Positioning empty containers under dependent demand process

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ABSTRACT

Owing to trade imbalance, shipping companies position empty containers between ports or depots periodically. The most difficult problem for positioning is that it is not possible to know the exact amounts of empty containers required in the future. The paper deals with the problem of positioning empty containers in a port area with multiple depots. Customer demands and returning containers in depots per unit time period are assumed to be serially-correlated and dependent random variables. Three options are considered to prepare the required extent of positioning: positioning from other overseas ports, inland positioning between depots, and leasing. The policies for empty-container management consist of three parts as follows: a coordinated, (s, S) inventory policy for positioning from other ports, (r, R) policy for inland positioning between depots; and a simple leasing policy with zero lead-time. For inland positioning policy, four heuristic methods are proposed to reposition empty containers between depots. The objective is to obtain optimal policies corresponding to different methods of inland positioning in order to minimize the expected total costs. A genetic-based optimization procedure is developed to find the optimal parameters (s, S) and (r, R). Some numerical examples and sensitivity analyses are given to demonstrate the results.

1. Introduction

Empty containers are important logistical resources in the light of changes in the international logistics environment; shipping companies wish to manage and operate them efficiently. Owing to the trade imbalance, empty containers must be positioned from surplus areas to shortage areas periodically and shipping companies have inventory policies to allocate empty containers. They reposition empty containers among hub areas, ports and depots. Hence, the efficient management of empty containers becomes a source of competitive advantage for shipping companies to improve their customer-service levels and productivity.

In empty-container allocation problems, some researchers paid attention to deterministic systems (e.g. Choong, Cole, & Kutanoglu, 2002; Di Francesco, Manca, & Zuddas, 2006; Moon, Do Ngoc, & Hur, 2010; Olivo, Zuddas, Francesco, & Manca, 2005; Shintani, Imai, Nishimura, & Papadimitriou, 2007). Stochastic problems have been studied since late 1990s. Cheung and Chen (1998) considered a two-stage stochastic network model for the dynamic empty-container allocation problem. Li, Liu, Leung, and Lai (2004) and Li, Wu, and Liu (2007) developed a new (u,d) policy for the empty container allocation problem between ports. Lam, Lee, and Tang (2007) proposed a dynamic stochastic model for a simple, two-port two-voyage system to prove the effectiveness of the approximately optimal results from a two-port two-voyage model that utilized linear approximation architecture. Song and Dong (2008) dealt with an empty-container management problem in a cyclic route to seek the optimal repositioning policy in a dynamic and stochastic situation. This study was then extended by Dong and Song (2009) in case of the joint container fleet sizing and empty container repositioning problem in multi-vessel, multi-port and multi-voyage shipping systems with dynamic, uncertain and imbalanced customer demands. Hwang (2008) established a simulation model by Arena that took account of positioning empty containers between multi-ports under uncertain factors such as shippers’ demand and navigation time of vessels. Che (2009) also developed an Arena simulation model to analyze empty container management considering uncertain demands and supplies in a two-depot system. Yun, Lee, and Choi (2011) considered an inventory control problem of empty containers in an inland transportation system with a simple policy to reposition containers from other hubs. Most of those research studied empty container repositioning between seaports; however, little research has been reported on the coordinated optimization of empty-container positioning from overseas ports and positioning between depots in an inland multi-depot system under dependent demand process, to which we attempt to contribute. Furthermore, short-term leasing of empty containers is also taken into consideration.

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The rest of this paper is organized as follows: in the next section, we address the model description and propose replenishment policies for empty containers in the inland multi-depot system. In Section 3 and 4, a solution method to obtain optimal policies is discussed. The structure of the simulation model is extensively described. A simulation-based optimization procedure is then developed based on genetic algorithm (GA) to find the optimal parameters. In Section V, numerical examples and sensitivity analyses are given to demonstrate the results. Finally, conclusions are made in Section VI.

2. Inventory control model

2.1. Model description

Fig. 1 shows an inland transportation network. Shipping companies have inland depots to store empty containers and to provide them for transportation of freights across/to terminals, depots, and customer locations. Due to the imbalance in trade, some ports accumulate a large number of empty containers, while other ports are often faced with a shortage of empty containers. The imbalance problem occurs across depots in an inland transport system.

To solve the imbalance problem, three options for replenishment are taken into account. Firstly, short-term lease with zero lead-times is available. A simulation-based optimization procedure is then developed based on genetic algorithm (GA) to find the optimal parameters. In Section V, numerical examples and sensitivity analyses are given to demonstrate the results. Finally, conclusions are made in Section VI.

2.2. Assumptions

In this paper, an inventory control problem for empty containers is studied under the following assumptions:

- There are \( n \) inland depots.
- Single commodity, 40 feet containers, is considered.
- The demand and supply of empty containers per unit time are serially-correlated and dependent variables.
- The lead-times for overseas orders are independent and identically distributed random variables.
- All lead-times for inland positioning across multiple depots are identical and constant.
- The lead-times for distributing empty containers from the terminal to depots are constant.
- Short-term lease with zero lead-times is available.

2.3. Empty-container replenishment policies

2.3.1. Coordinated overseas positioning policy

A discrete-time \((s, S)\) policy (refer to Silver, Pyke, & Peterson, 1998) is considered to decide on ordering time and quantity of empty containers based on the estimated total inventory position of all depots in the future, as shown in Fig. 2 below.

At the starting point of each time-period \( t \), we estimate the inventory position \( I_t \) of all depots \( i \) following a time-lapse of \( E(L_d) + d \), which is the sum of expected average lead-time for overseas positioning, denoted by \( E(L_d) \), and the smallest value among the lead-times for distribution from terminal to the depots, denoted by \( d \). If the estimated inventory position is less than \( s \), we order empty containers up to \( S \) in advance; otherwise, we do not place any order of empty containers from overseas ports.

An autoregressive model (AR(1)) is used to forecast the customer demands and the amount of returning containers during the expected lead-time for overseas positioning. The AR(1) is defined as:

\[
D_i^t = \xi D_i^{t-1} + \theta D_i^{t-1} + \epsilon_D
\]

(1)

\[
V_i^t = \xi V_i^{t-1} + \theta V_i^{t-1} + \epsilon_V
\]

(2)

where \( \theta \) is the parameter of the model, the constant term, \( \xi \), equals \( \mu_d (1 - \theta) \) and the error term, \( \epsilon \), is assumed to be a
normally distributed variable with zero mean and constant variance (with $\mu_{Di}$, $\sigma^2_{Di}$ are mean and variance of customer demands of depot $i$). Analogical definitions are used for $\theta_{Vi}$, $\delta_{Vi}$ and $\lambda_{Vi}$ in case of returning containers.

2.3.2. Distribution policy

A distribution policy is proposed for delivering empty containers from the terminal to the depots. Because the lead-time for overseas positioning is a source of randomness, the inland multi-depot system checks for each time-period $t$ whether or not overseas orders placed in the previous periods will arrive. As soon as empty containers that are positioned from overseas ports arrive at the terminal, they are divided into lots in proportion to net demands of the depots. These lots are then transported to the corresponding depots in the system.

2.3.3. Inland positioning policy

Inland positioning is an alternative approach for replenishing empty containers with shorter lead-times. Hence, at the starting points of all time periods, we also make another important decision about the inland positioning of empty containers across multiple depots.

The inventory position, $I_{t-1}^{i+1}$ of depot $i$ after the lead-time for inland positioning, $\delta_i$, is estimated. The AR(1) model is also applied to
forecast the customer demands and the supply by returning containers during the lead-time for inland positioning. Next, we classify depots into three sets.

- The first set includes depots which can position out empty containers.

\[ A^1 = \{ i | \mathcal{R}_i^o - (D_i^{o+4} - V_i^{o+4}) > R_i \} \]

\[ \mathcal{R}_i^o = (D_i^{o+4} - V_i^{o+4}) - R_i \text{ and } \mathcal{R}_i^o = 0 \]

- The second set includes depots which can position in empty containers.

\[ B^2 = \{ i | (D_i^{o+4} - V_i^{o+4}) < r_i \} \]

\[ \mathcal{R}_i^e = 0 \text{ and } \mathcal{R}_i^e = r_i - (D_i^{o+4} - V_i^{o+4}) \]

- The final set includes depots which neither position out nor position in empty containers.

\[ r_i < (D_i^{o+4} - V_i^{o+4}) < R_i \text{ for any } i \]

\[ \mathcal{R}_i^e = 0 \text{ and } \mathcal{R}_i^e = 0 \]

To define an origin depot in set \( A' \) and a destination depot in set \( B' \), in other words, to define the flow of repositioning empty containers, four methods of inland positioning policy are proposed using cost factors which are \( C_i^1, C_j^1, C_j^2, C_j^3 \). Holding costs for \( \mathcal{R}_i^o \) containers are considered at origin depots in set \( A' \) while leasing costs for \( \mathcal{R}_i^e \) containers are taken into account at destination depots in set \( B' \). In addition, inland positioning costs for \( \min(\mathcal{R}_i^o, \mathcal{R}_i^e) \) containers are considered from origin depots in set \( A' \) to destination depots in set \( B' \).

Four heuristic methods of inland positioning policy are described as follows:

(1) Method 1

Empty containers are repositioned from that depot \( i \) in set \( A' \) which has the highest holding cost, \( C_i^1 \times \mathcal{R}_i^o \), to depot \( j \) in set \( B' \) which has the highest leasing cost, \( C_j^1 \times \mathcal{R}_j^e \). If there is merely one depot in set \( A' \) (\( B' \)), the leasing cost of \( \mathcal{R}_i^o \) (the holding cost of \( A' \)) is considered only.

(2) Method 2

Empty containers are repositioned from depot \( i \) in set \( A' \) which has the lowest cost of inland positioning, \( C_i^1 \times \min(\mathcal{R}_i^o, \mathcal{R}_i^e) \). In case that there is merely one depot in set \( A' \) (\( B' \)), the cost of inland positioning (the holding cost) is considered only.

(3) Method 3

Empty containers are repositioned to that depot \( j \) in set \( B' \) which has the highest leasing cost, \( C_j^1 \times \mathcal{R}_j^e \), from depot \( i \) in set \( A' \) which has the lowest cost of inland positioning, \( C_i^1 \times \min(\mathcal{R}_i^o, \mathcal{R}_i^e) \). In case that there is one depot in set \( A' \) (\( B' \)), the leasing cost (the cost of inland positioning) is considered only.

(4) Method 4

Empty container repositioning from depot \( i \) in set \( A' \) to depot \( j \) in set \( B' \) is performed if the tuple \((i, j)\) obtains the most benefit among all tuples of depots. The benefit is defined as follows:

\[ \text{Benefit} = (C^1_i + C^1_j - C^1_j) \times \min(\mathcal{R}_i^o, \mathcal{R}_j^e) \]

Empty containers are progressively repositioned one-by-one from an origin depot in set \( A' \) to a destination depot in set \( B' \) based on one of four methods of inland positioning policy. After positioning one empty container from the origin to the destination, the costs used to determine the flow of inland positioning are updated. The inland positioning procedure continues until either the sum of all \( \mathcal{R}_i^o, \mathcal{R}_j^e \) (\( i \in A' \)) or the sum of all \( \mathcal{R}_i^e, \mathcal{R}_j^o \) (\( i \in B' \)) equals to zero.

2.3.4. Leasing policy

Short-term leases with a zero lead-time are employed to meet the shortage in a depot at once. Leased empty containers must be returned to the leasing companies after a leasing period \( L_c \). The leasing periods of empty containers at all depots are the same.

3. Simulation model

The flowchart of the simulation model in Fig. 3 describes the sequence of replenishment policies proposed.

![Fig. 3. Flowchart of the simulation model.](image)
4. Genetic algorithm

This section presents a simulation-based GA to seek the optimal policies minimizing the long-run expected total cost per unit time in an inland multi-depot system. The main idea of the optimization algorithm is introduced and we will show how we apply the GA to optimize the parameters (i.e. the values of the decision variables $s_i$, $S_i$, $r_i$ with $i \in I$) for all inland positioning methods presented in Section 2. The overall strategy is explained as follows:

- Gene representation
- Initialization
- Evaluation via simulation
- Selection
- Recombination, mutation and adjustment

4.1. Gene representation

The proper representation of a solution plays a key role in the development of a GA. For the system under consideration, a solution can be represented by a chromosome of non-negative integers $(s, S, r_1, \ldots, r_n)$ where $s$ and $S$ are the order-up-to level and reorder point for overseas positioning, $R_i$ and $r_i$ are inland-positioning-out and –in levels of depot $i$ for the inland positioning policy. A valid chromosome should satisfy two mandatory conditions. First, the value of $S$ is not less than that of $s$ ($S \geq s$). Second, the value of $R_i$ is not less than that of $r_i$ for each depot $i$ ($R_i \geq r_i$).

4.2. Initialization

For the initial generation, each gene of an individual is uniformly generated within a predetermined range of value. After initialization, for the pair of $(s, S)$ and each pair of $(r_i, R_i)$ for any $i$, if $S < s$ or $R_i < r_i$, then positions of genes in that pair are swapped with each other to satisfy two mandatory conditions. The initial individuals are evaluated through the simulation model.

4.3. Evaluation via simulation

Each individual in the population represents a potential strategy for solving the inland multi-depot system and, of course, a solution when applying this strategy. The elements in the chromosome of the individual are used for decision making of overseas positioning and inland positioning empty container during the simulation. The simulation is then run to evaluate the expected total cost per unit time for each individual and to give a feedback on its performance which is used as a fitness value of the individual in the GA.

4.4. Selection and population management

Various evolutionary methods can be applied to this problem. We use $(\mu + \lambda)$ selection for selecting individuals for reproduction. Under this method, $\mu$ parents and $\lambda$ offspring compete for survival and the $\mu$ best out of the set of offspring and old parents, in other words, the $\mu$ lowest in terms of total cost, are selected as parents of the next generation.

4.5. Crossover, mutation and adjustment

In the crossover operation, two individuals have to be selected from the parent population to generate an offspring. The Roulette-wheel selection is used in our algorithm, which probabilistically selects individuals based on their fitness values. There are many different crossover methods that can be performed on the real-value chromosome. In this paper, we produce offspring with uniform crossover which is described as follows. After having selected two parents for applying the crossover operator, an offspring is generated by copying the alleles of the genes of the parent chromosomes randomly from one of the two chosen parents. However, as shown in Fig. 4, a pair of elements corresponding to the same depot is copied into the offspring together as a block because of their close relationship. The uniform crossover acts with probability $P_c$.

Whenever an offspring is produced, mutation is applied with probability $P_m$. The operation of mutation changes a randomly chosen gene on a chromosome by assigning to its allele a new number uniformly generated in a predetermined range. If, after mutation, $S < s$ or $R_i < r_i$ for any $i$, then we swap the positions of $S$ and $s$ or $R_i$ and $r_i$ respectively, in order to ensure that the two above-mentioned mandatory conditions are always satisfied.

5. Numerical example and sensitivity analysis

In this section, a numerical example is explored to illustrate the procedure of the simulation-based genetic algorithm. To obtain best possible inventory policies based on the four proposed methods for inland positioning, the decision variables $(s, S, r_1, \ldots, r_n)$ must be specified in order to aspire the goal of minimizing the expected total cost per unit time which is used as the optimization criterion. More numerical computations are then conducted to examine the sensitivity of the results with respect to the system parameters such as lead-time and unit costs. The data for this example which takes account of four identical depots are given in Tables 1–4. The simulation length and warm-up period for the model are 730 and 120 days, respectively.

5.1. Numerical example

For this example, to determine the appropriate values of GA parameters, several experiments were conducted. Three different population size (10, 20, and 50), three different $P_c$ (0.4, 0.6, and 0.8), and three different $P_m$ (0.05, 0.1, and 0.15) are checked. The population size of 50, $P_c$ of 0.4 and $P_m$ of 0.1 gave the best performance with the lowest total cost. Hence, the population size, $P_c$, and $P_m$ are set to 50, 0.4, and 0.1, respectively. The termination rule is to stop when reaching the maximum of 200 generations. The best inventory policies with respect to the four heuristic methods ($M$) of inland positioning under dependent demands are shown in Table 5 and the relevant costs are similarly shown in Table 6 and Fig. 5.

---

**Table 1**

<table>
<thead>
<tr>
<th>Depot</th>
<th>Demand</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NORM (100, 20²)</td>
<td>NORM (100, 20²)</td>
</tr>
<tr>
<td>2</td>
<td>NORM (120, 20²)</td>
<td>NORM (110, 20²)</td>
</tr>
<tr>
<td>3</td>
<td>NORM (260, 30²)</td>
<td>NORM (200, 30²)</td>
</tr>
<tr>
<td>4</td>
<td>NORM (300, 30²)</td>
<td>NORM (220, 30²)</td>
</tr>
</tbody>
</table>
Table 6 shows that holding costs as well as overseas positioning costs of the solutions are almost the same for all methods $M$. It is consistent with the stabilities of $S$ and $s$ in Table 5. This demonstrates that the different heuristic methods of inland positioning do not affect the coordinated overseas positioning policy. Moreover, it is interesting to observe that for all methods $M$ the values of $R_i$ and $r_i$ of depots 3 and 4 are greater than those of depots 1 and 2. This can be explained by the fact that depots 3 and 4 have much more shortage of empty containers than depots 1 and 2, so they would rather to receive than position out empty containers. The higher the value of $R_i$ is, the less empty containers are positioned out by depots 3 and 4 and the higher $r_i$ is, the more empty containers are received by depots 3 and 4 from the other depots (an opposite explanation is applied to the case of depots 1 and 2).

Table 6

<table>
<thead>
<tr>
<th>Method</th>
<th>Relevant costs.</th>
<th></th>
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<tbody>
<tr>
<td>$M$</td>
<td>Holding cost</td>
<td>Overseas cost</td>
</tr>
<tr>
<td>1</td>
<td>2349</td>
<td>524</td>
</tr>
<tr>
<td>2</td>
<td>2408</td>
<td>503</td>
</tr>
<tr>
<td>3</td>
<td>2438</td>
<td>509</td>
</tr>
<tr>
<td>4</td>
<td>2307</td>
<td>504</td>
</tr>
</tbody>
</table>

Fig. 5. Comparison of relevant costs of numerical example.

Fig. 6. Trends of costs with varying lead-times for inland positioning.

Table 7

Near-optimal policies under changing $L_r$

Table 8

Relevant costs under changing $L_r$
It can be observed from Table 6 and Fig. 5 that method 4 has the lowest expected total cost, in other words this method is the most effective approach to manage and reposition empty containers between multi-depots (hence, method 4 will be used in all cases of sensitivity analysis). This can be explained by the fact that method 4 considered holding cost at surplus depots, leasing cost at shortage depots and cost for inland positioning from surplus to shortage depots.

5.2. Sensitivity analysis

5.2.1. Lead-time parameter

In this analysis we vary the lead-time for positioning empty containers from one depot to another. The best found policies for method 4 and the relevant costs under four different lead-times $L_r$ for inland positioning are shown in Tables 7 and 8, while trends of relevant costs are displayed in Fig. 6.

From the results, we can find that when lead-time for inland positioning increases, the values of $S$ and $s$ become larger and larger in order to hold more empty containers. This leads to the increment of holding cost while overseas positioning cost almost does not change. In addition, the values $R_i$ and $r_i$ have upward trends while cost of inland positioning and leasing cost increase. As a result, the total cost increases.

5.2.2. Unit cost parameters

5.2.2.1. Leasing cost per unit per day. We increase the leasing cost per unit per day of all depots in steps of 20%. The parameters of the found policies and the relevant costs are shown in Tables 9 and 10, while trends of costs are displayed in Fig. 7 below.

From the results, it can be seen that when unit leasing cost increases under the same other unit costs, the values of $S$ and $s$ steadily increase so as to hold more empty containers, while $r_i$ follows an upward trend. The growing values of $r_i$ lead to a slight increase of holding cost and cost of inland positioning. In addition, cost for overseas positioning almost does not change and cost for leasing substantially rises due to the upward trend of its unit cost. As a result, the total cost goes up.

5.2.2.2. Cost of inland positioning per unit per day. We increase the cost for inland positioning per unit per day of all depots in steps of 20%. The best found policies and the relevant costs are shown in Tables 11 and 12, while trends of costs are displayed in Fig. 8 below.

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Near-optimal policies under changing $C_i^l$.</th>
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<tr>
<td>$C_i^l$</td>
<td>$S$</td>
</tr>
<tr>
<td>0</td>
<td>554</td>
</tr>
<tr>
<td>20</td>
<td>554</td>
</tr>
<tr>
<td>40</td>
<td>559</td>
</tr>
<tr>
<td>60</td>
<td>564</td>
</tr>
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<table>
<thead>
<tr>
<th>Table 10</th>
<th>Relevant costs under changing unit leasing cost $C_i^l$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i^l$</td>
<td>Holding cost</td>
</tr>
<tr>
<td>0</td>
<td>2307</td>
</tr>
<tr>
<td>20</td>
<td>2386</td>
</tr>
<tr>
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<td>2396</td>
</tr>
<tr>
<td>60</td>
<td>2388</td>
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</table>

<table>
<thead>
<tr>
<th>Table 11</th>
<th>Near-optimal policies under changing $C_i^r$.</th>
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</thead>
<tbody>
<tr>
<td>$C_i^r$</td>
<td>$S$</td>
</tr>
<tr>
<td>0</td>
<td>554</td>
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<tr>
<td>20</td>
<td>550</td>
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<tr>
<td>40</td>
<td>549</td>
</tr>
<tr>
<td>60</td>
<td>549</td>
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</table>

<table>
<thead>
<tr>
<th>Table 12</th>
<th>Relevant costs under changing $C_i^r$.</th>
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</thead>
<tbody>
<tr>
<td>$C_i^r$</td>
<td>Holding cost</td>
</tr>
<tr>
<td>0</td>
<td>2307</td>
</tr>
<tr>
<td>20</td>
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</tr>
<tr>
<td>40</td>
<td>2334</td>
</tr>
<tr>
<td>60</td>
<td>2397</td>
</tr>
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</table>

Fig. 7. Trends of costs with change of unit leasing cost.

Fig. 8. Trends of costs with change of unit inland positioning cost.
From the results above, it can be observed that when cost of inland positioning per unit per day increases under unchanged other unit costs, the values of \( S \) and \( s \) are quite stable whereas \( R_i \) increases and \( ri \) decreases to diminish inland positioning activity. The holding cost and overseas positioning cost have no change, while inland positioning cost increases because of the upward trend of its unit cost. Hence, the total cost goes up.

6. Conclusion

In this paper, we studied a replenishment problem for empty containers in an inland multi-depot system. For serially-correlated and dependent demands and supplies of empty containers, a coordinated \((s,S)\) ordering policy is used to position empty containers from overseas ports. In addition, a new inland positioning \((ri,Ri)\) policy along with four heuristic methods to position empty containers between depots is proposed. Also, short-term leasing is available if a shortage of empty containers occurs. A simulation model and a genetic algorithm based heuristics are developed to find optimal inventory policies corresponding to those methods of inland positioning so as to minimize the expected total cost per unit time. Some computational experiments are then done to validate the model and examine the sensitivity of results with respect to system parameters.

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