been, to this reader’s knowledge, the only persons in mathematics education who have been analysing schools and mathematics education as part of capitalist economics (Baldino, 1998a, 1998b; Baldino & Cabral, 1998, 1999, 2006). Their suggestion is that we should look at school not so much as a place of knowledge but as a place of production. Very briefly, if we consider salary as school credit, and work as the presence of students in school, then, taking into consideration that students who fail do not receive any credit (diploma), the salary/school credit is not equivalent to the student’s work in school. There is something that is missed by the student. Even though he spent all the year in school, went to classes, carried out all the regular activities, if after everything he does not get approved, he will not receive anything for all the work he did:

Only students who get certificates recapture their labour force. This labour force embodies the work done by all, by those who flunked, by those who abandoned the course, by those who could not buy a higher education and remained at the lower levels of the pyramid. Graduates get higher salaries because their labour force embodies more value, more work done by themselves but, mainly, by others who were left behind. (Baldino, 1998a, p. 43)

From this perspective, failure is a school necessity. It is because some of us fail that others can achieve higher positions in social hierarchies. The value of the ones who flunk is appropriated by the ones who pass as surplus value. At school the stu-

dent learns, above all, to participate in and accept the conditions of production and seizure of surplus value. Failure is posited as a *necessary* condition for schooling: “in order to perpetuate the process of production/seizure of surplus value, a certain amount of failure is necessary” (Baldino, 1998a, p. 5). Therefore, “failure of students means success of the institution” (Baldino & Cabral, 2006, p. 34).

Some people justify inequity by saying that since some perform better than others, they should be compensated for it. This argument presupposes that schools are places where equal students meet freely, and where some kind of “invisible hand” guarantees that the competition of individuals’ egotisms works for the common good. What such an approach makes invisible is that such merit is possible only by the demerit of others, i.e., the notion of personal merit is only possible as long as others fail. This is the capitalist ideology at work, by means of making individuals recognize their choices as their own, as free choices that they took – especially when these choices imply failure.¹⁸ All the work a failed student produced, all the time he spent in school, is not his (since he will not receive the diploma at the end of the year) but without producing it would be impossible for him to keep living.¹⁹ Individuals must realise failure as the result of a wealth competition among equals, and repress the traumatic truth that they fail so that others can succeed.

The key element to be noticed here is that schools need this subversive supplement in order to retain their indispensable role in maintaining our democratic and inclusive society. In order for school to be the most important ideological apparatus, to function as a credit system, it is not productive for it to be presented as an exclusionary institution. That would cause criticism from the whole of society, and would be unbearable from an educational or political point of view. In order to perform well in the role of credit systems, schools need to be presented as inclusionary and emancipatory places, places where phenomena such as exclusion and failure are seen not as necessary parts of the same system which purports to be trying to abolish them, but as contingent problems, malfunctions of an otherwise good system.

The materiality of exclusion

In the article *Inclusion and diversity from Hegel-Lacan point of view: Do we desire our desire for change?* Baldino and Cabral (2006) create a parody concerning where one can find exclusion in school. The authors suppose that we enter an elementary school and ask the staff where the so-called “exclusion” is happening. Who will be able to answer such a question? Where to locate exclusion in schools?

¹⁸ People accept this inequality because the dominant ideology conceives them as self-conscious subjects (Althusser, 2000). That is, the worker who works all his life and ends up with nothing sees his misfortune as a natural consequence of the way economical relations are built. He can even blame himself for not having worked enough, for lacking initiative, for being an unlucky guy; or blame some “sublime” (Žižek, 1989) group such as immigrants or Jews in Nazism – he naturalizes his poorness. Capitalist ideology must conceive subjects as owners of their own actions, as individuals completely conscious that their misfortune cannot be imputed to anything other than their own lack of capacity to succeed.

¹⁹ We should keep in mind that schooling is not free but compulsory.

It seems as if exclusion has no “materiality”, no precise site where it is happening. It seems as if it is a name to represent some structural impalpable reality, resulting from several complex factors, having to do with teacher engagement with the students, with the quality of the mathematics learning, with issues of race, gender, and social class, with lack of resources, and so on. Equity is understood as a complex phenomenon involving several dimensions, not identifiable in some place or in some practice. From this perspective, achieving equity means to fight in different battles (for groups of people considered to be in disadvantage, inequity of resources, teacher formation, mathematical content for social justice, etc.).

I argue that such dissemination of the problem of inequity disavows its materiality. Although exclusion may be related with all these different aspects, one should insist that exclusion has a materiality visible in assessment. In the story of Baldino and Cabral (2006) we have the chance to meet a special girl:

Suppose we enter an elementary school and ask the staff where the so-called “exclusion” is happening. We will get no answer, but if we are lucky to meet the child who told us that the king was naked she will take us directly into Mr. Smith’s office where the teacher is grading students’ final exams. (p. 33)

Let us take a similar situation, more akin to my own experience. The girl will take us to the final evaluation meeting of the year where all teachers of the class will present their grades and decide who will pass and who will flunk. From my experience as a teacher, I felt the final moment of evaluation as a disheartening one. It is a commonplace among teachers to wail about their vain efforts to promote success among their pupils: “I don’t know what else I could have done?” A sense of disbelief falls on teachers. But the year is at the end, and vacations are imperative now. So, they forget all the unrewarding strategies and, at the beginning of the next year, they appear again optimistic, with lots of new ideas to promote success among their students. Next year, touched off by a kind of compulsive bias for repetition, they start all over again. As Baldino and Cabral (2006) suggest, we can distinguish between two kinds of teachers. The “healthy-but-not-ethical” one, who continues his work trying to improve teaching, alleviating the suffering of the most oppressed, mitigating segregation, but who realises that the problem of inequity will not be solved by such actions: “it does not solve the problem, but, they say, it is the most we can do” (Baldino & Cabral, 2006, p. 31). And the “obsessive” one who emphatically defends that mathematics should be for all, assuming the role of students’ saviour from the dark reality of failure in mathematics. This teacher assumes that it is possible to solve the problem of inequity through his practice, by endowing students with sufficient mathematical skills as to lead to full social equity – he completely endorses the (hysterical) societal claim that mathematics is the key for full citizenship. However, at the end (in the traumatic moment of final evaluation) he fails again in providing success to all students, and repeats the process again. About them, Baldino & Cabral (2006) state that:

We might, perhaps, be tempted to praise these people as true heroes of mathematics education, the ones who refuse to lose their hope. However, if equity is an economical problem that escapes school, at least in the short run,

the society's demand is really the demand of the hysterics and, as such, impossible to satisfy. (p. 31)

This desire for repetition reflects the desire for not knowing the deep roots of failure. The demand of society upon school is a hysterical demand, because it is impossible to satisfy. To such impossible demand the teacher responds with an obsessive behaviour of repetition of failure. This demand of society conceals a specific purpose: "the pleasure of this gaze [society; super-ego's gaze] in looking at the obsessive teacher's failure is that, in so doing, it is able to avoid looking at other issues" (Baldino & Cabral, 2006, p. 31).

These "other issues" are the facts that assessment is exclusion and these evaluation meetings are places of judgement. In them, teachers decide which students will continue to the next year, and who will be left behind, to repeat the same year sometimes three, four times²⁰. In such places we witness all the materiality of exclusion. However, this materiality is not experienced as such. It is unbearable for the teacher to conceive himself as entrusted with the smooth functioning of school as an excluding machinery. Teachers cannot live with the guilt feeling of perceiving evaluation as judgement, as pure exclusion/inclusion. Here begins all the process of "disavowing" (we can hear things like: "I would like to pass him but all his test grades were negative" or "he didn't acquire the competences necessary to proceed to the next year"). What these arguments disguise is the fact that in evaluation we are not evaluating content or competences, but people. This displacement is the fetish of evaluation, whereby we have to forget that we are evaluating people and act as if we were evaluating objects (contents, competences) in order to avoid the guilt feeling. This fetichistic attitude²¹ enables the teacher to avoid the confrontation with the trauma of acknowledging that assessment is exclusion.

Concerning the absence of research that specifically addresses assessment as exclusion, one can find support in the study of Marshall and Thompson (1994, quoted in Baldino, 1998b) that surveyed six at-the-time recent books on assessment. There was no reference in those books about the implications of assessment for social promotion and selection. Baldino (1998b) conjectures that this absence is due to a belief that social selection is a natural consequence of the various evaluation processes incorporated in society. That is, even though we acknowledge the role of selection perpetrated in school we address it as an anomaly, as something that, with

²⁰ Martin Willis called my attention to the fact that exclusion is also happening when students aren't barred each year. For instance, in the United Kingdom education system there is no opportunity to repeat the same year. Those students who fail still progress to the next year along with those who pass. However, as mentioned by Martin, exclusion is still happening: by allowing students to 'move up' to the next year without having succeeded in the previous year it sets them up to fail again and excludes them from the beginning of the new school year. And, we can argue, this exclusion works in a more efficient way since we are dealing here with "exclusion without excluding", a veiled exclusion and, as such, more effective.

²¹ Namely, forgetting that through assessment we are deciding students' lives; that we are dealing not with students' knowledge but with people; conceiving assessment as a treatment or as a measure for merit and knowledge.

time and research improvements, will be solved by constructing the evaluation instruments that guarantee success to all children:

They [studies on assessment] seem to feed the hope that trustful evaluation procedures in mathematics could contribute to the edification of a just society: to each according to his/her merit. In fact, an ideology of justice and an implicit validation of instructional objectives is observable at the basis of most research about evaluation. (Baldino, 1998b, p. 1)

It is beyond the scope of this chapter to present an analysis of how studies on assessment do not address assessment as promotion. However, I suggest looking at *Topic Study Group 36* in the *11th International Conference in Mathematics Education* (ICME11), dedicated to *Research and development in assessment and testing in mathematics education*.²² The reading of the aims and focus of this study group shows that the emphasis of research is not on assessment as a promotional mechanism but on assessment as part of the learning process. The text emphasizes the importance of moving from assessment *of* learning to assessment *for* learning. According to the team members of this study group, in the last fifteen years, assessment and testing have been evolving in the direction of addressing the need of the student, to help him or her learn better, rather than making judgments on the achievement of the student. The challenge for the mathematics education community is how to get social recognition for this new role of assessment.

However, in the last years we have been witnessing an increasing concern with measurement within education. Biesta (2009) argues that we are living in an age of measurement in which pressure is put on teachers, schools, and governments to increase educational results measured by mass-scale comparative studies such as the *Trends in International Mathematics and Science Study* (TIMSS) and the OECD's *Programme for International Student Assessment* (PISA). These international, comparative studies are to an increasing extent brought into the political sphere, placing pressure on national governments to regulate their educational systems according to the standards stipulated by those tests (Biesta, 2009; Wilson, 2007). This is what has been happening in the last eight years in very many developed countries where education tends to be transformed, by the pressure of politicians' demands for accountability, into an evidence-based profession. Consequently, political measures contribute to formatting teaching and learning of mathematics in a clear and crude way. Teachers tend to tailor their instructional practices to the format of the test out of concern that if they design their teaching differently, their students will fail. Although they might know all the didactical novelties and methods to promote learning in a way meaningful to the students, if what counts is to pass the test, that is how they will "educate" their students (Lerman, 1998; Wilson, 2007).

This scenario contrasts with the scenario imagined by the text of the ICME11 study group. On the one hand we have mathematics education research's call to reduce the importance of making judgments on the achievement of the student. On

²² The work developed by this group is available at <http://tsg.icme11.org/tsg/show/37>

the other hand, we have a societal trend that puts pressure on teachers and schools to reduce the educational process to a promotional process. I argue that studies on assessment should make this antagonism visible, and avoid engaging in discourses that, by harmonizing assessment and promotion, ended up disguising the promotional role that assessment has today.

I agree that assessment is indispensable for learning. A teacher needs to constantly assess student's learning so that both can move forward in the educational process. However, assessment is just one part of evaluation. The other part is promotion, which is related with what we value when we decide the grade that should be attributed to a student. To grade a student is to materialize in a mark the (school) value of that person. That mark will have consequences for the student's future, both in terms of school and professional life. This part of evaluation is not about assessment, but about promotion. When we "blur" assessment and promotion, and start conceiving of promotion as the "fair" consequence of a good assessment, then we no longer have to be worried about exams or other forms of classifications, since students are being evaluated strictly for their mathematical knowledge, that has been developed by means of assessment for learning. As a consequence, school as a credit system is completely buried in oblivion in the large majority of the studies on assessment in mathematics education.

WHERE DOES THIS TAKE US, IN TERMS OF EQUITY?

Reaching equity

De Freitas (2004) has been an important voice in mathematics education research working in the field of teacher education. She does not take for granted the work of teachers in schools, and she is aware of all the structural constraints that can compromise good intentions of promoting mathematical success among students. She is also aware of the connections between school and political ideologies, and how the problem of inequity extends beyond the educational sphere:

The goal of learning is no longer elite class privilege and intellectual awakening. The goal is market expertise and infinite consumption. (...) Mathematics becomes the handmaiden to yet another industrial agenda, an agenda outlined by aspiring, managerial, corporate needs. (...) Mathematics becomes a means of transforming experience into bits of information, and concurrently mutates learners into consumers. (pp. 264-265)

In de Freitas and Zolkower (2009), the authors address the issue of how to prepare prospective mathematics teachers to teach social justice. Although the authors acknowledge all the social and political dimensions of the problem of social justice within education, they engage in an obsessive task, stating that: "our central argument in this article is that a social semiotic theory of learning, in the hands of teachers, contributes to interrupting the cycle of inequity reproduced through education" (p. 191). The problem of inequity, previously described as a political prob-

lem, can be solved by changing teacher education. I criticize this minimal political reduction that posits a solution to the problem of equity both in the hands of teachers and in better theories for learning. I must say that I found the research conducted by De Freitas and Zolkower of high value, especially the way in which they deconstruct teacher discourses, showing how they can convey social prejudices. However, there is the risk of falling into salvationist discourses that displace the problem from its political dimension. Especially that is so, if we bear in mind the huge gap that exists between research and practice, and how most of the time this research comes to the teacher already “institutionalised”, and is seen by teachers as another research novelty.

In the research carried out by Nolan and previously described, this obsession to promote equity and the repeated failure that follows it becomes visible. As Nolan (2009) insistently admits, her effort to promote a teacher education for social justice suffers from repeated failure. What she seems not to realise is that this repeated failure, cause of anxiety, is originated by a fundamental antagonism that remains unaddressed in her work: the antagonism between the societal demands for equity and the role of schools as exclusionary institutions. In the last paragraph of the article, after describing all the failed attempts to educate students in her own image (as people concerned and willing to implement social justice within their classrooms), something uncanny but highly revealing occurs:

Finally, a break-through moment. On one final course evaluation, a prospective teacher wrote:

I finally get that the way [the instructor] taught her class WAS about social justice... that teaching mathematics about, or though, social justice isn't just about poverty statistics and world population figures... it's also in the thoughts and actions of the teacher towards his/her students and in the thoughts and actions of students toward each other. It's about feeling safe to be who I am and, at the same time, to critically question who I want to become and what (and who) I value. And, most of all, I think it's also about opening up the content of mathematics (what and how we teach) to this same kind of critical questioning. (pp. 214-215, my emphasis).

What appears to be a light at the end of the tunnel, a sign of hope that gives strength for the researcher to continue her work towards social justice is ... a dream: “Ok, so that’s a lie. No one actually wrote that on the course evaluations. This is the fictional part of my story – it’s the opportunity I am seizing to convey my dream for mathematics in and through social justice” (p. 215). What if the whole truth of her story is precisely this dream? In these last words we realize all the phantasmatic support that grounds Nolan’s desire to educate teachers for social justice – the total fantasy that disables us from approaching directly the core of the problem, that is, the traumatic kernel of school inequity.

But the solutions presented by research in equity are not restricted to teacher education. Gates and Zevenbergen (2009) identify a common basis for such measures:

What might we all agree on then as fundamentals of a socially just mathematics education? Perhaps we can list: access to the curriculum; access to resources and good teachers; conditions to learn; and feeling valued. (p. 165)

The first thing that is evident here is the complete absence of a political conceptualization of equity. What is recognized as an economical and political problem, ends up being addressed in a technical fashion: better ways to teach and learn mathematics for all students. Another thing that looms up is the complete obliteration of the role of assessment in exclusion as was previously described.

We tend to see the problem of inequity as a problem of achievement gaps, of teacher education, of curriculum applicability, and other scientific categories. These ideas are well established in research. According to Schoenfeld (2002, quoted in Langrall, Mooney, Nisbet, & Jones, 2008, p. 127), to achieve equity requires four systematic conditions to be met, namely, 1) high quality curriculum; 2) a stable, knowledgeable, and professional teaching community; 3) high quality assessment that is aligned with curricular goals; and 4) stability and mechanisms for the evolution of curricula, assessment, and professional development. Lubienski (2002) claims that, as far as the issue of equity is concerned, the goal is to learn more about the complexities of successfully implementing meaningful instructional methods equitably with students who differ in terms of social class, ethnicity, and gender. Or, according to Goldin (2008) “to create teaching methods capable of developing mathematical power in the majority of students” (p. 178). The problem of equity is reduced to a problem of developing the best “instructional methods” to allow mathematical success to all students.

Langrall *et al.* (2008, p. 118) take a more sophisticated approach, which they generalize to the whole mathematics education community:

Mathematics educators today (...) recognize clear discrepancies among the *desired curriculum* – as it exists in a national goal statement or a ministry of education syllabus, the *implemented curriculum* – as it plays out in classrooms, and the *achieved curriculum* – in terms of what children learn. Ultimately, while these inconsistencies remain, we cannot guarantee that all elementary students will have access to powerful mathematical ideas. (p. 118)

The idea that we will achieve equity when these three phases of curriculum implementation coincide presupposes the idea of society as an organic whole, within which people become what Society stipulates as the “ideal citizen”, propagated by the curriculum. The discrepancy between the three levels is seen as a malfunction of the system. When we manage to fix this, all students will have access to powerful mathematical ideas. Equity will be achieved when those three curriculums become one and the same. What I think is highly problematic in this approach²³ is the evolutionistic thesis that it conveys. The idea is that inequity and social exclusion

²³ But also the fact that authors are trying to close the irreducible gap that exists in all social structure. The existence of this fissure – between what society wants us to be, and in what we actually become – is Real in the Lacanian sense, and cannot ever be closed. What it can be is disguised, disavowed. These three levels only coincide in an absolutely neurotic society.

are *still* associated with school mathematics and that, through research and practice, it will be possible to overcome these problems: we already know the right path (make the three levels coincide), and what remains is a particular question of how to do it. In these circumstances, the question of equity is not a political, economical question. It is transformed into a “technical question” – how to reduce the gap between the three levels of curriculum implementation, so that students act and think as stipulated by Society. The fact that equity is a political problem is dismissed.

This refusal to confront the real core of the problem of equity can be seen as the result of an ideological injunction that systematically leads us to repeat the same “abstract” discourses – school as a place for emancipation, mathematics as a powerful knowledge and competence, mathematics for all, etc. In order to critically analyze such discourses we should replace the abstract form of the problem with the concrete scenes of its actualization within a life-form (Žižek, 1991, p. 145). That is, in order to understand which are the real aims for school mathematics, or the real motives that students have for being in school, we must not repeat ideologically loaded discourses conveyed by the curriculum, by political statements, and even by research, but rather look the “negative” (schools sorting future people for the labour market by means of credit accumulation) in the face and convert it into research problems. Following Žižek’s (1989) thought, this implies research should pass from the notion of crisis (in this case, the fact that people fail in school mathematics therefore creating exclusion) as an occasional contingent malfunctioning of the system to the notion of crisis as the symptomal point at which the truth of the system becomes visible. In this case, what is revealed is the inconsistency of a system that, on the one hand, demands mathematics for all and, on the other hand, uses school mathematics as a privileged mechanism of selection and credit.

I argue that research in mathematics education is destined to repeated failure if it continues to avoid facing exclusion in its materiality, and restricting research to a “technical” enterprise. Although issues of equity, social justice, inclusion/exclusion, can be addressed in a multitude of forms, its fundamental structure has to do with the process of schooling itself. It is the very “nature” of schooling that carries the germ of exclusion. In the first pages of this chapter I told an anecdote about a guy who lost a needle. Why is the guy searching for the needle in the kitchen if the needle is in the bathroom? Because in the bathroom, since there is no light, he can’t “work”. Therefore (because he has to work) he goes to the kitchen where there is light. Of course no needle will be found there, but does he really want to find the needle, or is the whole purpose of his “work” just to keep him occupied by repeating again and again the same procedure? From what has been said in this chapter, the analogy between this guy and mathematics education research seems obvious. As Baldino and Cabral (2006) suggest, the purpose of our desire is not really to solve the problem of equity (since we know that its causes lie elsewhere) but to keep us occupied, thus guaranteeing that things remain the same: “It appears that the true goal [of research in mathematics education] is repetition: repetition of teaching attempts, repetition of research issues, repetition of explanations; in one word, repetition of failure” (p. 30). The authors raise the question: do we desire our desire for change?

A dialectical materialist approach to the problem of equity

In the remaining pages, a philosophical background to the problem of equity within mathematics education will be suggested. I shall give consistency to the claim that exclusion is not some marginal problem of schooling, but its constitutive element. This approach can be called dialectical materialist, having as reference the philosophical work of Slavoj Žižek.

Mathematics education research acknowledges that schools can represent the opportunity to succeed in life, and can also represent the place that marks you as “disposable” (Skovsmose, 2006). And mathematics is right at the centre of this selective process. Assuming this background, we have two ways of contextualizing the problem of exclusion and the aim of equity in mathematics education. For the first one, in line with the evolutionistic world-view addressed at the beginning of this chapter, school is a necessary institution in our societies and mathematics, being one of the biggest achievements of mankind and the basis of our high-tech world, that should be accessible to all people. Assuming this premise, the problem of inequity is seen as a malfunction of the school system. The fact that school mathematics is dishonourably involved in processes of social exclusion is understood as an obstacle for the full actualization of an equitable society. The focus of research and politics is then, first, to improve the studies on how mathematics excludes people from social life, by studying the peculiarities of some groups of people (based on gender, ethnicity, socioeconomic status, etc.) in their relation with school mathematics. The premise is that when we understand better the mechanisms involved in the exclusion of groups of people we can implement strategies that will allow them to achieve success in mathematics. Second, in a political dimension, we intend to create an international mechanism that could measure the performance of each student, so that we can address the problem of exclusion properly. The idea is that we *still* have inequalities in school mathematics. Nevertheless we are on the right path, which just has to be polished.

I suggest another approach. What if exclusion is not an occasional and contingent phenomenon that will be ameliorated by research and political efforts, but a symptom in which the “truth” of the system becomes visible? That is, what if the exclusion associated with school mathematics is not a particular negativity, a vicissitude of a “good” system, but, on the contrary, represents a glimpse of what the school system really is: a credit system with the main goal of social selection by means of deciding who is capable and who is disposable? This picture of schooling is in line with broader liberal-democratic politics:

All phenomena which appear to liberal-democratic ideology as mere excess, degenerations, aberrations – in short: signs that the liberal-democratic project is not yet fully realized – are *stricto sensu* its symptoms, points at which its hidden truth emerges. (Žižek, 1991, p. 270)

To better explain this conflict between the *particular* – school exclusion – and the *universal* – school equity – I present some considerations using Žižek’s philosophical theorizations, especially his analysis of the Hegelian concept of *negative particularity*. The universal law dictates that school is a benefit: all children should

go to school and experience success. The negative particularities are particularities that do not follow this universal rule; they are seen as exceptions, aberrations of the system, such as, for instance, the fact that school provokes exclusion. There are two different ways of dealing with the dialectics of the universal (schooling) and the particular (exclusion). The first one, which Žižek (1991) calls traditional, faces exclusion as a passing moment of a universal process of schooling: “a passing moment of the law’s mediated identity-with-itself” (p. 33). It has been argued in this chapter that mathematics education research conceives the problem of equity in this way, as is evident in the research discourses about equity. These discourses convey the idea that we still have inequity, which is understood as a malfunction of a system that aims at universality – mathematics for all. Exclusion is perceived as an “error”, something that defies the universal rule and, as such, something that needs to be repressed for the universality to constitute itself. What follows is a multiplication of efforts to insert exclusion (the traumatic kernel of social reality) into our symbolic order by means of conceiving it as achievement gaps, possible to solve by means of better teacher education, as a characteristic of certain groups of people, by improving classroom practices, etc.

The second way, according to Žižek (1991)²⁴, states that universal schooling is nothing but universalized exclusion, exclusion brought to its extreme, “to the point of self-negation” (p. 38), whereby the distinction between exclusion/schooling collapses into exclusion. Schooling or inclusion “dominates” exclusion when some “absolute exclusion” particularizes all other exclusions, converts them into “mere particular exclusion”, in a gesture of universalization by means of which an entity turns into its opposite. That is, the logic of the universal always carries with it some fundamental exception which is a precondition for its existence. This was first acknowledged by Marx:

Marx’s key theoretical achievement, which allowed him to articulate the constitutive imbalance of capitalist society, was his insight into how the very logic of the Universal, of formal equality, involves material inequality – not as a remainder of the past to be gradually abolished, but as a structural necessity inscribed into the very formal notion of equality. (Žižek, 2005, p. 183)

In other words, (school) inclusion and (school) exclusion are not two opposite poles, “struggling” with each other, in which we take the side of inclusion. Inclusion already presupposes exclusion – it is only by means of exclusion that the notion of inclusion can be made effective: “An entity is negated, passes over into its opposite, as a result of the development of its own potential” (Žižek, 1991, p. 180). In the case of equity and exclusion, we can say that equity is negated, passes over into its opposite – exclusion or inequity – as a result of the development of its own potential. It is the development of a school aimed for inclusion and equity that produces inequity. This dialectical approach brings into the light of day the forgotten reverse of equity – the way equity itself coincides with supreme exclusion: “the Universal is the domain of Falsity *par excellence*, whereas truth emerges as a par-

²⁴ The example he uses as illustration is law (universal) and crime (particular).

ticular contingent encounter which renders visible its ‘repressed’ ” (Žižek, 1991, p. 196).

Žižek calls this kind of dialectical twist – from contingency (exclusion as an obstacle to equity) to necessity (exclusion as inherent to equity) – an *ideological anamorphosis* (1997, p. 97). In the case of equity, what first appears as an external obstacle to the universal idea of equity – exclusion – is revealed to be an inherent hindrance, i.e., an outside force turns into an inner compulsion. We do not have two different and opposite identities – exclusion and inclusion – so that by eliminating the former we can achieve the later. Rather, both exclusion and inclusion are part of the same identity, they coexist, not because we are doomed to live in a society that excludes people in it, but because to stop (school) exclusion means to put an end to a society wherein school, by means of universalized exclusion, is universal itself. The negative force exclusion has in school (seen as a place for inclusion) is its essence – universalized exclusion. The true antagonism is not between school exclusion and school inclusion, or between equity and inequity, but between this antagonism itself and the end of school as a credit institution.

Against this background, exclusion is not a stumbling block to the emergence of a fully inclusive school but a necessity, insofar as it maintains the illusion that it is exclusion itself that prevents the establishment of a fully equitable school. Exclusion functions not as an obstacle to full schooling, but as a “filler”, as something school needs in order to affirm its identity. The traditional way of positing the problem of inequity conceals the fact that achieving equity is not about dealing with some particular forms of exclusion, but to a matter of addressing school as a capitalist state apparatus.

If one assumes the traditional explanation, then our work as mathematics educators worried about the issue of inequity will be to develop strategies that promote a better teaching and learning of mathematics for all people. But if we assume that the role of selection performed by school mathematics is at the core of schooling, as a universal particularity (in which we can see the whole purpose of the system), then we must think more broadly and address social and political structures that are usually taken for granted. Žižek suggests that “one undermines a universal ‘thesis’ by means of exhibiting the ‘stain’ of its constitutive exception” (Žižek, 1991, p. 160). This gives us some guidance for addressing the problem of equity. In order to undermine the universal thesis that provides the apologia of a school for all, we must exhibit how exclusion exists at the core of the universal rule. We need to posit exclusion as an intrinsic quality of school.

FINAL COMMENTS

Some will say that such an awareness of the problem takes us to a deadlock. Indeed, by realising that exclusion is something inherent to the school system we realise that to end exclusion means to end schooling as we know it. In the current matrix of world social organization this does not seem possible. Thus, as raised in discussion with the editors of this book during the review process, what should be done?

I would like to think that in our day and age, in which universities are being transformed into service provider companies, there can still be a place in them for contemplation. In our frenetic academic life, it is not easy to find the time and the will to contemplate. Partly because one of the necessary requisites for contemplation is the absence of a concern with the applicability of our thoughts, in these days in which time is money, some will ask: why lose money with all this philosophical/contemplative waste of time? The ethos of scientific research today makes plain that empty words are not enough; we must set to work, do it instead of just talking about it. What we need, some say, is engagement in action, quick solutions ready to be implemented, evaluated and, eventually, discarded, so that the entire process can start again. I argue that this pressure to produce “solution-based” research is part of an ideological injunction to keep us occupied with specific research, while neglecting research that is not immediately concerned with providing solutions but rather to complicate the usual ways we approach problems. Some would say that such an approach to research will lead us into a state of paralysis, lost in an endless discussion from which no practical solutions, no “insights for action” will emerge. My response is that the true act sometimes could be a purely “inactive” one. I strongly believe that sometimes the best way to act is to stop “acting” – in the sense of doing research that immediately implies some kind of action – and ruminate. Žižek (2006) expresses this attitude as follow:

The threat today is not passivity but pseudo-activity, the urge to “be active”, to “participate”, to mask the Nothingness of what goes on. People intervene all the time, “do something”; academics participate in meaningless “debates”, and so forth, and the truly difficult thing is to step back, to withdraw from all this. Those in power often prefer even a “critical” participation, a dialogue, to silence – just to engage us in “dialogue”, to make sure our ominous passivity is broken. (p. 334)

In this chapter, I sought an analytic understanding of how the issue of equity is being addressed in mathematics education research. In order to carry out such an analysis, I mostly deployed the recent revitalization of Hegel and Marx by Slavoj Žižek. My goal was to redefine the theoretical and political coordinates mapping the way mathematics education research engages in the issue of equity and to give a different explanation for the existence of inequity within a system (school) which stands for inclusion and democracy. If the theorization I advanced leads to paralysis it will not be the worst of evils. It would be worse to keep the current state of affairs, wherein huge amounts of resources are disbursed in innocuous research, which has not been proved to have the solutions for the core problems of the field. Indeed, if teachers refuse to participate in school promotion, and if researchers reserve more time for contemplation instead of complying with market demands for *fastresearch*, perhaps paralysis would have a very disruptive effect. As put by Žižek above, the worst threat for the system today is not “activism”, but passivity: the refusal to comply with more of the same.

The same capitalist system that needs social exclusion to reproduce itself stimulates the frenetic production of research aimed to eliminate such exclusion. Such

research is usually performed as a design of what to do (e.g. Cobb, 2007), planning an entire set of “guidelines for action” both for researchers and for teachers.²⁵ My call is that if we really desire our desire for change, the task ahead is much more painful than following “What to do?” answers. And the first step is, in line with a long critical tradition from Marx through Adorno and Foucault, thinking of the present as a way of thinking about change. From the moment we critically think about the present – in the sense of suspending what exists so that a sense of strangeness towards the present can appear – we are already changing it. This kind of posture is usually marginal in research. Most of the times we accept the rules of the game (we know the needle is not in the kitchen!), and avoid a questioning of the whole game. My intention was precisely to question the whole game, that is, to realise that all the particular problems we are trying to solve within mathematics education don’t change if the school credit system as a whole remains unaddressed. We realise the wilderness of the task ahead!

It appears that mathematics education is confronted with a challenge similar to that of psychoanalysis.²⁶ The latter challenge is ultimately the theory of why its clinical practice is doomed to fail (Lacan, 2008). And, along the same lines, perhaps mathematics education should be the field of research of why people are doomed to fail in school mathematics, and not so much, as it is today, the field of research on how people should achieve success. If we assume that mathematics education as a field of research exists with the purpose of facilitating the universal goal of mathematics for all, then it would become obsolete if school mathematics ceased to be the headache that it is. In this sense, mathematics education research should be concerned not so much with integrating and domesticating the excess (the ones who fail) that resists a society of full democracy and citizenship, but in developing research around the fact that mathematics education systematically fails in achieving the universal goal. That is, to study why inequity persists²⁷, to take it *not* as a contingency but as a *necessity* of the same system which so eagerly tries to eliminate it.

Against this background, perhaps I can risk a suggestion for research in order to satisfy the demand for action. Instead of leaving the field and become a political activist, or remaining within the field and treating it as one arena within which the political struggle for equity can be waged (“equity through mathematics education”), I suggest that more attention should be given to *failure*. That is, more research efforts to study failed attempts to promote a meaningful mathematics educa-

²⁵ Thomas Popkewitz, in a conversation, called my attention to the modern spirit of individualism behind this posture, reflected in the growing idea in the population that there is someone who does the work of thinking for us, and we just have to apply it.

²⁶ Indeed, Freud considered psychoanalysis, education, and politics the three impossible societal tasks.

²⁷ Or, to use one of Lacan’s (1998) neologisms, *ex-sists*, to signal the paradoxical nature of failure, that is both extrinsic to the system of equality and freedom upon which liberal capitalist society is based and, at the same time, necessary, that is, intrinsic, to the reproduction of this same system.

tion for all.²⁸ In a way analogous to the psychoanalytical interpretation, I argue that the community should take the difficulties and impediments in achieving a meaningful mathematics education for all not as particular obstacles to get rid of, but as central issues for educational research. It is my contention, to paraphrase Žižek (1994, p. 7), that such “failures” have the potentiality to point towards the system’s antagonistic character, and thus “estrangle” us to the self-evidence of its established identity. This “estrangement” takes place when we no longer take as given the liberal-democratic capitalist view of school as a neutral environment purged of ideology. Instead, we realize the purely manufactured, contingent nature of such discourse. Ultimately, this discourse functions as an ideological mechanism aimed at obliterating the traces of its traumatic origins, that school is a place of social promotion. There is no Supreme Good waiting for us at the end of the tunnel. Inequity is a necessary condition for the consistency of the same system which strives to abolish it. The conclusion to be drawn is that if our purpose is to extinguish school inequality then the whole ideological fantasy structuring schooling as a place of inclusion, freedom and equality must go altogether. Borrowing an expression from Hegel, which gives the title to one of Žižek’s books, we are faced with the challenge of *tarrying with the negative*, that is, of assuming that there is no safe constellation of values (such as equality, freedom, etc.) from which we can stand and preach the Good. We are compelled to tarry with the fundamental social antagonism that perpetrates the very universality of our social formation.

ACKNOWLEDGEMENTS

This article is part of my PhD project, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007. It is also part of the Project LEARN, funded by the same foundation (contract PTDC/CED/65800/2006). I am grateful to Roberto Baldino who first read this text and pointed out crucial misunderstandings regarding the dialectical materialist approach I sought to deploy.

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²⁸ I leave to the reader the exercise of noticing how mathematics education research is mainly targeted at analysing and reporting successful experiences. Some exceptions are Vithal (2000) and Baldino and Cabral (2005).

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Criticisms and contradictions of ethnomathematics

Educational Studies in Mathematics

An International Journal

ISSN 0013-1954

Volume 76

Number 2

Educ Stud Math (2010)

76:209-230

DOI 10.1007/

s10649-010-9289-7

EDUCATIONAL STUDIES IN MATHEMATICS

AN INTERNATIONAL JOURNAL

EDITOR-IN-CHIEF: NORMA PRESMEG

Volume 76 No. 2 March 2011

 Springer

ISSN 0013-1954

 Springer

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Criticisms and contradictions of ethnomathematics

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Published online: 22 December 2010
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Abstract This article aims to contribute to the ongoing discussion about the epistemology and philosophy of ethnomathematics, and to debate its educational implications. It begins by identifying in recent literature two categories of criticism of ethnomathematics: epistemological, related with the way ethnomathematics positions itself in terms of mathematical knowledge; and pedagogical, related to the way ethnomathematical ideas are implicated in formal education. After a description of both of these categories, the pedagogical implications of ethnomathematics are considered by means of confronting the criticisms of recent research in the field. Ethnomathematics research conceives its pedagogical implications in different ways, some of them contradictory. Such contradictions are related with the societal role of school, with the idea that we can “transfer” knowledge from one setting to another and the tendency to reduce ethnomathematics to a ready-to-apply “tool” for the school-learning of mathematics. The author discusses the first two criticisms in the light of recent research concerned with the social and political dimensions of mathematics education. Concerning the latter, a typical case of an ethnomathematical research study looking at bringing local knowledge into school in the name of promoting diversity is analyzed. It is the author's contention that ethnomathematical research runs the risk of conveying an idea of culture where the Other is squeezed from its otherness. The article concludes by arguing that a deeper theoretical discussion is needed in the majority of the research currently done in ethnomathematics so that well-intentioned actions do not end up having a result opposite to their aims.

Keywords Ethnomathematics · Philosophy · School mathematics · Multiculturalism · Criticism

1 Introduction

To associate the prefix “ethno” to something so well defined, exact, and consensual as mathematics may cause surprise. The idea of a science that is human-proof, as mathematics

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has been conceived along modernity, is splintered when we associate it with the prefix “ethno”. “Ethno” shifts mathematics from the places where it has been erected and glorified (university and schools) and spreads it to the world of people, in their diverse cultures and everyday activities. An ethnomathematical program sullies mathematics with the human factor, not an abstract human, but a human situated in space and time that implies different knowledge and different practices. Ethnomathematics as a research program is less a complement to mathematics, than a critique of the knowledge that is valorized as being mathematical knowledge.

Despite this all-embracing enterprise, ethnomathematics is often confused with an ethnic or indigenous mathematics. In this article, I take the position that ethnomathematics does not restrict its research to the mathematical knowledge of culturally distinct people, or people in their daily activities. The focus could be academic mathematics, through a social, historical, political, and economical analysis of how mathematics has become what it is today. With this background, research on ethnomathematics brought to the mathematics education field new and refreshing insights not just about ethnic or local mathematical knowledge, but also in terms of philosophical, historical, and political approaches involved in mathematics and its education. Ubiratan D'Ambrosio (2002) defines ethnomathematics as “a research program in the history and philosophy of mathematics, with obvious pedagogic implications” (p. 27) and points out some of the dimensions involved in the ethnomathematical research: historical, cognitive, epistemological, political, and educational. Although I agree with D'Ambrosio that ethnomathematical research could provide us with a deeper understanding of the history and philosophy of mathematics, I contest the idea that the pedagogical implications of ethnomathematics are obvious. In this article, I will focus on the epistemological and educational dimensions of ethnomathematics that, in my view, cannot be fully understood without a focus on the social and political dimension of knowledge and education. My main focus will be educational criticism, concerning the way ethnomathematical ideas are applied in formal educational settings.

A significant part of ethnomathematics research has educational aims (Borba, 1990; Gerdes, 1995; Barton, 1996; Powell & Frankenstein, 1997; Knijnik et al. 2004), seeking to bring to the schools or other formal educational environments (like indigenous schools) the knowledge and the mathematical practices of cultural groups of people. This makes ethnomathematics research part of a multicultural approach in education that during the last 30 years has aimed to open schools to the cultural diversity that characterizes our current societies. Together with globalization, the concern with diversity is currently considered to be one of the two main educational functions (Izquierdo & Mínguez, 2003). While globalization refers to the social need to respond to market globalization which imposes a convergent education by training individuals to perform a role in the global society, diversity demands an integration of different cultures in a model of divergent education, able to educate citizens in what has been called equity within diversity. It seems that there is nothing wrong with the valorization of different cultures, by contemplating them in the educational context. However, I shall argue that a poor theoretical stance on the way we insert (mathematics) diversity into schools, by obliterating the tension existing between globalization and diversity, ends up conveying an idea of culture where the Other is squeezed from its otherness. This is when a multicultural approach becomes racist, when we are willing to accept the Other as long as it is deprived of all the vicissitudes that characterize its otherness—what the Slovenian philosopher Slavoj Žižek (1992, 2009) refers to as the *desubstantialized* Other.

In the final section of this article, I will develop further Slavoj Žižek's critique of multiculturalism, in connection with the exploration of a typical case of an ethno-

mathematical research study, which looked at bringing local knowledge into school in the name of promoting cultural diversity. Nonetheless, the same critique has been pointed out both within education and mathematics education. Kincheloe and Steinberg (2008), in the article *Indigenous Knowledges in Education*, make a review of the complexities, dangers, and benefits involved in the way indigenous knowledge is handled in education. As the authors acknowledge, “[W]estern scholars dedicated to the best interests of indigenous people often unwittingly participate in the Western hegemonic process” (p. 141). The authors criticize the way researchers and teachers engage in multicultural education, revealing some common problems with the way Western scholars engage in helping indigenous people, such as what the authors calls “vampirism”—Western researchers that do not simply want to work with indigenous people but also to transform their identity and become indigenous persons themselves—or the way Western scholars promote a decontextualization of indigenous knowledge—conceptualization of indigenous knowledge in contexts far removed from its production. Paul Dowling (1998) developed a similar critique regarding ethnomathematics. According to him, “[t]o varying degrees, all of this work [studies in ethnomathematics] succeeds in celebrating non-European cultural practices only by describing them in European mathematical terms, that is, by depriving them of their social and cultural specificity” (Dowling 1998, p.14). It is in this sense that I understand Žižek’s critique of multiculturalism: the Other is accepted, even celebrated, as long as it is the Other of our gaze.

In order to develop my argument, I shall start by bringing into discussion important criticisms made of ethnomathematics. Indeed, given the radicalism of the ethnomathematical program (at least as it is put by D’Ambrosio (2002)), it is not surprising that its emergence has been the target of strong criticism. Current ethnomathematics studies are numerous and scattered around the world¹, and it is difficult to gain an international perspective on how ethnomathematical research is being done. Hence, to criticize something with such different practices and discourses can result in an unreal chimera, if we do not take into consideration the different contexts in which research is conducted. A way to overcome these difficulties requires us to criticize ethnomathematics as a well-defined research program and by analyzing the work of the most important ethnomathematical researchers. That was the path chosen by Rowlands and Carson (2002, 2004) and Horsthemke and Schäfer (2006), in their epistemological and educational critique of ethnomathematics. This critique, I argue, although apparently pedagogical, is an epistemological critique that claims to highlight academic mathematics as one of the biggest achievements of mankind. As concerns the pedagogical critique made by the latter researchers, and also by Skovsmose and Vithal (1997), I shall articulate it with the contradictions raised by ethnomathematical researchers. Even among these researchers, there are contradictions in how they understand the pedagogical implications of ethnomathematics.

Three of these four articles were published in *Educational Studies in Mathematics*, where a discussion about the epistemological status and educational implications of ethnomathematics has been developed. My discussion will consist of the analysis and confrontation of the criticisms raised in these articles with recent research in ethnomathematics that highlights the contradictions involved when we try to implement mathematical ideas in educational settings. The comprehension about the nature of mathematical knowledge and how this comprehension is fundamental when we engage in

¹ For updated information on the international research on ethnomathematics I suggest using the Website <http://www.ethnomath.org/> (Ethnomathematics Digital Library).

educational tasks, together with the aim of equity desired for mathematics education, justifies the importance of exploring how in our current society mathematics education deals with the situated character of knowledge as it is presented from an ethnomathematical perspective. As schools are considered places in which pupils have access to a set of practices and knowledge indispensable for their formation as citizens, the question of what mathematics should pupils learn is not consensual and, as we shall see, could be a source of conflict that makes visible the political dimensions involved in the aims of school mathematics.

2 Epistemological criticisms

In 2002, Rowlands and Carson wrote an article published in this journal, in which they carried out a critical review of ethnomathematics, by comparing the ethnomathematical program to the curriculum of school mathematics. This article was subsequently answered by Adam, Alangui, and Barton (2003), which Rowlands and Carson (2004) later responded to in turn. As raised above, this article also draws on arguments by Horsthemke and Schäfer who wrote two articles presented at the International Congress on Ethnomathematics in 2006, where they followed most of the arguments presented by Rowlands and Carson. Those two sources of criticism present themselves as an educational critique on ethnomathematics but, in the way I analyzed the texts, they are, above all, an epistemological critique, especially the articles from Horsthemke and Schäfer.

Against a non-essentialist posture assumed by ethnomathematics, Rowlands and Carson (2002, 2004) and Horsthemke and Schäfer (2006) advocate an essentialist position, based on the idea that, although knowledge is constructed by humans, it remains beyond them. This is to say, there is some kind of invariant (an essence) which is repeated in all mathematical knowledge; whether this knowledge is developed in a Mongolian tribe or in a European university, the mathematics involved is the same. As the authors state: “[m]athematics is universal because, although aspects of culture do influence mathematics, nevertheless these cultural aspects do not determine the truth content of mathematics” (Rowlands & Carson, 2002, p. 98).

The authors position themselves against the politicization of science, claiming that, “mathematics is a science, and its laws, principles, functions and axioms have little to do with issues of social justice” (Horsthemke & Schäfer, 2006, p. 9). Or, as mentioned by Rowlands and Carson (2002) “rationality may be the preserve of an oppressive cultural system but that does not necessarily mean that rationality is in itself oppressive” (p. 82). Represented very strongly in this sentence is the idea that rationality exists *per se*, i.e., as something disconnected from the social and political environment. In that sense, mathematics is taken by the authors as a piece of truth and neutral knowledge that could be used for good or evil, although mathematics itself is free from judgement: “the odious use of something does not make that something odious” (p. 98).

These authors embraced academic mathematics as a universal human good, shared by all people and considered to be one of the biggest achievements of mankind. This universal knowledge is presented as being the climax of a human evolution and clearly more precious than others: “[t]he reason we are attempting to ‘privilege’ modern, abstract, formalized mathematics is precisely because it is an unusual, stunning advance over the mathematical systems characteristic of any of our ancient traditional cultures” (Rowlands & Carson, 2004, p. 331).

Finally, the authors adopted an epistemological position in which the genesis and consolidation of knowledge must be understood by analyzing the internal logic of that knowledge and its pragmatic value, suggesting that social and political aspects have no influence in that genesis:

[M]odern conventions of mainstream mathematics have become ‘privileged’ (i.e. accepted by the world’s mathematical community and numerous secular societies) for reasons that have little if anything to do with the politics of nations or ethnic groups, but have much to do with their pragmatic value. (Rowlands & Carson, 2004, p. 339)

3 Educational criticisms

The catalyst for the educational critique developed by Horsthemke and Schäfer is the way the application of ethnomathematical ideas into South African schools contributed not to the inclusion, but to the exclusion of children. Ten years before, Skovsmose and Vithal (1997) had developed the same critique, although in a more constructive way. They called our attention to the way ethnomathematical ideas are implicated in schools of countries suffering from ethnic and racial tensions. In the case of South Africa, we can see how those ideas contributed to the creation of a lighter mathematical curriculum (based on students’ backgrounds) for those students considered “ethnic”.² As a consequence of these politics, those students were systematically excluded from access to academic mathematics then aimed at the white student: “in South Africa bringing students’ background into the classroom could come to mean reproducing those inequalities on the classroom” (p. 146).

This critique of the way ethnomathematical ideas in school could be inadvertently used to deny access to academic mathematics is also made by Rowlands and Carson. These authors emphasize the dangers involved in not considering formal mathematics as an important part of all students’ education. According to the authors, it is formal mathematics that gives access to a privileged world and all students should know how to appreciate that knowledge:

There is every danger that mathematics as an academic discipline will become accessible only to the most privileged in society and the rest learn multicultural arithmetic within problem solving as a life skill or merely venture into geometric aesthetics. (2002, p. 99)

What the authors criticize is the idea that school-learning should be centered in the development of local and practical knowledge of the students. According to them, such a utilitarian education will limit students’ life chances. In this sense, the introduction of ethnomathematical ideas in school can function as a factor for exclusion because, whereas the students from the “dominant culture” continue to learn the academic mathematics that allows them to compete in a more and more *mathematized* world, students from other cultures will only learn a local and rudimentary knowledge that scarcely contributes to their emancipation. This tension reflects the ambivalence between the two functions of current education: globalization and diversity. In this sense, the authors defend a clear distinction between the local culture of a student and the scientific and school culture: “to preserve American Indian cultures, African tribal cultures, traditional cultures of Asia and elsewhere,

² Black students in the context of the apartheid regime.

their uniqueness must be recognized, not collapsed into a dreary and illusory sameness with scientific culture” (2002, p. 91).

Rowlands and Carson are against the use of ethnomathematical knowledge in the classroom, arguing that there may be incommensurable ways of understanding and perceiving mathematics. It is that incommensurability that could make an artificial endeavor out of the attempt to articulate ethnomathematical knowledge with school knowledge. They argue that people can master more than one culture, and school should be the place where people have contact with the more universalized culture, that is, the Western culture.

Finally, Rowlands and Carson consider mathematics to be a foreign language to all students before they go to school. Contrary to the ethnomathematical stance, which argues that students already have non-formalized mathematical knowledge before they start school, these authors argue that *protomathematical* knowledge is not important for learning school mathematics because all students are equally positioned to learn new knowledge:

We go to great lengths to point out that children of traditional cultural backgrounds are probably not at any significant disadvantage when it comes to learning mathematics, since it is a ‘foreign language’ to all novices, regardless of their cultural background. (2004, p. 335)

Skovsmose and Vithal (1997) acknowledge the importance of ethnomathematical ideas for a critical mathematics education. They identify four trends in the ethnomathematical research and stress that it is in the confrontation with school mathematical curriculum that ethnomathematics finds its greatest challenge and also the possibility of critique. Firstly, the authors stress the fact that research in ethnomathematics does not usually specify much about the relation between culture and power. Secondly, they identify a problem with the definition of “ethnomathematics” and pose the question: how can someone educated in formal mathematics identify other mathematics? According to them, ethnomathematics only makes sense through the perspective of academic mathematics. Thirdly, the authors argue that ethnomathematics lacks a critique about how mathematics formats reality (Skovsmose, 1994). Finally, as mentioned before, Skovsmose and Vithal (1997) think it necessary to *problematize* the idea of students' background and not just in terms of the actual culture of students but also about the aspirations and desires that students have of emancipation. This is what they call the students' *foreground*, by which they mean: “the set of opportunities that the learner's social context makes accessible to the learner to perceive as his or her possibilities for the future” (p. 147). According to Skovsmose (1994), all the importance given to students' background could inhibit them from emancipation, and more attention should be paid to the opportunities that the social, cultural, and political context could bring to students. By emancipation, Skovsmose means the access to and participation in a world where mathematical knowledge is central.

4 Some comments on epistemological criticisms

Before entering into a discussion of the epistemological criticisms made of ethnomathematics, I take the position that the interpretation of ethnomathematics carried out by Rowlands, Carson, Horsthemke, and Schäfer is misleading. These authors understand ethnomathematics as an ethnic or indigenous mathematics. In fact, there is a vast diversity of studies in ethnomathematics, and some of them assume that ethnomathematics research consists of understanding, with the tools of academic mathematics, the mathematical ideas

of culturally distinct people.³ In that sense, ethnomathematics is indeed the study of an “ethnic” mathematics:

The prefix *ethno* refers to ethnicity, this is, to a group of people belonging to a same culture, sharing the same language and rituals, in other words, culturally well delineated characteristics so we can recognise it as a specific group. (Ferreira, 2004, p. 70, my translation from Portuguese)

In this perspective, the educational implications of ethnomathematics are focused on “how to bring ethnic knowledge to the classroom to allow for a meaningful education; how to establish the bridge between ethnic and institutional knowledge” (Ferreira, 2004, p. 75, my translation from Portuguese). But, there are other ways of addressing ethnomathematics. For instance, D'Ambrosio (2004) clearly says that his “view of ethnomathematics tries to avoid confusing ethnomathematics with ethnic mathematics, as understood by many” (p. 286). That is why D'Ambrosio prefers to talk about an “ethnomathematics program”, as something more than the study of the ideas and uses of non-academic mathematics. I understand this program as a radical one in the sense that it endeavors to criticize not just mathematics and mathematics education but social orders and ideologies that feed our current world. As mentioned by D'Ambrosio (2004), “the ethnomathematical program focuses on the adventure of human species” (p. 286). Others like Knijnik (2004) and Powell and Frankenstein (1997) also criticize the idea of ethnomathematics as an ethnic mathematics and have developed investigations where the theme of power and politics are taken seriously.

The epistemological discussion carried out by Rowlands, Carson, Horsthemke, and Schäfer is an echo of a bigger philosophical discussion about the nature of knowledge that was intensively debated in recent decades under the label of “science wars”. As with any philosophical question, there are different ways of analyzing it, and everyone has the right to choose the one that better fits his or her interests. I shall not enter into such a discussion here; rather, I just want to call attention to the pertinence of having philosophical thought in such discussions. In a philosophical line, in which we can include Nietzsche, Kierkegaard, Durkheim, Wittgenstein, Althusser, Foucault, Derrida, Lacan among many others representatives of continental philosophy, knowledge is perceived from a non-essentialist perspective, i.e., as something whose creation, maintenance, valorization, or disqualification has nothing to do with its intrinsic or essentialist value, but with the way knowledge is exercised, whether it is in a language game (Wittgenstein, 2002), in the webs of discursive modalities involving power relations (Foucault, 2004), or as an ideological interpellation (Althusser, 1994). The meaning and the knowledge we have of something is always contingent, full of historicity, and involved with power relations. As mentioned by Amâncio (2004) the idea of knowledge as something universal, with an existence *per se*, is itself a very ideologically loaded position. Hence, the important aspect of this epistemological discussion is less a discussion on whether knowledge is itself universal or situated. Rather, this discussion is concerned with matters as mentioned by Foucault (2004): what intentions, what politics, allow us to claim that some knowledge (like academic mathematics) is universal?

Therefore, if we take support from these philosophers mentioned above, the essentialist position of Rowlands, Carson, Horsthemke, and Schäfer regarding mathematical knowledge is hardly sustainable. Against the idea that our words, texts, language, or discourses describe some given reality (the “truth” of things, the “objective” world), authors such as

³ See, for instance, the work of Bill Barton, Sebastiani Ferreira, Paulus Gerdes, and Marcia Ascher.

Lacan, Derrida, and Žižek argue that rather than just describing reality, our words constitute reality as such:

It is the world of words which creates the world of things. (Lacan, 2001, p. 155)

The outside, “spatial” and “objective” exteriority which we believe we know as the most familiar thing in the world, as familiarity itself, would not appear without the gramme. (Derrida, 1976, p. 42)

The discourse of ontology is thus sustained by an “indirect speech act”: its assertive surface, its stating that the world “is like that”, conceals a performative dimension, i.e., ontology is constituted by the misrecognition of how its enunciation brings about its propositional content. (Žižek, 1992, p. 114)

This is what in psychoanalysis is termed the *performative* power of the word: reality is something which is constituted, posited by the subject. When we say that the world is written in mathematical language—the idea that mathematics is everywhere—we are not asserting some ontological truth about the world or about mathematics, rather, it is by means of declaring it that the world becomes “written” in mathematics. The truth claim of a statement cannot be authorized by means of its inherent content. The omnipresence of mathematics relies on its own act of enunciation. This is why Foucault says that the epistemological discussion should be centered not in finding the intrinsic truth of knowledge, but which are the positions of enunciation (the politics) that vouch for the veracity of some knowledge.

This same idea is also present in the way some ethnomathematics researchers understand mathematics. For instance, when Adam, Alanguai and Barton (2003) talk in terms of “recognis[ing] mathematics as part of their [students'] everyday life” (p. 332), we sense as if students already are using mathematics before they learn it in school: in school they just “recognize” it. However, I prefer to conceive this “act of recognition” as a performative one, in the sense that students do not recognize something that was always there, rather, mathematics starts being there from the moment someone performatively asserts that what students were doing was mathematics.

The authors of the essentialist perspective position themselves as the guardians of academic mathematics that fuels this modern world and which is seen as being superior to any existing society, “the beliefs and practices of other societies are epistemic and vertically inferior to our own” (Horsthemke & Schäfer, 2006, p. 12). From their perspective, we are living the climax of a human evolution, in which academic mathematics is the substrate of a society based on humanistic ideals. This universal society is however problematic. Part of the research on ethnomathematics has been concerned to understand how these universal images of society are generated through history.⁴ As mentioned by the Spanish philosopher Lizcano Fernández (2004), the idea of such a universal society was possible through “the development of a set of formalisms characteristic of a peculiar way that has a certain tribe, of European origin, to understand the world” (p. 126). In other words, the universal society (capitalist society) based on universal knowledge (mathematics and science) suggested by Rowlands, Carson, Horsthemke, and Schäfer is a very particular way of understanding time and space, of classifying and ordering the world, and of understanding economic and social relations, in short, of conceiving what is possible and impossible to think and do.

⁴ See, for instance, the book edited by Powell and Frankenstein (1997), which collects a set of articles in which these ideas are deconstructed.

5 Criticisms and contradictions of the educational implications of ethnomathematics

5.1 On the neutrality of school mathematics

One of the claims made by Rowlands and Carson (2002) and Horsthemke and Schäfer (2006) has to do with the alleged neutrality of school mathematics. As we saw, these authors defend the idea that every student is from the beginning equally positioned to learn this completely new language called mathematics. Their arguments are based on the idea that mathematics is a universal body of knowledge, accessible to every human being through cognitive work. We can glimpse in these positions the way psychology continues to be a main influence in the field of mathematics education. Despite all the research done in the last 20 years criticizing the idea of the child as a “cognitive subject” (Valero, 2004), as an idealized student willing to learn and reduced to his cognitive dimension,⁵ the prevailing theoretical ideas in mathematics education continue emphasizing the psychological dimension of the student (Lerman, 2000; Valero, 2004).

On the other hand, Rowlands and Carson (2002) and Horsthemke and Schäfer (2006), and also Skovsmose and Vithal (1997), to some degree, understand school as a place for emancipation, in the sense that it is through school that someone can become a fully active and participative member of our society. This approach to mathematics education is in line with the discourse underlying most mathematics education research, which establishes mathematics as one of the biggest achievements of humankind, considered to be the main pillar of our technological society and an indispensable tool to becoming an active participant in a more and more mathematized world. Learning mathematics is a critical requisite to being a citizen. According to the National Council of Teachers of Mathematics (NCTM) (2000), it gives power to participate in the world.

Using the theoretical framework developed by Skovsmose and Valero (2008) to deal with the issue of power in mathematics education, we can say that mathematics education can empower people through the intrinsic characteristics of mathematics itself (logical thinking, abstraction); by providing students with psychologically meaningful experiences (solving problems, metacognition); by enhancing the relation between cultural background and foreground therefore allowing students to learn “in context” (connection between everyday practices and school mathematics; providing opportunities to envision a desirable range of future possibilities); and finally, students can get empowered through school mathematics by exploring situations of “mathematics in action”, which makes visible the way mathematics formats reality (exploring real mathematical models in a critical way). What is missing in these four perspectives of conceiving the way mathematics empowers people is the fundamental one—mathematics empowers people not so much because it provides some kind of knowledge or competence to them but because it gives people a value. Mathematics empowers people because it is posited as a socially valuable resource.

To my knowledge, it was Shlomo Vinner (1997) who, within mathematics education, first addressed schools as credit systems. According to him,

Students have very good reasons to study mathematics. It is not the necessity of mathematics in their future professional life or their everyday life. It is because of the selection role mathematics has in all stages of our educational system. (p. 3)

⁵ According to Valero (2004), this research approach conveys neoliberal perspectives of school, by putting the emphasis on the individual subject barred from the social and political context in which the learning occurs.

Contrary to the NCTM assertion that mathematics empowers people because it provides some kind of knowledge or competence to them, Vinner argues that this empowerment has instead to be understood in the field of value. Mathematics allows students to accumulate credit in the school system that will allow them to continue studying and later to achieve a place in the sun. Mathematics empowers people because it is posited as an economically valuable resource. As Bishop and Forgasz (2007) put it, “in every country in the world mathematics now holds a special position, and those who excel at it or its applications also hold significant positions in their societies” (p. 1149).

This political dimension of mathematics education has been addressed within the research community. For instance, Skovsmose (2005), in his latter writings, moves towards a more critical view of the role of school in our society, in particular, the role of school mathematics. According to Skovsmose, schools can represent the opportunity to succeed in life, or they can represent the place that marks you as “disposable”, and mathematics is right at the core of this selective process, as has been shown by research in mathematics education (e.g., Atweh, Forgasz, & Nebres, 2001; Burton, 2003; Secada, Fennema, & Byrd, 1995; Valero & Zevenbergen, 2004; Bishop & Forgasz, 2007). We have studies today that analyze the way school mathematics functions as a gatekeeper (e.g., Stinson, 2004), is involved in processes of social exclusion (e.g., Knijnik, 2004), fosters social injustice (e.g., Gutstein, 2003), or, within a Foucaultian perspective (Popkewitz, 2004), how school mathematics constructs a set of learning standards that are more closely related to the administration of the children than with an agenda of mathematical learning.

These studies allow us to understand how school mathematics is not a neutral subject, equally available to everyone. At the same time, they invite us to posit the importance of mathematics not so much in terms of knowledge and competences, but in terms of accreditation through a credit system conveyed by school. However, despite the awareness that school mathematics performs a strong economical and political role, there is a lack of studies in mathematics education that seek to understand school mathematics as part of dominant economical systems such as capitalism (Gutiérrez, 2007; Pais, 2011). Indeed, elsewhere (Pais, 2011), I argue that there is a tendency to face the problem of equity as being strictly from the domain of mathematics education understood as the developing of better ways to teach and learn mathematics in school environments (Cobb, 2007; Silver & Herbst, 2007). Gates and Zevenbergen (2009) state that “mathematics and social justice has been the focus of much research—however this has largely focussed on such issues as the process of learning, the content of the curriculum and its assessment” (p. 162). They also make a very suggestive point. They argue that it is common in mathematics education research to discard such “political” questions since it is not the responsibility of mathematics education to address them (p. 165). This picture is worrisome if we accept that the school system, as a rule, is *overdetermined* by late capitalist economics and ideology (Althusser, 1994; Jameson, 1991).⁶

⁶ Contrary to the assumption defended by many economists (most notably Daniel Bell's *post-industrial society*) that we have arrived at a new type of society, where the new social formation in question no longer obeys the laws of classical capitalism, “late capitalism” (Jameson, 1991) signals instead that this “new society” is a *purser* stage of capitalism than any of the moments that preceded it. In this sense, every position on postmodern in culture “is also at one and the same time, and *necessarily*, an implicitly or explicitly political stance on the nature of multinational capitalism today” (Jameson, 1991, p. 3). Frederic Jameson, together with Slavoj Žižek, are arguably the two most powerful contemporaneous theoreticians interested in scrutinizing the ways in which capitalism has become the “concrete universal”, as Žižek (2004, p. 3) calls it, of our historical époque: “what this means is that while it remains a particular formation, it overdetermines all alternative formations, as well as all noneconomic strata of social life”. It is in this sense that I argue that education is overdetermined by capitalism.

Ethnomathematics carries within it a critique of school mathematics.⁷ D'Ambrosio (2003), for instance, compares the current school with a factory, where people are components of big machinery that aims for uniformity. Others, such as Powell and Frankenstein (1997) or Bishop (1995), try to understand how mathematics in society conveys hegemonic discourses and oppressive educational practices that promote exclusion and domination. In school, as mentioned by Rowlands and Carson (2002, 2004), we are introduced to a certain society. And if we are satisfied with our current society, as apparently is the case of Rowlands, Carson, Horsthemke, and Schäfer, then we must prepare students the best we can to be full members of that society. But, authors of some of the studies in ethnomathematics do not share this optimistic view of current society.⁸

Society should be problematized and not taken for granted, especially when we are aware of the economical politics based on market priorities and all the ideologies that fuel our way of living. What does it mean to educate people to be participative, active authors in a more and more merchandized society? Do we all want “schooling to serve the needs of industry and commerce?” (Rowlands & Carson, 2002, p. 85). Hence, a problematization of society and the role of school in society is, in my opinion, a priority in a research program like ethnomathematics. But, that is far from happening.

5.2 School knowledge is a particular form of knowledge

Critics do not deny the existence of different cultures, where different knowledge has been developed. However, they clearly say that this knowledge is inferior to the knowledge developed by the dominant Western culture, since it is the latter that allows people to become full citizens in current society. In order to allow everyone this opportunity, school should provide students with this universal knowledge, the base of our high-tech society. This position does not assume that other knowledge and practices different from the ones conveyed by school curriculum are not important and should be forgotten. On the contrary, they acknowledge the importance of this different knowledge. But, they argue that it should remain in the places where it makes sense and not in school designed to be the place for learning the official knowledge. This official mathematical knowledge is presented as being universal in the sense that it can be learned without context.

Although this critique may sound like a prepotent and exclusionary act, it calls attention to some problematic theoretical assumptions in ethnomathematics research. If we consider the research on the socio-cultural aspects of knowledge and learning, we can assert that all knowledge is eminently situated in the places where it is used, whether these places are a workplace or an indigenous community (Lave, 1988). The meaning of some practice and knowledge is deeply involved in the community of practice where it is exercised and developed (Lave & Wenger, 1991). There is no guarantee that people transfer knowledge from one practice to another without some kind of “misrecognition”. School mathematics, although it can explore “real” situations, will always be school knowledge, learned in a specific place called school where students are not necessarily concerned with learning.⁹

Based on Wittgenstein's philosophy, some researchers (e.g., Gottschalk, 2004) have argued that meaning is just possible from the point that we get appropriated of the rules of

⁷ See, for instance, the work of Ubiratan D'Ambrosio, Gelsa Knijnik and Alexandrina Monteiro.

⁸ In Powell & Frankenstein (1997), we can find a set of articles that articulate a critique of mathematics with a critique of society. See also the most recent writings of Ubiratan D'Ambrosio (for instance, D'Ambrosio, 2007) where he developed a social critique, based on the idea of peace.

⁹ For a clear understanding of how students in school engage not in learning but in passing, see Baldino and Cabral (1998).

the game, of the constituent language that allows us to speak with meaning. In the case of mathematics, only when the pupils get a set of (mathematical) rules can they play the game of school mathematics. In school, we learn “conventional techniques”. The constructivist idea that a student constructs the meaning by himself is, if we take the work of Wittgenstein seriously, a fallacy. In school, students get enculturated into a set of conventions that allow them to be part of that community. Therefore, we can say that there is no emergence of meaning without the transmission of a set of conventions and rules, that is, without the appropriation by the student of the conditions of meaning that will determine his thought.

We can then assert that knowledge is not independent from the contingency of its emergence. When we bring local knowledge to school, whether it is some practical knowledge or “ethnic” knowledge, what happens is a *decontextualization* of the conditions that justify the emergence and use of this knowledge. If knowledge and learning are not purely cognitive processes happening in the heads of individuals, but socially situated practices, deeply connected with context, then it is not without trouble that we can assert the advantages of linking local and school knowledge and practices. Like Rowlands and Carson said, there can be incommensurability between these two dimensions.

When some researchers went to indigenous tribes to research how indigenous people use mathematics, for instance, in the construction of houses, and then brought this knowledge to the school system in order to take advantage of it to teach them the school mathematics knowledge, something was already lost. The construction of houses in a tribe is a communitarian activity, involving rituals and knowledge only identified as being mathematical by people outside the community. When this knowledge is brought to school to teach, for instance, the relation between the sides of a rectangular triangle, students are in a completely different setting and only in a forceful and artificial way can we say that the previous knowledge about house construction will be helpful to learn the Pythagorean theorem in a formalized or *scholarized* way. If, on the other hand, we take the option of incorporating this knowledge into the school curriculum by itself¹⁰ (i.e., as construction of houses as it is done in the indigenous community) we end up, as the critics notice, allowing students only a local and practical knowledge that will only be useful to them in the context of the community, which has other ways of transmitting this knowledge that have nothing to do with school. D'Angelis (2000) argues that an indigenous school should be concerned in teaching not what has always been taught inside the community with their own methods, but what indigenous people want: mathematics and language of the white people (as is noted by Scanduzzi, 2004, p. 373). According to D'Angelis, the content of the school subjects should not conflict with the proper and particular forms of education characteristic of some community (whether this community is an indigenous village or a work place). That is, school should not take spaces that belong to other forms of culture, by *scholarizing* those contents that are specific to a non-school environment. This will to bring to school local knowledge can entangle a prejudice, since it seems like we are avoiding a prejudice—the one that says that knowledge developed by people in their local practices is not knowledge—by fuelling another—the one according to which local knowledge is only considered to be true knowledge if it is taught and evaluated in school. The result of such a strategy of bringing local knowledge to school, by transforming it in curriculum content, takes the risk of not being an indigenous school but a “Westernized” indigenous school.

Particularly important was the eye-opening work done by Dowling (1998), who revealed some of the pitfalls of what Bernstein called *recontextualization*. For him, “the

¹⁰ This is possible in the context of indigenous schools that have been developed very strongly in Brazil during the last 10 years.

recontextualization of everyday life material into the curriculum ends up by being neither 'real maths' nor 'real life'" (Evans, 1999, p. 27). Something is always already lost when we transpose some everyday activity into school, and vice-versa. What Dowling (1998) calls the myth of participation—the idea that mathematics is a necessary feature of everyday practices—ends up creating a school curriculum where mundane activities are *mythologized* in a way that "privileges" mathematical rather than everyday principles. That is to say, everyday activities, in order to be introduced in school, need to be amputated of all the complex vicissitudes which makes them what they are. This amputation—the result of casting the mathematical gaze on to public domains—privileges what Dowling (2001, p. 22) calls the "esoteric domain", while, at the same time, concealing the purely fictional status of the importance attributed to mathematics.

It would seem that it is not intrinsically "beneficial" to bring local knowledge to school. On the one hand, we have the position defended by most of the ethnomathematics educators that ethnomathematical knowledge could help students understand the formal mathematics, by incorporating this knowledge in the curriculum. On the other hand, we have researchers that challenge this idea of adapting or incorporating local knowledge into the curriculum. According to them, school should be reserved for the learning of the official knowledge and leave the local knowledge developed in the communities. There is no easy way out of this paradox: whether school should be reserved for the learning of the "globalized" knowledge allowing everyone to participate in our high-tech world, or a school that incorporates diversity but runs the risk of *domestication* of the Other.

5.3 Ethnomathematics in school settings: dealing with contradictions

In response to the article of Rowlands and Carson (2002), Adam, Alangui and Barton (2003) identify several possibilities for an ethnomathematical curriculum. Ethnomathematics can appear in schools as an approach to mathematics; as a particular content distinct from the conventional mathematical concepts taught in schools; as a stage in the progression of mathematical thinking; and as awareness that classrooms are situated in a cultural context. However, the approach favored by these authors is:

An integration of the mathematical concepts and practices originating in the learners' culture with those of conventional, formal academic mathematics. The mathematical experiences from the learner's culture are used to understand how mathematical ideas are formulated and applied. This general mathematical knowledge is then used to introduce conventional mathematics in such a way that it is better understood, its power, beauty and utility are better appreciated, and its relationship to familiar practices and concepts made explicit. In other words, a curriculum of this type allows learners to become aware of how people mathematise and use this awareness to learn about a more encompassing mathematics. (Adam et al. 2003, p. 332)

This approach, concerned in establishing a "bridge" between local and school knowledge, is prevalent in ethnomathematics research, as we understand by reading the book entitled *Ethnomathematics and Mathematics Education* (Favilli, 2007), which collects research in ethnomathematics from all around the world as a result of the works presented and discussed in the Discussion Group 15 at the 10th International Congress in Mathematics Education (ICME10). Many authors of the studies in ethnomathematics advocate the importance of bringing to the classroom the local knowledge of students. This "making the bridge" between local mathematical knowledge and school mathematics knowledge is seen as a way of valorising students' cultures and at the same time allowing

students to better understand the formal mathematics from their own not yet formalized knowledge. However, this approach is not consensual among ethnomathematical researchers.

To speak to one of the criticisms made by Rowlands, Carson, Horsthemke, and Schäfer regarding the use of ethnomathematical knowledge in regular schools, we can identify a contradiction in how ethnomathematicians understand its pedagogical implications. On the one hand, as mentioned before, some researchers, such as Adam et al. (2003), defend the idea of using students' ethnomathematical knowledge to construct a bridge for the learning of formal mathematics. But, on the other hand, researchers like Knijnik (2004) clearly state that: "it's not a matter of establishing connections between school mathematics and mathematics as it is used by social groups, with the purpose of achieving a better learning of school mathematics" (p. 228).

Implicit in these two views is the way researchers understand the role of mathematics and school in our society. The problem with the first one, characterized by the "bridge metaphor", is the reinforcement of the hegemony of school mathematics because the Other is valorized only as a way to achieve the true knowledge. Thus, it contradicts the critique that ethnomathematics makes to the hegemony of academic mathematics. The same problem identified by the critics regarding the valorization of background instead of the foreground is also raised by Knijnik (2004), Monteiro (2004), and Duarte (2004). These authors raise questions about the usually folkloric way ethnomathematical ideas appear in the curriculum. According to them, the use of local knowledge as a curiosity to start the learning of school mathematics could be the cause of social inequalities, as mentioned by the critics.

Duarte (2004) argues that there is an abundance of cases in ethnomathematics research where the generous act of integrating local knowledge into the school curriculum ends up contributing to its marginalization. The author uses the expression "include to exclude" to illustrate how the result of some inclusive actions consists in accepting diversity but keeping it at a safe distance by means of integrating different cultures not in their totality, but as a curiosity, an illustration, a "starter" to the real mathematics. According to her:

This distance has been kept when, in the curriculum, in the task of rescuing knowledge from subordinated groups, we treat them as something exotic, as a souvenir or simply as starting points to the academic knowledge. In this sense, rescuing popular savoir becomes a trap that ends up producing and reinforcing social inequalities. (Duarte, 2004, p. 188, my translation from Portuguese)

But, truly, to include ethnomathematical ideas in the curriculum is no less problematic. If we focus on a regular school and take into account its role in preparing students for a globalized market-orientated society, with all the pressure to learn the mathematics of the standard curriculum that will be essential to students' approval in the high stakes tests, we can ask ourselves if there is a place for ethnomathematical knowledge (or other local, non-scholarly knowledge)? My opinion, according to my review on ethnomathematical research, is that these educational implications of ethnomathematics (in a regular school) end up being co-opted by a school that, as Rowlands, Carson, Horsthemke, and Schäfer would agree, is worried with the uniformization of knowledge. In that sense, I agree with them and also with Skovsmose and Vithal when they say that focusing the learning of mathematics on students' local knowledge could be a factor for social exclusion. But, the problem is not just in ethnomathematics, but in school itself. Monteiro (2004), a Brazilian ethnomathematician, poses the definitive question: "Is it possible to develop ethnomathematical work in the current school model?" (p. 437, my translation from Portuguese).

According to her, an education based on ethnomathematical ideas demands a deep transformation in school organization.

6 A critical approach to diversity and schooling

6.1 A typical example of the “bridge metaphor”

In the remaining pages, I will use Slavoj Žižek's critique of multiculturalism (Žižek, 1992, 2009) to analyze how research on ethnomathematics can convey ideologies contrary to the ones it praises. I will look at two articles published in the book edited by Franco Favilli and mentioned previously, which I consider typical of the “bridge metaphor” approach in the way ethnomathematics research conceives its educational implications. The first one (Favilli & Tintori, 2007) is part of a bigger project called *Innovation in Mathematics Didactics and Technological Aids for Multicultural Context with Immigrant and Minority Pupils*, a 3-year trans-national project funded by the European Union. In this particular article, authors present considerations made by mathematics teachers and their pupils after completing an experimental, intercultural, and interdisciplinary didactic proposal. The second one (Fiorentino & Favilli, 2007) consists of the exploration of an ancient Inca mathematical artifact called *yupana*, in an electronic version. The aim of the authors is to explicitly make the bridge between several dimensions that opponents are unlikely to see as being compatible: “link tradition and modernity, indigenous and scientific knowledge, poor and rich cultures” (p. 49).

In both articles, authors note that classrooms are now, more than ever, multicultural sites, implying a need for a multicultural approach by the teacher. They also note that, in such multicultural classrooms, pupils bring extraordinarily diverse cultural experiences in which we can identify mathematical knowledge apart from mathematical programs that inform school work. This idea, however, contrasts with the example Favilli and Tintori (2007) explore with the teachers—the construction of a *zampoña* (Andean flute or Pan Pipes)—since no evidence in the article is provided that those Italian students were descended from South American people. Although we can accept the argument that all students should be confronted with practices and knowledge from different cultures, the idea of bringing to the classroom local knowledge of students is not visible in this example. The construction of the *zampoña* served as background to the learning of curricular mathematical content like proportionality, functions, and the concept of ratio, and, at the end, “results in the test that followed were quite good” (p. 46, speech of one of the teachers).

What is the problem with this almost idyllic example of a multicultural approach? Apparently researchers and teachers are valorizing other cultures, the manual work, the discussion among students, and the curricular mathematical content. No major problems were raised either by students or by teachers, at least, from the transcription of their opinions present in the article. But, are they really “valorizing” other cultures? What was the role of the Andean *zampoña* in these mathematical classes? Do teachers explore with students the Andean understanding of music and its meaning in the Andean culture? Do students (or teachers and researchers) acknowledge the social context involved in the local construction and use of a *zampoña*? For instance, the fact is that, in such a rugged environment as the Andes, the sense of community is absolutely integral to the concept of survival, and the way Andean people play the *zampoña* reflects the community spirit. Is this spirit compatible with the realization of standard tests designed to evaluate individual

achievements of mathematical knowledge raised by the *zampoña* exploration in the classroom?

In Fiorentino and Favilli (2007), the example changes—now the didactical tool is a reinvented yupana, an Inca abacus. The idea of the authors is to “reinvent the yupana to obtain an easy and solidly founded didactic tool” (p. 55). They construct an electronic yupana aimed at providing a classical base ten numeric feedback by continuously showing all relevant numeric information along the border. As far as I know, up to now nobody has been able to completely explain the meaning of the Inca geometrical tablets.¹¹ I suggest that the difficulty in deciphering these tablets is due to the fact that it is not just a mathematical matter. It involves a deep contextualization of the social, political, economical, and religious system in which these artifacts made sense. To transmit the idea to students that what Inca people did is similar to what they are doing in modern classrooms could end up reducing the meaning of yupana in the Inca culture for the purpose of learning school mathematics.

I argue that this “microscopic” exploration of a cultural artifact serves as no more than a “motive” or an “illustration” to learn the mathematics of the official curriculum. Culture is reduced to a musical object that, for that purpose, could be any other. The Other is desubstantialized (Žižek, 2009), deprived of all the social, political, economical, or religious dimensions that characterized their culture, when it is presented in the classroom as a folkloric example that neglects all the complexity that surrounds it. It is just a way of teaching mathematics.

6.2 Multiculturalism within capitalism

These examples make visible the sometimes shallow way in which the educational implications of ethnomathematics are thought and practiced. It becomes clear in these examples how powerful ideas, such as ethnomathematics' critique of academic and school mathematics, end up being co-opted by school practices, which do not sit well with the broader ethnomathematics critique of schooling. As we saw, the ethnomathematics program encompasses a critique of society and school. Thus, it is not enough to have a didactical approach to ethnomathematics or schools. The insertion of ethnomathematical ideas into regular schools needs to be theorized. In particular, I argue that we need studies that extol not so much the potentialities and benefits of multicultural education, but above all the obstacles and the inconsistencies involved in such education.

According to Žižek (1997), such inconsistencies make visible the way ideology functions today. Ideology becomes effective precisely in the discrepancy between the official discourse—which exalts teachers to work with students' topics of ethnomathematics, and schools to promote cultural diversity—and its actualization into a “life-world context” (p. 93). What, at the level of the enunciated content, runs smoothly—practically nobody within ethnomathematics research contests the importance of cultural diversity—when actualized in a specific practice (in our case, school practice) often encounters a series of obstacles which ends up perverting the core goal of ethnomathematics. This way, a potentially emancipatory enterprise such as ethnomathematics is transformed into what Žižek (1997, p. 76) calls an “inherent digression”, i.e., a change that is already predicted and even promoted (many curricula around the world already refer to the importance of bringing local cultures into the classroom) by the same system it tries to change.

¹¹ See <http://www.abc.net.au/science/news/stories/s1036168.htm> for a discussion of this issue.

This critique is not new. Already, Paulo Freire, four decades ago, called our attention to what he called “superficial transformations”: “the elites are anxious to maintain the status quo by allowing only superficial transformations to prevent any real change in their power of prescription” (Freire 1998, p. 508). Žižek gives us theoretical tools to understand this process of *domestication* (Pais, Fernandes, Matos & Alves, 2011) within the broader fields of politics, namely, within the hegemonic capitalist economics and ideology.

Although I will not have the opportunity to address in this article the full potentiality of Žižek's political analysis of multiculturalism as the cultural logic of multinational capitalism (Žižek, 2009), I shall call readers' attention to the way in which multicultural ideas can convey the spirit of “late capitalism” (Jameson, 1991). As I mentioned, today, ideology functions by means of making effective what officially conceals. In our case, we can say that other cultures are allowed to come into school, as long as they become part of the school culture. The system satisfies the societal demand of a meaningful education for all, by importing local cultures into the curriculum, while assuring that such “insertion” will not actually change any of the core features of the school system. It is in this sense that Žižek says that today's capitalism needs to promote constant reforms and innovations to conceal the crude reality that core choices (such as a radical transformation of the school system as advocated by some ethnomathematicians) are not available. As Žižek (2006, p. 348) puts it, “[t]his appearance of choice, however, should not deceive us: it is the mode of appearance of its very opposite: of the *absence* of any real choice with regard to the fundamental structure of society”.

Without a doubt, we—fed, washed, and scented people—live in a world where the choices available are numerous, in virtually all dimensions of life. Regarding education and ethnomathematics, there is already a considerable array of didactical proposals aimed to nurture a multicultural education. Indeed, if we take the various texts listed in the website of the Ethnomathematics Digital Library (a program aimed to develop resources for education and learning) we could, without difficulty, prepare a full mathematical curriculum around the exploration of local uses people do of mathematics. As it was mentioned by one of the reviewers of this text, in South Africa, a whole knowledge industry is developing around the idea of Indigenous Knowledge Systems of which ethnomathematics is one component. The power of capitalism to produce variety is at work in the educational applications of ethnomathematics. It is in this sense that Jameson (1991) calls our attention, in a somehow severe way, to the dangers of what he calls “neo-ethnicity”: “Ethnicity in the postmodern, in other words—*neo-ethnicity*—is something of a yuppie phenomenon, and thereby without too many mediations a matter of fashion and the market” (p. 341).

This incorporation of ethnomathematical ideas into capitalist dynamics is made possible through the deployment of an ideological injunction where we are willing to accept the Other deprived of its otherness (Žižek, 1992). That is, we are willing to accept the Other as long as it fits into our symbolic order¹²; as long as it is kept at a safe distance, the distance that prevents us from reaching its non-symbolic dimension. I love the Other (the poor, the indigenous) precisely because he is poor, oppressed, and utterly helpless, needing protective care. Žižek puts this way:

It is easy to love the idealized figure of a poor, helpless neighbour, the starving African or Indian, for example; in other words, it is easy to love one's neighbour as long as he stays far enough from us, as long as there is a proper distance separating

¹² I am referring to what Jacques Lacan (2001) called the Symbolic: the intersubjective symbolic network that structures our sense of reality.

us. The problem arises at the moment when he comes too near us, when we start to feel his suffocating proximity—at this moment when the neighbour exposes himself to us too much, love can suddenly turn into hatred. (1992, p. 9)

This critique, as we saw in the beginning, is also made by Kincheloe and Steinberg (2008) regarding indigenous education and Dowling (1998) regarding ethnomathematics. When we claim that we must “give voice” to the oppressed (poor students, ethnic students, minorities, and so on), we always take the risk, behind the appearance of a legitimate will for valorizing the voices of oppressed people, of stipulating how this voice should be, by positing the Other in our symbolic order, constructing him as innocent, in need of help, oppressed people who are seen as victims of our consumerist and racist society. They have voice as long as their voice is the voice of the oppressed, the voice asking for help, the voice we expect to hear. When the poor student admits that he just wants to be rich, or the “ethnic” student says that he only wants to learn the mathematics of the white people, we feel deceived, because we encounter the real Other. It is as if there was an underlying desire to keep someone in the status of a victim, so that we can enact in ourselves the desire for helping: “the saintly person uses the suffering of others to bring about his own narcissistic satisfaction in helping those in distress” (Žižek, 1997, p. 101). This “saintly” spirit, in all similar to the one of charity and philanthropy, completely endorses the spirit of capitalism. It allows people to ease their consciousness, while at the same time assures that no fundamental change in schools or in the economical organization of society occurs.

I want to make clear that I do not consider the kind of educational approach promoted by the example I explored to be representative of the way ethnomathematics conceives its educational implications. However, I do not see it as an extreme case either. On the contrary, situations such as this one—the application of ethnomathematical ideas into formal educational spaces without a critical reflection on schooling and society—are common in ethnomathematical research, as we saw previously. On the other hand, by criticizing the insertion of cultural artifacts into regular mathematical classrooms I do not pretend to diminish the value of the research or the goodwill of the researchers and teachers who are attentive to the diversity of mathematical practices. Both the *zampoña* and the electronic yupana can be good didactical tools (such as modern calculators, polydrons, 24game, or other of the many currently available tools to learn mathematics), depending on the way they are used in the classrooms. What I want to highlight is the assumption that by simply using these tools, presented as “multicultural” artifacts, we guarantee the promotion of what authors call a multicultural education. In fact, I argue that such activities, disguised as multicultural, end up conveying the idea that culture can be reduced to the exploration of an artifact, as, when we visit some foreign country, the idea that we get in touch with local culture by seeing some folkloric dance. Culture and cultural diversity are something much more complex and, sometimes, what we call local or foreign cultures are something much closer to our own culture than we are willing to accept.

7 Conclusion

In this article, I have tried to promote a deeper theoretical approach to the pedagogical implications of ethnomathematics. I have confronted some of the most important criticisms made of current research in ethnomathematics, showing that the educational proposals raised by ethnomathematics research are not consensual even among ethnomathematicians. Where some see as unproblematic the “making of the bridge” between local and school

knowledge, others criticize this learning strategy, claiming a place for a more serious understanding of the role of school and how local knowledge is inserted into it.

One of the main features of ethnomathematics research consists in developing a critique of what is accepted as being mathematical knowledge, by the confrontation of knowledge from different cultures. The existence of different ways of dealing with quantity, space, and patterns are now well documented, and it is not possible to deny them. But, to pass from this acknowledgement to the aim of inserting it in a school setting in order to be disseminated through school education is problematic because schools are not open spaces of shared knowledge. On the contrary, curricular changes, especially when the subject is mathematics, are very strict. Whether we choose to use this different knowledge as a curiosity, an illustration or a “starter” to the formal mathematics of the curriculum, or to develop a curriculum where one of the topics is local knowledge per se, the result may not be students' emancipation or the valorization of different cultures. On the contrary, the process of bringing diversity and ethnomathematical ideas into the classroom may end up conveying practices opposed to the benevolent multicultural ideas these researchers want to enforce, by promoting a desubstantialized view of Other's culture.

Therefore, unlike D'Ambrosio, I do not think that the educational implications of ethnomathematics are obvious. Although I do believe in the good will of teachers, politicians, or researchers in trying to bring to school knowledge and practices different from the ones conveyed by the official school curriculum, I have called attention to the sometimes naïve way in which this bridging is made. I challenge research in ethnomathematics to develop a stronger theoretical analysis of the social and political strands within which its research is carried out. Confronting different criticisms and contradictions of ethnomathematics and bringing the work of the Slovenian philosopher Slavoj Žižek to the field was my way of contributing to this theoretical discussion. It is my contention that such a discussion cannot avoid addressing the educational implications of ethnomathematics in the context of the two antagonistic functions of education in current societies: the necessity to preserve knowledge and practices from different cultures and, at the same time, the concern with the appropriation by all people of the global knowledge conveyed by school. Paying attention to the contradictions involved in this antagonism is needed so that well-intentioned actions do not end up having results opposite to their aims. Ethnomathematics, as a research field, takes advantage by rejecting any dogmatic position and being aware of contradictions implicated in their pedagogical aims.

Acknowledgments This article is part of my PhD project, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007. It is also part of the Project LEARN, funded by the same foundation (contract PTDC/CED/65800/2006). I am grateful to Maria do Carmo Domite who received me during the year of 2008 in São Paulo and introduced me to a field of literature and research without which this article would not be possible.

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METHODOLOGICAL ISSUES IN CRITICAL MATHEMATICS EDUCATION¹

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INTRODUCTION

During the last century, mathematics has been posited as a fundamental knowledge for scientific, technological and social development. Considered by many as the dorsal spine of our high-tech world (e. g. D'Ambrosio, 1993; Ernest, 1991; Skovsmose, 1994), during the twentieth century mathematics has become a cornerstone of progress, and its education synonymous with economical and social achievements. Today mathematics stands for far more than simply the scientific research produced by mathematicians. Mathematics stands for the rational power of the human mind, precision, rigor, trust in human consciousness, material success, economic benefits, technological progress, scientific development, school achievements, models to compare people and countries, social and political administration, citizenship, professional qualification, democracy, and so on. These different social arenas in which mathematics is involved poses important challenges

¹ A first version of this text was presented at the Mathematics Education and Society 6th International Conference, which took place in Berlin, Germany, from 20th to 25th of March, 2010.

² The research reported in this paper was prepared within the Project LEARN funded by Fundação para a Ciência e Tecnologia under contract # PTDC/CED/65800/2006. This paper is also part of Alexandre Pais's PhD study, supported by the same foundation, grant SFRH/BD/38231/2007.

for mathematics education that cannot be understood independently of the multiple social functions that it is expected to fulfil. If the purpose of education is to allow societal development in a democratic way, then it is not enough for people to learn a ‘petrified’ mathematics, isolated from the social implications that this science has in the world. The recognition that mathematics is present in many of our daily activities and decisions has been pushing the mathematics education community to rethink the aims of school mathematics. Since most of the mathematics ‘ruling’ our world is ‘under the veil’, students need to critically deconstruct the way in which mathematics formats reality, so that they can socially participate as informed and critical citizens in the construction of a democratic, social just society. This has been the call of critical mathematics education (CME).

To our knowledge, it was Marilyn Frankenstein the first person to use the term “critical” in connection with mathematics education in her 1983 text “Critical mathematics education: An application of Paulo Freire’s epistemology”. As expressed in the title, the conceptualization of “critique” in the work of Frankenstein comes from Paulo Freire’s liberatory pedagogy, where the notions of *conscientização* (critical consciousness) and *transformation* are crucial to thinking about educational practice. By *conscientização*, Freire (1998) understands the process in which humans, not as recipients but as knowing objects, achieve a deeper awareness both of the sociocultural reality that shapes their lives and of their capacity to transform it. Transformation is then the process undertaken by humans to change the oppressive situation in which they are caught. Freire is well aware that transformation without a critical consciousness of the real-life situation can lead to the dehumanization of people (p. 502). Thus the importance for Freire to merge into the educational practice a critical analysis of the conditions that oppress people so that the path for emancipation could be designed in a way that truly changes their actual life situations. As emphasized by Freire several times in his work: “The elites are anxious to maintain the status quo by allowing only superficial transformations designed to prevent any real change in their power of prescription” (p. 508).

Marilyn Frankenstein developed her work in CME mainly working in adult education, where she developed the “critical mathematical literacy” program. This approach sought to increase the mathematical confidence of adult students through a collaborative work where political and social issues are directly related with the learning of mathematics. Students are asked not just to solve some particular mathematical problem, but a mathematical problem potentially coming from all areas of everyday life—political, economical and social—as a way to critically analyse how mathematics is used to manipulate people’s decisions and how they can use it to interpret information, make informed decisions and transform their (often oppressed) realities.

Ten years after Frankenstein has coined the term “CME”, Ole Skovsmose published his book “Towards a Philosophy of Critical Mathematics Education”, where he launches the philosophical sparks for a CME utilising Critical Theory developed by the Frankfurt School. Skovsmose (1994) understands critical education as one that addresses the conflicts and crisis in society by means of disclosing inequalities and oppression of whatever kind (p. 22). In such a task, there is a desire for emancipation, whereby a critical education must not simply contribute to the prolonging of existing social relations. Skovsmose substantiates his idea of emancipation in the work of Habermas, who connected emancipation with a critique of the positivist way of researching in social sciences, and the need for social sciences to be founded on an interest in emancipation. According to Habermas (Skovsmose, 1994), without an awareness of the social and political role science plays in society, scientific research will be colonized by the technical-manipulative research paradigm, which makes science blind to its social impact. For Skovsmose, addressing the critical role played by mathematics in society implies to understand the risks and uncertainties that mathematics and societal progress conveys. In the field of mathematics education, this can be done by confronting students with situations in which mathematics seems to “format” the way we understand and act upon reality.

As can be noticed from the outset, both Skovsmose and Frankenstein go beyond educational views that don't take into account the social and political dimension of mathematics and its education. Also, within the theoretical frameworks informing their work there is, despite the differences³, a strong affinity. The presence of Paulo Freire's theory in Skovsmose's notion of *mathemacy* as well as the latter notion of "dialogic learning and teaching" developed together with Helle Alrø; and the fact that Paulo Freire himself was informed by theories coming from the Frankfurt School of Critical Theory, most notably Erich Fromm; are probably the most visible qualities. Together, Frankenstein and Skovsmose, can arguably be considered the authors of the seminal and most influential work within CME.⁴

These perspectives have spread their influence through mathematics education, until the point of being adopted by teachers in schools that somehow felt the need to change their practices towards a more significant and socially relevant learning of mathematics. This has been the case of Ana, a Portuguese mathematics teacher. In this article we shall explore Ana's attempt to bring into her classroom topics of critical mathematics education, by highlighting not the successful features of such experience but, instead, what went 'wrong'. That is, the problems Ana encountered in her practice when trying to implement topics of CME into a regular school. In analysing these 'obstacles' we will develop considerations about the proper

³ Frankenstein follows a Marxist, class centered approach, informed by Freirean pedagogy and critical pedagogy. While Skovsmose, follows a Habermasian critique of the "technological society". In the last years Skovsmose has been broadening his theoretical landscape, including insights from poststructuralist literature. See for instance Skovsmose (2006).

⁴ In the last two decades research within a social, cultural and political perspective has been increasing, leading some researchers (e. g. Lerman 2000) to speak about a "social-turn" to signal the entrance into mathematics education research of theories which recognize the cultural, social and political as influential dimensions when learning mathematics. Many of these researchers touch in one way or another concerns related with CME, without explicitly adding the prefix "critical" to mathematics education. Among the most important are perhaps the work developed by Eric Gutstein on issues of "social justice" in the United States, Gelsa Knijnik and her experience with the landless movement in Brazil, or Renuka Vithal and her work around methodological issues in CME in South Africa, but many others could be mentioned. In the book recently edited by Alrø, Ravn and Valero (2010), entitled "Critical Mathematics Education: Past, Present and Future" the reader can have a glimpse of the different research being done worldwide around the topic of CME. We cannot also fail to mention the entire field of ethnomathematics, and its struggle to understand how mathematics in society conveys hegemonic discourses and oppressive practices that promote exclusion and domination. For a discussion on the convergences and divergences between CME and ethnomathematics see Pais, Lima and Gerardo (2003).

methodological approach to be adopted if research is concerned in positing these problems in the ideological space that colours the dynamics of schools and mathematics education. We will take support from the work of Slavoj Žižek, which allows us to situate the discussion within the philosophical sphere⁵, where we intend to confront the theoretical assumptions behind CME and its actualization within a life-world context (in our particular case, Ana's experience as a teacher).

WHERE DO WE STAND?

Some researchers have been recognizing that the problems faced by the mathematics education community (most notably, the persistent failure in school mathematics worldwide, the senseless mathematics instruction which reduces mathematics education to the apprehension of a set of routine procedures to be reproduced in exams, and the problem of inequity and how school mathematics is associated with process of social exclusion) cannot be fully grasped within the strictly didactical perspective that animates the majority of the research being done in the field (Pais, 2011; Pais, Valero & Stentoft, 2010). A paradigmatic example of such awareness is the raising concern with *equity*, which led to the appearance of several publications in the last decade addressing this issue.⁶ For instance, Nasir and Cobb (2007) state that all the contributors to their book “view equity as situated and relational and as being informed both by local schooling practices and by practices and ideologies that transcend school” (p. 5). However, when reading the contributions in the book, we find that all the research reported is centred on improving the process of teaching and learning mathematics. Although politics is acknowledged as determinant in equity, and some authors explore the connections between mathematics education and politics (e.g., Gutiérrez, 2007), the contributions lack a theoretical analysis on how these “ideologies that transcend school” influence what happens in schools, and its contribution—or not—to equity. As mentioned by Gutiérrez (2007), “little has been

⁵ We understand philosophy in the Althusserian way, that is, as politics in the field of theory.

⁶ For a critical review of the research being done in the last decade around issues of equity in mathematics education, see Pais (2011).

written in mathematics education that addresses how mathematics might play a role in broader politics” (p. 38).

This article intends to address precisely this role politics has in the so-called “micro context” of schools and mathematical learning. This kind of analysis demands looking at research from a socio-political perspective (Valero, 2004) that explicitly aims at connecting the role of research—in particular in mathematics education—to the discourses and ideologies that fuel our current society. In order to understand the dynamics of the teaching and learning of mathematics and the way research results influence what is happening in mathematics classrooms, we need to contextualize these practices within the social modes of living that characterizes the world today.

The theory informing our approach comes from the philosophical work of Slavoj Žižek, which in the last few years has been actively engaged in recovering the so-called outdated notion of *ideology* as a crucial concept to understand the dynamics of our current capitalist society. Particularly important for our argument will be the notion of *symptom*. Despite being originally a clinical/psychoanalytical notion, Lacan already noticed how, in historical terms, it was Marx who invented it. If we are addressing a person, then a symptom is some disruption that perturbs the homeostasis of his or her physical or mental state. In psychoanalysis, the proper way to address a symptom is not to try to ‘heal’ it, that is, to erase its trace from the body of the patient, but to ‘make it talk’. In other words, instead of getting rid of these dirty malfunctions in order to keep the healthy Ego, the psychoanalytical method seek to suspend the patient’s Ego in order to confront the patient with his or her ‘dirty water’, that is the dysfunctionalities, which inhibit him or her from achieving a psychic equilibrium. Thus it is the symptom, and the process of its interpretation, which allows us to grasp the functioning of the person’s entire psychic system.

Why, then, was it Marx who invented the symptom? In *The Ethics of Psychoanalysis*, Jacques Lacan (2008) describes the ideology of *evolutionism* as implying a belief in a Supreme Good, in a final goal of evolution which guides its course from the very

beginning. This perspective is notably evident in the influential works of John Rawls and Jürgen Habermas. Despite their differences, they share the assumption that a theory of the social should be primarily concerned with the delineation of a set of universal principles that should guide our action towards a better society. According to Chantal Mouffe (2005), who developed a powerful critique on such approaches to democracy, Rawls and Habermas don't deny that there will be obstacles to the realization of the ideal discourse, but those obstacles are conceived as *empirical* ones. That is, they are not seen as being intrinsic to the conceptualization of democracy involved in the evolutionistic thesis, but as practical and empirical limitations of social life that, with time and effort, will naturally be surpassed. What in this thesis are seen as subsidiary problems of a 'good' system, are in Marxian theory the points at which the 'truth', the immanent antagonistic character of the system, erupts (Žižek, 1989, p. 114). Capitalism and its ideology posit progress, equality and freedom as natural ideals shared by all humankind. These are presented as the goals we have to strive for—we know what we want, so the question is how to achieve it. The fact that we are today (still) living in an unequal society, where democracy has been reduced to elections and our freedom to freedom to select between a set of pre-given conditions, is seen by today's liberal-democracy as simple deviations, contingent deformations and degenerations of the normal functioning of society, and as such abolished through amelioration of the system. By inventing the symptom, Marx called our attention to the fact that such 'empirical obstacles' are the necessary conditions for the maintenance of the system which generates them, and it is through them that we can perceive the antagonistic structure of society:

for the standard capitalist view, crises are 'temporary, correctable glitches' in the functioning of the system, while from the Marxist point, they are its moment of truth, the 'exception' which only allows us to grasp the functioning of the system. (Žižek, 2004a, p. 6)

Our challenge in this article will be to conceive the everyday problems that a teacher lives in her work when trying to implement a CME in a regular school not as

temporary, correctable glitches, but as core points where is possible to see the entire purpose of the school system: sorting people by means of (school) credit accumulation (as first pointed out by Vinner (1997), and further developed by Baldino (1998) and Baldino and Cabral (1998)). For this purpose, we take the standpoint that a critical methodological approach in research in education has not just to do with the way the researcher engages with the participants, but also the way the researcher makes sense of the empirical reality addressed. Reality is seen as contradictory, full of curves and spins, and a critical methodology is the one that tries to find a language to express these contradictions in a way that does not neglect them, nor clean the research from them, but takes them as part of the core focus. In order to enlighten this tension between a research that ‘cleans’ reality of contradictions and a critical one, we will bring in the example of the work of Ana, a mathematics teacher in a Portuguese secondary school. In the empirical part of her study Ana was confronted with several difficulties while trying to implement critical mathematics education in the context of a mathematics class. For the sake of the research, she decided to obliterate them from the final report (a Masters thesis), concluding that despite all the constraints she felt, it is possible and fruitful to bring critical mathematics education into the mathematics classroom. We see the difficulties faced by Ana not as marginalities, things to be avoided, details of a school system, but as core problems of the current school systems and societies that keep suspending what could be a radical emancipatory mathematics education. Therefore, in an analogous way to the psychoanalytical interpretation, we take the difficulties and constrains of a life-world situation not as particular vicissitudes to get rid of, but central issues for educational research.

A STUDY IN CRITICAL MATHEMATICS EDUCATION

The interest of Ana in critical mathematics education is partly the result of a concern with the way mathematics is traditionally taught in schools: as something disconnected from students’ reality. Being committed to pupils’ education for

citizenship, Ana sees her role as a mathematics teacher as an important factor in allowing her students to become participative, active, competent, critical citizens. The ways she found to accomplish this aim are diverse, one being the development of activities with students in which they can uncover and understand the role of mathematics in different social situations. She adopts Skovsmose's idea of mathemacy, as the competence to analyse and reflect upon the mathematics behind a world strongly structured around mathematical modelling (Skovsmose, 1994). As a final product of Ana's experience (which took place during the first period of 2006, with a class of 9th graders), she developed a Masters thesis where she explored the implementation of this critical mathematics education experience.

The Portuguese curriculum explicitly mention that "mathematics education has the purpose of helping students to uncover the mathematics behind the more diverse situations, promoting the education of participative, critical and confident citizens" (ME-DEB, 2001, p.58, our translation from Portuguese), thus apparently incentivizing Ana's effort to implement a CME. However, Ana had to *recontextualize*⁷ the official discourse of the curriculum in order to be able to conceptualize and implement a CME. Such recontextualization followed many of the assumptions behind a CME: choosing tasks which bring to light the social dimension of mathematics, making sure that the theme of the task is relevant to students' social reality, and assuring that the task will allow a critical analysis of the mathematics behind the modelling of a specific social activity. For this to be possible, Ana carried out informal conversations with her students in order to know better their concerns and social interests. She soon realised that students' concerns were strongly

⁷ Ana draws on official documents, such as curriculum guidelines, subjects programs and others, produced by official recontextualizing agents and recontextualizes them in a way that allows her to introduce CME on the classroom. To do that, Ana takes advantage of a set of discourses and practices, available within the field of recontextualization, and subsumes them under their aims and purposes (Morgan, Tsatsaroni e Lerman, 2002). Among such discourses are those produced by teacher training courses and educational masters and circulated within the Unofficial Pedagogic Recontextualizing Field (UPRF) (Bernstein, 1996). Elements of these are appropriated by official agents, often constituting central elements of the official discourse. Thus, official discourse consists of a variety of elements drawn from heterogeneous discourses, some of which form unofficial, sometimes oppositional educational discourses (Tsatsaroni, Ravanis and Falaga, 2004).

connected to daily life activities ranging from the use of public transportation to eating habits and media. After this casual survey Ana spent time studying the best way to construct a task that brought together the development of mathematical competences and the possibility to engage in a critical discussion about the way mathematics formats some of students' daily activities. As a result, two tasks were construed and explored with Ana's students: "Supermarket promotions" and "A taxi trip".⁸

Therefore, everything seems prepared and even willing to implement a critical mathematics education in the classroom. What issues are involved here?

The first issue is the decision of Ana of not implementing her critical mathematics education experience in the regular schedule of the mathematics class. She decided to invite some students to form a club, outside the hours designated for mathematics, where they could develop the tasks designed by her. Ana justifies this decision because students of the 9th grade will have a final exam at the end of the year, on which their final grades will depend and the approval to enrol in 10th grade the next year. Here we can notice the contradiction between the official discourse (present in the curriculum—involving students with topics of critical mathematics education) and the real practice where it is the exam which delineates the teaching content and form. Ana is well aware of this contradiction:

It is though visible that, even being a teacher concerned with critical mathematics education and conscious that uncovering mathematical structures present in social phenomena is a way of learning potentially more meaningful to the majority of pupils, the pressure of the school system (school, parents, pupils) took me to decide for this alternative [critical mathematics education

⁸ The former confronted students with some of the mathematical models informing the way we participate in our current society as consumers, by exploring how students engage in shopping when going to the supermarket. On the other hand, "A taxi trip" tried to bring together important social issues such as the recent raise in the price of fossil energy and all the social consequences that follow. In Portugal, for instance, the rise in the price of fuel has led the population to use more and more public transportation, which, in principle, should be more economic. The task explored by Ana was intended to critically analyse the advantages (or disadvantages) of using public transportation (in this case, taxis) instead of private modes of transportation.

developed not in the regular class but in a club] showing the first kind of difficulties that a teacher faces when one wants to implement this kind of work in the context of school mathematics. (Alves, 2007, p. 57, 58, our translation from Portuguese)

The idea conveyed here is that although it is good and innovative to implement such topics, there is an inner and rather invisible pressure into conformity that the teacher is aware of and that makes her put into practice activities that do not directly challenge the school system (and do not change any core features of the school structuring activities). On the other hand, it makes explicit the fact that critical mathematics education is not part of the curriculum and in pupils' minds creates the idea that perhaps is not really mathematics.

Another aspect of the research of Ana that we want to highlight is the criteria that she used to choose the students to interview. She opted for those who had shown more interest and enthusiasm within the sessions, and justifies this choice by mentioning the visibility – “choosing those who appeared more involved and participative in the sessions was a way of guaranteeing the collection of data (...) I choose the students who gave more visibility to their involvement” (Alves, 2007, p. 66, our translation). This is an option that most researchers choose (finding the ‘best’ informants) as they need to provide clear evidence of their claims. In the case of Ana, what did she want to make visible in her research? She wanted to highlight the potentialities of critical mathematics education for developing citizenship. Therefore, it was not appropriate to choose students who in one way or another did not engage so enthusiastically with such experiences. On the other hand, the selection of the students was also related to the aim of her research. This type of ‘selection’ is a case of what Vithal & Valero (2003) call the ‘cleaning’ of research—putting aside conflicts and constraints so that research is presented in a harmonious and positive way.

Finally, Ana justifies the lower involvement of some students because, on the one hand, they were not familiarised with the way they could work in the club (which was more unstructured and ‘free’ than in the classroom environment) and, on the other

hand, they were still attached to a vision of mathematics as a static science having nothing to do with real life situations. Although these arguments could be true, we suggest that other issues are at stake. We take the risk of saying that the lower involvement of the students could be due to the fact that they knew that these activities would not contribute directly to prepare them for the tests and to get a good mark at the end of the year. Using Vinner's (1997) description of school as a credit system, we could say that students felt that those activities would not give them much credit.⁹ Just remember how many times a teacher who wants to proffer some explanation (a little bit of history, an application, a connection with other themes, a more insightful explanation) heard the students promptly ask "will that show up in the test, teacher?" And the teacher is forced to say "well, yes" if maintaining students' attention is on the agenda.

LOOKING AT THE SYMPTOMS

The research developed by Ana shows methodological concerns that are characteristic of a critical methodology. The most evident one is the assumption by the researcher of her subjectivity. Ana expresses her concerns about the difficulties of implementing critical mathematics education in schools, the resistance of the students to such topics, the pressure to fulfil the entire disciplinary program, the need to prepare students for the final exam. This is an example of what Valero (2004) calls "making the researcher visible" (p. 19), which opens to the critical examination of the reader the products of the research process, the intentionality of the researcher, and the paths that the researcher decided. It was this openness in the work of Ana that allowed us to develop such a critique on her work.

But, despite all the difficulties, Ana assumes that it is possible and desirable to develop tasks of CME with students, and suggests that this could be a way of promoting a bigger societal transformation. That is, Ana endorses the evolutionistic

⁹ It is always useful to remember the research carried out by Baldino & Cabral (1998), where they show how students in school are primarily worried about passing (and not necessarily to learn).

thesis and conceives these difficulties as contingent obstacles which, although conditioning the implementation of CME, do not compromise its global purpose. However, if the goal of CME is not merely a “didactical” one, but above all a “societal” one, having to do with emancipation from social forms of oppression, then a closer look should be given to the way in which these obstacles can be seen not as a contingency but a necessity of the same system which promotes the implementation of CME in schools.

In the case of the research developed by Ana, one of these symptoms is the fact that CME collides with the assessment system, which forced her to implement the critical mathematics tasks outside of the mathematics classroom. Ana sees this contradiction as a difficulty, as a problem she had to surpass in order to open a space to promote critical education to her students. But what this contradiction shows is that what the system points as the most important role of the teacher is in fact to prepare students for the final exam. It is good to work with students on these ‘radical’ topics as long as they do not change the smooth functioning of schools as credit systems. This way a potentially emancipatory attempt to educate students according to a critical education is completely inserted and transformed into what Žižek (1997, p. 76) calls an “inherent transgression”, that is, a change which is already predicted and even promoted (the curriculum exalts teachers to work such topics) by the system it tries to change.

From the point of view of the Marxian theory explored above, these ‘marginal’ problems make visible the inconsistency of a system that, on the one hand, demands mathematics for all and, on the other hand, uses school mathematics as a privileged mechanism of selection and credit. If we recover the Marxian ideas which were at the core of Critical Theory as it was developed by the Frankfurt School and, to some extent, by Paulo Freire, we can say that all these ‘phenomena’¹⁰ are necessary

¹⁰ Such as testing and selection, the lack of students’ engagement with issues that will not give them school credit, or the need to develop critical mathematics education outside the regular math schedule.

products of the system itself. That is, the discontentment felt by CME regarding school mathematics is a discontentment that is consubstantial with school and, more broadly, with society. If the aim of critical mathematics education is emancipation from forms of social organization based on inequity, exclusion and non-democratic tenets, then it becomes paramount to acknowledge that these disruptions are not marginal to our society but the very core of its functioning and reproduction. We understand critical methodology as explicitly addressing these borderline problems, which truly connects mathematics education to the political sphere in which we live.

RECOVERING THE CORE MEANING OF “CRITIQUE”

In these last pages we would like to address some philosophical issues involved in our discussion, by recovering what we consider to be the core meaning of critique both in Freirean Pedagogy and Frankfurt School. We will argue that the notion of critique suffered from a kind of “domestication” designed to keep at a safe distance the radical emancipatory potentiality of this notion.

We should start by clarifying how the Critical Theory developed within the Frankfurt scholars’ conceived society and the existing social relations. In other words, which was the core focus of the social and political critique developed by these scholars? The answer is capitalism. Despite major differences between members of the Frankfurt School in their assessment of the development of capitalism, it may be noted from the outset that their respective analyses were informed by Marxian tenets (Held, 1980). According to Benhabib (1994) the core feature of Critical Theory, as it emerged in the works of Horkheimer, Adorno, Marcuse, Löwenthal, Pollock and Benjamin, was the realisation that a revolutionary transformation of capitalism from within capitalism itself was doomed to fail. Critical Theory was confronted with the enterprise of thinking a “radical alternative”.

Although initially the critique was focused on political economy, with time it gave place to a critique of instrumental reason, as a response to a positivist paradigm which restricted research to the activity of outlining correlations between well-

defined phenomena. These two critiques did not coincide; rather the critique of instrumental reason surpassed the critique on political economy:

The transformation of the critique of political economy into the critique of instrumental reason signals not only a shift in the *object* of critique, but, more significantly, in the *logic* of critique. (Benhabib, 1994, p. 79)

The work of Habermas exemplifies this shift on the logic of critique. In his work, political economy is not so much a matter of *superstructure*, of class struggle, but a matter of administration and technique, due to a change in which politics becomes the sphere for the technical elimination of dysfunctions and the avoidance of risks threatening “the system” (Held, 1980). This split provoked a displacement of the way the political was conceived: capitalism became “naturalized” and accepted, and transformation started to be conceived inside capitalism. Capitalism is no longer seen as the fundamental core of the problem, as the system we have to emancipate from, but the social and political background in which emancipation can take place. This disavowal of Marxism in theory was accompanied by the discarding of communism as a political system. After the fall of the Berlin Wall and the dismantling of the Soviet Union, communism is doomed to be associated with a dark, totalitarian past and the end of history, to use Fukuyama’s words, was proclaimed: capitalism becomes the global social, political and economical system, to which there seems to be no alternative.

However, every day it becomes more difficult to accept this verdict. The recent economical crisis seems to have aroused a sense of discomfort with capitalism; a sense that there is something profoundly wrong with a system that enables the obscene accumulation of wealth in a minority group of people. Without surprise, the measures taken to deal with the so called “crisis” are cosmetic operations—the injection of huge amounts of public money into the banking system, increasing taxation of private profit, governmental limitations on capital speculation, the criminalization of individual magnates of which Murdoch in the USA is the best known example—and will just repress a problem that we have known since Marx is

necessary to capitalism: inequality. A clarification should be made here. It is not just that capitalism produces inequality as a residue that can be managed through, for instance, philanthropy or charity. Inequality is a necessary condition for capitalism to emerge and to reproduce. Thus, and this is one of the central Marxian insights to think today's economics, it is counterproductive trying to eliminate inequality within capitalism. If the purpose is to get rid of inequality, then capitalism, as the whole encompassing economical system mapping our lives, must go altogether.

Although not so explicit in Paulo Freire's work¹¹, the Marxist vein is easily visible in the way he conceives the thematic of change. Freire was well aware that no matter how intense and spread the local struggles in which we are involved could be, without a change in the "superstructure", these local struggles easily end up being co-opted by the dominant ideology:

Cultural action occurs at the level of superstructure. It can only be understood by what Althusser calls "the dialectic of overdetermination". This analytic tool prevents us from falling into mechanistic explanations or, what is worse, mechanistic action. An understanding of it precludes surprise that cultural myths remain after the infrastructure is transformed, even by revolution. (Freire, 1998, p. 480)

These "mechanistic actions" that Freire mentions are nothing less than the capitalist demands for perpetual reforms by means of integrating what could be new and potential emancipatory acts into well established social structures. This paradox of capitalism—the constant need to change something so that nothing really changes—leads Žižek (2006) to argue that the threat today is not passivity but pseudo-activity, "the urge to 'be active', to 'participate', to mask the Nothingness of what goes on" (p. 334).

¹¹ Which is justified partly because at the time when Paulo Freire developed his work Stalinism was at its peak, and the reference to the Marxist insignia could easily lead to the automatic discard of the one who endorses it, or even more severe persecutions (which is still more accentuated by the fact that Freire at that time was living in a dictatorial regime in Brazil)—Paulo Freire was indeed arrested and detained for seventy days in 1963 accused of developing a teaching method "similar to that of Stalin, Hitler, Péron and Mussolini" (McLaren, 2000, p. 144).

Despite the apparent fall of capitalist principles in the last two years, we are facing the emergence of reforms that keep unaddressed the core of the societal problems. All emancipatory actions are thought and put into action within capitalism. If we recover the critique of political economy developed in the first years of the Frankfurt School, we can say that emancipation from capitalism failed completely. No radical alternative was made. But this fact contrasts with the proliferation of the idea of critique, especially in education. In curricular guidelines all around the world the word “critique” is being used to enhance the importance of allowing students a critical education and to become critical citizens. It is in that sense that we argue that the word critique has become “domesticated”, it has lost its most radical meaning. It is a case of what Žižek (1995) calls “progressive amnesia” (p. 9): we recover Critical Theory but it is deprived of its true transformative core. It is fine to take a critical stance as long as you do not raise questions that could undermine the foundations of society—we are allowed to be critical as long as we are not actually critical...

At this point a question should be raised: in which way does this disavowal of “changing the totality” reproduce capitalist ideology? For sure it has to do with the fact that capitalism today *is* a totality. That is, capitalism is the all-inclusive economical system that ends up subsuming the cultural, social and political spaces of our lives. It has become common sense to argue against the increasing capitalization of the academy, technology, education, media, leisure and, of course, politics, where the logic of the market dictates the, many times invisible, rules for act and be acted in these social spaces. Although we live in a world of multiple social, cultural and political realities, we must ask what, in all these different sets, remains unchangeable, that is, necessary for the existence of this multiplicity as such. In cultural and social terms, there is no doubt that the world is diverse. European culture and sociability, despite all the similitude, is different from North American culture, as we can easily notice in literature, cinema or philosophy. Not to mention the perhaps even deeper differences between these and Asian and African cultures. Also in political terms, we have around the world all sorts of political organizations: neo-liberal American

ideologies, European social-democracy, China's (capitalized) communism, Arabic religious states, etc. However, when talking about economy, we get stuck. Can we say that we have a plurality of different economical systems around the world? No, *the* global economical system present in all this multiplicity of cultural and social formations is capitalism.¹² Even communism, whether we are talking about state communism as in China or North Korea or the communist parties that subsist within capitalist democracies, follows the rules of capital. Capitalism dresses in diverse clothes in order to keep reproducing, and no matter how different the "philosophies" of political organization could be around the world (monarchy, socialism, religious fundamentalism, dictatorship, neo-liberalism, etc.) what is common in all of them is that, despite the apparently different "clothes", the human relations are based on capital. By explicitly mentioning capitalism we want to point to the very core of the problem—this (so often) unaddressed reality that permeates all social relations and for which we seem to have no alternatives.

Making use of Hegel's dialectics, Žižek (2004b) poses as follows this "overdetermination" carried out by capitalism:

More than ever, capital is the 'concrete universal' of our historical epoch. What this means is that while it remains a particular formation, it overdetermines all alternative formations, as well as all noneconomic strata of social life. (p. 3)

Being a "concrete universal", capitalism ends up co-opting the multitude of local struggles that animate, for instance, the CME program. In this sense, we risk saying that, concerning emancipation, the CME plea is a failed one from the beginning—it is ignited by the renunciation of changing not just some variables of the 'game', but the whole system of coordinates that maps the way we play the game. The giving up of a

¹² Not all societies are at the same level of capitalist development, and, evidently, some are still indeed in pre-modern systems of production. However, with the advent of global, multinational capitalism, the way to get into the "market" is through capitalism. For instance, we can say that indigenous tribes in Amazonia don't live in a capitalist economy. However, when exchanges with the "outside world" are needed (and they are increasingly needed since the neo-colonization taken care of by capitalist expansion obliges these people to search for products that cannot be produced within the community) they are automatically inside the capitalist mode of production. So the choice is between accepting capitalism or to perish.

common struggle against capital in favour of a multitude of particular struggles is the result of an accommodation to or a *naturalization* of capitalism: we are no longer able to change as a whole, and the least we can do is to content ourselves with our private struggles.

As Adorno and Horkheimer (2002, p. 135, 136) brilliantly posed, the thematic of choice within capitalism is reduced to choose between a set of pre-given and non crucial features:

All are free to dance and enjoy themselves, just as they have been free, since the historical neutralisation of religion, to join any of the innumerable sects. But freedom to choose an ideology—since ideology always reflects economic coercion—everywhere proves to be freedom to choose what is always the same.

The endless myriad of choices available today conceals the fact that the “true” choices are not accessible. The impossibility of choosing the core features that map our lives must be disguised by the availability of a multitude of choices that don’t change anything. As Žižek (2006, p. 348) put it, “[t]his appearance of choice, however, should not deceive us: it is the mode of appearance of its very opposite: of the *absence* of any real choice with regard to the fundamental structure of society”.

CONCLUSION

The realisation that the problems faced in mathematics education transcend the boundaries of the field is well acknowledged in critical mathematics education research. It is common to recognize that critical mathematics education research requires social and political approaches that commonly situate the problem of “change” in a broader context than the classroom or schools (Gutstein, 2003; Gates & Zevenbergen, 2009; Valero, 2004). Although studies in a critical trend acknowledge this social and political dimension of emancipation (especially in the beginning and the end of the texts), we argue that they manifest signs of persistence as if the problem of allowing students a “critical education” could be understood and solved within mathematics education. It is as if we realize that the problem has a social and

political nature well beyond the classroom, but, since we are mathematics educators, we should investigate it in the classroom.

It is our contention that without deepening the theoretical understanding of such problems, by means of broadening the way we conceive and study them—namely by positing them not as “didactical” ones but as truly social and political problems that require perhaps different theoretical approaches—we run the risk of moving blindly. Our aim was precisely to call the reader’s attention to the way ideology is at work in CME in the way potentially emancipatory actions intended to produce change (such as the one carried out by Ana) get caught in a system that uses them to assert its intention to change, while the main features of it remain unchangeable. This was the way we found it productive to posit the problem of CME, and the change it wants to carry out in mathematics education, namely through the introduction of topics of CME in regular classrooms. The introduction of such topics, we argue, although potentially emancipatory, can easily end up reinforcing the same system it criticizes, by being caught in the dynamics of capitalist ideology which we characterized as “evolutionistic”.

Against the belief that fuels CME that we can retain the consistency of school mathematics by means of getting rid of the embarrassing symptoms which disturb this consistency, we sought to argue that such symptoms are indeed intrinsic and necessary to the reproduction of the system which posits them as contingent phenomena. Our proposal is that such embarrassing symptoms should be made the core focus of a CME research (instead of, for instance, research only concerned in discussing the potentialities and the positive experiences carried out under the insignia of CME). That is, it is our contention that the symptoms are to be addressed not as a minus detail, to be rapidly discarded in a footnote, but as the core of a system in which the “ideal discourse” only serves to conceal the pure antagonistic nature of social reality. It is the antagonism itself which must be addressed. For instance, when we read an abstract ‘ideological’ proclamation such as ‘mathematics for all’, we

should be aware that people's experiences are different. For teachers and students know and experience that in any mathematics class there will always be some—or many—who fail. The official discourse functions not as some kind of utopian state to be achieved, a desired good to strive for, but rather as a pure mechanism to conceal the fact that mathematics is not for all. The obliteration of the “background noise”—the voices of those who will always fail—is the very core of utopia. The “background noise” conveys “the obscenity of barbarian violence which sustains the public law and order” (Žižek, 2010, p. 10). In the case of mathematics education, the obscenity of the barbarian violence that school exercises year after year when it throws to the garbage bin of society thousands of people, under the official discourse of an inclusionary and democratic school. If, for some uncanny reason, suddenly all students become successful in school, the entire system which relies on schools to perform their role as credit systems will collapse.¹³ The symptoms cannot be erased without questioning the school system as a whole.

Some will say that such an awareness of the problem takes us to a deadlock. Indeed, by realising that schools are overdetermined by capitalist ideology, we are faced with the monstrous task of—if the purpose is radical emancipation—ending schools as we know them. In the current myriad of world social structure this does not seem possible. However, what dooms us to repeated failure is precisely experiencing the change as impossible—we acknowledge that to achieve the desired emancipatory goals of CME requires a fundamental societal change which we experience as impossible. There is no easy way out. On our part, we adopt the old Pascalian maxim: “Be optimistic in practice, while pessimistic in theory”. While in practice, we completely support the struggle of many teachers (although they are a minority—how many teachers worldwide have ever heard about CME?) to promote activities that raise a critical consciousness on students on the role of mathematics in society, we argue that what should also be part of this consciousness is the way school and its

¹³ Certainly, within capitalist ideology, other selective mechanism will occupy the place of school.

ideology co-opt such practices so that they will, in the end, reproduce what they criticize. Thus, although in practice we should be optimistic, our role in theory (that is, when thinking deeply about the problems involved) is to have a broader understanding of the dynamics of change, precisely by positing mathematics and its education not merely as school subjects responsible for the acquisition of knowledge and competences by the students, but also as “disciplinary mechanisms” to use a Foucaultian language, or ideological state apparatus if we follow Althusser. This is the arena in which our paper was written. And this is a theoretical manuscript, in which we dare to be pessimistic about the belief that, with time, and through our local struggles we will achieve the desirable change. This jump from quantity to quality is based on a “leap of faith”: there are no guarantees that such a change will occur. We think that more easily CME becomes one of the dozens of “school mathematics contents” completely inserted into the maintenance of school as an institution at the service of capital.¹⁴ And this “phenomena” needs to be understood if we really want to address radical emancipation. That is, teachers need to be confronted not just with good examples of activities to be explored in a critical way with students, but also to the bigger macro dynamics that colors their work as teachers. Without this broader awareness, we risk moving blindly even though we seem illuminated in pursuing the right path.

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¹⁴ An exemplary case of such domestication is the way ideas coming from CME have been inserted in the examination system itself. The importance to teach mathematics around its applications has in the last decade considerably changed the mathematics curriculum for the secondary school (10th to 12th grade) in Portugal. As part of the final evaluation and in order to get into university, students of the 12th grade are obliged to do national exams. Real problems are now an integral part of the mathematics national exam. Students are called to address complex social problems—such as modelling the AIDS spreading in Africa, or minimizing the effects of global warming—in the setting of a final examination from which his or her future depends. This real problem is proposed to students as one of the 10 or 11 problems of the exam, leaving students with more or less 12 minutes to solve it. The question is, of course, where is the mind of the student when solving this problem? Is he or she truly engaged in AIDS or global warming or does he or she just wants to solve the question and move on to the next?

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A CRITIQUE OF IDEOLOGY ON THE ISSUE OF TRANSFER

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The idea that mathematics is needed for our mundane everyday activities has stirred up a concern with how people deal with mathematics outside school's walls. Much has been written in mathematics education research about the possibility of transferring knowledge from and into school. While the majority of this literature commends the possibility of transfer, thus assuming both the desirability of transfer and the importance of school mathematics for the professional and mundane lives of individuals, I am interested in developing an ideology critique on the belief sustaining the research investigating this issue. It will be argued that the use-value attributed to school mathematics disavows its value as part of a political and economical structure, which requires school mathematics to perform other roles than the one related with utility. This critique will be illustrated through the exploration of a typical transfer situation between school and workplace.

Keywords: Economy, ideology, transfer, use-value, Slavoj Žižek.

1. Introduction

The importance attributed to the *use-value* (Dowling, 1998) of school mathematics—the utility of mathematics in optimizing the mundane activities of people—is widespread both in research and in curriculum making. The importance of exploring with students ‘real-life’ situations is acquainted with a more meaningful learning of mathematics, the development of competent citizens, the enhancement of general problem solving competencies and attitudes, and the use of mathematics when solving problems of everyday life (e.g., Freudenthal, 1973; Frankenstein, 1983; Niss, Blum & Huntley, 1991; Boaler, 1993; Skovsmose, 1994; de Lange, 1996; Brenner, 1998; Chapman, 2006). In this realistic perspective, the importance of mathematics comes not so much from its ‘internal’ characteristics (as was the case with the *new math movement*), but from the possibility mathematics offers for people to understand and act within real world situations. This view has influenced curricular changes all around the world (Blum & Niss, 1991; de Lange, 1996; Dowling, 1998), which can be easily noticed by reading curriculum guidelines from countries so disparate as Colombia, Portugal, South Africa or New Zealand. Evaluations of what is considered to be a quality mathematics education seriously take into account the capacity of students to use mathematics as workers and citizens (Bishop & Forgasz, 2007; Atweh, Graven, Secada & Valero, 2011). And a global assessment program known as PISA (OECD, 1999) has been designed, and implemented in

the principal industrialised countries, around the idea that students should be proficient not merely in terms of mastering the school curriculum, but in terms of important knowledge and skills needed in adult life. Both in research, curriculum making and evaluation, the importance of mathematics to understand and deal with real situations—its use-value—has become a leading reason sustaining the learning of mathematics in schools.

The consideration that students should be able to apply school mathematics in real life situations is supported by a considerable amount of research exploring the relation between school and out-of-school mathematics; whether these are ‘crystallized’ mathematics which the person needs to formally learn, or ‘local’ mathematics which need to be recognized and worked within school mathematics. This brings up the issue of *transfer*: the use of ideas and knowledge learned in one situation in another (cf. Evans, 1999). Such loose definition suits very well the metonymic effect I intend to operationalize with the word “transfer”. In this article, the word transfer is intended to resonate on other signifiers that somehow attribute importance to the use-value of mathematics. In general terms, research developed under the label of *real life/world problems* (e.g., Freudenthal, 1973; Gravemeijer, 1994; de Lange, 1996), *word problems* (e.g., Wyndhamn & Säljö, 1997; Verschaffel, Greer & De Corte, 2000; Chapman, 2006), *applications* (e.g., Niss, Blum & Huntley, 1991) *transition* (e.g., Abreu, Bishop & Presmeg, 2002), *context* (e.g., Boaler, 1993; Wedege, 1999), or *workplace/adult* (e.g., Bessot & Ridgway, 2000; Coben, O’Donoghue & Fitzsimons, 2000), address people’s use and transfer of mathematics in different situations. Within these areas, a set of important theories has been used and developed: theories of situated cognition/learning (e.g., Lave, 1988; Lave & Wenger, 1991; Watson & Winbourne, 2008), poststructuralist developments (e.g., Walkerdine, 1988; Fitzsimons, 2002), critical education (e.g., Frankenstein, 1983; Skovsmose, 1994; Gutstein, 2003), and activity theory (e.g., Mellin-Olssen, 1987; Williams & Wake, 2007; Jurdak, 2006), are the ones I found of interest for this article. Although the metonymic effect involved in the way I am using the word transfer can make tabula rasa of the different angles, theories and methodologies involved in these studies, it allows the possibility of detuning a sense of *universality* (Butler, Laclau, Žižek, 2000; Žižek, 1989, 1997, 2006). I will be not so much interested in the differences within the studies on transfer, and they are immense, but in what remains the same despite all the differences. As the Slovenian philosopher Slavoj Žižek has been showing, this is the field of *ideology*, of an “unconscious fantasy” (Žižek, 1989, p. 33) structuring our belief. In this case, the fantasy is the belief in the importance of the use-value school mathematics has for people’s lives.

In this article, by taking advantage of the recent revitalization of the notion of ideology by Žižek, and through the exploration of a transfer situation between school mathematics and workplace, I shall argue that the importance given to the use-value of mathematics functions as the necessary ideological ‘double’ disguising the role school mathematics plays within capitalist economics and ideology. It may be that people in mathematics classrooms learn mathematics, but they also learn the necessary conditions for the reproduction of a certain kind of political and economical organization, as authors such as Bourdieu and Passeron (1977) and Althusser (1994) had demonstrated. By positing schools as places of economical and ideological (re)production, I seek to address how ideology operates in the way teachers and students convey discourses about the importance of mathematics for daily life activities. Although asserting the importance of mathematics for their work, students, as we shall see, clearly do not transfer any of the mathematics learned in school. Such incongruence between what people say and what people do is, in line with Žižek’s theorization, the proper ideological mechanism assuring that, under the guise of a useful mathematics education, the role of schools as part of capitalist economics remains unaddressed.

I will start by discussing the importance of considering the fields of ideology and economy with respect to what Gutiérrez (2010) recently called the “sociopolitical turn in mathematics education”. It is my contention that the postmodern spirit animating this turn disavows a broader comprehension of schools as places of economical production. Afterwards I will review the predicaments around the research on transfer, emphasizing studies that have shown the difficulties or even the impossibility of transfer, in order to argue for the fictional character of the importance attributed to the use-value of mathematics. The second part of the article will deal with the exploration of a typical transfer situation between school and workplace: young students attending a blacksmith technical course with mathematics as a compulsory school subject designed to be useful for their practice. I will draw on the research developed by Elsa Fernandes (2004, 2008), who worked closely with the participants in this study during her PhD dissertation. As we shall see, although the course is designed so that students can learn meaningful mathematics for their work, what they end up learning is that in order to get approved in the course they have to pass in a school subject called mathematics which, although useless for their lives, is presented as the most important qualification to become a blacksmith. That is, they learn how to engage in a discourse without really believing in it. Even more, they learn that in order to pass the course this facade must be kept. Ideology appears then in a new light: it is no longer a superstructure dominating individuals, but individuals themselves cynically handling ideology for their own interest. Thus, the

proper answer to such cynical position is not to say: “it may seem to you that mathematics is important for your daily life and work, but in reality, it is completely useless”; but “it may seem to you that mathematics is useless for your daily activities and work, but in reality, its magic properties make mathematics crucial if you want to achieve a proper life and work”. These magic properties are the ones that inexplicably make mathematics such an efficient component of the school credit system. Indeed, I shall argue that such properties are the ones that make mathematics economically relevant, while the importance of mathematics for daily life and work functions as the necessary “fantasy-scenario” (Žižek, 1997, p. 6), preventing us from dealing with the political economy of schooling.

The reader should keep in mind that my critique is pointed towards the *research* done in mathematics education. Thus it should be read as “research on research” (Pais & Valero, forthcoming). Although focused on the issue of transfer, this analysis aims to have implications for how the community of researchers situate their work within the political spectrum of our times. By articulating the topics of *transfer*, *ideology* and *economy*, I seek to address what Tony Brown (2008a, p. 239), drawing on Jacques Lacan, calls the big Other of Mathematics Education. That is, I seek to question the entire system of beliefs—the fantasy—sustaining the discourse around the importance of mathematics as use-value. It is my contention that many of the problems related to school mathematics cannot be fully conceptualized without some kind of ‘political mapping’ that situates them not at the level of ‘learning’ but as part of the political and economical landscape of our times. The recent research done under the label sociopolitical (cf. Gutiérrez, 2010) falls short in providing an account of the system as a whole. By reducing the Political to identity and power issues—what Laclau (Butler, Laclau & Žižek, 2000, p. 202) calls “issue-oriented politics”—it leaves us without theoretical tools to address the global economical system of capitalism, and the role of school mathematics within it. It involves, as we shall see, a tacit acceptance of capitalist economics as the unquestioned framework structuring our social life. Perhaps the field of mathematics education research is dismissing too quickly theories which allow us to grasp what Butler, Laclau & Žižek (2000) call a sense of universality; of what is this big Other that gives meaning to our daily efforts to make school mathematics more meaningful for people.

2. Ideology, economics and politics of mathematics education

“It is clear, even if one admits that Marx will disappear for now, that he will reappear one day.” (Foucault, quoted in Cole, 2003, p. 489)

Before entering into the issue of transfer properly, I shall start by saying some words on ideology, and by suggesting a political reading of mathematics education. This is important since it will allow me to explain how I intend to operationalize the notion of ideology, and to bring forth the importance of considering the economical role of schools.

2.1. Between politics and the Political

Within a field historically centred on the specificity of mathematics and having as main theoretical informant the discipline of psychology, it is not surprising that ‘macro’ level issues related to formal structures of schooling have been a matter of little concern in research (Valero, 2004; Brown, 2008b; Gates & Visstro-Yu, 2009; Klette, 2004). As argued by Brown (2008b, p. 255), when talking about change and reform, the emphasis is posited on individual action rather than on structural adjustments. However, in the last few decades the field has been through a process of ‘socialisation’, adopting broader theoretical landscapes and a growing concern with the way mathematics is implicated in the welfare of society (Lerman, 2000; Valero, 2004; Gutiérrez, 2010). According to Gutiérrez (2010), such a trend tends to move from Marxist views of society, where structures are conceived as determining individual agency, to critical and poststructuralist approaches where the focus is on *identity* and *power* issues. Therefore, a shift has occurred “from examining school structures and institutions to examining discourses and social interactions” (p. 3), and “educators who take a socio-political perspective stance recognize that mathematics education is identity work” (p. 17), engaged in transforming mathematics education in ways that privilege more socially just practices towards marginalised students. This turn is in line with postmodern educational discourses which conceives change not as a change in the totality—a change in the global mode of production, for instance—but as being based in local struggles which take into account the complexities of particular contexts (Cole, 2003; Peters & Burbules, 2004). Such a position, that Gutiérrez (2010) summarizes as comprising both issues of identity and power, has been growing within mathematics education research in recent years (Brown, 1994; Tymoczko, 1994; Fitzsimons, 2002; Valero, 2004; Walshaw, 2004), and is a constitutive part of what Gutiérrez calls the “sociopolitical turn” in this field.

The adoption of critical and poststructuralist approaches in mathematics education has given researchers a language to address the problems involved in the teaching and learning of mathematics in a way that takes into account the identity of students and teachers—as opposed to their strictly psychological dimension—and the important Foucaultian notion of power in the process

of forming particular subjectivities in and through school mathematics. However, I do not feel at ease with the way these approaches discard the possibility of analysing school mathematics as part of global systems such as capitalism. Particularly, I contest the way in which critical (Pais, Fernandes, Matos & Alves, forthcoming) and poststructuralist theories (Pais & Valero, forthcoming) disavow the economical dimension of schools. This debate, opposing the Marxian notion of *class antagonism*—concerning the relations of production within capitalism—and what is usually called *identity politics*, is becoming a central one in contemporary theory.¹ While authors such as Laclau or Butler (Butler, Laclau & Žižek, 2000) argue that class struggle is just one species of identity politics, and one which is becoming less and less important in the world in which we live, Žižek (1989) or Jameson (1991), in accordance with Marx, defend the opposite: class *qua* structuring principle of the social totality colours all the other particular struggles. According to Žižek (Butler, Laclau & Žižek, 2000), it is not a matter of neglecting the achievements produced by what he calls the “politics of recognition”, but to supplement them with a critique of capitalism that, in current postmodern politics, is clearly disavowed:

Postmodern politics definitely has the great merit that it ‘repoliticizes’ a series of domains previously considered ‘apolitical’ or ‘private’; the fact remains, however, that it does *not* in fact repoliticize capitalism, because *the very notion and form of the ‘political’ within which it operates is grounded in the ‘depoliticization’ of economy.* (p. 98)

Such *naturalisation* of capitalist economics is visible in the way Gutiérrez (2010) describes the sociopolitical turn in mathematics education. Although she acknowledges that “[t]he intention of creating solidarity can be carried out without ignoring the material realities of society and a schooling system based on capitalism” (p. 13), when reading the article the image we get is that achieving equity can indeed be possible *by ignoring* the material reality of a capitalist school based on exclusion. All the change is thought of in terms of identity and power, and the field of the Political—the moment of openness, of undecidability, when the very structuring principal of society is called into question, as elaborated by Lefort (1986) or Laclau (1990)—is reduced to the one of politics—a positively determined subsystem of social relations in interaction with other subsystems (culture, institutional, religious, ethnic, ecological, gay, etc.). A broader conceptualization of change

¹ For an account of the terms in which this discussion is carried out see, for example, Butler, Laclau & Žižek (2000) or Eagleton (2001); and within education, Cole (2003).

that takes into account the economic—to posit equity not as the goal but as the point of departure², for instance—is disavowed. This disavowal involves what Laclau (Butler, Laclau & Žižek, 2000, p. 202) calls “an unconscious acceptance” of the dominant logics of the system. As Wendy Brown (1995, p. 60) noticed, within the political purchase of contemporary American politics nobody seems to question capitalism as the neutral economical and ideological matrix organizing our social relations.

2.2. *Ideology*

The postmodern disavowal of theories that seek to address the totality of a system has been effacing from research notions such as ideology. With its Marxist connotations, ideology is not considered to be important when thinking today’s world of multiple identities, of radical contingency, of an irreducible plurality of struggles (Eagleton, 2001; Brown, 1995; Žižek, 1989). Analysis of the system as a whole are either seen as ‘ideological’—that is, biased, exclusionist—discarded as essentialist metanarratives, or delivered through statistics—sometimes the only language available to address society as a whole. What we should bear in mind is that the gesture of rejecting ideology remains an ideological one. As argued by Jameson (1991, p. 350), “the need to avoid evaluations of the system as a whole is now an integral part of its own internal organization as well as its various ideologies”.

In mathematics education, despite often being acknowledged as an ‘issue’ to take into account (e.g., Nasir & Cobb, 2007), the concept of ideology has only sporadically been used in a consistent way (consider, for example, Baldino & Cabral, 2006; Baldino, 1998b; Brown & McNamara, 2011; Brown, Hanley, Darby & Calder, 2007). It is my contention that, when used in a proper way, this concept gives us the possibility to articulate two often unequal levels of analysis: the level of the classroom—teachers and students ‘interacting’ around mathematics—and the level of what Lacan called the big Other³: the socio-symbolic substance generating meaning about who we are and whom we are supposed to be. Roughly, what has been called in mathematics education research the macro-level (e.g., Valero, 2004). The proper way to carry a critique of ideology is precisely to

² For an analysis of research in equity in mathematics taking this assumption, see Pais and Valero (2011).

³ There are many ‘others’ in Lacan. The one I will be referring to in this text stands for the Symbolic order: the entire set of legalities, norms of conduct, policies, which bound our sociability. This Other is not ‘outside’ the individual. Rather it speaks through him. As Lacan pointed out after Freud, *the self is an Other*, that there, where the subject speaks, *an Other speaks for him*. Lacan’s assertion that “the unconscious is politics” means precisely that what we think to be the innermost core of our being—the level of desire—is not only unconscious but schematized by politics. Žižek concludes that it is precisely ideology which organizes our desires, which teaches us what and how to desire.

confront what is usually called ‘ideology’—the macro—with its (failed) actualization within a *life world context*:

In order to pass from abstract propositions to people’s ‘real lives’, one has to add to the abstract propositions the unfathomable density of a life world context—and ideology are not the abstract propositions in themselves, ideology is this very world density which ‘schematizes’ them, renders them ‘livable’. (Žižek, 2010, p. 6)

Mathematics education is full of abstract propositions that, schematised, enter the lived world of people⁴. I focus my attention here on the fantasmatic importance of mathematics as use-value. When actualized in a concrete practice, such importance turns to be a farce. I will explore this incongruence in the way teacher and students convey discourses about the importance of mathematics for their work, despite all the evidence that students do not use school mathematics in their out-of-school practice. There is no support in the real for such assumptions. Nevertheless, the belief persists. The question is: who is this Other who sustains people’s beliefs in the use-value of mathematics? By taking a certain distance towards this ‘fundamental fantasy’, by refusing to stage it directly, I intend to point to the system’s own inconsistency, to show that this Other is no more than the result of the activity of all of those who believe in it. It is a pure fiction that simultaneously conceals its own inconsistency and allows it to endure.

2.3. *The economy of school mathematics*

Elsewhere I have explored the relation between school mathematics and capitalism.⁵ For our purposes here, since we are centring our discussion around the use-value of mathematics, it should be enough to call the reader’s attention to what Marx (1976) explored as the *exchange-value* of a commodity, to refer to one of the two values a commodity can take. While the use-value of a commodity is strictly related with the concrete use someone makes of such commodity—the mathematical know-how necessary to perform a profession, for instance—the exchange-value posits this commodity in relation to all the others, that is, as part of a structure of equivalences where its value can be gauged. Thus exchange-value has a purely *relational* status: is not inherent to a commodity. It expresses the way this commodity relates to all the others. When looking at a commodity, say a table, we see its use-value. What we cannot see is its exchange-value, which

⁴ Lundin (forthcoming), Knijnik and Wanderer (2010), and Popkewitz (2004) are examples of research following this line of analysis.

⁵ In Pais (forthcoming) I develop this relation in connection with the issue of equity.

remains invisible. If we transpose this line of thought to school mathematics, we can speculate how the gesture of positing the value of mathematics in its use hides its exchange-value. That is, the formal place school mathematics occupies within capitalism. Vinner (2007, p. 2) expresses such incongruence between use-value and exchange-value as follows:

I suggest that the students have very good reasons to study mathematics. It is not the necessity of mathematics in their future professional life or their everyday life. It is because of the selection role of mathematics has in all stages of our educational system.

Due partly to the above mentioned disavowal of Marxism in the research produced in social sciences in the last three decades, the educational field lacks research that seeks to understand school as an endemic place of economic production. Two exceptions are the work of Pierre Bourdieu, who showed how the school system functions as an “immense cognitive machine which continually redistributes students submitted to its examination according to their previous positions in the system of distributions” (Bourdieu, 1989, p. 1), and Althusser (1994) and his analysis of school as the central ideological state apparatus. However, even their work is in our days often discarded as an example of a structuralist/deterministic point of view, as if a critical analysis of a current situation should be committed with the demand for a solution.

Other exceptions that I have found is the extensive article published by Jean Lave and Ray McDermott (2002), entitled *Estranged Learning* and, within mathematics education, the work of Julian Williams (2011)⁶, and Roberto Baldino and Tânia Cabral (Baldino, 1998a, 1998b; Baldino & Cabral, 1998, 1999, 2006). In these works, it is assumed that education has been institutionalised under advanced capitalism as an integral part of the political economy. They use the categories of use-value, exchange-value and surplus-value—the credentials which represent the surplus products of learning—to show how in schools students learn, above all, to participate in and accept the conditions of production and seizure of surplus value. The value of the ones who flunk is appropriated by the one who pass as surplus value. Failure is posited as a *necessary* condition for schooling: “in order to perpetuate the process of production/seizure of surplus value, a certain amount of failure is necessary” (Baldino, 1998a, p. 5). Therefore, “failure of students means success of the institution” (Baldino & Cabral, 2006, p. 34). As Williams (2011, p. 17) put it:

⁶ I thank one of the anonymous reviewers for this reference.

Though in a sense Bourdieu and Passeron (1977) got there first, Lave & McDermott (2002) convince us that the hierarchical consumption of education and consequentially the distribution of labour power by the education system to the labour market takes place through alienation, via ‘failure’ and ‘success’, that is self justifying; access to privilege is justified by success in a meritocratic system just as exclusion is justified by failure.

These studies call our attention to the primacy of what Marx called the *base*—the economic structure of society—over the *superstructure*—the legal, political, intellectual life. Marxism is primarily concerned not with the end of a certain political thinking—a “frame of mind”, as Gutiérrez (2010, p. 4) puts it—but with the end of an economic system. Regarding school mathematics, I suggest that the problem of failure can be broadly understood if we do not take it as a problem affecting particular groups of people. Instead, we should take it to be an economical problem, involving the way schools are structured as credit systems; thus involving *all* students. Apparently, there is no way of getting out of such an accreditation system—where year after year teachers are asked to mark students with a grade that will determine (sometimes in quite severe ways) their future possibilities—and mathematics education research ends up taking it for granted. The struggle against inequity is then elaborated in terms of what I called “identity politics”, concerning the emancipation of particular groups of people considered to be in disadvantage; or by addressing issues of power. That is, the problem of inequity is largely thought only in terms of changing the superstructure. The base, that is, schools as places of production, is taken for granted. However, it is my contention that we cannot dismiss as a remainder of the past what are effectively the positive conditions of today’s schooling.

3. The impossibility of transfer

It is beyond the scope of this article to do a review of the literature addressing the thematic of transfer. I will be instead interested in noticing research that in diverse ways points towards the difficulties or even impossibility of transfer.

Traditional views of transfer that assume the *continuity* between school and out-of-school activities have been giving way to investigations who suppose the *contextuality* of mathematical reasoning, challenging the dominance of strictly cognitive perspectives in mathematics education (Abreu, Bishop & Presmeg, 2002; Boaler, 1993; Evans, 1999). The seminal work of Jean Lave (1988) and Valerie Walkerdine (1988) have been particularly important for criticizing the idea that

we can abstract from the context some ‘mathematical task’ as if the mathematics involved were the same and thus susceptible of being transferred from one context to another without any kind of ‘misrecognition’. Regarding the work of Lave, to assert that learning is situated (Lave, 1988) means precisely to accept the fundamental difference between school and out-of-school learning. If knowledge has no ‘essence’ which can be transferred from one situation to another, that is, if knowledge ultimately is determined by the situation, then we will encounter problems when attempting to ‘harness’ local knowledge in schools, while at the same time maintaining its localised character. From the moment we bring the local into school, it becomes a ‘scholarized local’, even if the problems addressed are out-of-school problems (Lave & McDermott, 2002; Gerofsky, 2010; Pais, 2011).

Research developed around the relation between school mathematics and the workplace (e.g., Riall & Burghes, 2000; Williams & Wake, 2007; Hudson, 2008), and dealing with real/word problems (e.g., Boaler, 1998; Molyneux & Sutherland, 1998; Jurdak, 2006; Brenner, 1998) seems to corroborate these interpretations. Hudson (2008), for example, who developed research similar to Elsa’s research that I am going to address in a moment, found that the people in his study did not transfer what they learned in mathematics’ classroom to their daily work activities; rather they develop their mathematical skills on the workplace. Williams and Wake (2007), within the theoretical framework of activity theory, in a similar way conclude:

while it is true that some mathematics can readily be identified by academics in workplace practices, we find that workplace mathematics has its own distinct genre, inflected by the local practice and its activity system, its instruments and division of labour and power, as well as the productive goal of the whole activity. (p. 336)

Ernest (2007), when discussing epistemological issues in the internationalization and globalization of mathematics education, generalizes these findings by arguing that the mathematics behind our high-tech society is just a small part of the huge amount of research being done in mathematics. Some kind of applied mathematics that ends being *routinised* and used in a ‘technical’ way. This mathematics is learned by people in practice, outside school:

It is not academic mathematics which underpins the information revolution. It is instead a collection of technical mathematized subjects and practices which are largely institutionalised and taught, or acquired in practice, outside of the academy. (p. 31)

Nevertheless the belief that students profit from being confronted with real life situations endures as an important argument justifying the teaching and learning of mathematics in schools. For instance, instead of talking about a “disjunction”, Evans (1999) suggests that we should talk in terms of “distinction”, as to avoid the pessimism of “impossibility” (p. 40). In some studies (e.g., Williams & Wake, 2007; Jurdak, 2006; Brenner, 1998) the “faith” (Lundin, forthcoming) involved in the importance given to the use-value is exposed wide open. For instance, Jurdak (2006), after concluding that

the activity of situated problem solving in the school context seems to be fundamentally different from decision-making in the real world because of the difference of the activity systems that govern them. (p. 296)

And that students “define their own problems, operate under different constraints, and mathematics, if used at all, plays a minor role in their decision making” (p. 296), still insists on the importance of confronting students with real-life situations:

simulations of such authentic real life situations as embedded in situated problem solving may provide a plausible option to develop appreciation of the role, power, and limitations of mathematics in real-world decision-making.

And he adds,

though *quite different* in real life from that in school, the process of mathematization is *essentially the same* and having experience in it in a school context may impact on mathematization in real life. (Jurdak, 2006, p. 297, my italics)

Saying that the process of mathematization is the same, no matter the context, does not sit well with the socio-cultural approach from which Jurdak writes. It is impossible to find support in the research done and reported in Jurdak’s text for such statements. The belief that the exploration of real life situations in school will impact the way people use mathematics in real life is based on a ‘leap of faith’, thus, ideology at its purest.

The belief sustaining the importance for school mathematics to work with real problems has not been without its criticisms, and some authors have been arguing that transfer is not always desirable and could in fact be impossible (for instance, in the field of ethnomathematics [Knijnik, 1993;

Skovsmose & Vithal, 1997; Pais, 2011] and situated cognition [Stech, 2008]). Of interest for our discussion are studies that take a more sociological or philosophical approach. For instance, Dowling (1998) showed how the recontextualization of everyday material into the curriculum ends up being neither ‘real maths’ nor ‘real life’. Something is always already lost when we transpose some everyday activity into school, or vice-versa. What Dowling (1998) calls the myth of participation—the idea that mathematics is a necessary feature of everyday practices—ends up creating a school curriculum where mundane activities are *mythologised* in a way that privileges mathematical rather than everyday principles. That is to say, everyday activities, in order to be introduced in school, need to be amputated of all the complex vicissitudes, which makes them what they are. This amputation—the result from casting the mathematical gaze onto public domains—privileges what Dowling calls the *esoteric domain*, while, at the same time, concealing the purely fictional status of the importance attributed to mathematics.

Such fictional status of the importance of mathematics to deal with real life situations was recently taken further by Lundin (forthcoming) and Gerofsky (2010). The latter argues that most researchers in mathematics education working with word problems continue to work within Modernist paradigms that assume “an unproblematic transparency of language, and one-to-one matching or mapping models of the relationship between mathematical representations and ‘reality’” (p. 63). By bringing into the discussion the work of contemporary theorists Bakhtin, Lacan, Žižek and Baudrillard with regard to the relationships between language, representation, simulation and the ‘real’ as these are played out in mathematical word problems, Gerofsky shows the constitutional *impossibility* of ‘real-life’ word problems according to these theorists. On the other hand, Lundin shows us that the simultaneous formation of competence to understand and master the world and the formation of a perspective which makes the world appear in such a way as to make this competence relevant *is peculiar to and characteristic of mathematics education*. His investigation bears witness to the fact that the importance of mathematics as use-value does not reside within the object ‘mathematics’ itself. It is, instead, the result of the subjective activity of all of those who assert its importance.

The studies of Gerofsky and Lundin point towards what in contemporary theory is called the *performative*⁷ power of the word: reality as something which is constituted, posited by the subject. When we say that the world is written in mathematical language—the Galilean idea that mathematics is everywhere—we are not asserting some ontological truth about the world or about

⁷ Especially important are the works of Jacques Derrida and Judith Butler, or, in a psychoanalytical mode, the ones of Jacques Lacan and Slavoj Žižek.

mathematics, rather, it is by means of declaring it that the world becomes ‘written’ in mathematics. The truth claim of a statement cannot be authorized by means of its inherent content, but results from the “‘rationalization’, the enumeration of a network of reasons, masking the unbearable fact that the Law is grounded only in its own act of enunciation” (Žižek, 1997, p. 100). It is the proper gesture of positing mathematics as important for mundane activities that makes us believe that mathematics is indeed important for such activities.

4. Between school and workplace

I will now analyze a typical transfer situation involving workplace and school mathematics. The research reported here was initially developed by Elsa Fernandes⁸ in her PhD thesis. Hence, this particular research was already completed when I started my analysis. My purpose here is to analyze in a new light the data and the conclusions taken by Elsa.

During her PhD, Elsa collected data in a vocational school, in two different learning contexts: a mathematics classroom and a blacksmith’s workshop (where students learned to be a blacksmith from a master-blacksmith). She was concerned with identifying and characterizing mathematical activity that students used in practices not socially defined as mathematics and attempted to understand how that activity can be linked to the mathematics curriculum and to their development. The theoretical framework of this research was composed by Bernstein theory and situated learning theory.

The technical blacksmith course planned by the vocational school was attended by a group of youths, all boys, between 16 and 21 years old, who will receive the minimum Portuguese salary to attend the course. At the end, if they successfully finish the course, they will receive the diploma equivalent to the 9th grade (the compulsory schooling in Portugal). They have to attend different classes during the week (such as Mathematics, English, Technical Design, Computing, etc.), from 9h to 18h, and every Friday or Saturday, alternatively, they had “Blacksmith practices” in a real blacksmith workshop. The mathematics class was designed so that the students could use the mathematical knowledge learned in the classes in their blacksmith practices, or, conversely, give

⁸ Elsa Fernandes participates, just as the author of this text, in the project LEARN, which is one of the activities of the Technology, Mathematics and Society Learning Research Group of the Centre for Research in Education at the University of Lisbon. One of the purposes of this project is to analyse, from a different theoretical perspective, data already collected in previous research work done by the participants in the project.

mathematical meaning to some particular aspect of their blacksmith practice. Approval in this subject was necessary for the conclusion of the course.

All participants (students, teacher and master) assume the importance of mathematics as use-value:

However, at the end of the course, when I spoke with them [the students] again about the importance of mathematics as a part of the curriculum, all of them were peremptory in affirming that mathematics was very important. (Fernandes, 2004, p. 217)

Teacher: But a blacksmith that doesn't know how to calculate the necessary material is a false blacksmith. (ibid., p. 307)

Master: They [the students] already told me: Well, yes! We already learned this. So, don't you know that? I knew, but I didn't know that it was here where we can use it. Didn't you know? So now you know! It comes from the school [the mathematics]. (ibid., p. 266)

However, a closer look makes us suspicious about whether the students actually *use* the mathematics they learn in school while working as a blacksmith. Elsa's remarks clearly point to the mismatch between school mathematics and the mathematics they use while performing their blacksmith work. Even though apparently we are dealing with the same knowledge, the mathematics that emerge from school practice is not recognized as being the same mathematics involved in the blacksmith activity, and vice-versa:

When students were performing their blacksmith activity they didn't make any connexions between the mathematics that they learn in this practice and the mathematics that they learn in school. (Fernandes, 2004, p. 296)

Apprentices learn the mathematics they use while performing the blacksmith activity, with the other more experienced blacksmiths, and I don't have evidence that they did connexions between the mathematics they learn in school and the mathematics they use as blacksmiths. (ibid., 368)

The speech of Alberto, one of the students, is exemplary:

Alberto confessed that he was surprised and at the same time horrified when he saw that he will have to study mathematics in the course. However, when I asked him about the importance of mathematics in the course, he promptly answered:

“mathematics is very important and it will help me a lot in this course. I think it makes sense to have mathematics in the course”. (ibid., p. 203)

One of the students, with lots of difficulties in learning mathematics, Alberto, said very often that it was easier to construct the object referred to in the text of the mathematical exercise presented in the classroom, than making the calculations asked by the teacher to do. He also said that in blacksmith practice he didn't need to do such calculations; he just had to build the object. (ibid., p. 217)

Alberto hesitates about the importance of mathematics. On the one hand, while being interpellated by the researcher (whom he knew to be a mathematics teacher), he promptly states the importance of mathematics for the blacksmith's practice. However, during his work in the blacksmith workshop he often noted, in a tone of outburst, the complete obsolescence of school mathematics for what he was doing. Although when questioned students seemingly acknowledge the importance of mathematics for their work, when they are actually performing their work they solve the problems without making reference to school mathematics. The question to be posed is: if students show that they do not need school mathematics to perform their work as blacksmiths well, why do they say mathematics is important?

What we realise from the contradictory discourse of the participants and the observations made by Elsa is that students do not really need school mathematics to perform well as blacksmiths. What they need is to get approved in a school subject called mathematics because without this approval they will flunk the course. However, none of the participants articulate like this the justification for the presence of mathematics in the course. All participants assume the “social fantasy” (Žižek, 1989; Lundin, forthcoming) that school mathematics is a subject of great utility for our lives. Mathematics conveys a highly symbolic value, evident in the following dialogue between Alberto and his mathematics teacher:

Alberto: It is easier to build the stair.

Alberto: I only know how to construct the stair.

Teacher: But a blacksmith that doesn't know how to calculate the necessary material is a false blacksmith. (Fernandes, 2004, p. 12)

That is to say, even if a blacksmith were able to construct their products effectively, if he does not know mathematics he is not really a true blacksmith. For the teacher, it is mathematics that legitimizes being a blacksmith.

This symbolic importance attributed to mathematics is visible also in the way students engage in learning. Despite the efforts of the teacher to make class exercises resemble blacksmith practices, what mobilizes students to learn mathematics is not the use they will make of it, but a necessity imposed by the structure of the course. This feature is acknowledged by Elsa:

Mathematics was important because apprentices needed mathematics to obtain the course diploma. The relevance attributed by these students to mathematics didn't depend on any justification intrinsic to mathematics. It depended on the role that such experience [learning mathematics] had within students' future possibilities. (Fernandes, 2004, p. 200)

For the students, to be engaged in school mathematics is not related to the mathematical activity in itself; it depends more on the predisposition of students to engage themselves in this kind of activity—this is, on the predisposition to have success in the subject (to be approved). The motivation for this predisposition comes from the desire of the students in becoming blacksmiths and, for that purpose, they have to achieve success in all the school subjects that compound the course. (ibid., p. 365)

The engagement of students in the subject derives from a will to pass and not necessarily to learn. As a result the mechanisms put in motion by students are not aimed at learning mathematics, but to learn the best strategy which could enable them to get approved. This *subsidiary promotional criterion*, as Baldino & Cabral (1998) call it, “validate[s] non-learning strategies to get credit, to the benefit of those students for whom the learning-based strategy is impossible” (p. 5). In this process, the teacher is many times an accomplice to students' strategies, ending up adopting evaluation mechanisms that reward not the learning of mathematics but the learning of approval mechanisms. In the words of Baldino and Cabral (1998), the ideological relation between teacher and students can be described thus:

The teacher must not know that he is there to promote this passing without knowledge. Students know many things, specially they know how to pass. However what they know still better is that the teacher wants the game to go on and things to work well. They also know that the teacher does not know that this is what he wants. In order to be there, the teacher has to inebriate himself with his phantasm of minister of knowledge. (p. 5)

The way the teacher construes the exams, a central element for students' final grades, reveals this:

This is, in this activity [exam] students didn't need to memorize the procedure because they had their notebooks at hand for consultation. Besides that, the final exam was made with their notebooks and the tasks presented were similar to the ones students performed in class. (Fernandes, 2004, p. 335)

Everything seemed to be orchestrated so that students couldn't fail no matter if they really learned any mathematics. Indeed, taking into account that this is a highly expensive course for the government (it involved learning spaces both in schools and in the workplace, students were getting paid to do the course, many of them have a history of repeated failure in regular schooling, teachers were recruited especially to teach these students), it was not good for business to flunk these people.

5. Critique of ideology

5.1. *The primordial lie*

Mathematics is posited as a crucial knowledge to be learned by blacksmith apprentices so that they can become skilled workers and competent citizens. The learning of mathematics is supposed to occur in a meaningful way, by working with students 'realistic' situations connected with their blacksmith practice. At the end, there is an assessment which will dictate who achieved these aims, and who did not. So goes the public rule. However, it appears as if there is a set of *unwritten rules* which, although not openly spoken, dictates the activity of teachers and students. For instance, it seems as if there was something preventing the teacher from flunking these students.⁹ It is as if deep down the teacher knew that some of the students would never achieve the mathematical learning officially required, thus lowering down what students need to learn by doing, for instance, exams that reproduce exercises done in class or allowing students to use their notebooks during the exam. These unwritten rules are, according to Žižek (1997), fundamental to sustaining a social edifice: "Obscene unwritten rules sustain Power as long as they remain in the shadows; the moment they are publicly recognized, the edifice of Power is thrown into disarray" (p. 93). If the official rule was to be kept—students really have to learn mathematics, as well as to use the mathematics learned—the majority of students will flunk, and their work as blacksmiths will most likely become mischaracterized with the introduction of formal mathematics. However, it is not only 'forbidden' to flunk these students, it is even more forbidden to announce this very prohibition. That is, to publically state the unwritten rule. The school system needs to maintain the appearance that the

⁹ They all got approved; despite the flagrant difficulties some of them, such as Alberto, have in the subject.

teacher is allowed to flunk the students; that the absence of failure simply shows that students effectively learned important mathematics for their lives.

Instead of representing simple deviations from the formal ones, unwritten rules are indeed the fundamental elements sustaining the symbolic field of a given community. Žižek (1995, p. 54) calls it the *primordial lie*: that what must be concealed so that the community can constitute itself as a positive entity. In the case of the community of blacksmith apprentices, that which should remain unspoken is the unimportance of school mathematics for their work.¹⁰ When asking blacksmith apprentices if mathematics is important for their work, although we explicitly give the opportunity to students to say no, we already presuppose their affirmative answer—they are expected to say yes. If, by chance, any of them have said “no” (“No, mathematics is not really important, its presence in the course has just to do with its credibilization”; or even: “I am aware that I don’t even have to learn mathematics, I just have to reproduce the solved exercises in the exam”), the social link between students, teacher and curriculum will be broken. Students learn that in order to become a blacksmith they have to pass in a school subject called mathematics, which ends up to be of no utility for their work. However, this unimportance should remain in the shadow; officially all the students assure the importance of mathematics for their work.

In reality, students do not have a choice regarding the presence or not of mathematics in the course. From the moment they choose to do the course, mathematics is compulsory. However, the argument used to justify the presence of mathematics in the course is not based on traditional authority (for instance, by calling into play tradition—“It has always been like this”—or simple by saying: “Shut up and eat!”), but in a more refined demand, which, although remaining authoritarian, it disavows authority by positing the demand as if it were a demand of the subject for himself or herself. Students are led to ‘accept’ the introduction of mathematics in the course not as an imposition but as meaningful and necessary for their work as blacksmiths. As argued by Žižek (1997, p. xiii), in our contemporary society, “subjects are no longer interpellated on behalf of some big ideological identity, but directly as subjects of pleasures, so that the implied ideological identity is invisible”. It is for the students’ own good to have mathematics in the course. They should *see* and *acknowledge* its importance (instead of accepting it as a pure authoritarian act). In order for students to belong to the social sphere of ‘blacksmith schooling’ they have to ‘freely’ embrace the assumption that mathematics is important for their work. They have to secretly accept the lie.

¹⁰ But also that in order to pass in mathematics they do not really need to learn mathematics, only to reproduce in the exam the resolution that the teacher performed during class; they learn the correct way to answer their teacher’s questions, and how to appear busy in order to avoid extra work (Fernandes, 2004).

5.2. *Between knowing and believing*

If students know they do not use any school mathematics in their practice, why do they believe it is important for their work as blacksmiths? At issue here is the ‘deferential’ nature of belief (Žižek, 1997): belief is never my own belief but *the belief of the Other*, the belief that is supposed to be believed. In our case, the belief of students in the applicability of school mathematics is not really their belief, but the belief of the “subject suppose to believe” (p. 136), which in this case is partially performed by the teacher who is supposed to believe in the importance of mathematics for blacksmith practices. While students know that they do not use any school mathematics in their practice, they believe that mathematics is important.¹¹ This belief is not corroborated by their experience, but this does not prevent it from functioning because, in order to function, it is only necessary to suppose that others believe.

The crucial remark to be made is that this “Other who believes” has no positive existence. It is a pure fiction resulting from the frenetic activity of all those who believe in it. Both teachers and students (and even the master) endorse and reinforce the discourse around the importance of mathematics for daily life—their discourse is part of the same network of arguments and reasons deployed by mathematics education research to justify the importance of mathematics as use-value. This is the fundamental level of ideology: “not of an illusion masking the real state of things but that of an (unconscious) fantasy structuring our social reality itself” (Žižek, 1989, p. 30). This fantasy, as Žižek calls it, is what enables us to make sense of our daily lives, and is thus necessary. However, its status is purely fictional. That is, although it stands for the generative matrix that regulates our awareness of things, it is a purely formal structure without any ontological existence—it is no more than a collective fiction. The fact that Modernity sees mathematics in the world results not from real features of the world itself but from the workings of modern institutions such as mathematics education. Although the belief that school mathematics is useful in professional or mundane activities seems to fall when we get a closer look at the types of activities in which people are involved outside school, it is fuelled by teachers, researchers, politicians and, ultimately, by students themselves. Lundin (forthcoming) calls it an “engagement *en masse* in

¹¹ An important distinction should be made between believing and knowing: “I *believe* through the other, but I cannot *know* through the other” (Žižek, 1997, p. 138). When students say they need mathematics, this assertion belongs to the sphere of the (Lacanian) Symbolic: what it really means is that students believe that others believe mathematics is important. While the knowledge they have of the useless character of school mathematics in their practice nobody can have for them: they experience it in the (Lacanian) Real. Our everyday ideological attitude consists, according to Žižek (1997), precisely in the gap between (real) knowledge and symbolic (belief).

pretending that work on ‘word problems’ in some way corresponds to real use of mathematics” (p. 6).

Ideology has thus this paradoxical assemblance: it does not exist out there, orchestrating our lives as if we were puppets, but is inherently dependent on the subjects who, while not really believing in it, keep acting as if they do. And the system needs these subjects who ‘do not really believe’ but continue to act as if they did. This is clearly the case within the community of “blacksmith apprentices”. Students know the falsehood of school mathematics very well, they can even be aware of a particular interest—the credibilization of the course through the insertion of mathematics, for instance—hidden behind an ideological universality—the importance of mathematics for their work—but still they do not renounce it. A renunciation will imply tearing apart the entire universality sustaining the professional course. Nobody has interest in that, especially students who see in this course an opportunity to finish the compulsory education. The way students find to engage in classes is by means of adopting what Lundin (forthcoming, p. 7) calls a “cynical distance”, by actively distancing themselves from the activity in which they participate. If we recall the research developed by Jurdak (2006), despite all the evidence that students do not use school mathematics outside school, the author still *believes* in the use-value of mathematics. Jurdak abstains from taking what would appear to be the obvious consequence of his research: school mathematics is useless in terms of giving people competences to perform well in out-of-school activities, thus implying a questioning of the entire argument justifying the importance of mathematics as use-value. On the contrary, the author sticks to the fantasy. Doing differently would perhaps jeopardize the central role mathematics has in education, with obvious implications for those of us who rely on the social importance given to mathematics. It seems that, although we may know the falsity of the use-value of school mathematics, we continue to play the game and to obey it in order not to disturb the usual run of things. What ultimately legitimates the system is not what we know about it, the truth of it, but simply that it works: “ideology can lay its cards on the table, reveal the secret of its functioning, *and still continue to function*” (Žižek, 1996, p. 200).

Therefore, when researchers are confronted with the difficulties in transfer, they usually proceed by eliminating the obstacles, so that the official discourse on the importance of mathematics as use-value could be sustained: better teacher formation (e.g., Chapman, 2006), a more refined theorization of transfer (e.g., Evans, 1999), a greater understanding of the relation between school and workplace (e.g., Williams & Wake, 2007), more realism (e.g., Brenner, 1998), etc. However, I

argue that such mismatch—between what we know and what we believe—is the proper ideological mechanism which makes effective the presence of mathematics in school. While presenting school mathematics as an important subject in terms of knowledge and competence—i.e., in terms of its use-value—the other, surreptitious, functions of mathematics—its *exchange-value*—can, actually, become operative.

6. Final Remarks

In the introduction I raised the question: What is this big Other which gives meaning to our daily efforts to make school mathematics more meaningful for people? Against all the odds, I suggest that this all-encompassing totality mediating our engagement in research is the system of Capital. Capitalism, far from being a static hegemonic system, is forced to revolutionize its material conditions precisely in order to maintain the same fundamental relations of production (Jameson, 1991; Žižek, 1989, 2006). To acknowledge that school is more about production than learning is too deterministic, too pessimist, some would say: “Okay, we get it: school is about credit! But we cannot change that; so there should be other ways to keep working. What we cannot do is to stop”. However “To stop is to die!” is indeed the first rule of Capital, which needs to be in constant movement, creating novelties, new products containing the flash of redemption. Capital needs to constantly change something so that nothing really changes. This way, I suggest that it is for Capital that we keep ‘revolutionaizing’ mathematics education, with new insights for action, new theories, and new turns. The fact that the growth of research—we never had so many and so diverse studies in mathematics education as we have today—is not followed by an amelioration of the worldwide problem of failure in school mathematics (and some, as Gates & Visstro-Yu (2003) or Baldino & Cabral (2006) would even say that it is getting worse) is seldom posed. The final goal of research seems not to be the acclaimed slogan “mathematics for all”, but the perpetual production of research. And this serves very well the logic of Capital as today’s non-questioned totality: by avoiding analysis of the system as a whole, we assure that nothing changes at the *base*.

This power of Capital—a strong machine of deterritorialization that generates new modes of reterritorialization, as Deleuze & Guattari (2004) put it—to produce variety is coupled with its power to co-opt what in principle are resistant forces against it. Within today’s capitalism, as emphasized by Massumi (2002, p. 224), it seems that there has been a certain kind of convergence between the dynamic of capitalist power and the dynamic of resistance. By taking Capital as the *ontologised* substrata of our society, what prevents sociopolitical approaches in mathematics

education of becoming one of the many “inherent transgressions” (Žižek, 1997, p. 76) that, far from posing a resistance to the system, allows it to endure?

By enacting the concept of ideology I sought to elaborate the argument that the belief in the importance of mathematics as use-value functions not as the ultimate truth sustaining the learning of mathematics in school, but as the ultimate founding *lie*. To acknowledge this feature does not sit well for us who think of mathematics as a powerful knowledge, through which people can get emancipated and a better world becomes possible. However, it is my contention that without adopting a proper distance and developing consistent theoretical frameworks to appreciate our practice, we risk moving blindly. If the purpose of the sociopolitical turn is “deconstructing the taken for granted rules and modes of operating and making the familiar seem strange” (Gutiérrez, 2010, p. 20), we need to designate the elements within the community which point towards the system’s antagonistic character, and thus ‘estrangle’ us to the self-evidence of mathematics education its established identity. As we saw, what, at the level of the official discourse, runs smoothly—the argument of the utility of mathematics in becoming a worker or a citizen is well established—when actualized in a specific practice encounters a series of obstacles which ends up perverting the official intention. My suggestion is for research to bring into its gaze these obstacles—what I called the unwritten rules—and investigate them not as correctable deviations from the official rule, but as crucial conditions of today’s schooling. This implies research to be concerned in apprehending school mathematics in its totality, that is, as involving both the official learning of mathematics and the alienated (Lave & McDermott, 2002) character of this learning from the moment it occurs within the frame of capitalist schooling.

My plea is thus directed towards the way researchers give meaning and choose the theories informing our research. Maybe capitalism is here to stay. In this case, taking into account its self-revolutionizing ideology, perhaps the best way to counteract it is not by asking for more of that which cannot be said to have produced results for change so far, but in a refusal to participate in more of the same. Some would say that such an approach to research will lead us to a state of paralysis; lost in an endless discussion from which no practical solutions, no ‘insights for action’ will appear. My reproach is that the true act sometimes could be a purely ‘inactive’ one. I strongly believe that sometimes the best way to act is to stop ‘acting’—in the sense of doing research that immediately implies some kind of action—and chew over. As posed by Žižek (2006), “the threat today is not passivity but pseudo-activity, the urge to ‘be active’, to ‘participate’, to mask the Nothingness of what goes on” (p. 334). It is my contention that mathematics education research

needs some kind of “cognitive mapping” (Jameson, 1991, p. 51) informing its position within the Political. It needs research which is not primarily concerned in offering answers for “What should be done?” questions, but to *complicate* the usual way we approach problems. In line with a long critical tradition from Marx through Adorno and Foucault, I see thinking of the present as a way of thinking about change. From the moment we critically think about the present—in the sense of suspending what exists so that a sense of strangeness towards the present can appear—we are already changing it. What we cannot do, especially within a sociopolitical approach, is leave unaddressed crucial problems of school mathematics by shearing away apparent abstractions as the economic system and the social totality itself.

7. Acknowledgements

This article is part of my PhD project, supported by the Foundation for Science and Technology of Portugal, grant SFRH/BD/38231/2007. It is also part of the Project LEARN, funded by the same foundation (contract PTDC/CED/65800/2006). I thank Paola Valero for the final ‘cleaning’ of the text, but above all for the thrilling discussions during its writing.

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Addenda

Here there are the declarations of co-authorship for the papers that are part of the thesis.

ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: Broadening the role of theory in mathematics education research

Place of publication: In C. Bergsten & E. Jablonka (Eds.), *Skrifter från Svensk förening för matematikdidaktisk forskning MADIF 7*. Stockholm: Stockholm University.

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ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: The specificity of mathematics learning and the disavowal of the political in research

Place of publication: Educational Studies in Mathematics

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ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: Methodological issues in critical mathematics education

Place of publication: For the Learning of Mathematics

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ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

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Paper title: Methodological issues in critical mathematics education

Place of publication: For the Learning of Mathematics

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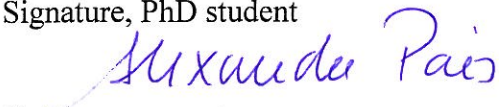
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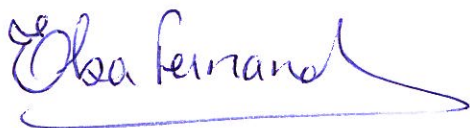
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ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: Methodological issues in critical mathematics education

Place of publication: For the Learning of Mathematics

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ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: Beyond disavowing the politics of equity and quality in mathematics education.

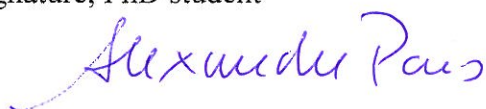
Place of publication: In B. Atweh, M. Graven, W. Secada & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 35-48). New York: Springer.

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ATTACHMENT 2

Co-author statement in connection with submission of PhD thesis

With reference to Ministerial Order no. 18 of 14 January 2008 regarding the PhD Degree § 12, article 4, statements from each author about the PhD student's part in the shared work must be included in case the thesis is based on already published articles.

Paper title: Broadening the role of theory in mathematics education research

Place of publication: In C. Bergsten & E. Jablonka (Eds.), *Skrifter från Svensk förening för matematikdidaktisk forskning MADIF 7*. Stockholm: Stockholm University.

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