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Robust Parametric Fault Estimation in A Hopper System

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Abstract: The ability of diagnosis of the possible faults is a necessity for satellite launch vehicles during their mission. In this paper, a structural analysis method is employed to divide the complex propulsion system into simpler subsystems for fault diagnosis filter design. A robust fault diagnosis method, which is an optimization based approach, is applied to the subsystems of the propulsion system. The optimization problem has been solved within two different tools and the results are compared with two other optimization based approaches. The turbo-pump system is used to illustrate the employed methods and obtained results.

1. INTRODUCTION

Reliability is a highly desired topic in many industrial applications, particularly in aerospace. The mission objectives of a spacecraft may not be disrupted by any possible fault. A fault diagnosis system is able to monitor the system performance and alert the control system when a fault has occurred. In this regards, the problem of model-based fault diagnosis has been receiving increasing attention from the research communities (Willsky, 1996).

By the early 90’s, the paradigm of the conventional methods for fault diagnosis problem, which included annihilating the matrices, was substituted by the methods based on norm minimization. This phenomenon opened the doors of the $\mathcal{H}_\infty$ and other optimization approaches to the field of fault diagnosis (Frank and Ding, 1994; Mangoubi et al., 1995; Edelmayer et al., 1996; Edelmayer and Bokor, 2000). Most of those FD approaches (except parameter identification methods) have considered the models with additive fault input to the system. In the other words, they are modeled as exogenous perturbations to the system (Basseville, 1988; Chen and Patton, 1999; Frank, 1990).

In this paper, the fault is modeled as a parameter, since the nature of many faults are parametric. Indeed, an exogenous essentially bounded input cannot de-stabilize a linear system.; whereas a parameter change might do so. A fault diagnosis approach for systems with parametric fault which has been proposed by (Stoustrup et al., 1997; Niemann and Stoustrup, 1997) is used here. The optimization problem has been defined in the so-called standard set-up for robust control based on LFT. In this approach, the residual is in fact an estimation of the fault. A "Hopper", which is a horizontally launched and horizontally landing rocket-propelled launch vehicle comprising a non-disposable primary stage and one expendable upper stage, is under consideration as a reusable launch vehicle to replace the existing expandable launch vehicles in ESA (European Space Agency) in the future. The advantages include: reduction of transportation cost to orbit, return capability from orbit, and less environmental pollution.

The main engine is a complex system with various subsystems. Designing a filter for this system, which is capable of determining faults in a reliable manner, is shown to be a nearly impossible task. To address and solve this problem, a structural analysis approach was employed. The structural analysis of the system leads to identifying subsystems with inherent redundant information required for designing appropriate filters.

The contributions of this paper are two-fold: 1- illustrating the advantage of combined utilization of qualitative as well as quantitative methods to design a fault diagnosis system. Structural analysis method, which is a qualitative method, is used to analyze the system and divide the system into manageable (and monitorable) parts; whereas, quantitative (here optimization based robust methods) are used for the detailed design. 2 - the application of the parametric fault diagnosis filter design based on the $\mathcal{H}_\infty$ as well as the $\mu$ synthesis, in the set-up presented in (Stoustrup and Niemann, 2002; Niemann and Stoustrup, 2000; Soltani et al., 2011). In addition, the main results of the designed filter has been compared with two other optimization based methods (Khosrowjerdi M.J., 2005; Zhong M., 2003).
This paper is organized as follows: In Section II, an structural analysis on the propulsion system is presented while the turbo-pump has been chosen as a subsystem. The fault diagnosis method has been presented in Section III and the fault estimator design procedure has been described. In Section IV, the results of fault estimation are illustrated and compared with two other methods, and eventually, the conclusions are brought in Section V.

2. STRUCTURAL ANALYSIS OF THE PROPULSION SYSTEM

2.1 Motivation

The overall nonlinear system of the considered engine model is translated to a model block diagram. The blocks in the diagram, which is shown in figure 1 on the following page, represent the functionalities of the engines main parts; valves, pumps, combustion chamber, and the generator. The considered plant has 14 independent inputs, 18 outputs, 14 intermittent (nonmeasurable) variables, and 6 dynamic/continuous states.

2.2 Structural Analysis

Structural analysis is concerned with the properties of the system structural model, which is the abstraction of its behavior model in the sense that only the structure is considered. A case in point, only the existence of relations between variables and parameters is taken into account (Blanke et al., 2006). The links are represented by a bi-partite graph, which is independent of the of the values of the variables and the parameters. Hence, the structural model is a qualitative, very low level, easy to obtain, model of the system behavior. The structural analysis provides the following information:

- the subset of the components, in which the faults can be detected and isolated;
- the possibility of designing residuals to meet specific requirements;
- the existence of reconfiguration possibilities.

To demonstrate the use of the structural analysis, we have taken the results for the liquid oxygen LOX turbo-pump. The structural model of the LOX pump is shown in table 1. The constraints are \( C = \{ c_1, c_2, c_3, c_4, c_5, c_6 \} \), the unknown variables and (intermittent) parameters are \( X = \{ R_o, p_{13}, p_{14}, p_{15}, p_{16}, m_6, m_{14}, m_{15}, m_{16} \} \), and the known (measured) variables are \( M = \{ m_1, m_4, m_6, m_{14}, m_{15}, m_{16} \} \).

<table>
<thead>
<tr>
<th>( C )</th>
<th>( X )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_6 )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Structural model of the LOX pump.

A matching between an unknown variable/parameter and a constraint, denoted by a \( \circ \) in the cross section between the variable column and the constraint row, indicates that the matched variable can be calculated/computed through the corresponding constraint. For instance, \( p_{13} \) can be calculated through \( c_1 \) when the values of \( R_o \) and \( m_6 \) are known. The constraint \( c_1(R_o, p_{13}, m_6) = 0 \) represents the following dynamical behavior:

\[
R_o = a_5 Q_o^2 - b_o T_o + c_o Q_o R_o + d_o R_{0h} R_o^2
\]

where \( a_o, b_o, c_o, \) and \( d_o \) are constant coefficients depending on the design of the turbo-pump and \( T_o \) is the LOX turbine torque. The pump speed is represented by \( R_o \), the pump flow by \( Q_o \), and the mixture ratio by \( R_{0h} \).

3. FAULT ESTIMATION METHOD

3.1 Robust Parametric FDI in A Standard Set-up

A general concept of parametric fault detection architecture in a turbo-pump standard set-up is proposed in (Stoustrup and Niemann,
Fig. 1. Modular decomposition of engine system. (Known) Inputs to each block are shown by green color, faults have red color, Blue color represents the measured outputs, and black color represents internal variables which are not known (not measured).

The approach is to model a potentially faulty component as a nominal component in parallel with a (fictitious) error component. Subsequently, the optimization procedure suggested here estimates the ingoing and outgoing signals from the error component. This works only well in cases where the component is reasonably well excited, but on the other hand, if the component is not active at all, there is absolutely no way to detect whether it is faulty. The considered plant is described by the model

\[
\begin{align*}
\dot{x} &= A_A x + B_A u \\
y &= C_y x + D_y y u
\end{align*}
\]

(3)

where \( A_A \) is the deviated matrix from the nominal value \( A \) by a dependency to the fault where the dependency can be nonlinear. The possibly nonlinear parameter dependency of \( A_A \) is approximated with a polynomial. Therefore, \( A_A = A + p(\delta)A \), where \( p \) is a polynomial function of the parameter \( \delta \) satisfying \( p(0) = 0 \) (the non-faulty operation mode). Finally, the model (3) is written in linear fractional transformation form. As a result we get a system of the form

\[
\begin{bmatrix}
\dot{x} \\
z
\end{bmatrix} =
\begin{bmatrix}
A & B_f & B_a \\
C_f & D_{zf} & 0 \\
0 & C_y & D_{zu}
\end{bmatrix}
\begin{bmatrix}
x \\
f \\
u
\end{bmatrix}
\]

(4)

where \( z \) is the external output, \( f \) is the fault input signal, the matrix \( D_{zf} \) is well-posed (LFT’s are normally used), and the connection between \( z \) and \( f \) is given by

\[
f = \Delta_{par} z,
\]

(5)

where \( \Delta_{par} \) is a diagonal matrix \( \Delta_{par} = \delta I \).

The next step in setting up the fault estimation problem as a standard problem is to introduce two fault estimation errors \( e_f \) and \( e_z \) as

\[
\begin{align*}
e_f &= f - \hat{f} \\
e_z &= z - \hat{z}
\end{align*}
\]

(6)

where \( \hat{f} \) and \( \hat{z} \) are the estimation of \( f \) and \( z \) to be generated by the filter respectively. Fig. 2 shows the setup for this approach. To design a filter \( F \) such that applying \( F \) to \( u \) and \( y \) provides the two desired estimates \( \hat{f} \) and \( \hat{z} \), one additional step is required. To this end, we introduce a fictitious performance block \( \Delta_{perf} \); suggesting that the input \( u \) was generated as a feedback \( \Delta_{perf} \) from the outputs \( \begin{bmatrix} e_f \\ e_z \end{bmatrix} \)

\[
u = \Delta_{perf} \begin{bmatrix} e_f \\ e_z \end{bmatrix}.
\]

(7)

Therefore, two filters \( (W_f(s) \text{ and } W_z(s)) \) are introduced to make sure that the norm of \( \|e_f\| \) is minimized in the frequency area of interest. (For incipient faults a low frequency filter is used.) In fact, we introduce these filters to handle the high excitation level of the inputs. Finally we introduce
3.2 Design of The Fault Detector for Turbopump

As an example of the fault estimation method, we brought one of the subsystems in the propulsion system (Soltani et al., 2008). The Oxygen turbopump subsystem is actually the combination of the RTO and PUMP O-1 blocks in Figure 1. The dynamic model of this block is written as following

\[
\dot{x} = (-ax - cQ_o)(1 - p(\delta)) + bT_o, \tag{12}
\]

where \(a, b,\) and \(c\) are constants from the linearization, \(x\) is the shaft speed, \(Q_o\) is the pump flow, and \(T_o\) is the turbine torque and

\[
p(\delta) = \lambda \delta^3 + \delta^2 - \lambda \delta \tag{13}
\]

is the parametric fault model with some constant \(\lambda\).

The system is formulated in a standard form as

\[
\begin{bmatrix}
\dot{x} \\
\dot{\hat{x}}_f \\
\dot{\hat{x}}_e \\
\end{bmatrix} = \begin{bmatrix}
A & B & C \\
D & E & F \\
G & H & I \\
\end{bmatrix} \begin{bmatrix}
x \\
\hat{x}_f \\
\hat{x}_e \\
\end{bmatrix} + \begin{bmatrix}
u \\
0 \\
0 \\
\end{bmatrix},
\]

where \(W\) is a relatively big constant used to include the actuator fault in the state variables. The standard model (with \(D_{ef} = 0\)) becomes

\[
\begin{bmatrix}
\dot{x} \\
\dot{\hat{x}}_f \\
\dot{\hat{x}}_e \\
\end{bmatrix} = \begin{bmatrix}
A_1 & B_1 & B_2 \\
D_1 & D_{1f} & D_{1e} \\
C_1 & D_{2f} & D_{2e} \\
\end{bmatrix} \begin{bmatrix}
x \\
\hat{x}_f \\
\hat{x}_e \\
\end{bmatrix} + \begin{bmatrix}
u \\
0 \\
0 \\
\end{bmatrix},
\]

where the matrix values can be found in Appendix 5. Finally, a \(\mathcal{H}_\infty\) filter \(F\), which estimates \(\hat{f}\) and \(\hat{z}\) and takes \(u\) and \(y\) as inputs, is designed using hinfsyn in MATLAB. This filter results in \(e_f\) and \(e_z\) vanishing to zero as time goes to infinity.

4. RESULTS

4.1 Comparable fault estimation/diagnosis algorithms

To evaluate the described algorithm in the previous section, three other suggested optimization based robust methods are
employed. These methods are described in the following subsections.

**μ-Synthesis** The optimization problem in Theorem 1 can be solved using D-K iteration as a numerical method for μ-synthesis. The set-up, should be formulated in a way so that δ is augmented in the set-up and considered to be in the unit circle of the complex plane.

\[
\begin{align*}
\dot{x} &= -ax - x_u + b_{T_x} + (ax + x_u)\lambda_1 \delta_1 \gamma - (ax + x_u)\lambda_2 \delta_1 \\
\dot{x}_u &= -Wx_u + WcQ_o \\
\dot{\hat{x}} &= A_{\hat{x}}\hat{x} + B_{\hat{x}}(\hat{x} + x_u)\lambda_1 \delta_1 \gamma - (ax + x_u)\lambda_2 \delta_1 - \hat{f} \\
\dot{\hat{e}} &= C_{\hat{e}}\hat{x} + D_{\hat{e}}\hat{e}_f \\
\dot{e}_f &= C_{ef}\hat{x} + D_{ef}e_f \\
\dot{e}_c &= C_{ec}\hat{x} + D_{ec}\hat{e}_c \\
y_1 &= x \\
y_2 &= T_o \\
y_3 &= Q_o
\end{align*}
\]

(16)

**Mixed \(H_2/\infty\) Fault Diagnosis** In (Khosrowjerdi M.J., 2005), the residual for the system

\[
\begin{align*}
\dot{x} &= Ax + Bu + B_f f_u + B_d d_s \\
y &= Cx + Du + D_f f_s + D_d d_s
\end{align*}
\]

is given by

\[
\begin{align*}
\dot{\hat{x}} &= (A - KC)\hat{x} + [B - KD K][u \ y_m]^T \\
\dot{\hat{\delta}} &= -C\hat{x} + [-D \ 1][u \ y_m]^T
\end{align*}
\]

(18)

where the gain \(K\) is obtained through solving the convex optimization problem in (Khosrowjerdi M.J., 2005) and \(\hat{\delta}\) is the estimated residual here.

**Mixed \(H_\infty/\text{LMI}\) Fault Diagnosis** In (Zhong M., 2003), a model-matching problem is solved by minimizing the \(H_\infty\) norm of the difference between the residual reference model and the real residual. In this method, the residual for the system 17 is given by

\[
\begin{align*}
\dot{\hat{x}} &= (A - HC)\hat{x} + [B - HD \ H][u \ y_m]^T \\
\hat{\gamma}_m &= Ci + Du \\
\hat{\delta} &= V[y_m - \hat{y}_m]
\end{align*}
\]

(19)

where \(\hat{x}\) and \(\hat{\gamma}_m\) are the estimates of the state and measurement output vectors and the filter gains \(H\) and \(V\) are designed according to theorem 2 in (Zhong M., 2003).

### 4.2 Comparison of Estimation Results

Figure 3 shows the comparison of the different designs for \(H_\infty\) and μ-synthesis. By reducing the γ of \(H_\infty\) optimization the estimation becomes more robust to the disturbances. The comparison of the \(H_\infty\) with μ-synthesis shows that in the no-fault interval (0-25s), the estimation has lower amount of...
fluctuations and is more robust. However, in the fault interval (25s-50s), the residual generated by $H_\infty$ design is more robust to the disturbances.

Figure 4 shows the output of the estimated residual generated by the mixed $H_2$/$H_\infty$ design method. As the change in the parameter results in instability of the system, the estimated residual is also unbounded. Consequently, the estimated residual does not estimate to the injected fault, but it determines the existence of a fault.

In figure 5, a similar type of output (unbounded) is observed. The residual is the result of the mixed $H_\infty$ LMI design method which does not represent the estimation of the fault, though it is less robust to the disturbances compared to $H_2$/$H_\infty$ design method.

In figures 6, 7, 8, 9, 10, 11, 12, and 13, the output of all four different FD filters are illustrated for different injected faults $\delta$. These results show that the filter designed through $\mu$-synthesis approach gives the best estimation of the injected fault in different scenarios.

### 4.3 Structural Analysis Results

The structural analysis, carried out on the propulsion engine model, identified 11 independent subsystems with inherent redundant information. Hence it is possible to derive 11 different and linearly independent residual expressions. 6 of these subsystems exhibit dynamic behavior while the other 5 are of algebraic nature. 12 different faults were considered in this system. A preliminary analysis of the fault impacts on each subsystem (represented by a corresponding residual) suggested that all faults were detectable. In addition, 7 faults were isolable while the other faults were group-wise isolable, i.e. a group of 2 faults and a group of 3 faults were isolable, but with no possibility of isolating the faults from each other in each group (Soltani and Izadi-Zamanabadi, 2007). Detailed design of fault diagnosis algorithms for each subsystem (in particular those with dynamic behavior) were carried out, and the results showed an exact match between the detected/isolated faults and the detectable/isolable faults determined in the structural analysis. Despite being a simple qualitative method the structural analysis showed to be an extremely powerful tool for developing health monitoring systems in complex dynamical systems.

### 5. CONCLUSION

The structural analysis approach was applied to identify the monitorable parts/subsystems of a complex propulsion system and provide information about the possibility of detecting and isolating the considered faults in the system. In this paper, the process of using the structural analysis was briefly illustrated by applying it on a turbo-pump subsystem.

The obtained filter was based on the parametric fault diagnosis filter design approach based on the $H_\infty$ as well as the $\mu$ synthesis, where the chosen turbo-pump subsystem was used as the benchmark. Eventually, the results of the designed fault estimator have been compared with that of two other optimization based methods.

### APPENDIX

**Matrices Values**

$$A_1 = \begin{bmatrix} -a & -1 & 0 & 0 \\ 0 & -W & 0 & 0 \\ 0 & 0 & A_f & 0 \\ a\lambda B_{ec} & \lambda B_{ec} & 0 & A_{ec} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} T_e \\ 0 \\ 0 \end{bmatrix}$$

$$B_f = \begin{bmatrix} \lambda & -\lambda \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -B_{ef} & 0 \\ -B_{ec} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} a & 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{ef} \\ 0 & 0 & C_{ec} \end{bmatrix}$$

$$D_{11} = 0_{3x2},$$

$$D_{1f} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$D_{12} = 0_{5x2},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D_{2f} = 0_{3x3},$$

and $D_{22} = 0_{3x2}$.

### REFERENCES


A. M. Edelmayer, J. Bokor, and L. Keviczky. h-infinity filter design for linear systems: comparison of two approaches. *in*
Fig. 6. Fault injected as a step (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$/LMI design.

Fig. 7. Fault injected as a fast ramped-raising step (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$/LMI design.

Fig. 8. Fault injected as a slow ramped-raising step (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$/LMI design.

Fig. 9. Fault injected as rectangular pulses (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$/LMI design.

Fig. 10. Fault injected as triangular pulses (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$/LMI design.

Fig. 11. Fault injected as sine with the frequency of $2\pi$ rad (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$/LMI design.
Fig. 12. Fault injected as sine with the frequency of 1 rad/s (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$ / LMI design.

Fig. 13. Fault injected as sine with the frequency of 0.3 rad/s (a) the injected $\delta$, (b) the residual provided by $\mathcal{H}_\infty$ design, (c) the residual provided by $\mu$ design, (d) the residual provided by $\mathcal{H}_2/\mathcal{H}_\infty$ design, and (e) the residual provided by $\mathcal{H}_\infty$ / LMI design.