Abstract—In group-oriented applications for wireless networks, reliable multicast strategies are important in order to efficiently distribute data, e.g. in Wireless Mesh Networks (WMNs) and Mobile Ad-hoc NETworks (MANETs). To ensure that developed protocols and systems will operate as expected when deployed in the wild, a good understanding of several factors such as packet loss characteristics is necessary. In this paper the correlation of erasures in a cluster of receiving mobile devices is measured and analyzed. In the considered scenario, a source node broadcasts packets to a cluster of receivers located relatively far away. To ensure that the obtained data can easily be applied in analysis, we introduce the cluster erasure transition matrix. We then analyze a simple broadcast and cooperative scheme, and show that the assumption of independent packet erasures unfairly favors the cooperative scheme according to the obtained measurements.

I. INTRODUCTION

The correlation of erasures among the devices in a receiving cluster is of great importance, when investigating strategies that exploit the diversity stemming from packet erasures [1]. The area of reliable multicast has attracted significant attention as it can provide an efficient utilization of the available bandwidth [2], [3]. Cooperative strategies are of particular interest as nodes can cooperate if they hold different packets. In analytic work, erasures are often assumed to be independent which simplifies analysis significantly. If this assumption is not accurate, it can have a substantial impact on the performance [4]. In particular it is important to notice that this is a worst-case assumption for reliable multicast, but a best-case assumption for cooperative strategies. In this paper these two strategies are investigated in the basic scenario illustrated in Figure 1. Here a single source broadcasts data that is received by a cluster of receivers.

Fig. 1: The investigated scenario, a source multicasts to $n$ nodes

Most Wireless Local Area Network (WLAN) measurements in the current literature are carried out using a low number of laptops with an attached WLAN card. In this paper measurements are carried out using 802.11 capable mobile devices, and the size of the receiving cluster is relatively big. In [4] loss correlation between a small number of WLAN equipped laptops is investigated. The results are used to compare video streaming using leader-based Automatic Repeat-reQuest (ARQ) schemes and legacy multicast. Unfortunately the paper do not describe how the correlation was calculated making it difficult to verify the results. In [5] several error control mechanisms are evaluated using packet loss profiles built from real traces. In [6] WLAN measurements show that packet losses at several devices are not fully uncorrelated and the authors propose a new approach for simulating packet losses. Unfortunately these two works do not consider losses that occur before the traffic is in the air, e.g. packets dropped from the sending queue. In [7] packets are transmitted to a heterogeneous cluster of different types of WLAN cards. The authors conclude that the correlation between losses is low.

The primary contribution of this paper is an evaluation of reliable multicast and cooperative strategies based on both the common assumption of independent erasures, and using the data obtained from our measurements. The results show that the gain of using user cooperation instead of an ARQ scheme, is significantly smaller when the analysis is based on the measurement data, compared to when erasures are assumed to be independent. However, user cooperation is still able to provide a significant gain. Another contribution is the measurements on the correlation of packet erasures in a 802.11b/g cluster of mobile devices. The size of the cluster considered is significantly higher than any in the existing literature. Additionally several different devices are tested simultaneously which is necessary to ensure the general validity of the results. We also present an approach, the cluster erasure transition matrix, that enables us to condense the measurement results and easily use them in analytical work.

The remainder of the paper is organized as follows. In Section II the test setup and data processing are described. Section III presents the obtained measurement results. In Section IV reliable multicast and cooperation are evaluated based on the obtained results and the assumption of independent erasures. The final conclusions are drawn in Section V.
II. TEST SETUP AND DATA PROCESSING

The measurements were carried out at Aalborg University, Denmark, in a reception area surrounded by offices. The setup consisted of one laptop connected to a wireless Access Point (AP), and clusters of four, nine and 15 mobile devices, and was similar to that used in [8]. Figure 2 shows a 28 × 53 m rectangle of the ground floor around the measurement location. The mobile devices were placed in a square grid, and the sending AP was located 25 m from the center of the grid.

Fig. 2: Position of mobile devices and AP used in the measurements. The distance from the AP to the receiving devices is 25 m.

The following equipment was used in the measurement.

- Access Point: A Cisco Arionet 1100 series AP was used as it provided the necessary control over transmission power. It can also monitor transmitted packets to ensure that all packets are transmitted from the AP.
- Laptop: An IBM T40 running Ubuntu 10.04, was used as packet generator and connected to the AP with an Ethernet 10/100 TBase connection.
- iPod 3rd gen: 15 3rd generation Apple iPod Touches.

In each test the AP broadcasted 100,000 packets that included a sequence number and some dummy data. The length of each packet on the application layer was 1400 B. Every 4 ms a packet was generated at the laptop and transmitted by the AP in order to avoid dropped packets due to congestion at the sender and/or the receivers. This value was also used for the N95’s in [8], and it was verified that the erasure rate of the iPods did not decrease if the packet spacing was increased to more than 4 ms. The transmission power on the AP was fixed to 100 mW, and the transmission rate was 54 Mbs. A native application was running on each of the receiving devices. For each received packet, a time stamp, sequence number, and some additional information were saved to a trace file. The trace files were retrieved from the mobile devices after the test was completed and processed using a Python script.

For each cluster, the total number of devices is denoted $N$. When a subset of nodes in the cluster is considered, the size of this subset is denoted $n$. From the trace files, the erasure rate of each device, $\epsilon_i$, was calculated. The vector containing the erasure probabilities of all nodes is denoted $\epsilon$. The mean node erasure probability $\tau$ is the mean of $\epsilon$. The cluster erasure Probability Mass Function (pmf), $\kappa_n$, defines the probabilities that $x \in [0, n]$ nodes experience an erasure, and has length of $n + 1$. Thus $\kappa_{n,0}$ denotes the probability that no nodes in the cluster experience an erasure, and $\kappa_{n,n}$ denotes that all nodes experience an erasure and denoted the cluster erasure probability. We only have measurements from a cluster of size $N$. So to obtain results from clusters of size $n < N$, we chose all possible combinations of size $n$ and calculated the corresponding mean pmfs.

Table I: Notation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Total devices in the cluster</td>
</tr>
<tr>
<td>$n$</td>
<td>Considered devices in the cluster</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Node erasure probabilities</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Mean node erasure probability</td>
</tr>
<tr>
<td>$\kappa_n$</td>
<td>$n$-Cluster erasure pmf</td>
</tr>
<tr>
<td>$\kappa_{n,n}$</td>
<td>$n$-Cluster erasure probability</td>
</tr>
</tbody>
</table>

In order to condense the extensive measurement data, we introduce the cluster erasure transition matrix which we denote $T$. This matrix can be used to perform Markov chain evaluations as it defines the transition probabilities that any number of nodes experience an erasure for a single packet. $T$ is an upper triangular matrix and is constructed from the $N$ cluster erasure pmfs as defined by Equation 1. Note that zero padded versions of $\kappa_n$’s are used, where $N - n$ zeros are appended to $\kappa_n$. The input is defined by the column index, where the $n$th column denotes that $n$ nodes experience an erasure, and the first column has index zero. The output is defined by the row index, where the $n$th row denotes that $n$ nodes experience an erasure, and the first row has index zero. The zeroth column is added to specify that a cluster of size zero always experiences zero erasures.

$$
T = \begin{bmatrix}
1 & \kappa_{1,0} & \kappa_{2,0} & \cdots & \kappa_{N,0} \\
0 & \kappa_{1,1} & \kappa_{2,1} & \cdots & \kappa_{N,1} \\
& \kappa_{2,2} & \cdots & \kappa_{N,2} & \\
& & \cdots & \\
& & & \kappa_{N,N}
\end{bmatrix}
$$

The observed erasure rates are reported as otherwise the results can be difficult to interpret, see Table II. For the N95’s the difference in erasure rate between the worst and best device is approximately a factor of two. For the iPods the factor is approximately 3.5. We note that the erasure rates for the N95’s are comparable to those reported in [8].

Table II: Sorted node erasure rates, and mean node erasure rate.

<table>
<thead>
<tr>
<th>Device</th>
<th>$\epsilon$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia N95</td>
<td>0.219 0.227</td>
<td>0.342 0.374 0.391 0.393</td>
</tr>
<tr>
<td>iPod 3rd gen</td>
<td>0.053 0.058</td>
<td>0.101 0.113 0.113 0.114 0.181</td>
</tr>
</tbody>
</table>
III. CORRELATION RESULTS

In Figure 3, the measured cluster erasure probability is plotted as a function of the cluster size for the two types of devices. The figure is truncated on the y-axis in order to provide more details when the cluster is small. To compare with the assumption of independent erasures, the binomial distribution based on the mean erasure rate and the number of nodes is also plotted. The binomial distribution is a straight line when plotted against a semi-logarithmic y-axis. For the 3rd generation iPods it extends to $10^{-16}$ when the cluster size is 15.

![Fig. 3: Cluster erasure probability as a function of the number of cluster nodes. The dotted lines indicate calculated cluster erasure probabilities based on the independent assumption.](image1)

As the number of nodes in the cluster increases, the cluster erasure probability decreases. When the measured erasure probability and the erasure probability calculated with the binomial distribution is compared, it can be observed that the error for some cluster sizes is several orders of magnitude. This is most prominent for the two types of iPods, and the measured erasure rate for the 3rd generation iPod is more than 14 orders of magnitude higher when compared to the value obtained with the binomial distribution. This indicates that assuming independent packet losses among nodes in a cluster is not a valid assumption in the given measurement scenario. It is important to notice the dynamic range of the nodes' erasure rates, see Table II. Thus care should be taken when the measurements for each cluster are compared. One interesting observation is that the correlation of erasures is considerably lower for the N95's compared to the iPods.

To obtain a more detailed view on the measurements, we model the erasure pmf of the cluster with the binomial and the negative binomial distribution. These are discrete approximations of the normal distribution. Typically the binomial distribution is used when independence is assumed. Both pmfs are calculated based on the mean erasure rate $\tau$, and the number of devices in the cluster $n$. Thus they are both simple to determine as the necessary input values can be observed directly. The binomial distribution is defined as.

$$X_1 \sim Bi(n, \tau)$$ (2)

We use a truncated and normalized version of the negative binomial distribution as the number of erasures cannot be higher than the number of receivers. First $Bi(n = i)$, where $i \in [0, N]$, is calculated, see Equation (3) and then the result is normalized to obtain a valid pmf.

$$X_2 \sim NB \left( n, \frac{1}{1 + \tau} \right)$$ (3)

The cluster erasure pmf for the Nokia N95 cluster with nine devices is plotted on Figure 4. In addition the binomial and negative binomial models are plotted. The x-axis indicates the number of nodes in the cluster that experienced an erasure. Thus zero on the x-axis indicates the mean probability that all nodes in the cluster received a given packet.

![Fig. 4: Cluster erasure pmf for a cluster of nine Nokia N95's.](image2)

We observe that generally the binomial model does not predict the measured cluster erasure pmf. The two most important points are the two extremes, where all nodes lose or receive the packet respectively. Unfortunately the binomial model does not provide an accurate prediction in these cases. Interestingly we observe that the probability that all nodes in the cluster receive a packet is much higher than predicted by the binomial model.

The reason to include the negative binomial distribution should become apparent when Figure 4 is observed. This model fits the erasure pmf with surprising accuracy, and thus indicate that the negative binomial model should be used instead. Due to space constraints we do not include pmfs for smaller clusters. But these plots also show that the negative binomial distribution predicts the measured pmfs much more accurately than the binomial distribution. Except when the cluster size is two or three in which case both perform equally well. To verify the results we repeated the test three times.
They all produced a similar result, and these plots can be obtained from [9].

The same models are applied to the iPod 3rd generation measurements. To make the figures more comparable, we first consider a cluster size of nine, see Figure 5.

![Cluster erasure pmf for a cluster of nine 3rd generation iPods.](image1)

Neither of the models fits the measured data. The error is largest for the probability that all nodes lose the packet. In this case, it is eight and four orders of magnitude for the binomial and negative binomial model, respectively. It can also be observed that the shape of the cluster erasure pmf is significantly different compared to that of the Nokia N95’s. In particular it appears that a part of the erasures is due to a highly correlated process in the cluster, as the probability that all but one node experience an erasure is higher than the probability that all nodes experience an erasure is significantly different compared to that of the Nokia N95’s.

For completeness we also include the plot for a cluster of all 15 3rd generation iPods in Figure 6.

![Cluster erasure pmf for a cluster of 15 3rd generation iPods.](image2)

IV. IMPACT ON RELIABLE MULTICAST AND COOPERATIVE STRATEGIES

We now analyze the performance of a reliable multicast scheme and a cooperative scheme, assuming independent erasures and using the data obtained with the 3rd generation iPods. The performance metric is the expected number of retransmissions from the source. For the reliable multicast scheme the source retransmits until all nodes in the cluster have successfully received the packet. For the cooperative scheme the source retransmits until at least one node in the cluster has received the packet. For both cases we assume that a perfect orthogonal feedback channel exists from all receivers to the source, and thus the results are lower bounds on the number of necessary retransmissions. A script to evaluate the presented strategies can be obtained from [9].

As mentioned the assumption of independent erasures is a worst-case assumption for the multicast strategy, but a best-case assumption for the cooperation strategy. The reason is that this assumption gives the highest probability that at least one node in the cluster receives a given packet, but it also gives the highest probability that at least one node in the cluster does not receive a given packet. For cooperative schemes if any node in the cluster receives a packet it can distribute it to other nodes in the cluster. For multicast unless all nodes in the cluster receive a packet, the source must retransmit it.

For multicast where independent erasures are assumed the expected number of retransmissions necessary is given by Equation 4. This is the sum of probabilities that all nodes in the cluster have not received the packet. \(i\) is the number of retransmissions performed.

\[
B_{\text{independent},n} = 1 - \sum_{i=1}^{\infty} (1 - \tau^i)^n \quad (4)
\]

For multicast based on the measurements the transition matrix, \(T\), described in Section II is used as a Markov chain. We consider a cluster of size \(n\), where initially all nodes have an erasure for the packet. The starting probability where zero retransmissions have been performed, for a cluster of \(n\) nodes is a column vector with \(N + 1\) rows which we denote \(\sigma_n^0\). This vector is equal to the zero padded version of \(\kappa_n\). Thus the pmf of the number of nodes that experience an erasure in the cluster after \(i\) retransmissions is denoted \(\sigma_n^i\). The number of retransmissions is obtained as the summation of the probability that an additional retransmission is required, see Equation 5.

\[
B_{\text{measured},n} = \sum_{i=1}^{\infty} (1 - \sigma_{n,0}^i), \quad \sigma_{n}^i = T^i \cdot \kappa_n \quad (5)
\]

For the cooperative case when independent erasures are assumed, the cluster erasure probability is simply given by \(\tau^n\). Thus the expected number of retransmissions is obtained as the sum of the probabilities that no nodes in the cluster have received the packet after \(i\) transmissions.

\[
C_{\text{independent},n} = \sum_{i=1}^{\infty} (\tau^{i-1}) = \frac{\tau^i}{1 - \tau^n} \quad (6)
\]
For cooperation based on the measurements the probability of a cluster erasure, $\kappa_n$ is known, and is used directly.

$$C_{\text{measured},n} = \sum_{i=1}^{\infty} \left( \frac{\kappa_{n,i}}{1 - \kappa_{n,i}} \right) = \frac{\kappa_{n,n}}{1 - \kappa_{n,n}} \quad (7)$$

On Figure 7 the expected number of retransmissions by the source is plotted as a function of the number of nodes in the cluster, for the four cases. As the number of nodes increases the number of retransmissions increases for the reliable multicast case, and decreases for the cooperative case.

For reliable multicast the independent assumption significantly overestimates the number of retransmissions. The reason is that the probability that all nodes in the cluster either receive or lose the packet is higher compared to that obtained when independent erasures are assumed. In these two cases the source either retransmits to all or none of the nodes and thus multicast performs optimally. For the cooperative case the number of retransmissions approaches zero very fast when independent erasures are assumed. When based on the measurements this number quickly plateaus because an additional node does not significantly change the cluster erasure probability, unless the cluster is very small. Table III presents the reduction in retransmissions when cooperation is used instead of reliable multicast, calculated as $(B - C)/B$. Both the reduction using independent erasures and the measurements are shown.

TABLE III: The reduction in overhead in percent when independent erasures are assumed (top), and when based on the obtained results (bottom).

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia</td>
<td>86.4</td>
<td>97.0</td>
<td>99.2</td>
<td>99.8</td>
<td>99.9</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>N95</td>
<td>81.7</td>
<td>93.7</td>
<td>97.1</td>
<td>98.4</td>
<td>99.1</td>
<td>99.4</td>
<td>99.6</td>
<td>99.7</td>
</tr>
<tr>
<td>iPod</td>
<td>95.7</td>
<td>99.7</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>3rd gen</td>
<td>60.0</td>
<td>72.5</td>
<td>78.4</td>
<td>81.9</td>
<td>84.3</td>
<td>86.0</td>
<td>87.4</td>
<td>88.5</td>
</tr>
</tbody>
</table>

Fig. 7: Comparison of reliable multicast and cooperation assuming independent erasures and using the measurements from the 3rd generation iPods.

V. Conclusion

The obtained measurement results are used to evaluate the necessary retransmissions from an AP to a cluster using a reliable multicast and a cooperative strategy. The same evaluation is conducted based on the assumption of independent erasures, and the results show that the gain of cooperation over reliable multicast is unrealistically high when independent erasures are assumed. This is not surprising as this is a worst-case assumption for reliable multicast, and a best-case assumption for cooperation. However, the cooperative strategy still performs significantly better than the multicast strategy when the evaluation is based on the obtained data.

Based on the obtained measurements, packet erasures appear to be neither independent nor fully correlated, but somewhere in between. Furthermore the investigated devices exhibited significantly different erasure properties, both in terms of erasure probability and erasure correlation. Therefore we conclude that assuming independent erasures in this type of scenario is not valid in general.

In the future, the measurements should be conducted using other devices, and in different scenarios. In particular the dynamic range of the observed erasure probabilities is wide, thus it is difficult to directly compare the measured devices. Additionally the erasure correlation within a cluster should be investigated in order to allow for a realistic evaluation of the local cooperation phase of cooperative strategies.

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REFERENCES