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# Recovery of Compressively Sampled Sparse Signals using Cyclic Matching Pursuit

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**Abstract**—We empirically show how applying a pure greedy algorithm cyclically can recover compressively sampled sparse signals as well as other more computationally complex approaches, such as orthogonal greedy algorithms, iterative thresholding, and  $\ell_1$ -minimization.

## I. INTRODUCTION

Under certain conditions, we can recover a vector  $\mathbf{x} \in \mathbb{R}^N$  from measurements  $\mathbf{u} = \Phi \mathbf{x}$  created by a matrix with unit-norm columns  $\Phi \in \mathbb{R}^{m \times N}$  ( $N > m$ ). Here we focus on a cyclic application of the pure greedy algorithm matching pursuit (MP) [1]. Given the index set  $\Omega_k \subset \Omega = \{1, 2, \dots, N\}$  (indexing the columns of  $\Phi$ ), MP augments this set by  $\Omega_{k+1} = \Omega_k \cup \{n_k\}$  using

$$n_k = \arg \min_{n \in \Omega} \|\mathbf{r}_k - \langle \mathbf{r}_k, \varphi_n \rangle \varphi_n\|_2^2 = \arg \max_{n \in \Omega} |\langle \mathbf{r}_k, \varphi_n \rangle| \quad (1)$$

where  $\varphi_n$  is the  $n$ th column of  $\Phi$ ,  $\mathbf{r}_k = \mathbf{u} - \Phi \mathbf{x}_k$  is the residual, and the  $n_k$  row of  $\mathbf{x}_{k+1}$  is defined

$$[\mathbf{x}_{k+1}]_{n_k} = [\mathbf{x}_k]_{n_k} + \langle \mathbf{r}_k, \varphi_{n_k} \rangle. \quad (2)$$

Cyclic MP (CMP) [2], [3] runs as MP at each iteration, but includes a model refinement. Define the  $i$ th value of  $\Omega_k \subset \Omega = \{1, 2, \dots, N\}$ ,  $\Omega_k(i)$ ,  $1 \leq i \leq k$ . First, CMP finds

$$n_i = \arg \min_{n \in \Omega} \|\mathbf{r}_{k \setminus i} - \langle \mathbf{r}_{k \setminus i}, \varphi_n \rangle \varphi_n\|_2^2 = \arg \max_{n \in \Omega} |\langle \mathbf{r}_{k \setminus i}, \varphi_n \rangle| \quad (3)$$

where  $\mathbf{r}_{k \setminus i} = \mathbf{u} - [\Phi \mathbf{x}_k - \varphi_{\Omega_k(i)} [\mathbf{x}_k]_{\Omega_k(i)}]$ . Then CMP updates  $\Omega_k$  such that  $\Omega_k(i) = n_i$ , and the solution  $[\mathbf{x}_k]_{n_i} = \langle \mathbf{r}_{k \setminus i}, \varphi_{n_i} \rangle$ . Finally, CMP augments  $\Omega_k$  as in MP, and refines again.

Figure 1 shows the probability of exact recovery ( $\|\mathbf{x} - \hat{\mathbf{x}}\|_2 / \|\mathbf{x}\|_2 < 0.01$ ) for vectors of varying sparsity  $k$  with elements drawn from two distributions, for six undersampling ratios  $m/N$  with no noise, using both CMP and Orthogonal MP (OMP). For these experiments, we make  $N = 400$ , sample  $\Phi$  from the uniform spherical ensemble, and average the results over 100 independent trials for each sparsity and number of measurements. In our implementation, we make CMP run the refinement procedure a max of five times, or until  $\|\mathbf{r}_k\|_2^2 / \|\mathbf{x}_k\|_2^2 > 0.999$ , where  $\mathbf{r}_k$  is the residual after refinement. It is clear that CMP can perform just as well as OMP at this task without matrix inversions. Our final work will include comparisons with other methods, such as iterative thresholding [4],  $\ell_1$  minimization [5], and two-stage thresholding [6], as well as an analysis of the algorithm.

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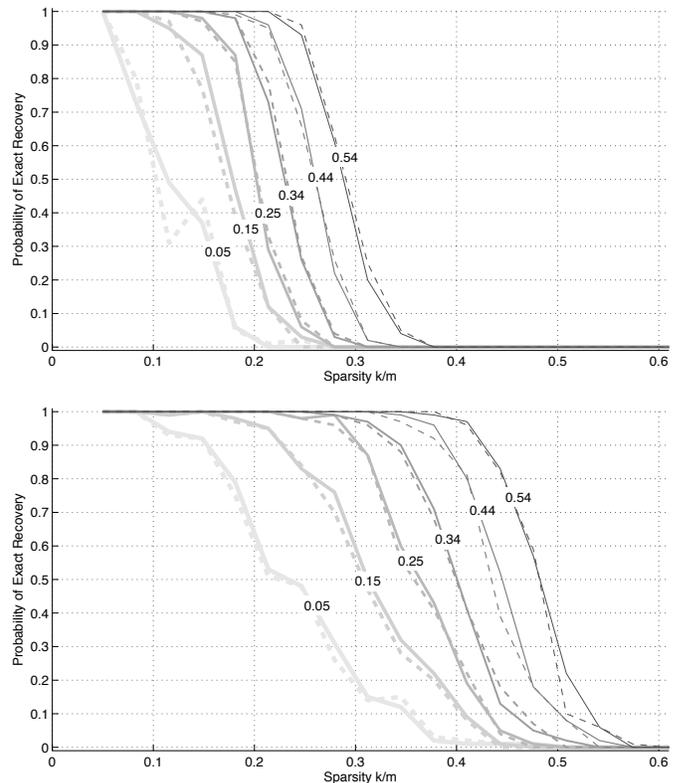


Fig. 1. Probability of exact recovery using CMP (solid) and OMP (dashed) at several undersampling values  $m/N$  (labeled). Top: Active elements distributed Normal. Bottom: Active elements distributed Constant Amplitude Random Signs [6].

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