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On the Citizen-Candidate Model with Electoral and Ideological Motivated Candidates and Consensus Outcomes

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Abstract: Recently the notion of citizen-candidates have received considerable interest in models of electoral equilibrium. Building on the models developed by Osborne and Slivinski (1996) and Hamlin and Hjortlund (1998) this paper investigates equilibrium conditions in the citizen-candidate model when the outcome of the political process is a convex combination of candidates' policy-announcements. In particular we generalise the work of Hamlin and Hjortlund (1998) to include non-uniform distributions of citizens' ideal policies and to include a variety of distribution functions regarding candidates' rents from office holding. The main results following from this set up are that; (i) candidates need not obtain equal support in equilibrium, and (ii) equilibria exist for any distribution of citizens ideal-points in which the implemented policy is different from the policy preferred by the median voter.

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1. Introduction

The spatial electoral equilibrium model has attracted considerable interest since it first was suggested by Hotelling (1929). Downs (1957) set up the model which has become known as the starting point of analyses of spatial electoral equilibrium. The Downsian model introduce a set of crucial assumptions regarding four basic characteristics which can be seen to constitute the backbones of the spatial electoral equilibrium model: (i) Political competition is set in a single dimension. (ii) Two parties compete for office. (iii) Policy is chosen by the plurality winner. (iv) Voters have single peaked preferences over a unidimensional issue space and voting is costless, thereby implying that all voters vote sincerely for their most preferred candidate. This set up generate the well-known convergence result that in equilibrium both parties will announce the policy preferred by the median voter. Unfortunately, the assumptions used to generate this result are rather restrictive. Thus, the vast majority of subsequent research on the spatial electoral model seems to have evolved around the introduction of weaker assumptions of the model. The political issue space has been expanded to include more issues and dimensions,1 the behaviour of voters have been altered in order to allow for strategical or expressively motivated voting2, parties have been modelled as ideologically motivated rather than just vote-maximising3, the number of parties has been changed4, and the voting rule has been altered5.

One of the more well known results is reached by Calvert (1985) in his multi-dimensional analysis. Calvert shows that the convergence result is robust to changes in either candidate motivations or to the introduction of uncertainty about the distribution of voters' preferences and

1See for example Enelow and Hinich (1984) or Calvert (1985).

2A host of articles use strategical voting, see for example Austen-Smith and Banks (1988) or Besley and Coate (1997). Others have allowed citizens to take into account their marginal influence on the final election result, see for example Brennan and Lomasky (1983) or Brennan and Hamlin (1998).


4Palfrey (1984) studies a Stackelberg game between two dominating parties and an entrant, and Austen-Smith and Banks (1988) develop a negotiating game between three parties. Osborne and Slivinski (1996), Besley and Coate (1997) and Hamlin and Hjortlund (1998) endogenise the number of candidates in the model by allowing citizens to become candidates at some positive cost.

that, in the presence of both, non-convergence is moderate. Another recent model which has received considerable attention is the Citizen-Candidate model first presented by Osborne and Slivinski (1996). This model changes the conception of candidates by replacing the traditional assumption that there is an exogenously given number of parties, usually two, who compete against each other, with an assumption that all citizens can become candidates at some positive cost. Instead of the strategic game between two given agents, we have a n-person entry game, where the central choice now becomes who (which candidate positions) will run for office in a plurality rule contest. Although the set up of the citizen-candidate model is very different from the traditional model, the system continues to predict some results that can be attributed to the choice of voting rule rather than the conceptualisation of the political process in itself. One specific feature of the original model which remain true in the Citizen-Candidate model with plurality rule is the result that candidates obtain equal electoral support in equilibrium. Thus, for a distribution of citizens ideal points which is symmetric around the median citizen, the ex ante expected policy continues to be equal to the median citizens preferred policy. Unsatisfied with this result, Hamlin and Hjortlund (1998) present a simple citizen-candidate model with proportional representation. The most striking difference between the models suggested by Osborne and Slivinski (1996) and Hamlin and Hjortlund (1998) is that while policy in the first is determined by the position of the plurality winner, it is defined as a vote-weighted average of candidates announced policies in the latter. Hamlin and Hjortlund show that candidates need not obtain equal support in equilibrium. From this it follows that even in the case of a symmetric distribution of citizens ideal points, implemented policy as well as expected policy can deviate quite substantially from the median position. This is a genuine new result, not only in comparison to plurality voting models but also in comparison to proportional representation models with two exogenously given parties.

Hamlin and Hjortlund analyse situations where the distribution of citizens ideal points is uniform and where rents from office holding are allocated to the plurality winner. In present paper we

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Osborne and Slivinski (1996) present some multi-candidate equilibria in which a third candidate does not obtain a winning vote share. But the presence of this "spoiler" candidate assures that the two other candidates each receive a winning voteshare. The intuition behind this is of course that without candidacy of the third candidate, the candidate of the remaining two candidates who is least preferred by the third candidate will win the election.

build on the citizen-candidate model presented by Hamlin and Hjortlund (1998) to investigate the effects of introducing a proportional representation voting scheme in the citizen-candidate model where we allow for non-uniform distributions of citizens' ideal points and where rents from office holding can be distributed in different ways rather than just occurring to the plurality winner.

The basic intuition behind the citizen-candidate model goes like this: A society is made up of citizens who have to democratically choose a unique policy from the set of attainable policies. Citizens are characterised by their preferences over policies. The political process can be characterised as a three stage game. In the first stage citizens simultaneously decide whether they will become candidates or not. In the second stage citizens vote for their preferred candidate among the set of candidates and in the third and final stage policy is implemented on the basis of the electoral outcome and according to a particular mechanism which transforms election results into a unique policy.

The main results that we are able to present following this set up is that equilibria of the game exist with either one, two or more candidates. Furthermore, we are able to show that equilibria with two or more candidates in which candidates receive different support exist, regardless of the particular distribution of office rents and citizens' ideal points. As a consequence of this result, both expected and implemented policy can deviate from median voter policy in equilibrium.

2. The model

Society is made up by a continuum of voters who have Euclidean preferences over the outcome of a legislative process. For future purposes we denote the set of citizens I. Policy is unidimensional and taken to be given by the segment of the real line $[0,1]$. Thus, the generic citizen $i$ with ideal-point $x_i$ will obtain the pay-off from the policy $P$:

$$u_i = -|P - x_i|$$

Citizens are distributed over the political issue space according to an absolutely continuous
distribution function \( f(x) \) with associated cumulative distribution function \( F(x) \). \( f(x) \) is assumed to be common knowledge. Hamlin and Hjortlund use a uniform distribution function in their analysis. In the present paper we expand the possible distributions of citizens' ideal points. Actually, as we mentioned in the introduction, this is one of the central scopes of the paper.

Policy is the outcome of a three-stage game. In the first stage citizens simultaneously decide whether to become candidates or not. Given a set of candidate decisions, an election takes place in which all citizens vote for their preferred candidate. Finally, policy is implemented on the basis of the election result.

We make some crucial assumptions concerning each stage of the game. We try to present and discuss these assumptions as we define the model. We present the model in reverse order of the timing of events.

Policy

Given a set of candidate decisions, from here on denoted \( J \), in which there is a positive number of candidates, and a voting result given \( J \), a unique policy always exist.

When at least one citizen declare herself as a candidate, the resulting policy is a combination of candidates' policy announcements. If no candidates run for office we assume that a (inferior) default state of the world, \( P_0 \) is implemented. In order to keep things simple we assume that the implemented policy is based on pure proportional representation, ie. policy is given by a vote-weighted average of candidates' announced policies. In our notation that is:

\[
(1) \quad P_j = \sum_j x_j v_j
\]

Where \( x_j \) is the announced policy by the \( j \)’th candidate and \( v_j \) is the vote-share of that candidate.

We shall refer to this way of seeing the outcome of politics as a proportional representation policy implementation mechanism, or for short simply PR. Following Ortuno-Ortin (1997) we can elaborate on PR relative to the plurality voting policy implementation mechanism that is used
so frequently in the literature. We believe that proportional representation is an equally (un)-realistic assumption to impose on the system as is plurality rule. Both policy implementation mechanisms, however, can be seen as benchmark cases of a more general mechanism in which the implemented policy is a combination of candidates’ announced policies. We could for example assign a weight function, $G$, that distributes weights to different vote shares, such that

$$P = \sum_j G(v_j)x_j \quad \text{where} \quad G(v_j)x_j + G(v_{j-1})x_{j-1} = 1$$

It is easy to see that both plurality rule and proportional representation is special cases of this conceptualisation. In the case of proportional representation we simply define $G_{pr}(v_j) = v_j$. In case of plurality rule we define $G_{pl}(v_j)$ as a simple step function. Let $(J-j)$ be the candidate set less $j$. then $G_{pl}(v_j) = 1$ for $v_j > \max v_{i,j}$, $G_{pl}(v_j) = 0$ for $v_j < \max v_{i,j}$ and $G_{pr}(v_j) = 1/n$ for $v_j = \max v_{i,j}$, where $n-1$ candidates in $(J-j)$ obtain winning vote shares. Modelling the game with realistic assumptions we should expect a weight-function in between the two extremes.

**Voting**

As voters, citizens’ display honest (non-strategic) behaviour. Thus for any given set of candidates all voters vote honestly. If two or more candidates are located equidistant from any voter, she splits her vote for these candidates (votes with equal probability for each of such candidates). The assumption of honest voting goes back to Downs (1957) and have been used frequently in the literature on political equilibrium. It is however a rather problematic assumption and it has been altered in order to allow the voter to behave strategically in many studies. In the Citizen-Candidate model the assumption of honest voting seems to be even stronger than in traditional models of political competition, because all voters prior to undertaking voting have considered whether to become candidates and done so using strategical considerations. Thus, by imposing honest voting we implicitly impose on individuals that they behave strategically and non-

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8 There are other conceptions of proportional representation in the literature. Austen-Smith and Banks (1988) model proportional representation as a voting rule for parliament and then they let actual policy be decided in a coalition game between candidates. Hjortlund and Zambelli (1998) and Kollman, Miller and Page (1997) both set up a multiple issues model in which policy on each issue is decided using majority rule in a legislative body elected through proportional representation.

9 See for example Austen-Smith and Banks (1988). For a version of the strategic voter in the Citizen-Candidate model, see Besley and Coate (1997).

6
strategically within the same game. A closer look into the rationale behind voter behaviour seems to be a major scope for further work on spatial electoral equilibrium models in general. In the present study, however, we will continue to use the assumption of honest voting.10

Candidates
Most studies of electoral equilibrium model the strategical choice of two exogenously given parties. Parties may be ideologically motivated or not,11 and they may be constrained or unconstrained in their strategical choice of platform.12 Nevertheless very few studies have considered equilibrium with more than two parties.13 The Citizen-Candidate model changes all that. The central idea of this type of model is that all citizens have the opportunity to become candidates at some positive cost c. Also, following the work of Osborne and Slivinski (1996), Besley and Coate (1997) and Hamlin and Hjortlund (1998), we assume that candidates must announce their ideal policy. This assumption is quite plausible in the case of plurality rule. Following the argumentation of Alesina and Rosenthal (1995) We simply need to assume that; (i) voters know the ideal points of all candidates and (ii) that candidates cannot commit to any policy ex ante. Given these assumptions it naturally follows that voters regardless of candidates announcements will vote on the basis of candidates’ true ideal policies rather than their policy announcements. When we are dealing with proportional representation things are somewhat more complicated, because the implemented policy is not determined by any single candidate alone. This is the reason why we have assumed earlier that policy is a vote weighted average of candidates’ announcements. Thus we cannot postulate that the PR policy implementation rule used here is more than one of many explicit ways of translating a sophisticated legislative process.

10 We use the honest voting assumption partly because of notational convenience and partly because it proves extremely difficult to reach a unique voting equilibrium with strategical voting without resorting to very particular refinement criteria, see for example Besley and Coate (1997) or Austen-Smith and Banks (1988).

11 While the original work by Downs (1957) assumes that candidates are motivated by office holding, the predominant view in later studies seems to be either a combination of the two or purely ideologically motivated candidates. See Wittman (1983), Calvert (1985), Alesina and Rosenthal (1995), Roemer (1994) or Schultz (1996) for examples in which candidates are either fully or partly motivated by ideological concerns.

12 The predominant view seems to be the latter, but some papers model candidates’ policy announcements to be fixed in the issue-space. The rationale behind this idea is related to time-inconsistency arguments, see for example Alesina (1988). The Citizen-Candidate models of Osborne and Slivinski (1996), Besley and Coate (1997) and Hamlin and Hjortlund (1998) all use the assumption that candidates cannot commit to any policy which is different from their preferred policy.

into a tractable outcome.\textsuperscript{14}

Given a set of candidates \( J \), the \( j' \)th candidate in that set will have a pay off from candidacy equal to the policy that is implemented with \( x_j \) being a candidate plus the benefit associated with being elected as a candidate itself (office rents) less the cost of running.

\[
    u_j = -|P_j - x_j| + \delta r_j - c
\]

where we take \( P_j \) to be the policy when \( x_j \in J \), \( c \) to be the cost of running as a candidate, \( r_j \) to be the benefit (rents) candidate \( j \) receives from being elected in itself. Finally, we take \( \delta \) to be a parameter indicating the salience of \( j \)'s non-ideological concerns. We assume that \( c \) and \( \delta \) are equal across all citizens. Note that we have not stated any form of office rents \( r \). Hamlin and Hjortlund (1998) analyze the case in which \( r \) is distributed such that the plurality winner receive the full benefits from office holding, and their discussion make it clear that in order for \( r \) to be an interesting feature of the model at all, we need it to depend on the vote share received in the election. Thus we assume here that \( r_j = H(v_j) \), where \( H(v) \) is a weight function distributing the total office rents \( r \) across the set of candidates.

**Equilibrium**

Although we started out by describing the set up as a three stage game it follows directly from the formulation of the model that equilibrium is given by a set of candidate decisions. Once citizens have decided whether to become candidates or not, policy follows “mechanically” from the definitions of voting behaviour and policy implementation mechanism. Given a set of candidates, a unique voting equilibrium always exists and given any voting equilibrium a unique policy always exists. This means (i) that we can base our arguments on backward induction and (ii) that we define equilibrium as a set of candidate-decisions only. Notice that, from the definitions of pay-offs of voters and candidates, it immediately follows that the net pay off to the \( j \)'th candidate from candidacy, denoted \( B_j \) is given by:

\[
    B_j = |P_j - x_j| - |P_j - x_j| + \delta r - c
\]

\textsuperscript{14}The negotiation game suggested by for example Austen-Smith and Banks (1988) or Persson, Roland and Tabellini (1997) are other examples.
Where we now take $P_j$ to be the policy with $j$ removed from the candidate set. Now let us define equilibrium. We are interested in Nash-equilibrium in pure strategies. Thus we need a set of citizens decisions on candidacy such that no citizen would be better off by changing her decision. Letting citizens who are indifferent between candidacy and non-candidacy prefer to become candidates rather than abstain, and following the notation introduced above we let the set of citizens who have decided to become candidates be denoted by $J$, where $J = I$, and the set of citizen who have chosen not to become candidates be denoted by $M$, $M = I$, the following must be true in equilibrium:

**Definition one**

(i) For all $j \in J$, $|P_j \cdot x_j| - |P_j \cdot x_j| + \delta r_j - c \geq 0$, and
(ii) For all $i \in M$, $|P_i \cdot x_i| - |P_i \cdot x_i| + \delta r - c < 0$

It is quite easy to see why (i) and (ii) must hold in equilibrium. If for any $j \in J$, (i) is not true, $j$ will not wish to stay a candidate. On the other hand, if for any $i \in M$ (ii) is not true, such a citizen will wish to become a candidate. Definition one is the backbone of the subsequent analysis of the model in the sense that all results are derived directly from that definition.

3. Results

As mentioned earlier the model is, broadly speaking, a generalisation of the model presented in Hamlin and Hjortlund (1998). In the process of making the model more general we loose some precision. It is so because the equilibrium conditions of any model using proportional representation must rest on the particular distribution of citizens' ideal points. Thus, if we compare the results of this model to the results obtained by Hamlin and Hjortlund the equilibrium conditions presented here are quite broad. On the other hand, all results obtained by Hamlin and Hjortlund are contained within the present model. In order to stress our points we will describe equilibria conditions within an example in which voters' ideal points are distributed non-uniformly. Before proceeding to the results we should also mention that we state equilibrium conditions within an example in which voters' ideal points are distributed non-uniformly.

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conditions for a given number of candidates. \(^{16}\) Finally let us introduce some notation which makes it easier to present the content of the conditions. First of all we can define vote shares rather precisely because we have assumed that voting is honest. Thus, if we order candidates in the candidate set from left to right, such that for the candidate set \(J\) with \(m\) candidates we have that \(J=(x_1, x_2, \ldots, x_m)\), the vote share of the \(j\) th candidate is given by:

\[
(2) \quad v_j = \int_{(x_{j-1})/2}^{(x_j+1)/2} f(x) \, dx
\]

Where \(x_{1,1}=0\) and \(x_{(m+1)}=1\), and \(f(x)\) is the distribution function of citizens ideal points.

Also, following definition one, we have a finite number of candidates in equilibrium who are better off by running as candidates than they are from abstaining from candidacy, given the candidate decisions of all other citizens. On the other hand all citizens who are not candidates will be better off by abstaining from candidacy than by entering as a candidate, given the candidate decisions of all other agents. Thus, an equilibrium consist of a pair of conditions; the first being the one that secures that all candidates wish to remain candidates and the second securing that no other citizen wishes to run given the set of candidates. In establishing the equilibrium conditions we will use the fact that there is a unique citizen position in the set of non-candidates, which is occupied by citizens who will gain as least as much as any other citizen by entering. If entry is deterred for such a citizen, we have established entry proofness. This position, of course, depends on the positions of candidates and the distribution of citizens ideal points. In order to present this in an understandable way, we will introduce separate notation for candidates and for possible entrants. In particular we will use the following notation: Let \(J\) be a set of candidates. Then we denote the left most candidate in that set by “a” the second candidate from left by “b” an so on, where the ideal policy of candidate a is given by \(x_a\) etc. Let the candidate set be given by, say \(J=(a,b,c,d)\) with announced policies \((x_a, x_b, x_c, x_d)\). Then this informs us that there is four candidates in the set of candidates, where \(x_a\) is the position of candidate located at the extreme left, \(x_b\) is the position of the middle-left candidate, \(x_c\) is the position of the

\(^{16}\)This way of characterising equilibria is also used in Osborne and Slivinski (1996), Besley and Coate (1997) and Hamlin and Hjortland (1998).
middle-right candidate and \( x_d \) is the position of the most extreme candidate at the right of the political issue space. Now we divide the possible entrants up according to their location relative to the candidates in \( J \). We denote entrants to the left of \( x_a \) by 0 (with ideal policy equal to \( x_0 \)), entrant located between \( x_a \) and \( x_b \) by 1 (\( x_1 \)) and so on. Thus, for any given set of candidates with \( m \) candidates, possible entrants is divided in \( m+1 \) groups. Furthermore, in cases with strictly more than one candidate we wish to divide one of the sets of possible entrants into two subsets. We need this for the set of citizens who are located between the same two candidates as the resulting policy for that candidate set. Thus, if for example \( x_a < P_j < x_b \), such that the relevant set of possible entrants is denoted \( x_j \), we divide this set into two subsets; \( x_{1j} \in (x_a, P_j) \) and \( x_{2j} \in (P_j, x_b) \).

With this notation in mind we are able to present the first result of the model:

**One candidate equilibrium**

A one candidate equilibrium is given by the candidate set \( J = (a) \). \( J \) is a one-candidate equilibrium, if and only if:

(i) \( c - \delta r < \frac{1}{P_a \cdot x_a} \cdot \text{AND} \)

\[
\begin{align*}
(i) \quad c &> \max \left[ \frac{x_a - x_d^*}{(x_a + x_d^*)^2} \int_0^{(x_a + x_d^*)^2} f(x)dx + \delta H \left( \int_0^{(x_a + x_d^*)^2} f(x)dx \right) \frac{1}{(x_b - x_a)^2} \int_0^{(x_b - x_a)^2} f(x)dx + \delta H \left( \int_0^{(x_b - x_a)^2} f(x)dx \right) \right] \\
(ii) \quad c &> \max \left[ \frac{(x_a - x_0^*)}{(x_a + x_0^*)^2} \int_0^{(x_a + x_0^*)^2} f(x)dx + \delta H \left( \int_0^{(x_a + x_0^*)^2} f(x)dx \right) \frac{1}{(x_b - x_a)^2} \int_0^{(x_b - x_a)^2} f(x)dx + \delta H \left( \int_0^{(x_b - x_a)^2} f(x)dx \right) \right] \\
(iii) \quad \max \left[ \frac{x_a - x_0}{(x_a + x_0)^2} \int_0^{(x_a + x_0)^2} f(x)dx + \delta H \left( \int_0^{(x_a + x_0)^2} f(x)dx \right) \right]
\end{align*}
\]

Where \( x_0^* \) is given by:

\[
\frac{\delta H}{\delta H} \left( \int f(x)dx \right)
\]

and \( x_1^* \) is defined in a similar way.

Condition (i) which secures that \( a \) is willing to run as a candidate follows straightforward from the observation that the cost of running less the office-related benefits of running unopposed should be less than the default outcome (the outcome when there are no candidates). Since candidate \( a \) according to the policy implementation rule will implement her preferred policy...
unrestricted when she is the only candidate, the policy loss is equal to zero in this case. Condition (ii) secures that candidate a running unopposed is entry proof. The condition may seem rather complicated in its formulation. It is nevertheless a simple fact following directly from definition one. In order for no citizen who is not in the candidate set to prefer running as a candidate, we need that the cost of candidacy is higher than the maximum gain any citizen can receive by becoming a candidate. Thus, we need candidate a to deter entry from both sides. The first term secures that no citizen to the left of \textit{x}_a wants to run as a candidate, given that a does run. On the other hand the second term secures that no one to the right of \textit{x}_a wants to become a candidate, when a is a candidate. Since the two situations are similar to demonstrate we will explain the entry proofness condition for all citizens to the left of \textit{x}_a. The first part of the statement is the policy benefit that a candidate located at \textit{x}_0^* will gain by becoming a candidate and the second part of the statement is the office rents obtained by this citizen if she becomes a candidate. Condition (iii) simply define \textit{x}_0^* as the position to the left of \textit{x}_a in which a citizen will gain at least as much as any other citizen by becoming a candidate given \textit{I}=(a). Since we can treat \textit{x}_a as given, \textit{x}_0^* is the solution to a straight forward maximisation problem, which we can solve once we know the distribution of ideal points and the weight function used to allocate office rents.

The basic problem with proportional representation as a policy implementation rule becomes evident from looking at the conditions under which a single candidate running unopposed is an equilibrium. The problem is, of course, that the conditions for equilibrium depend crucially on the distribution of citizens ideal points. Thus analyses involving proportional representation are bound to be either more complicated or less general than analyses of systems guided by plurality rule. Hamlin and Hjortlund (1998) discuss this problem in greater detail, so we will not dwell with it here. Instead we will provide an example of the conditions for one candidate equilibrium for a specific functional form of the distribution of citizens’ ideal points. We shall return to the example later on, so we start out by describing it in detail.

\textit{Example one - one candidate equilibrium.}

In order to operationalise the conditions for one-candidate equilibrium we need to impose additional knowledge on the system in the form of: (i) a functional form specifying the distribution of voters’ ideal points and (ii) a specific functional form of office rents. Let us turn to the second part first. In this example, to keep things as simple as possible, we assume that \delta
= 0, i.e. citizens care only about policy. In section 4 we provide examples when δ>0. Second, let us turn to the distribution of voters ideal points. We assume that voters are distributed over the political issue space according to a quadratic distribution function; i.e. the distribution function is given by \(6(x - x^2)\). Then, the cumulative distribution function is given by \(F(x) = 3x^2 - 2x^3\). Note that \(f(x)\) in this case is symmetric around the median position, \(x_m = 1/2\). Starting with condition (i) we are now able to describe the equilibrium conditions. For candidate a to be willing to run as a candidate we simply require that \(c < |P_0 - x|\) or \(c < |z - x_a|\). Now, let us turn to the conditions for entry-proofness. We start out by identifying the position of the citizen who will gain more than any other citizen from becoming a candidate. Identifying the most likely entrants and their positions \(x_0^*\) and \(x_1^*\) is a straightforward maximisation problem. Solving (iii) for \(x_0^*\) and given the functional form of \(F(x)\) we obtain the following for:

\[
\frac{3}{4} x_a^2 - \frac{9}{2} x_a x_0 - \frac{12}{2} x_a + \frac{9}{2} x_0^2 + x_0^3 = 0
\]

yielding the solution (in the domain \(0, x_a)\):

\[
x_0^* = \frac{9}{8} - \frac{1}{4} x_a - \frac{1}{8} \sqrt{36 x_a^2 - 84 x_a + 81}
\]

Now, if we much the same way identify \(x_1^*\) and assign an ideal point to a, say \(x_a = 0.25\), we are able to obtain the “best” locations of possible entrants. In this case they are given by: \(x_0^* = 0.076\) and \(x_1^* = 0.836\). Thus by insertion of these values in the equilibrium condition we immediately obtain that:

\[c > \max [0.0124, 0.331], \text{ or } c > 0.331 \text{ in order for } x_a = 0.25 \text{ to deter entry.}\]

Thus we conclude that candidate a, located at \(x_a = (0.25)\), is a one-candidate equilibrium for c, such that \(0.331 > c > |P_0 - 0.25|\).

The notion of one-candidate equilibrium is at odds with the basic intuition of political competition in democracies, at least at a national level. Nevertheless, one-candidate elections
are quite common in many democratic institutions, such as elections for leadership in political parties, unions, (sports-) clubs etc. It follows directly from the conditions securing one-candidate elections, that for such a situation to be an equilibrium we require either that costs of running as a candidate are high and/or that office-rents are low relative to citizens' ideological concerns. From this viewpoint there is nothing unnatural about the existence of one-candidate equilibria in democratic institutions. It is clear though, that when r becomes too high, or c becomes too low, there cannot be any one-candidate equilibria. We shall return to the interpretation of the parameters in section four. Before doing so, let us have a look at the somewhat more relevant situation (for political competition on the national level that is) in which there are two candidates running against each other in equilibrium.

Before turning to the conditions for equilibrium we will make the expression of net-benefits from candidacy more simple. We simply state that the policy oriented net-benefit can be reduced to $| P_j - P_j |$. This simplification requires a distribution of citizens ideal points such that if $(P_{(a,b)} - x_j^*) > 0$ then $(P_{(a,b)} - x_j^*) > 0$. It is immediately clear that we only rule out cases in which the distribution function is both multi-modal and highly asymmetric around the position of the median citizen. Thus the statement seems rather sensible. The new expression of net-benefits of the j'th entrant then is $B_j = \{ |P_j - P_j | + \delta r_j - c \}$. As we shall see this will benefit our analysis greatly when we look at the conditions for entryproofness in cases with more than one candidate. We are now able to present our results concerning two-candidate equilibria.

Two-candidate equilibrium:
Let two candidates running against each other occupy the positions $x_a$ and $x_b$. Then this is an equilibrium if and only if the following conditions hold:

(i) $c \leq \min \left[ \{ (x_b - x_a) (v_a) + \delta H(v_a) \} , \{ (x_b - x_a) (v_b) + \delta H(v_b) \} \right]$, AND

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17 The model might also be able to describe one-candidate situations which de facto has been viewed non-democratic by outsiders. Not so many years ago, a quite large proportion of Eastern European countries were effectively one-party systems. The continuing existence of these systems might be explained in the present model using the fact that in such systems the cost of running could be interpreted to be enormously high.

18 Actually this assumption is very weak, because we only need it to find the maximand, $x_j^*$, and for this it cannot be true that if $P_{(a,b)} > x_j$ then $P_{(a,b)} < x_j$. To see this, note that if such a position exist, then by virtue of continuity there must exist a position to the left of $x_j$ say $x_k$, for whom it is the case that $P_{(a,b)} = x_k$. Since $x_k$ will gain more by entering than $x_j$, the latter cannot be the citizen who gains most by becoming a candidate.
(ii) \( c > \max \left\{ \left( x_a-x_{11}^* \right) v_0^* + \delta H (v_0^*) , \left( x_a-x_{11}^* \right) \left( v_a - v_{11}^{11} \right) + \left( x_b-x_{11}^* \right) \left( v_b^{11} - v_b \right) + \delta H (v_{11}^*) \right\} \)

\[ = \left\{ \left( x_{12} - x_a \right) \left( v_a - v_{12} \right) + \left( x_{12} - x_b \right) \left( v_{12} - v_b \right) + \delta H (v_{12}^*) \right\} \]

Where \( v_a \) and \( v_{a1}^{11} \) is \( x_a \)'s voteshare (see (2)) without and with \( x_{11}^* \) being a candidate, the vote shares for \( x_b \) is defined the same way, and where \( x_0^* \) is given by (iii):

\[
(iii) \quad x_0^* = \max_{x_0, x_0' \in [0, x_a]} \left( x_a - x_0 \right) \left( \int_0 f(x)dx \right) + \delta H (\int_0 f(x)dx)
\]

and \( x_2^* \) is formulated in a similar way, where \( x_2 \in (x_b, x_1^*) \). \( x_{11}^* \) is given by:

\[
(iiia) \quad x_{11}^* = \max_{x_{11}, x_{11} \in \left[ x_{12}^* \right]} \left( x_a - x_{11} \right) \left( \int f(x)dx \right) + \left( x_b - x_{11} \right) \left( \int f(x)dx \right) + \delta H (\int f(x)dx)
\]

and \( x_{12}^*, x_{12} \in (P_j, x_b) \), is formulated in a similar way.

Formulated this way we do not begin to have any idea of the particular characteristics of two-candidate equilibria. Nevertheless, conditions (i) and (ii) gives us an algorithm which can be used to find any two-candidate equilibrium once we know \( f(x) \) and \( H(r) \). We shall see how this is done when the distribution of citizens' ideal points is quadratic below. Before doing so, we should make some notes concerning the conditions. Condition (i) secures that \( x_a \) and \( x_b \) are willing to run against each other. The condition follows directly from condition (i) in the definition of equilibrium. For a to be willing to run against b, we require that

\[
|P_b - x_a| - |P_{a,b} - x_a| + \delta x_a - c \geq 0.
\]

With a little rearranging, and using the facts that; (i) \( r_e = H(v_a) \) and (ii) \( x_e > x_a \) we obtain from this condition that \( x_e - (x_a v_a + x_b v_b) + H(v_e) \geq c \) or simply \( (x_b-x_a) v_a + H(v_a) \geq c \). Solving for \( b \) in a similar way we obtain condition (i). Condition (ii) secures that no other citizen wish to run as a candidate given that citizens located at \( x_a \) and \( x_b \) are candidates.

From the notation introduced at the beginning of this section we denote the position of an entrant located to the left of \( x_a \) by \( x_0 \), an entrant located between \( x_a \) and \( x_b \) by \( x_1 \), and an entrant to the right of \( x_b \) by \( x_2 \). Given this notation we require that condition (ii) in definition one holds for all
(x₀,x₁,x₂). Now the analysis splits up in two different directions, one for extremist entrants and one for entrants located between the two candidates. If we look at the extreme entrants first we know that if P_{(a,b)} > x_j then P_{(a,b,0)} > x_j. If we solve for x₀ definition one can be rewritten as: \( |P_{(a,b)} - P_{(a,b,0)}| + δH(v_j) < c \), or, simply, as \( (x_j - x_0)v_a + δH(v_0) < c \). Now for this to be true for all \( x_0 \in (0,x_0) \) it is sufficient that it is true for \( x_0^* \) where \( x_0^* \) is given by (iii) in the equilibrium condition above. The same procedure is used on \( x_2 \in (x_0,1) \) so we will not concentrate on it here. Instead we turn to the slightly more complicated situation where an entrant is located between the two existing candidates. First, note that we started out by assuming that f(x) is such that if \( P_{(a,b)} > x_i \) then \( P_{(a,b,0)} > x_i \). Thus we can rewrite the equilibrium condition in the same way as we did when we explained \( x_0 \) above. But in this case we will have to take into consideration that entrants can be located on each side of the policy \( P_{(a,b)} \). It is so because we know that policy is a combination of \( x_a \) and \( x_b \). This is why we split the set of intermediate entrants up in two subsets, \( x_{11} \in (x_0,P) \) and \( x_{12} \in (P,x_0) \). Since the argument for \( x_i > P \) by the virtue of symmetry is similar to the argument for the case in which \( x_i < P \), we concentrate on the latter. Consider the net pay-off obtained by a citizen, \( x_{11} \), \( x_{12} \in (x_0,P) \) by becoming a candidate. It is equal to \( |P_{(a,b)} - P_{(a,b,0)}| + δr_1 - c \). The net payoff must be negative in order to avoid entering from such a candidate. By using the definition of \( r \) and the fact that \( (x_a + x_{11})/2 < (x_a + x_{12})/2 < (x_b + x_{11})/2 \), we obtain that:

\[
x_a (v_a - v_a^{11}) + x_b (v_b - v_b^{11}) - x_{11} v_{11}^{11} + δH(v_{11}^{11}) < c.
\]

Now notice that we can write \( v_{11}^{11} \) as \( ((v_a - v_a^{11}) + (v_b - v_b^{11})) \), so we directly obtain that

\[
\{ (x_a - x_{11})(v_a - v_a^{11}) + (x_b - x_{11})(v_b - v_b^{11}) \} < c - δH(v_{11}^{11})
\]

In order to avoid entry we need that no citizen in \( (x_0,P) \) want to enter. This is why we use the notation \( x_{11}^* \) in the condition, where \( x_{11}^* \) is defined as the value of \( x_{11} \) that maximises the expression.

The argumentation behind the conditions on existence of two-candidate equilibrium is quite subtle and it does not help us much in assessing the qualitative and quantitative characteristics of two-candidate equilibria. Before illuminating the equilibrium conditions with an example, we should make some points clear though. First of all, for any given pair of candidates there will
always be a threat of entry from candidates located at more extreme ideal positions than the two existing candidates (unless the two candidates are located at the endpoints of the distribution). It is so, because citizens located at more extreme positions than the two candidates always will gain some gross benefit by entering. The same is not generally true for citizens located between the two candidates. As Hamlin and Hjortlund (1998) show, there will be no policy effect for any entrant located between a and b, if the distribution of citizens ideal points is uniform. There can also be cases in which the policy effect will be negative for all intermediate entrants. It is quite easy to see that one example of such a distribution is a symmetric, U-shaped distribution function where candidates a and b are located such that $x_a + x_b = 1$. Thus in such cases, only the existence of office rents can motivate entrance from “middle of the road” citizens.

**Example two - two-candidate equilibrium**

As in example one, we assume that the distribution of citizens ideal points is quadratic and given by $f(x) = 6(x - x^2)$. To allow us to concentrate on the policy terms of citizens utility, and to keep things simple, we also assume that $\delta = 0$. Let us look at condition (i) first. For candidates at positions $x_a$ and $x_b$ to be willing to run against each other we require that:

$$c \leq \min \left\{ (x_b - x_a) \left[ 3 \left( \frac{x_a + x_b}{2} \right)^2 - 2 \left( \frac{x_a + x_b}{2} \right)^3 \right], \left( x_b - x_a \right) \left[ 1 - \left( 3 \left( \frac{x_a + x_b}{2} \right)^2 - 2 \left( \frac{x_a + x_b}{2} \right)^3 \right) \right] \right\}$$

Now let us try with a numerical example. Say that $x_a = 1/4$ and $x_b = 1$. We now obtain the following condition securing that a and b are willing to run against each other:

$$c \leq \min \left\{ \frac{525}{1024}, \frac{243}{1024} \right\} \text{ or simply } c \leq \frac{243}{1024} \approx 0.237.$$  

Note that the candidate who obtain the smallest vote share of the two (candidate b in this case) is the one who gains less by entering. Now let us turn to the condition on entry proofness. We are using condition (ii) in the equilibrium condition which in this case (with $J=(x_a, x_b)=(1/4, 1)$) reduces considerably because there exist no possible candidate to the right of $x_a$. Furthermore, we have already from example one, that in this case $x_0^* = 0.076$. For $x_1^*$ we simply solve the

\[19\] This can be seen by inserting the uniform distribution ($F(x)=x$) in condition (ii) above. Then the expression covering the policy effect for both $x_1$ and $x_2$ reduces to 0.

17
maximisation problem by inserting \((x_a, x_b) = (1/4, 1)\) in the maximisation problem for \(x_i \in (x_a, x_b)\). We find that \(x_1^* = 0.75\). Inserting the values of \(x_0^*\) and \(x_1^*\) back in the condition we obtain that \(c > \max\{0.0124, 0.0234\}\), or \(c > 0.0234\) in order for the candidate set \(J\) to avoid entry from a third candidate. We conclude that \(J = (x_a, x_b) = (1/4, 1)\) is a an equilibrium for \(0.0234 > c > 0.237\).

There are several interesting things to extract from the equilibrium described in example two. First of all, the equilibrium policy is equal to \(x_a v_a + x_b v_b = 0.487\) which is different from the median policy 0.5. Thus for this particular, symmetric distribution of citizens' ideal points we are able to replicate the result from Hamlin and Hjortlund (1998) that (i) candidate positions can be asymmetric around the median in two-candidate equilibria, (ii) The vote shares of the two candidates need not be equal and, most important, (iii) policy can deviate from the median policy, both when evaluated ex ante and ex post. First of all this shows us that the results obtained in Hamlin and Hjortlund are robust to changes in the distribution of citizens' ideal points. Second, the last point is crucial when we relate the results to the results obtained in related models. The citizen-candidate model with plurality voting suggested by Osborne and Slivinski (1996) predicts that candidates will be located equidistant from the median voter and that the ex ante expected policy will be equal to \(\frac{1}{2}\) when the distribution of citizens ideal points is symmetric around the median citizen. Also models with proportional representation produce these results under symmetric distributions. The traditional political competition model of Ortuño-Ortín (1997) where two strategic behaving parties compete by choosing platforms under proportional representation procedures suggests (i) that candidates are located equidistant from the median voter and (ii) that the ex ante expected policy is equal to 0.5. Thus we are inclined to conclude that both proportional representation and a deviation from the traditional conception of candidates are needed in order to destroy the median voter outcome.

It could of course be said that policy in our example does not differ much from the ideal point of the median voter and that the cost of candidacy necessary to avoid entry seems to be very low. The obvious answer to the first question is that the policy outcome is due to the particular distribution and the candidate positions we have chosen for our example. It is quite easy to find symmetric distributions and equilibrium candidate positions where the policy deviates substantially from 0.5. We will return to the second question and the interpretation of the parameter \(c\) in the next section, but note that examples of distributions of citizens ideal points
exist where the cost of candidacy would have to be substantially higher in order for the candidate pair \((1/4, 1)\) to be an equilibrium. Consider for example a normal distribution with its overwhelming mass concentrated close to the median position. Here we can end up in a situation, where the candidate pair \((1/4, 1)\) cannot be an equilibrium, because the costs necessary to hold an entrant out of the game are too high for \(x_b\) to wish to stay a candidate.

Now let us define the conditions for multi candidate equilibria. It is clear that the conditions are nothing more than a generalisation of the conditions for two-candidate equilibria. Let \((a, b, ..., m)\) be the candidates running for office. Then in order for this to be an equilibrium we require that:

(i) \(c \leq \min \left\{ \left| P_a - P_a \right| + \delta H(v_a), \left| P_b - P_b \right| + \delta H(v_b), ..., \left| P_m - P_m \right| + \delta H(v_m) \right\} \), AND

(ii) \(c > \max \left\{ (x_a - x_0^*) v_a + \delta H(v_a^*), (x_b - x_0^*) (v_b - v_b^*) + (x_b - x_0^*) (v_b^* - v_b) + \delta H(v_b^*), ..., (x_{m+1} - x_m^*) v_{m+1} + \delta H(v_{m+1}^*) \right\} \),

Where \(v_a\) and \(v_a^*\) is a’s voteshare with and without 1 participating as a candidate respectively, and where \(x_0^*\) is given by:

\[
(iii) \quad x_0^* = \max_{x_0, x_0^* \in [0, x_a]} \left( \frac{(x_a - x_0)^2}{x_0} + \frac{(x_a - x_0^*)^2}{x_0^*} \right)
\]

and \((x_1^*, ..., x_{m+1}^*)\) are given in the same way.\(^{20}\)

Again, straight forward calculation can be used to find examples of multi-candidate equilibria. One example is the candidate set \(J=(a,b,c)\) with announced policies \((x_a, x_b, x_c) = (0.1, 0.7, 1)\) which is a three candidate equilibrium for \(0.0096 < c < 0.027\) when the distribution of citizens ideal points is given by the quadratic function used in examples one and two, and there are no office rents to be distributed to candidates.

\(^{20}\)Still, for one set of possible entrants we need to divide it into two subsets, as we did when we obtained the conditions for two-candidate equilibria.

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4. Interpretation and discussion of the parameters of the model

It is clear from the examples in section three that we get very "small" maximum and minimum values of the cost of candidacy in equilibrium. This is so because c (and r) should be interpreted as a parameter which only has meaning relative to the span of the political issue space. Since we have normalised the political issue space the values of c (and r) are normalised as well. This has the implication that comparison between different political arenas cannot be easily interpreted within the model. Ideally we would "denormalise" the political issue space in order to do so.

While we might expect the absolute span of the political issue space in an election for the policy of a local sports club to be reasonable low, it might be quite large in parliamentary elections on the national level. Thus for comparisons without "denormalisation" we should expect c to be relatively high in elections defining policy of a local sports club and we should expect c to be relatively low in national elections of a legislative assembly.

The normalisation of the political issue space also has the self evident implication that when we consider office rents they should be related to the absolute span of the political issue space. Explicit introduction of office-rents gives us the opportunity to discuss the changes that occurs in equilibrium, when candidates receive benefits from being elected as well as from having an effect on the implemented policy. Thus, we will briefly consider the introduction of office rents in the context of the examples we have used in section 3. Hamlin and Hjortlund (1998) discuss the effects of office rents distributed to the plurality winner of the election. This combination of the proportional representation model with the more traditional plurality rule models of Osborne and Slivinski (1996) and Besley and Coate (1997) offers some very interesting insights into two inherently different types of candidate incentives which cannot be extracted directly from a model based only on plurality rule. Hamlin and Hjortlund find that asymmetric equilibria are a quite persistent result in the citizen candidate model with proportional representation as long as r is not to high relative to c. With r rising (relative to c) the results increasingly resembles the results that would be obtained in a model without proportional representation in the policy implementation mechanism. It should be clear from the presentation of the general equilibrium conditions presented here, that we can choose any distribution of office rents that we desire.

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21 In the latter case we could see the political issue space to cover the question of the size of budgetary means used for redistributive purposes (Roemer (1996)) or the total level of taxation.
this section we will restrict ourselves to present examples in which office rents are distributed according to a proportional representation rule. The reason for this is twofold. First, using the other benchmark case would not produce much new insight relative to Hamlin and Hjortlund (1998). Second, we feel that in many situations the proportional representation distribution of office rents is highly relevant. Although office rents should be interpreted in utility terms, we can report that in the Danish parliamentary system, for example, parties receive financial support (to pay professional staff etc.) relative to their voteshare. So much for the argumentation. Let us see what happens to the examples we have already looked at in section three when we introduce office rents. First, let us define the distribution of office rents. The j’th candidate now receive office rents proportional to her voteshare, ie. \( r_j = rv_j \), thereby implying that

\[
B_j = |P_j - x_j| - |P_j - x_j| + 6rv_j.
\]

We assume that \( 6 \) is equal to one. This simply means that candidates care as much for office rents as they do for policy. Second we assume that the total office rents to be distributed to candidates is equal to 1. Compared to the size of \( c \) that we find in the examples in section three, this is a rather high number, but since office rents are distributed through a proportional representation mechanism, they enter the individuals decision problem in an undramatic way (small changes in vote shares only result in small changes in allocated office rents), thus making too high values of \( r \) less costly to the analysis than in case of distribution of officerents via plurality rule, where it is the size of \( r \) relative to \( c \) that matters [eg. see Hamlin and Hjortlund (1998)]. Finally, by letting \( r \) be quite high we obtain enough effects of the introduction for office rents to analyse whether such rents alter the characteristics of equilibria.

**Example 3. One-candidate equilibrium with office rents.**

We keep everything unaltered compared to example 1 except from the fact that candidates have positive benefits from holding political office. Thus, the distribution of citizens ideal points are given by the quadratic distribution function \( 6(x -x^2) \). Now let \( 6=1 \) and \( r=1 \). What are the conditions under which \( J=(a)=(0.25) \) will be a one-candidate equilibrium? Inserting the facts of the example in condition (i) for one-candidate equilibria we obtain that for \( a \) to be willing to run unopposed we require that;
All that is altered in comparison with example 1 is that the value of \( r \) now has been added to the right hand side of the inequality. Since \( \delta = 1 \) and \( r = 1 \) and \( r \) is distributed according to a proportional representation rule, \( \delta r = 1 \).

Now to secure entry-proofness, we require that no citizen to the left or to the right of candidate \( x \), wishes to enter, given that \( a \) is running. First we identify the location of citizens who will gain more than any other citizen by entering. For \( x_0 \epsilon (0,a) \) there is no interior solution to the problem, implying that \( x_0^* = x = 0.25 \). For \( x \epsilon (a,1) \) we obtain that \( x^* = 0.397 \). Inserting these values in condition (ii) of the equilibrium condition, we obtain that:

\[ (ii) \ c > \max (0.5, 0.865) \text{ or } c > 0.865 \]

We conclude that \( J = (a) \), with \( x_a = (0.25) \) is a one-candidate equilibrium if and only if \( |P_0 - 0.25| + 1 \geq c > 0.865 \).

Comparing example three with example one, we see that all that has been altered by allowing for the existence of office rents is that the bounds of \( c \) for which \( x_a = 0.25 \) can be a one-candidate equilibrium has been pushed upwards. Thus in this particular example, the introduction of office rents have not been able to produce any new insight. Now let us turn to the example covering two-candidate equilibrium.

**Example 4: Two-candidate equilibrium with office rents**

We use the same assumptions as in example 3, only here we are interested in the potential two-candidate equilibrium with \( J = (a,b) = (0.25, 1) \). Inserting the values of \( a \) and \( b \) in condition (i) of the equilibrium conditions for two-candidate equilibrium we obtain that in order for \( a \) and \( b \) to be willing to run against each-other we require that:

\[ (i) \ c < \min (1.196, 0.5537) \text{ or } c < 0.5537 \]

Now for this to be an equilibrium, we require that no other citizen will be better off by entering. Using (iii) in the equilibrium conditions we obtain that the optimal positions of candidates in this
case is given by \((x_0^*, x_1^*) = (0.25, 0.3386)\). Note that the best possible citizen position in the interval \(x_0 \epsilon (0, x_1)\) is given by \(x_0^* = a = 0.25\) and that we have this directly from example 3. Inserting the citizen positions \((x_0^*, x_1^*)\) back in condition (ii) in the equilibrium condition for two-candidate equilibrium, we obtain that in order for \((a, b)\) to be an equilibrium we require that:

**(ii)** \(c \geq \max(0.342, 0.563)\) or \(c \geq 0.563\).

Thus we conclude that \((x_a, x_b) = (0.25, 1)\) cannot be a two-candidate equilibrium.

The failure of two-candidate equilibrium to exist in this case is caused by the ability of an intermediate candidate to obtain large office rents because of the large vote share she would obtain by entering. This immediately raises the question whether two-candidate equilibria exist at all for a quadratic distribution. It turns out that equilibria exist when (i) \(a\) and \(b\) is located not too far apart and (ii) neither candidate is located too near the endpoints of the distribution. We should also note that in the example above, office rents on its own are not high enough to exclude existence of a two-candidate equilibrium with \((x_a, x_b) = (0.25, 1)\). If there were no policy effect to be obtained for a citizen located at \(x_i = 0.3386\), she would wish to enter only if \(c \leq 0.5354\). But we should note here that there exist candidates who would obtain a higher voteshare than \(x_i = 0.3386\) if they entered. Indeed, the vote maximising position of \(x_i\) is 0.375. Inserting this value of \(x_1^*\) in condition (ii) we obtain that \((a, b)\) is a two candidate equilibrium with \((x_a, x_b) = (0.25, 1)\), when 0.5361 < \(c\) < 0.5537. Thus in example 4 policy oriented benefits enter individuals decision problems quite saliently; It is the policy oriented benefits which secure that the position of an intermediate citizen who would gain more than any other citizen by entering given \(a\) and \(b\) will be different from the vote-maximising citizen position.

What happens if we alter the size and distribution of office rents? It is immediately clear that, as we lower proportional distributed office rents, policy considerations become more salient, thus drawing \(x_1^*\) towards the position which is true for the case where there are no office rents. But it is also clear, that this will be done in a discontinuous matter in our example, since \(x_1^*\) in our original example without office rents is located on the opposite side of the policy that will be implemented when \(J = (a, b)\) where \((x_a, x_b) = (0.25, 1)\). It is also clear that rising \(r\) will pull \(x_1^*\) towards the vote-maximising position which in our example is equal to 0.375. What about
changes in the weight function that distributes $r$ then? Well, if we study the opposite benchmark case where office rents are allocated to the plurality winner, we can state the result that $x^*$ will be equal to the plurality winning position which will have the greatest positive policy impact. On the more general level we will observe that the size of $r$ relative to $c$ will become salient in the equilibria conditions. If $r$ is sufficiently higher than $c$ we will observe two types of equilibria: (i) All candidates obtain an equal and winning voteshare and (ii) some candidates obtain and equal and winning voteshare and some (often extreme) candidates obtain less than winning vote shares but remain candidates solely out of ideological reasons. Existence of the second type of equilibria with purely ideological motivated candidates, however, demand that $c$ is not too high in absolute terms.

5. Final remarks

We have used a generalised version of Hamlin and Hjortlund's (1998) citizen-candidate model with proportional representation to investigate the outcome of a political process in which citizens (in addition to their traditional role as voters) have to choose whether they want to become political candidates or not. In this set up, we are able to reach the result that for any continuous distribution function of citizens’ ideal points, political candidates need not obtain equal support in equilibrium. Furthermore, we have shown that in the case of a symmetric distribution function equilibria which are different from the median voters preferred policy will be possible both when policy is evaluated ex ante and ex post. To our knowledge this result, apart from Hamlin and Hjortlund (1998) who studies the benchmark case of a uniform distribution of citizens ideal points, is new in literature where the analysis does not rely on an explicit negotiation game. Finally we have shed light on the alterations to equilibrium in the citizen-candidate model when office rents are distributed according to candidates’ vote shares. This analysis shows that equilibria change substantially compared to the case where there are no office rents, but not as much as in the case where office rents are allocated to the plurality winner of the election. Furthermore we have established that as proportional distributed office rents become higher, the position of entrants who will gain more than any other by becoming candidates converge towards the citizen position that will gain most from entering when candidates care only about vote maximisation. Although we have been able to generalise the model first presented by Hamlin and Hjortlund much work on the model remains to be done. First,
assumptions on voter behaviour seem to be rather strict. Second, the proportional representation policy implementation mechanism remains a black box. One way of opening this black box could be by replacing the policy implementation mechanism with a voting rule based on proportional representation and letting policy be decided in a negotiation game a la Baron and Ferejohn (1989) and Austen-Smith and Banks (1988). Thus, further research on the model should be concentrated on these two issues.
References


