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Efficient Design of Super-Directive Antenna Array using Schelkunoff Method and Genetic Algorithm

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***Abstract*—Tightly packed super-directive antenna arrays with complex excitation functions are of recent interest in space communication. In this paper, Schelkunoff polynomials and genetic algorithms (GA) are used to formulate the super-directive array excitation functions. The proposed technique used to calculate the antenna properties considerably reduces solver time compared to professional simulators. A packed linear array with an antenna aperture of 2.85λ and element spacing of 0.3λ (centre-to-centre) is designed to demonstrate a 66.67% increase in directivity, reduced sidelobes, and improved null accuracy as proof of concept. The calculated antenna responses agree well with the results of a professional simulator (HFSS), where the proposed method requires 90% less calculation time compared to the simulator. The experimental results verify the predicted responses and demonstrate a 5 dB increase in the antenna directivity compared to a conventional array.**

***Index Terms*—Directivity, super-directive, microstrip array, Schelkunoff, genetic algorithm.**

I. INTRODUCTION

Miniaturized and directive antenna arrays are essential in current point-to-point communication and far-field power transfer devices. In a conventional antenna array, the gain is proportional to the number of radiating elements, separated by at least half a wavelength. However, by reducing the separation distance and using appropriate excitation functions, the mutual coupling effect between array elements can be compensated through the optimized excitations of the super-directive feed network. Composite polynomials have been widely utilized to generate efficient array excitation functions [1] - [5] that lead to a 20-30% reduction in the half-power beam width. However, for an N-element antenna array with an overall length of several wavelengths, the computational time and design challenges in realizing the array excitation functions have hindered the application of this class of super-directive arrays [6] - [14]. Low-cost and easily integrated microstrip antenna arrays are widely used in wireless communication devices [15] – [19] due to their tunable radiation pattern, polarization, impedance bandwidth and gain [20] - [23]. Several models exist in the literature to efficiently design microstrip antennas and arrays, such as the transmission-line model [24] – [27] and cavity model [28] – [36].

In recent years, full-wave electromagnetic solvers (HFSS) have been used to accurately model and optimize microstrip antenna arrays using an iterative process [37]. But for the larger array dimensions required in super directive antennas, the design and optimization using electromagnetic solvers become very computationally demanding and often require adopting Binomial or Chebyshev techniques [38] - [42]. However, these methods consider isotropic radiators and require complicated pattern synthesis to generate the required main beam position and side lobe levels. Alternatively, an optimized polynomial-based synthesis of super-directive radiation patterns can be used to design a specific array with a known number of elements and array length. This allows for generating and optimizing the array excitation function to yield the required radiation pattern and efficiency. In [43] - [49], evolutionary algorithms such as the genetic algorithm (GA) and Particle Swamp Optimization (PSO) are used with the Schelkunoff polynomial to achieve adaptive beam-forming and steering.

This paper addresses the generation and optimization of excitation functions for a linear antenna array to achieve a super-directive radiation pattern. Employing Genetic Algorithm (GA) and Schelkunoff theory, the excitation functions are formulated and refined. The method showcased herein exhibits a notable reduction in solver time, achieving a 90% decrease compared to conventional numerical techniques. Physical realization of the excitation functions necessitates the design of a densely packed microstrip array with reduced patch width while ensuring a total antenna aperture of approximately 2.85λ. To validate the super-directive response, a 7-element linear antenna array operating at 10 GHz is designed, employing shorted-to-ground patches with a reduced phase-centered separation distance of 0.3λ and a straightforward array feeder. The numerically simulated radiation patterns of the antenna array in the case of using a feeding network with uniform excitations and the proposed Schelkunoff excitations are compared to demonstrate the super-directive radiation performance. The designed antenna array is fabricated to experimentally verify the simulated super-directive radiation patterns. It's noteworthy that the complex and time-consuming optimization process of the super-directive excitation functions is significantly relaxed using the proposed technique with a directivity increase of 5 dB.

II. THEORETICAL FORMULATION

In this section, the Schelkunoff polynomials method and GA are utilized to calculate and optimize the excitation functions of a linear microstrip antenna array. It's important to note that

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conventional Chebyshev or Binomial techniques require considerably longer solver time to generate similar polynomials used for beam/null steering. The super-directivity excitation function of a linear array based on Schelkunoff polynomials can be expressed as [19]

$$F(x) = \sum_{p=0}^{M} w_{\mathrm{p}} x^{\mathrm{p}} \tag{1}$$

Equation (1) is the general form of the array factor (AF) of an antenna array with an odd number of elements, denoted by N, where M = (N-1)/2, the complex excitation coefficients are represented by w_p and $x^{\mathrm{p}} = e^{jp(kd\cos\theta+\beta)}$. Here, k represents the wave number for free space, which is equal to ($k = 2\pi/\lambda$), where λ is the free space wavelength, and β denotes the progressive phase shift. The angle θ is measured with respect to a plane normal to the array. Therefore, the array factor (AF) of form 2p+1 can be expressed as

$$AF = \sum_{p=1}^{M+1} w_p \cos\left[(p-1)kd\cos\theta\right] \tag{2}$$

Hence, from (1) and (2)

$$AF = F(x) = \sum_{p=0}^{M} w_{\mathrm{p}} x^{\mathrm{p}} \tag{3}$$

Applying the Schelkunoff polynomials method to (3) yields the AF of the odd symmetry as

$$\sum_{p=0}^{P} w_p x^p = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + ...w_P x^P \tag{4}$$

Expressing the polynomial in terms of its roots yields

$$\sum_{p=0}^{P} w_p x^p = (x-x_1)(x-x_2)(x-x_3)...(x-x_P) \tag{5}$$

Assuming L numbers of zeros are required at the desired locations θ_1,θ_2,...θ_L, (5) can be expressed as

$$\begin{gathered}\sum_{p=0}^{P} w_p x^p = w_P(x-x_1)(x-x_2)...(x-x_L) \\ \left[(x-x_{\text{L+1}})(x-x_{\text{L+2}})(x-x_P)\right]\end{gathered} \tag{6}$$

Simplifying the terms between the square brackets in (6) gives

$$\begin{gathered}\sum_{p=0}^{P} w_p x^p = w_p(x-x_1)(x-x_2)\dots(x-x_L)[x^{\text{P-L}} \\ +I_{P\text{-}L\text{-}1} x^{\text{P-L-1}} + I_{\text{P-L-2}} x^{\text{P-L-2}} 2... + I_1 x + I_0]\end{gathered} \tag{7}$$

where the elements set $[I] = [I_0, I_1, I_2, I_3 ... I_{P\text{-}L\text{-}1}]$ are the constants to be determined. Expanding (7) results in the following expression of the array factor

$$AF = f_0(I) + f_1(I)x + f_2(I)x^2 + ...f_{\text{P-1}}(I)x^{\text{P-1}} + x^{\text{P}} \tag{8}$$

where each $f_n(I)$ has its own array I for each p. Since all the required nulls are attained in (8), the array directivity can be optimized by calculating the best fit illumination function. Furthermore, the optimized variable I_n will be reduced by the number of zeros (L). Note that this novel process of calculating the coefficients of the super-directive illumination function needs a fraction of the computational time required by conventional methods, like uniform and Chebyshev antenna arrays [50].

Genetic Algorithm (GA) is a powerful method for optimizing the spacing and dimensions of super-directive array elements, enabling the achievement of the desired beam and null steering outcomes. The GA optimization process starts by creating a population of excitations, each assigned a specific fitness function known as a chromosome. Figure 1 depicts the flow chart of the Schekulnoff polynomial process integrated with the GA optimization. The GA optimization is based on Darwin's evolutionary theory, which favors the selection of the most skilled individuals for reproduction and survival through crossover and mutation. The fitness function formulation utilized in the Genetic Algorithm (GA) is described in [51] as follows.

$$Fitness = 20\log\frac{max \mid AF_m(x,y) \mid}{\Sigma(\mid AF_i(x,y) \mid)} \tag{9}$$

The objective function quantifies the fitness of the antenna design based on the amplitude excitation patterns across the array. It achieves this by comparing the sum of the absolute values of the amplitude excitations across all elements to the maximum amplitude excitation. The logarithmic scale (in decibels) is used to express the fitness metric, providing a clearer representation of the performance improvement achieved through optimization. By maximizing the ratio of the maximum amplitude excitation to the sum of the absolute values of the amplitude excitations, the objective function guides the optimization process toward configurations that exhibit enhanced directivity, reduced side lobes, and improved null accuracy. Specifically, maximizing this ratio corresponds to amplifying the dominant excitation while minimizing the contributions from other elements, thereby emphasizing the desired directional characteristics of the antenna array. Thus, the optimization process used by GA to reduce the number of array elements starts by optimizing the primary beam to achieve the specified directivity. Note that GA starts by forming a population of excitations, each with a specific fitness function, and passes them through the stages of selection, crossover, and mutation. Table I shows the resulting Schelkunoff excitation coefficients (I) obtained by optimizing the fitness function with GA.

TABLE I: Super-directive excitations of the 7-element compact antenna array.

I_1	I_2	I_3	I_4	I_5	I_6	I_7
2.46	-0.69	3.32	-1.0	3.32	-0.69	2.46

To verify the proposed solution, 7 element linear antenna array is analyzed with conventional Chebyshev and proposed

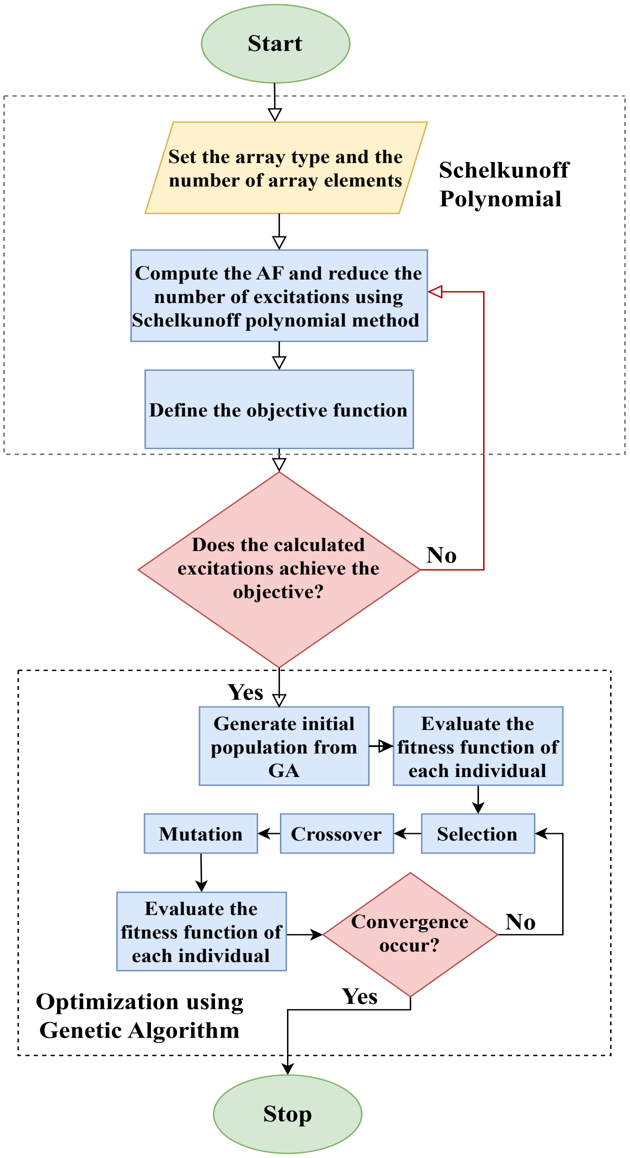


Fig. 1: Schekulnoff polyomial method integrated with GA optimization technique.

Schelkunoff excitation functions. Figure 2 shows the superimposed radiation patterns of the 7-element antenna array with conventional Chebyshev and optimized Schelkunoff excitations. Note that null accuracy is considerably improved by the proposed method. The conventional Chebyshev time is about 840 seconds, while the proposed Schelkunoff time is about 79 seconds. This resulted in 90% improvement in solution time.

III. DESIGN AND ANALYSIS

The main contribution of this paper is to demonstrate the application of Schelkunoff polynomials to significantly reduce the simulation time and optimization of a super-directive antenna array. When designing a super-directive antenna with a large number of radiating elements, simulation time can be extremely long, especially when optimizing the structure to avoid undesired responses. In this regard, The proposed method reduces the solver time to optimize the design of a super-directive antenna by 90%, which significantly benefits the design and optimization process. As a proof of concept, a 7-element packed linear array operating at 10 GHz with an antenna aperture of 2.85λ is used to demonstrate the calculated super-directive response. In a conventional design, the 10 GHz antenna array with an aperture of 2.85λ can accommodate three linear elements with a patch width and length of approximately 0.5λ and an element separation distance (center to center) of 0.5λ. However, to realize a 10 GHz super-directive response from this array, seven elements need to be tightly packed within the antenna aperture and excited with the derived super-directive functions amplitude and phase excitations listed in Table 1. Figure 3 shows the proposed 7-element super-directive antenna with tightly packed array elements. Note that due to the fixed antenna aperture of 2.85λ, the patch width is reduced by 50%, and the separation distance is also reduced to 0.3λ. Multiple shorting posts between the patch and the ground plane are used to reduce the patch size with minimal effect on the antenna impedance and radiation characteristics. Since the shorting posts can be modeled as cascaded inductance and parallel capacitance, they neutralized the antenna impedance mismatch due to shortening the width of the radiating patches [51]. However, the effective implementation of the shorting posts depended on parameters like; the number, location, and size of the posts and the thickness of the antenna substrate. The separation distance between the patches is also optimized to have the required reflection and radiation responses.

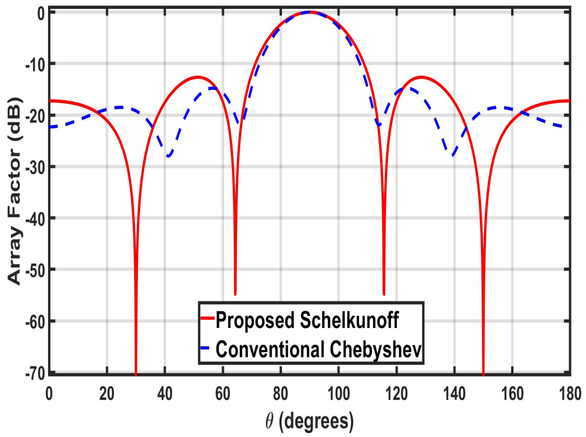


Fig. 2: Radiation patterns of the 7-element linear antenna array with conventional Chebyshev and super-directive excitations.

A rigorously designed 1:8 feed network is implemented to send power to the super-directive array elements, as depicted in Fig. 4. Professional software (HFSS) is used to individually optimize the array elements and the feed network. Figure 4 shows the optimized super-directive array feed with 50 Ω and 100 Ω input and output ports, respectively. Standard equations are used to calculate the dimensions of the microstrip lines of the feed network and the impedance transformers [15]- [19] for a Duroid substrate, ϵ = 2.2 and h = 1.6 mm. The delay lines are used to realize the output phase difference of 180^o in the consecutive patch excitation signals, whereas the lumped resistors are used to realize the tabulated magnitude of the

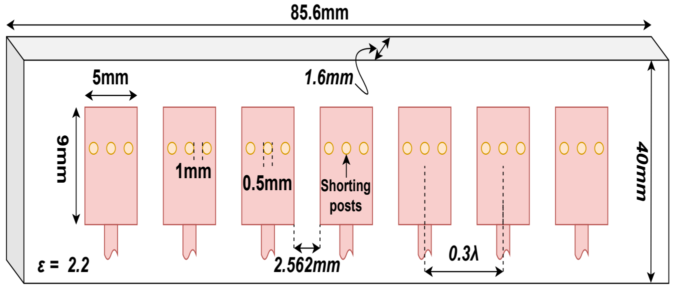


Fig. 3: Antenna array with seven packed radiators within an aperture of 2.85λ operating at 10 GHz.

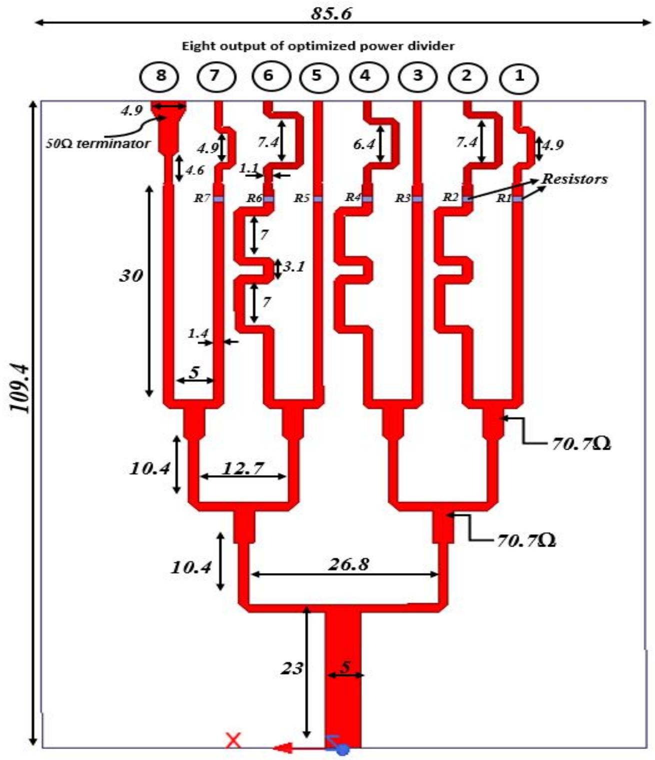
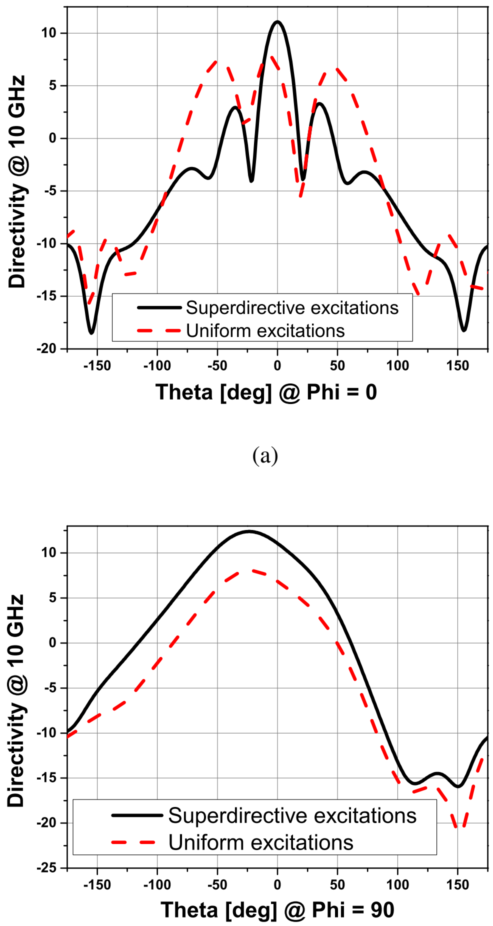


Fig. 4: Non-Uniform 1:8 feed network for the super-directive excitation of the 7-element array.



(a)

(b)

Fig. 5: Simulated directivity of the 7-element antenna array using uniform and super-directive feed networks at 10 GHz in the (a) E-plane and (b) H-plane.

excitation signals shown in Table I. The values of the used resistors are listed in Table II. It can be noted that there is an evenly distributed power divider feeding seven elements (odd number), which can cause mismatches. To effectively match the feeding network, the array feeder is terminated using a 50 Ω matched load. To compensate for the phase distortion introduced by the resistors, a phase compensation of 45^o is introduced at the end of each line of the array feeder.

This paper aims to reduce solver time to optimize the design of a super-directive antenna by 90% using the Schelkunoff method. As a proof of concept, a simple 7-element super-directive antenna is designed and excited. To radiate a super directive beam, the first condition was to reduce the size of the radiating elements by 50% and densely pack them within the array antenna aperture. The second condition was to excite the array elements with the derived super-directive excitation functions outlined in Table I. Due to the small array selected with only 7 elements, the derived super-directive functions required vast variations of excitation signal magnitude and phase from one array element to the next. The resistors, as tabulated in Table II, are used to realize the variation in excitation magnitudes, while the meander lines are employed to produce the required variation in the excitation phase. It is worth mentioning that the resistor values tabulated in Table II are large, which affects the efficiency of the array antenna. However, the practical design of a super-directive antenna involves a considerably larger number of radiating elements. This should significantly reduce the contrast of the required excitation magnitude and phase between a large number of array elements. Consequently, the required resistance values should decrease considerably, improving the trade-off between array directivity and efficiency. Since the computing facilities available to the authors could not handle the design of a larger array, we have selected a 7-element array as a proof of concept to demonstrate the solution efficiency of the proposed design technique.

TABLE II: Resistors needed for super-directive excitation

R_1	R_2	R_3	R_4	R_5	R_6	R_7
100Ω	1000Ω	0Ω	500Ω	0Ω	1000Ω	100Ω

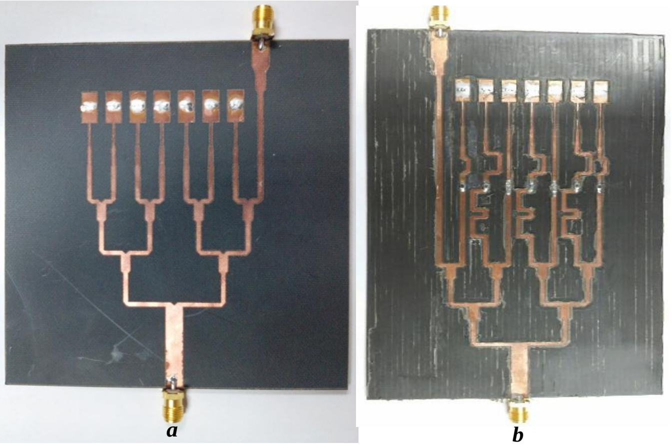


Fig. 6: The prototype of the 7-elements packed antenna array with (a) uniform feed-network and (b) super-directive feed-network.

The calculated and optimized excitations are fed into the designed seven element linear array patches without feeding network. The simulated E- and H-plane directivity patterns of the uniform and super-directive arrays are superimposed in Figs. 5 (a) and (b), respectively. Note that the super-directive array elements with ideal excitations exhibit a peak directivity of 12.5 dB, compared with 7.5 dB of the Uniform array, which is about 66.67% directivity improvement.

To accurately feed the excitations into the radiated patches, a uniform 1:8 feed network, seen in Fig. 4, is designed and optimized. To experimentally validate the simulated results, an antenna array with a uniform and super-directive feed network is also designed and fabricated. Fig. 6 shows the prototypes of the optimized 7-element linear antenna array with uniform and super-directive excitations operating at 10 GHz. The measured and simulated reflection coefficients (S_{11}) of the Antenna array are superimposed in Figs. 7 and 8 for uniform and super-directive excitation, respectively. Minor discrepancies between the simulated and measured responses are due to inaccuracies during the in-house fabrication process. The normalized measured and simulated E- and H-planes directivity patterns of the proposed antenna array for uniform and super-directive excitations with feeding network are shown in Figs. 9 (a) and (b), respectively. Note that the simulated responses agreed well with the measured values and exhibit 66.67% improved antenna directivity. The discrepancies between experimental and simulated responses are mainly due to in-house fabrication and soldering of components. Also, the tolerance of the used resistors may have played a part. The proposed method also demonstrated an increase in efficiency by reducing the solver's time by 90%. Table III compares our proposed antenna with recent literature. The proposed antenna, compared with [6] - [10], demonstrates significant advancements. It operates at a higher frequency of 10 GHz, which corresponds to a wavelength of 30 mm, and maintains a compact element spacing of 9 mm (0.3λ), achieving a better directivity of 12.5 dB and a directivity enhancement of 66.7%. These improvements position our design as a promising candidate for applications requiring high-performance antenna arrays at 10 GHz, such as radar systems, satellite communications, point-to-point wireless links, and millimeter-wave imaging, potentially pushing the boundaries of current technology in these fields.

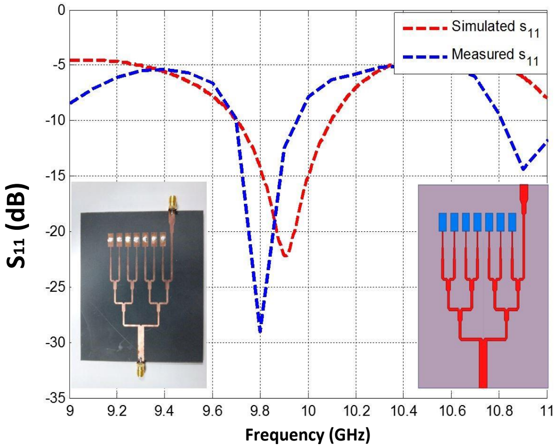


Fig. 7: Refection coefficient of the 7-element shorted patch antenna array with uniform feed.

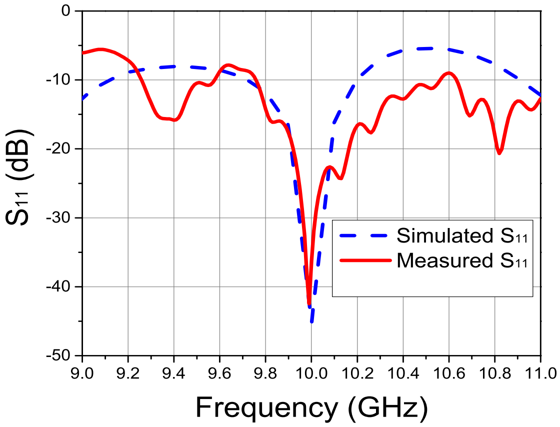


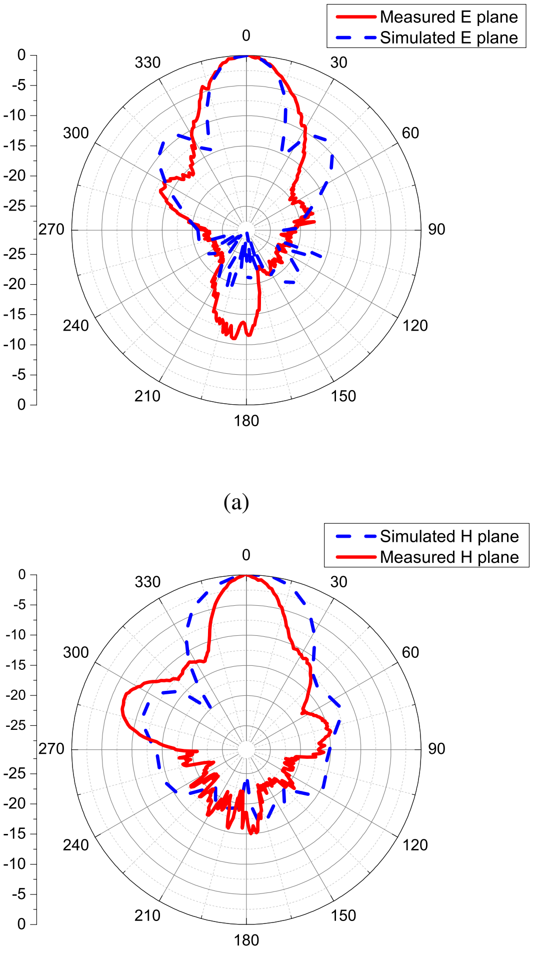
Fig. 8: Reflection coefficient of the 7-element shorted patch antenna array with super-directive feeding.

IV. CONCLUSION

An efficient technique to formulate the super-directive excitation functions for a compact antenna array was presented. The Schelkunoff method and genetic algorithm (GA) are adopted to minimize the number of iterations needed to optimize the excitation function of the super-directive antenna array. A 90% reduction in solver time (i.e., iterations) was demonstrated compared to conventional methods based on numerical simulation to derive the excitation functions. A 10-GHz, 7-element packed linear array with an antenna aperture

TABLE III: Proposed Antenna Compared with Relevant Literature

Ref./Year	Frequency (GHz)	Element spacing (mm)	Antenna Array Size	Directivity (dB)	Directivity Enhancement (%)
[6] (2024)	2.45	9	$0.21\lambda \times 0.18\lambda$	7.3	46.6
[7] (2023)	5	5.4	$3.92\lambda \times 3.92\lambda$	10	12.8
[8] (2023)	1.7	1.7	$0.80\lambda \times 0.34\lambda$	8.3	-
[9] (2018)	0.93	0.93	$0.40\lambda \times 0.30\lambda$	6.9	-
[10] (2010)	5.5	5.5	$6.20\lambda \times 6.20\lambda$	8.34	39
Proposed	10	9	$4.98\lambda \times 2.85\lambda$	12.5	66.7



(a)

(b)

Fig. 9: Measured and simulated radiation patterns of the super-directive antenna array at 10 GHz: (a) E-plane (b) H-plane.

of 2.85λ was designed as a proof of concept. This required 50% and 40% reduction of patch width and the element separation distance, respectively. To maintain impedance matching, HFSS was used to optimize the compressed 7-element array. A 66.67% increase in antenna directivity is demonstrated for super-directive excitations compared to uniform excitations of the feed network. Prototypes of the 7-element array with uniform and super-directive excitations were fabricated and tested. The measured reflection and radiation responses agreed well with the simulated results.

V. ACKNOWLEDGMENT

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